Assignment 5

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Outline

Problem Statement

(Papoulis chap-8 - 8.41) Let T(x) represent an unbiased estimator for the unknown parameter $\psi(\theta)$ based on the random variables $(X_1, X_2 x_n) = x$ under joint density function f(x,θ). Show that the Crammer-Rao lower bound for the parameter $\psi(\theta)$ satisfies the inequality

$$Var\,T(x) \geq \frac{[\psi'(\theta)]^2}{E\left\{\left(\frac{\partial \log f(x,\theta)}{\partial\theta}\right)^2\right\}}$$

Solution

$$E[T(x)] = \int_{-\infty}^{\infty} T(x)f(x;\theta)dx = \psi(\theta) \tag{1}$$

So after differentiating with respect to θ we get

$$\int_{-\infty}^{\infty} T(x)\frac{\partial f(x,\theta)}{\partial\theta}dx = \psi'(\theta \tag{2}$$

Replacing T(x) as $\psi(\theta)$

$$\int_{-\infty}^{\infty} \psi(\theta)\frac{\partial f(x,\theta)}{\partial\theta}dx = 0 \tag{3}$$

On subtrating equation 2 with 3 we get

$$\int_{-\infty}^{\infty}[T(x)-\psi(\theta)]\frac{\partial f(x,\theta)}{\partial\theta}dx=\psi^{'}(\theta) \tag{4}$$

we can say

$$\frac{\partial \log f(x,\theta)}{\partial\theta}f(x;\theta)=\frac{\partial f(x;\theta)}{\partial\theta} \tag{5}$$

$$\int_{-\infty}^{\infty}[T(x)-\psi(\theta)]f(x;\theta)\frac{\partial \log f(x,\theta)}{\partial\theta}dx=\psi^{'}(\theta) \tag{6}$$

From cauchy schwarz inequality we can give

$$E[[T(x)-\psi(\theta)]^2] \geq \frac{[\psi^{'}(\theta)]^2}{E\left\{\left(\frac{\partial \log f(x,\theta)}{\partial\theta}\right)^2\right\}}$$