

Assignment 8

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Outline

1 Problem

2 Solution

Problem Statement

(Papoulis chap-13- 13.5) Show that if
 $E[s(t+\lambda)|s(t), s(t-\tau)] = E[s(t+\lambda)|s(t)]$
 then $R_s(\tau) = Ie^{-\alpha|\tau|}$

Solution

Since,

$$E[x(t + \lambda)|s(t)] = as(t) \quad (1)$$

$$a = \frac{R(\lambda)}{R(0)} \quad (2)$$

It follows from the assumption that

$$s(t + \lambda) = as(t) \perp s(t - \tau) \quad (3)$$

Hence

$$R(\lambda + \tau) = \frac{R(\lambda)}{R(0)} R(\tau) \quad (4)$$

The only continuous function satisfying the above is an exponential . This is easily shown if we assume that

$$R(\lambda)$$

is differentiable for $\lambda \geq 0$.

Differentiating (4) with respect to λ and setting

$$\lambda = 0^+$$

$$R'(\tau) + \alpha R(\tau) = 0$$

$$\alpha = \frac{-R'(0^+)}{R(0)}$$

This yields

$$R(\tau) = Ie^{-\alpha\tau}$$

for $\tau \geq 0$.