Assignment3

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June 3, 2022

Outline

Problem Statement

(Papoulis chap-7 - 7.10) We denote by x_m a random variable equal to the number of tosses of a coin until heads shows for the mth time. Show that if P[h] = P. then $E_{x_m} = m/p$

Solution

As we know

$$1 + x + x^2......x^n = \frac{1}{1-x} \tag{1}$$

for

$$-1 < x < 1$$

Differentiating on both side with respect to x we get

$$1 + 2x + 3x^2 + 4x^3....(n-1)x^n.. = \frac{1}{(1-x)^2} \tag{2}$$

The random variable x1 equals the number of tosses until head shows for the time,
Hence, x1 takes the value 1,2..... with P(x1=k)=pq$^(k-1)$.
Hence,

$$E(x_1) = \sum_{k=1}^{\infty} kP(x1 = k) \tag{3}$$

$$E(x_1) = \sum_{k=1}^{\infty} kpq^(k - 1) \tag{4}$$

from equation 1 we can say

$$E(x_1) = \frac{p}{(1-q)^2} \tag{5}$$

as we know that p+q=1

$$E(x1) = \frac{1}{p} \tag{6}$$

Starting the count after the first head shows , we conclude that the random variable x2-x1 has the same statistics to x1
Hence,

$$E(X_2 - X_1) = E(x_1) \tag{7}$$

$$E(x_2) = 2E(x_1) = \frac{2}{p} \tag{8}$$

From induction

$$E(X_n - X_{(n-1)}) = E(x_1) \tag{9}$$

$$E(x_n) = E_{X_{(n-1)}} + E(x_1) = \frac{n-1}{p} + \frac{1}{p} = \frac{n}{p} \tag{10}$$

So

$$E(x_n) = \frac{n}{p} \tag{11}$$