Assignment3

MD GUFRAN ASHRAF (BT21BTECH11003)

June 2, 2022

Outline

Problem Statement

(Papoulis chap-7 - 7.10) We denote by xm a random variable equal to the number of tosses of a coin until heads shows for the mth time. Show that if P[h] = P. then $\mathrm{Ex}_m = m/p$

Solution

As we know

$$1 + x + x^2 ......x^n = \frac{1}{1 - x} \tag{1}$$

for

$$-1 < x < 1$$

Differentiating on both side with respect to x we get

$$1 + 2x + 3x^2 + 4x^3 ....(n - 1)x^n .. = \frac{1}{(1 - x)^2} \tag{2}$$

The random variable x1 equals the number of tosses until head shows for the time,
Hence, x1 takes the value 1,2..... with P(x1=k)=pq$^(k-1)$.
Hence,

E(x1)=$\sum_{k=1}^{\infty} kP(x1=k)$ (3)

E(x1)=$\sum_{k=1}^{\infty} kpq^(k-1)$ (4)

from equation 1 we can say

$$E(x1)=\frac{p}{(1-q)^2} \tag{5}$$

as we know that p+q=1

$$E(x1)=\frac{1}{p} \tag{6}$$

Starting the count after the first head shows , we conclude that the random variable x2-x1 has the same statistics to x1
Hence,

$$E(X2 - X1) = E(x1)$$

(7)

$$E(x2) = 2E(x1) = \frac{2}{p} \tag{8}$$

From induction

$$E(Xn - X(n-1)) = E(x1) \tag{9}$$

$$E(xn) = E(X(n-1) + E(x1) = \frac{n-1}{p} + \frac{1}{p} = \frac{n}{p} \tag{10}$$

So

$$E(xn) = \frac{n}{p} \tag{11}$$