Assignment 7

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Outline

Problem Statement

(Papoulis chap-12 - 12.2) Show that if a process is normal and distribution-ergodic as in (12-35), then it is also mean-ergodic.
Defination from 12-35

$$\frac{1}{T}\int_0^T F(x, x : \tau)d\tau \longrightarrow F^2(x)$$

for T $\rightarrow \infty$

Solution

The process x(t) is normal and such that

$$F(x, x : \tau) \longrightarrow F^2(x) \tag{1}$$

as $\tau \to \infty$
We shall show that it is mean - ergodic. It suffices to show that

$$C(\tau) \to 0$$

for $\tau \to \infty$We are assuming that $\eta = 0$
C(0)=1

$$f(x_1, x_2; \tau) = \frac{1}{2\pi\sqrt{1-r^2}} exp\left|-\frac{1}{2(1-r^2)}(x_1^2 - 2rx_1x_2 + x_2^2)\right| \tag{2}$$

$$= \frac{1}{2\pi\sqrt{1-r^2}} exp\left|-\frac{1}{2(1-r^2)}(x_1 - rx_2)^2\right| e^{\frac{-x_2^2}{2}} \tag{3}$$

Clearly, f(x,y) = f(y,x) , hence

$$F(x+dx, x+dx; \tau) - F(x, x, \tau) = 2\int_{-\infty}^{x} f(\epsilon, x) d\epsilon dx$$

$$= \frac{1}{\pi\sqrt{1-r^2}} \int_{-\infty}^{x} exp\left|-\frac{1}{2(1-r^2)}(\epsilon - rx)^2\right| d\epsilon e^{-x_2^2/2} dx \tag{4}$$

further

$$F^2(x+dx) - F^2(x) = 2F(x)f(x)dx$$

From above 1 it follows that G$\left\{\frac{x-rx}{\sqrt{1-r^2}}\right\} \longrightarrow G(x)$
for $\tau \to \infty$
Hence, r(τ) $\to$ 0
as $\tau \to \infty$