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**Generalized additive models via direct optimization  
of regularized decision stump forests**

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Table 1. Train and test RMSE, model size (number of parameters) and training time (average  $\pm$  standard deviation over 5 runs) for different GAMs.  $N$  refers to the dataset size,  $D$  is the feature dimension. Green color is the best test error, and blue is the second best.

Dataset		ORSF	GB	EBM	Splines	NAM	FLAM	FastSparse
<b>Cpuact</b> $N=8.2k$ $D=21$	train	2.12 $\pm$ 0.01	2.20 $\pm$ 0.04	2.19 $\pm$ 0.02	2.53 $\pm$ 0.02	3.38 $\pm$ 0.26	2.88 $\pm$ 0.01	2.76 $\pm$ 0.03
	test	2.37 $\pm$ 0.03	2.43 $\pm$ 0.06	2.50 $\pm$ 0.05	2.69 $\pm$ 0.06	3.41 $\pm$ 0.28	2.99 $\pm$ 0.05	2.91 $\pm$ 0.17
	size	642 $\pm$ 0	3.4k $\pm$ 133	16.6k $\pm$ 36	271 $\pm$ 3	134k $\pm$ 0	77.9k $\pm$ 123	119 $\pm$ 4
	time (s)	184 $\pm$ 25 9.4 $\pm$ 0.3	46 $\pm$ 17	39 $\pm$ 2	37 $\pm$ 0.03	99 $\pm$ 1	85 $\pm$ 2	3.8 $\pm$ 0.5
<b>Wine</b> $N=6.5k$ $D=11$	train $\times 10^{-2}$	65.70 $\pm$ 0.15	68.13 $\pm$ 0.27	66.73 $\pm$ 0.27	67.99 $\pm$ 0.29	74.40 $\pm$ 0.16	67.39 $\pm$ 0.21	68.01 $\pm$ 0.25
	test $\times 10^{-2}$	70.02 $\pm$ 0.66	70.92 $\pm$ 0.51	70.12 $\pm$ 0.39	71.79 $\pm$ 1.40	76.07 $\pm$ 2.11	70.19 $\pm$ 0.84	71.77 $\pm$ 0.63
	size	724 $\pm$ 12	770 $\pm$ 32	3.9k $\pm$ 11	197 $\pm$ 7	70.1k $\pm$ 0	5041 $\pm$ 11	182 $\pm$ 4
	time (s)	64 $\pm$ 3 6.0 $\pm$ 0.3	2.87 $\pm$ 0.58	4.44 $\pm$ 1.33	56 $\pm$ 16	64 $\pm$ 0	53 $\pm$ 3	0.57 $\pm$ 0.07
<b>Housing</b> $N=21k$ $D=8$	train $\times 10^{-2}$	51.84 $\pm$ 0.16	54.24 $\pm$ 0.27	52.70 $\pm$ 0.04	53.37 $\pm$ 0.21	71.56 $\pm$ 0.30	55.08 $\pm$ 0.20	54.62 $\pm$ 0.20
	test $\times 10^{-2}$	54.80 $\pm$ 0.65	56.15 $\pm$ 0.58	55.23 $\pm$ 0.68	55.49 $\pm$ 0.61	72.23 $\pm$ 0.88	56.24 $\pm$ 0.74	56.29 $\pm$ 0.65
	size	1.4k $\pm$ 20	2.4k $\pm$ 31	7.2k $\pm$ 8	528 $\pm$ 2	51.0k $\pm$ 0	118k $\pm$ 101	579 $\pm$ 9
	time (s)	600 $\pm$ 140 13.6 $\pm$ 0.4	42 $\pm$ 8	36 $\pm$ 2	37 $\pm$ 2	175 $\pm$ 2	73 $\pm$ 2	3.94 $\pm$ 0.73
<b>Diamond</b> $N=54k$ $D=26$	train $\times 10^2$	9.95 $\pm$ 0.02	10.07 $\pm$ 0.05	10.11 $\pm$ 0.03	10.02 $\pm$ 0.02	13.53 $\pm$ 0.22	11.75 $\pm$ 0.03	10.01 $\pm$ 0.02
	test $\times 10^2$	10.15 $\pm$ 0.08	10.19 $\pm$ 0.08	10.23 $\pm$ 0.06	10.96 $\pm$ 1.45	13.59 $\pm$ 0.25	11.70 $\pm$ 0.12	10.17 $\pm$ 0.09
	size	934 $\pm$ 16	1182 $\pm$ 81	3.4k $\pm$ 7	273 $\pm$ 24	86k $\pm$ 0	4139 $\pm$ 12	516 $\pm$ 11
	time (s)	648 $\pm$ 20 25.1 $\pm$ 0.9	140 $\pm$ 58	20 $\pm$ 2	42 $\pm$ 0.4	708 $\pm$ 2	805 $\pm$ 11	45 $\pm$ 10
<b>Year</b> $N=423k$ $D=90$	train	9.12 $\pm$ 0.03	9.30 $\pm$ 0.03	7.53 $\pm$ 0.02	9.14 $\pm$ 0.03	10.22 $\pm$ 0.05		9.14 $\pm$ 0.03
	test	9.30 $\pm$ 0.01	9.35 $\pm$ 0.00	9.82 $\pm$ 0.02	9.38 $\pm$ 0.03	10.22 $\pm$ 0.08	out of time	9.29 $\pm$ 0.01
	size	1379 $\pm$ 0.8	1490 $\pm$ 25	368k $\pm$ 0	2158 $\pm$ 55	573k $\pm$ 0	> 2 days	2601 $\pm$ 63
	time (s)	9681 $\pm$ 205 1402 $\pm$ 43	4368 $\pm$ 256	4262 $\pm$ 437	3618 $\pm$ 45	8858 $\pm$ 88		973 $\pm$ 65
<b>FPS</b> $N=401k$ $D=100$	train	55.40 $\pm$ 0.09	55.48 $\pm$ 0.09	55.42 $\pm$ 0.09	55.41 $\pm$ 0.09	56.23 $\pm$ 0.10		55.41 $\pm$ 0.09
	test	55.41 $\pm$ 0.34	55.45 $\pm$ 0.34	55.42 $\pm$ 0.34	55.42 $\pm$ 0.34	55.62 $\pm$ 0.24	out of time	55.42 $\pm$ 0.34
	size	983 $\pm$ 37	824 $\pm$ 57	2372 $\pm$ 12	411 $\pm$ 1	288k $\pm$ 0	> 2 days	1250 $\pm$ 17
	time (s)	6010 $\pm$ 314 798 $\pm$ 21	1803 $\pm$ 466	655 $\pm$ 84	2043 $\pm$ 2	4397 $\pm$ 10		625 $\pm$ 10

Dataset	$\lambda$	Train Error	Test Error	Leaves	Depth	Time
diamond	0.001	out-of-time				
diamond	0.005	out-of-time				
diamond	0.01	2615.15	2741.46	5	5	273.13
diamond	0.05	2806.17	2857.79	2	2	0.70
housing	0.001	0.83	0.96	42	9	0.62
housing	0.005	0.91	0.93	8	5	0.28
housing	0.01	0.90	0.95	5	4	0.20
housing	0.05	0.96	0.99	2	2	0.12
wine	0.001	0.60	0.89	120	11	29.20
wine	0.005	0.72	0.83	26	10	7.87
wine	0.01	0.80	0.80	5	4	2.68
wine	0.05	0.79	0.82	2	2	0.17
cpuact	0.001	out-of-time				
cpuact	0.005	out-of-time				
cpuact	0.01	8.58	19.50	24	11	1658.88
cpuact	0.05	12.39	16.19	8	7	11.03

Dataset	Alpha	Train Error	Test Error	Leaves	Depth	Time
bank	0.001			out-of-time		
bank	0.005			out-of-time		
bank	0.01			out-of-time		
bank	0.05	12.50	11.16	1	1	0.07
fico	0.001			out-of-time		
fico	0.005			out-of-time		
fico	0.01	30.40	30.35	2	2	363.54
fico	0.05	30.40	30.74	2	2	0.65
ijcnn	0.001	6.70	10.27	27	9	18.01
ijcnn	0.005	10.90	9.84	1	1	1.61
ijcnn	0.01	9.40	9.84	1	1	0.12
ijcnn	0.05	9.10	9.84	1	1	0.05
letter	0.001	10.30	23.72	88	12	1272.58
letter	0.005	22.40	25.85	10	6	166.80
letter	0.01	29.70	33.85	4	4	41.59
letter	0.05	33.20	34.08	2	2	0.19
telco	0.001			out-of-time		
telco	0.005			out-of-time		
telco	0.01			out-of-time		
telco	0.05	29.40	26.23	1	1	0.53

Table 2. As in Table 1 but for classification datasets. The error is a 0/1 misclassification (%).

Dataset		ORSF	GB	EBM	Splines	NAM	FLAM	FastSparse
<b>Letter</b> $N=20k$ $D=16$	train	15.94±0.14	16.38±0.17	16.12±0.20	15.87±0.14	21.54±1.1	17.94±0.18	15.88±0.14
	test	16.40±0.52	16.88±0.41	16.63±0.42	16.55±0.70	22.53±1.88	17.95±0.51	16.57±0.67
	size	403±13	420±15	502±2	224±1	68k±0	510±2	399±5
	time (s)	150±9 14.9±0.2	32±3	31±1	58±2	153±0	71±1	18±2
<b>Churn</b> $N=7.0k$ $D=45$	train	18.88±0.19	19.00±0.23	18.84±0.08	18.78±0.15	22.59±2.13	19.85±0.18	18.88±0.11
	test	19.28±0.29	19.32±0.37	19.47±0.51	19.32±0.48	21.69±2.02	20.30±0.88	19.87±0.36
	size	129±5	644±48	7292±11	40±0.04	120k±0	13.7k±15	105±8
	time (s)	36±8 6.8±0.4	3±1	15±1	0.5±0.03	120±2	113±2	0.59±0.07
<b>FICO</b> $N=10k$ $D=23$	train	24.86±0.13	26.54±0.15	26.37±0.10	26.79±0.15	28.23±0.41	27.15±0.21	25.87±0.16
	test	27.33±0.04	27.62±0.30	27.43±0.31	27.35±0.17	28.08±0.61	27.64±0.52	27.80±0.33
	size	550±28	1002±66	3680±9	83±1	130k±0	3791±11	196±10
	time (s)	231±18 7.8±0.1	1.6±0.6	7±0.2	1.96±0.10	180±1	61±1	1.74±0.12
<b>IJCNN</b> $N=50k$ $D=22$	train	4.42±0.05	4.56±0.07	4.51±0.03	4.44±0.04	7.51±0.44	6.86±0.08	4.84±0.16
	test	4.95±0.14	5.10±0.15	5.00±0.14	4.92±0.20	7.48±0.55	7.14±0.15	5.52±0.21
	size	414±23	918±21	12.3k±0	266±5	101k±0	828k±242	883±18
	time (s)	1090±200 46±1	148±24	19±0	153±40	501±1	249±6	47±1
<b>Covtype</b> $N=581k$ $D=54$	train	22.50±0.03	22.56±0.02	22.46±0.02	22.48±0.02	26.16±0.50		22.49±0.02
	test	22.71±0.11	22.77±0.10	22.68±0.12	22.72±0.10	26.08±0.54	out of time	22.68±0.10
	size	504±4	1090±32	6402±4	403±1	170k±0	> 2 days	841±15
	time (s)	4354±32 1091±16	1202±49	325±5	15624±84	5373±16		2763±177
<b>Bank</b> $N=41k$ $D=62$	train	9.81±0.04	10.00±0.03	9.75±0.05	9.79±0.04	10.09±0.08	11.27±0.04	9.79±0.04
	test	9.83±0.17	9.99±0.13	9.91±0.17	9.88±0.12	9.87±0.26	11.23±0.15	9.86±0.14
	size	231±4	530±15	1103±7	95±2	174k±0	1182±1	64±4
	time (s)	153±11 47±2	34±7	40±3	22±2	662±10	916±3	19.6±3.3

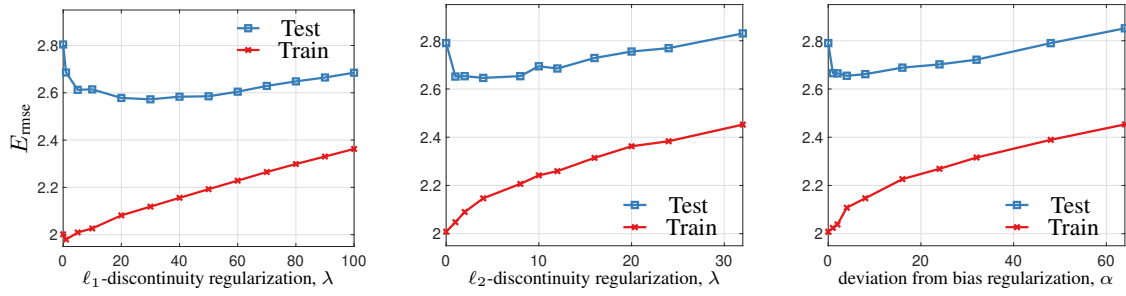


Figure 1. The effect of different regularization types in stump forests for the cpuct dataset. The number of stumps  $T = 200$ . For our final method we use the combination of  $\ell_1$ -discontinuity and the deviation from the bias regularizations.

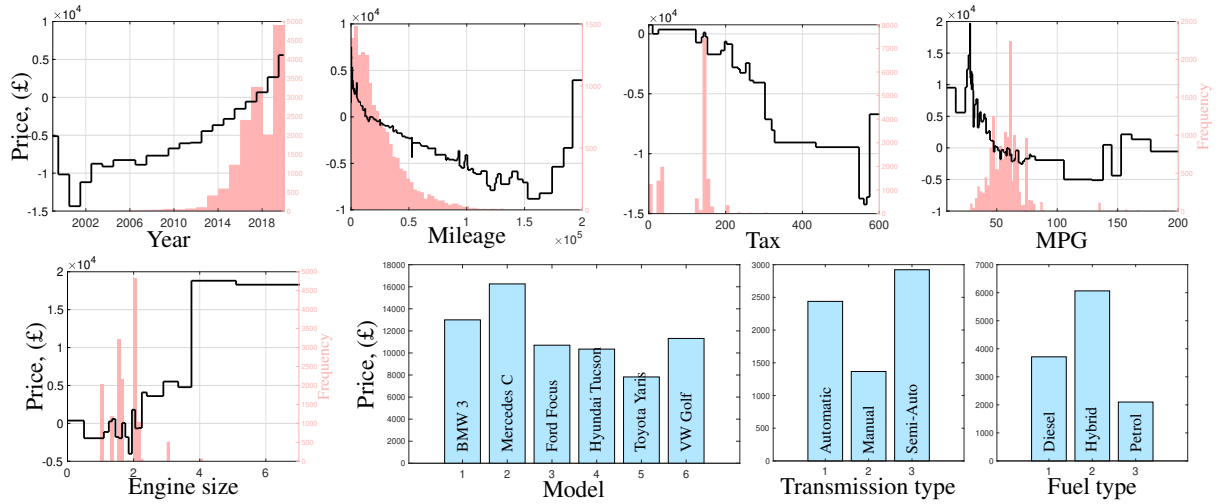


Figure 2. Visualization of the resulting additive model shape functions from our optimized stump forests for the UK used car dataset. For the numerical features, the light red bars show the histogram of the training points with the frequency values given on the right  $y$ -axis.

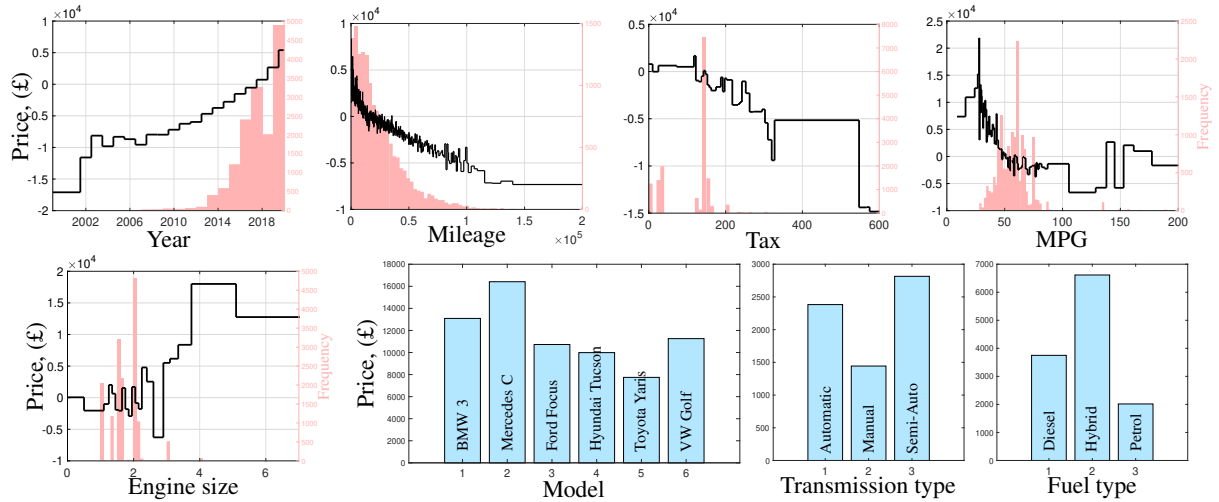


Figure 3. EBM shape functions.

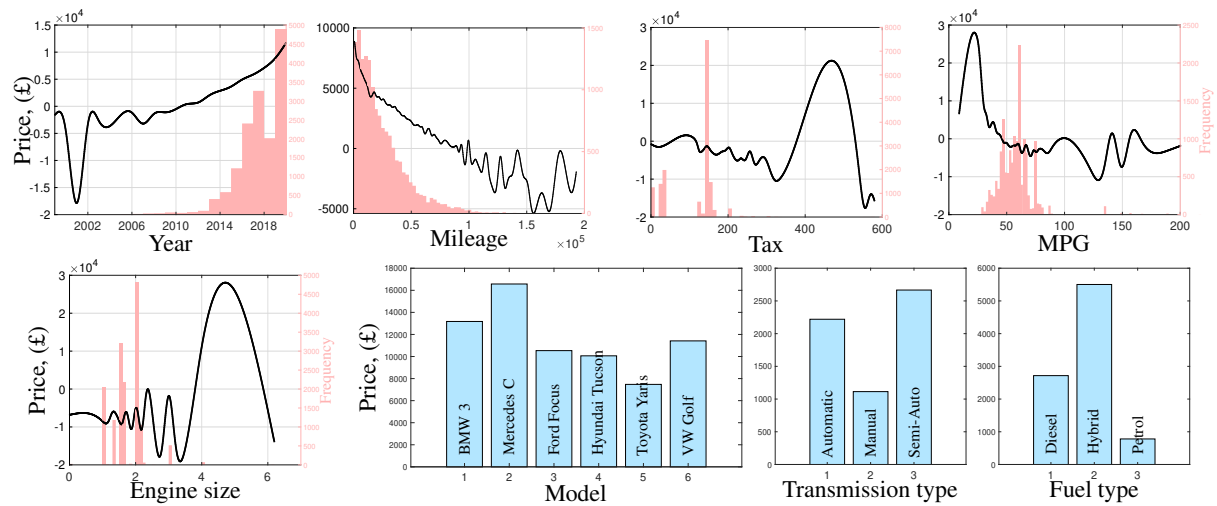


Figure 4. PyGAM shape functions.