Numerical Methods

Set 6: Numerical Integration

**Marios-Gavriil Petrakis**

# Problem 1

Combining Simpson’s 1/3 rule with the Romberg method.

Proving Simpson’s 1/3 rule:

If we have equidistant points from to , with step and values of a function at each point , then we interpolate each interval of 3 consecutive points (in non-overlapping intervals) with a forward-differences parabola, so with a Newton polynomial of 2nd order:

* In order not to have problems at the boundaries or with overlapping intervals, it’s best to have an even number of points (with convention of starting at ), unless of course we have a really small step h or equivalently a really large number of points and we can ignore some non-irregular interval of 1 step.

Let’s say we have the interval , then we find the polynomial as:

|  |  |  |  |
| --- | --- | --- | --- |
| *x* | y | Δy | Δ2y |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Then, to integrate , we can simply substitute x with s, dx = h ds and calculating the integrals:

and due to the “coincidence”, the error is known to be:

for the whole interval Δx = b- a, after the summation, or just for a single interval :

Applying Romberg’s method:

For a half-step optimization we need in total 5 points,

For a full step we have the numerical result:

And for a half step :

Since the global error in Simpson’s 1/3 method is , we apply Romberg’s method with n=4:

# Problem 2

For the Gauss-Legendre we can do the following rescaling for random limits of integration:

Where we want and :

# Problem 3

We use a nested method to compute the integral and we go from out to the inside. The code has been slightly modified to account for 2-variable functions. See .py file.

The third integral is: