Homework 8

2024-10-15

Question 11.1 Using the crime data set uscrime.txt from Questions 8.2, 9.1, and 10.1, build a regression model using: 1. Stepwise regression 2. Lasso 3. Elastic net For Parts 2 and 3, remember to scale the data first – otherwise, the regression coefficients will be on different scales and the constraint won't have the desired effect. For Parts 2 and 3, use the glmnet function in R

Load Data

```r
#housekeeping
library(pacman)
pacman::p_load(ggthemes, tidyverse, magrittr, TTR, tidyr, dplyr, lubridate,
ggplot2, plotly, fpp2, forecast, caTools, reshape2, psych, graphics, Matrix,
corrplot, mltools, fBasics, kableExtra, DMwR, caret, gridExtra, leaps, MASS,
glmnet, rio)

crime <- import("D:/…/uscrime.txt")
```

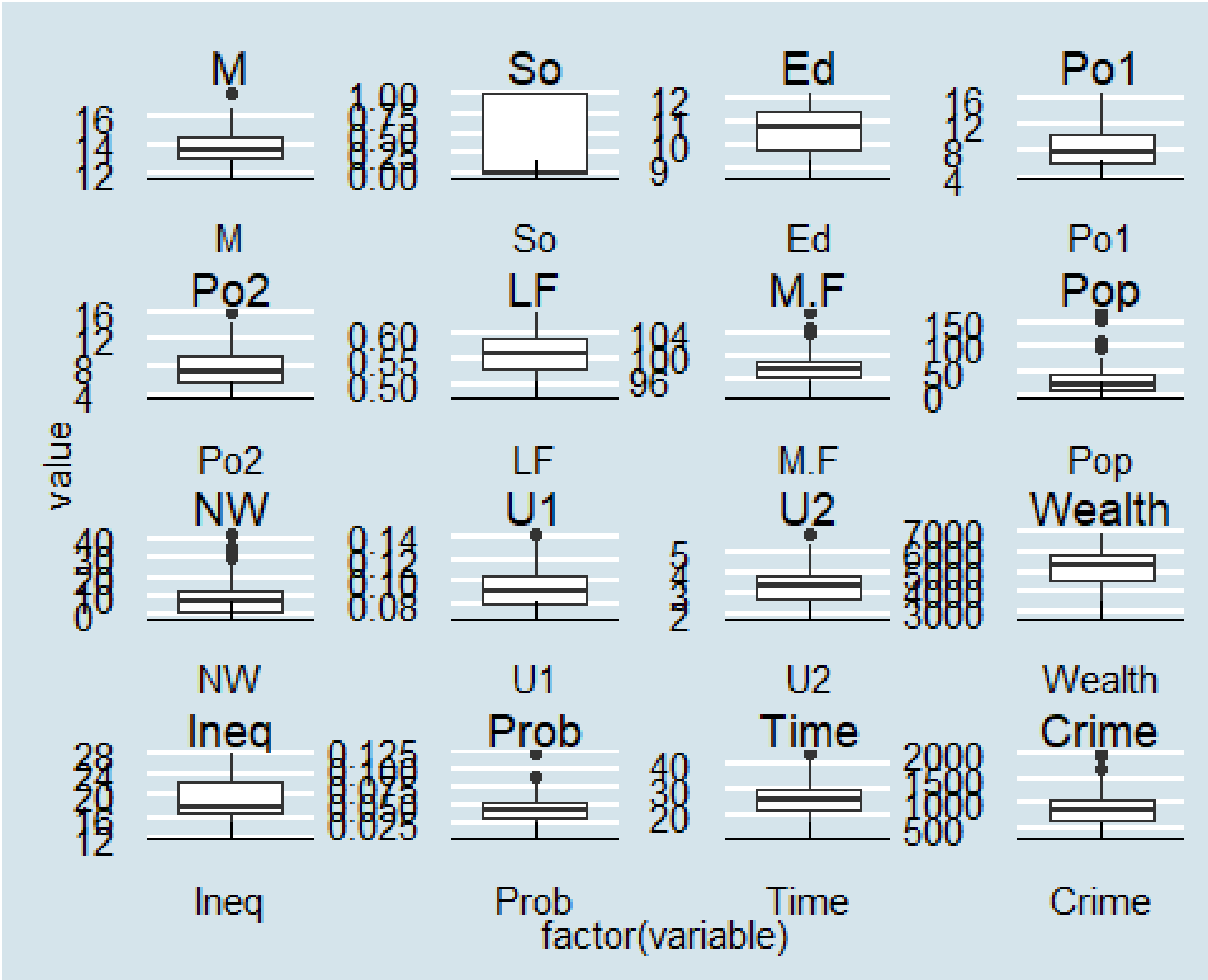
Data exploration and Simple visualization

```r
#Statistical Summary of original data
df_unistats<- basicStats(crime)[c("Minimum", "Maximum", "1. Quartile", "3.
Quartile", "Mean", "Median",
                                  "Variance", "Stdev", "Skewness", "Kurtosis"),
] %>% t() %>% as.data.frame()

kable(df_unistats)
```

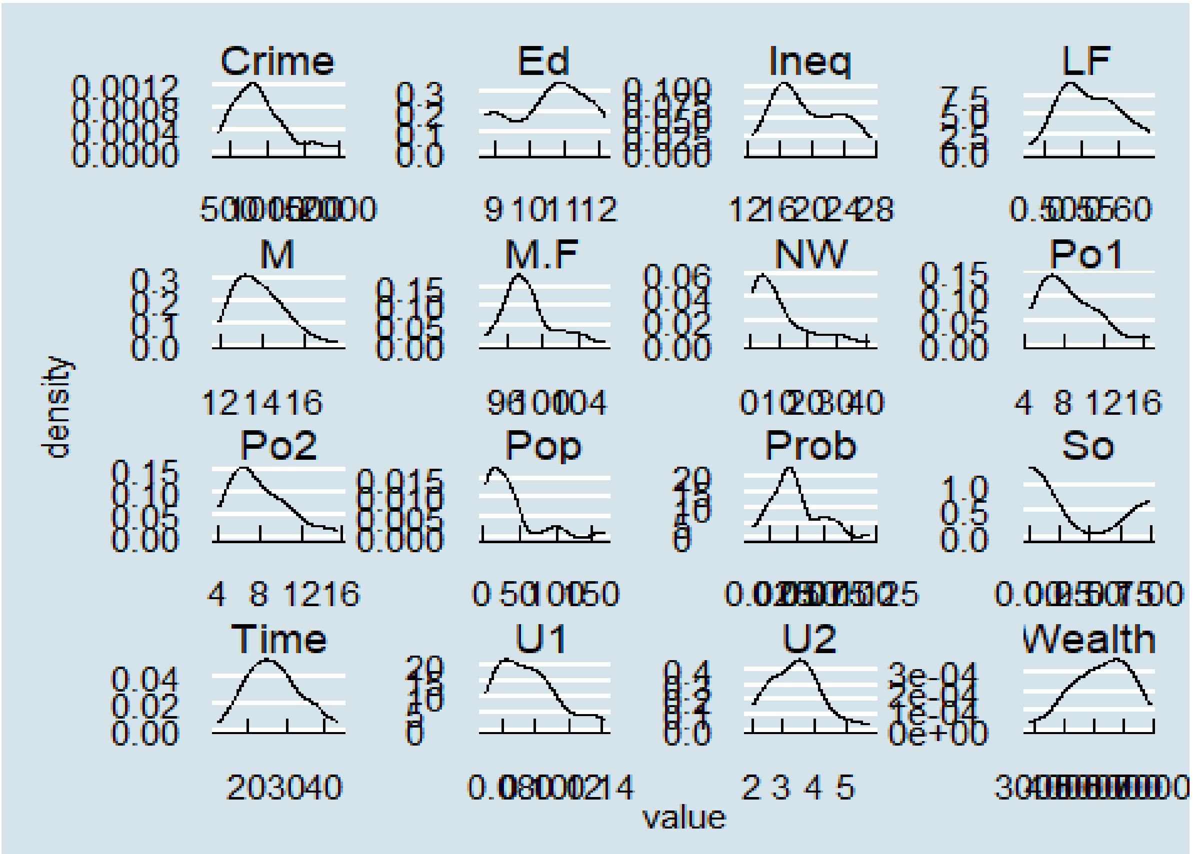
	Minimum	Maximum	1. Quartile	3. Quartile	Mean	Median	Variance	Stdev	Skewness	Kurtosis
M	11.9000	17.700000	13.000000	14.60000	13.857447	13.6000	1.579454e+00	1.256763	0.821917	0.377753
So	0.0000	1.000000	0.000000	1.00000	0.340426	0.0000	2.294170e-01	0.478975	0.652139	-1.607569
Ed	8.7000	12.200000	9.750000	11.45000	10.563830	10.8000	1.251489e+00	1.118700	-0.318987	-1.149253
Po1	4.5000	16.600000	6.250000	10.45000	8.500000	7.8000	8.832174e+00	2.971897	0.890124	0.162309
Po	4.100	15.700	5.8500	9.7000	8.0234	7.300	7.8183	2.796	0.844	0.008

	Minimum	Maximum	1. Quartile	3. Quartile	Mean	Median	Variance	Stdev	Skewness	Kurtosis
2	0	000	00	0	04	0	53e+00	132	367	590
LF	0.4800	0.641000	0.530500	0.59300	0.561191	0.5600	1.633000e-03	0.040412	0.270675	-0.888732
M.F	93.4000	107.100000	96.450000	99.20000	98.302128	97.7000	8.683256e+00	2.946737	0.993223	0.652010
Pop	3.0000	168.000000	10.000000	41.50000	36.617021	25.0000	1.449415e+03	38.071188	1.854230	3.078936
NW	0.2000	42.300000	2.400000	13.25000	10.112766	7.6000	1.057377e+02	10.282882	1.379966	1.077648
U1	0.0700	0.142000	0.080500	0.10400	0.095468	0.0920	3.250000e-04	0.018029	0.774876	-0.131208
U2	2.0000	5.800000	2.750000	3.85000	3.397872	3.4000	7.132560e-01	0.844545	0.542264	0.173008
Wealth	2880.0000	6890.000000	4595.000000	5915.00000	5253.829787	5370.0000	9.310502e+05	964.909442	-0.381952	-0.613169
Ineq	12.6000	27.600000	16.550000	22.75000	19.400000	17.6000	1.591696e+01	3.989606	0.367063	-1.138824
Prob	0.0069	0.119804	0.032701	0.05445	0.047091	0.0421	5.170000e-04	0.022737	0.883336	0.749579
Time	12.1996	44.000400	21.600350	30.45075	26.597921	25.8006	5.022408e+01	7.086895	0.371275	-0.413474
Crime	342.0000	1993.000000	658.500000	1057.50000	905.085106	831.0000	1.495854e+05	386.762697	1.053927	0.777628

```
# Visualizations via Box plots
meltData <- melt(crime)

## No id variables; using all as measure variables

p <- ggplot(meltData, aes(factor(variable), value))
p + geom_boxplot() + facet_wrap(~variable,
scale="free")+theme_economist()+scale_colour_economist()
```


M
So
Ed
Po1
Po2
LF
M.F
Pop
NW
U1
U2
Wealth
Ineq
Prob
Time
Crime
value
factor(variable)

```
#density plots of original data
density <- crime %>%
  gather() %>%
  ggplot(aes(value)) +
    facet_wrap(~ key, scales = "free") +
    geom_density()+theme_economist()+scale_colour_economist()
density
```



Stepwise Regression

```
# Set seed for reproducibility
set.seed(123)
#Generate a random sample of 90% of the rows
random_row<- sample(1:nrow(crime ),as.integer(0.9*nrow(crime),replace=F))
traindata = crime[random_row,]
#Assign the test data set to the remaining 10% of the original set
testdata = crime [-random_row,]
# Set up repeated k-fold cross-validation
train.control <- trainControl(method = "cv", number = 10)
# Stepwise regression model
# Train the model
step.model <- train(Crime ~., data = traindata ,
                    method = "lmStepAIC",
```

```
                    trControl = train.control,trace=F
                    )
#model accuracy
step.model$results

##   parameter     RMSE  Rsquared      MAE   RMSESD RsquaredSD    MAESD
## 1      none 285.9305 0.6092788 225.0695 105.3426  0.2387865 71.53864
```

Best Model # of Predictors Combo

```
# Final model coefficients
step.model$finalModel

##
## Call:
## lm(formula = .outcome ~ M + Ed + Po1 + M.F + U1 + U2 + Ineq +
##     Prob, data = dat)
##
## Coefficients:
## (Intercept)            M           Ed          Po1          M.F
U1
##    -6557.63        87.10       173.41        98.34        27.00     -
7394.41
##          U2         Ineq         Prob
##      206.41        56.57     -3489.88

# Summary of the model
summary(step.model$finalModel)

##
## Call:
## lm(formula = .outcome ~ M + Ed + Po1 + M.F + U1 + U2 + Ineq +
##     Prob, data = dat)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -439.19 -116.92   -4.76  127.15  474.12
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -6557.63    1359.17  -4.825 3.09e-05 ***
## M              87.10      34.76   2.506 0.017312 *
## Ed            173.41      55.87   3.104 0.003903 **
## Po1            98.34      16.22   6.064 7.99e-07 ***
## M.F            27.00      15.20   1.777 0.084851 .
## U1          -7394.41    3702.00  -1.997 0.054080 .
## U2            206.41      77.94   2.648 0.012312 *
## Ineq           56.57      15.02   3.766 0.000651 ***
## Prob        -3489.88    1578.65  -2.211 0.034100 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 200.7 on 33 degrees of freedom
## Multiple R-squared:  0.7873, Adjusted R-squared:  0.7358
## F-statistic: 15.27 on 8 and 33 DF,  p-value: 4.342e-09
```

Evaluation on train model

```
#setting a "full model"
full.model <- lm(Crime ~ M + Ed + Po1 + U2+M.F+U1+U2+ Ineq + Prob,data =
traindata) # on train data set
# Stepwise regression model- in both directions
stepfinal.model <- stepAIC(full.model, direction = "both",
                      trace = FALSE,k=2)
#model accuracy
summary(stepfinal.model)
```

```
##
## Call:
## lm(formula = Crime ~ M + Ed + Po1 + U2 + M.F + U1 + U2 + Ineq +
##     Prob, data = traindata)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -439.19 -116.92   -4.76  127.15  474.12
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -6557.63    1359.17  -4.825 3.09e-05 ***
## M              87.10      34.76   2.506 0.017312 *
## Ed            173.41      55.87   3.104 0.003903 **
## Po1            98.34      16.22   6.064 7.99e-07 ***
## U2            206.41      77.94   2.648 0.012312 *
## M.F            27.00      15.20   1.777 0.084851 .
## U1          -7394.41    3702.00  -1.997 0.054080 .
## Ineq           56.57      15.02   3.766 0.000651 ***
## Prob        -3489.88    1578.65  -2.211 0.034100 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 200.7 on 33 degrees of freedom
## Multiple R-squared:  0.7873, Adjusted R-squared:  0.7358
## F-statistic: 15.27 on 8 and 33 DF,  p-value: 4.342e-09
```

lesser significant predictors

```
#setting a "smaller leaner model"
full1.model <- lm(Crime ~ M + Ed + Po1 + Ineq + Prob,data = (traindata)) # on
train data set
# Stepwise regression model- in both directions
stepfinal1.model <- stepAIC(full1.model, direction = "both",
                      trace = FALSE,k=2)
```

```r
#model accuracy
summary(stepfinal1.model)
```

```
##
## Call:
## lm(formula = Crime ~ M + Ed + Po1 + Ineq + Prob, data = (traindata))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -520.80  -80.81   -9.98  156.62  511.52
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -3803.59     872.91  -4.357 0.000105 ***
## M              76.04      33.96   2.239 0.031423 *
## Ed            146.12      46.92   3.115 0.003604 **
## Po1           119.88      14.76   8.122 1.18e-09 ***
## Ineq           64.54      15.61   4.135 0.000203 ***
## Prob        -3622.43    1696.34  -2.135 0.039600 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 216.3 on 36 degrees of freedom
## Multiple R-squared:  0.7305, Adjusted R-squared:  0.693
## F-statistic: 19.51 on 5 and 36 DF,  p-value: 2.303e-09
```

evaluating on train and test data

```r
#create the evaluation metrics function
eval_metrics = function(model, crime, predictions, target){
    resids = crime[,target] - predictions
    resids2 = resids**2
    N = length(predictions)
    r2 = as.character(round(summary(model)$r.squared, 2))
    adj_r2 = as.character(round(summary(model)$adj.r.squared, 2))
    print(adj_r2) #Adjusted R-squared
    print(as.character(round(sqrt(sum(resids2)/N), 2))) #RMSE
}
predictions.train = predict(stepfinal1.model, newdata = (traindata))
predictions.test = predict(stepfinal1.model, newdata = testdata)
#model accuracy
eval_metrics(stepfinal1.model, traindata, predictions.train, target =
'Crime')
```

```
## [1] "0.69"
## [1] "200.27"
```

```r
eval_metrics(stepfinal1.model, testdata, predictions.test, target = 'Crime')
```

```
## [1] "0.69"
## [1] "162.05"
```

Observation #1:

Cross-validation to filter out unwanted predictors and fed that into our Stepwise (both directions) on training data

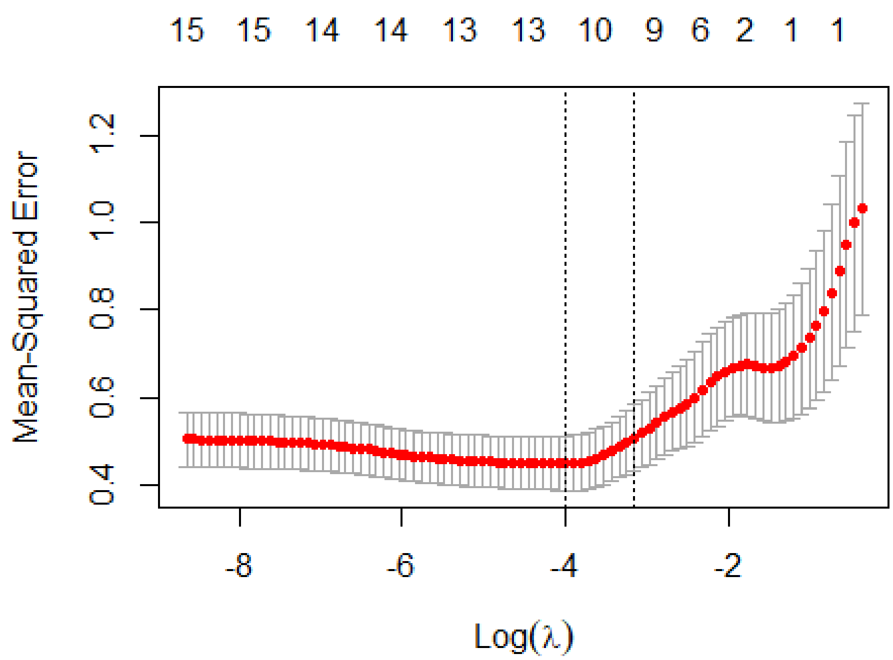
The R-squares for both training and test data is the same while RSME was surprisingly better in test set than training

Lasso:

```
#scale data set
xtrain<-scale(as.matrix(traindata)[,-16], center = TRUE, scale = TRUE)
ytrain<-scale(as.matrix(traindata)[,16], center = TRUE, scale = TRUE)
xtest<-scale(as.matrix(testdata)[,-16], center = TRUE, scale = TRUE)
ytest<-scale(as.matrix(testdata)[,16], center = TRUE, scale = TRUE)
```

defining the model

```
lasso_cv <- cv.glmnet(xtrain, ytrain, family="gaussian", alpha=1)
plot(lasso_cv)#plot lasso cv
```




```
coef(lasso_cv)

## 16 x 1 sparse Matrix of class "dgCMatrix"
##                        s1
## (Intercept) -1.278969e-16
## M            1.632944e-01
## So           1.039512e-01
```

```
## Ed           1.902793e-01
## Po1          7.972784e-01
## Po2          .
## LF           3.783644e-02
## M.F          1.205711e-01
## Pop          .
## NW           .
## U1           .
## U2           6.527132e-02
## Wealth       .
## Ineq         3.319624e-01
## Prob        -1.857967e-01
## Time         .
```

```r
best_lambda <- lasso_cv$lambda.min
cat(best_lambda)
```

```
## 0.01844124
```

Model using best lambda:

```r
lasso_mod = glmnet(xtrain, ytrain, family = "gaussian", alpha = 1, lambda =
best_lambda)
coef(lasso_mod)
```

```
## 16 x 1 sparse Matrix of class "dgCMatrix"
##                        s0
## (Intercept) -1.732407e-16
## M            2.248093e-01
## So           9.661256e-02
## Ed           3.637371e-01
## Po1          7.720783e-01
## Po2          .
## LF           2.177071e-02
## M.F          1.563956e-01
## Pop          .
## NW           5.887326e-03
## U1          -1.563660e-01
## U2           2.607918e-01
## Wealth       3.499338e-02
## Ineq         4.674476e-01
## Prob        -2.103825e-01
## Time         .
```

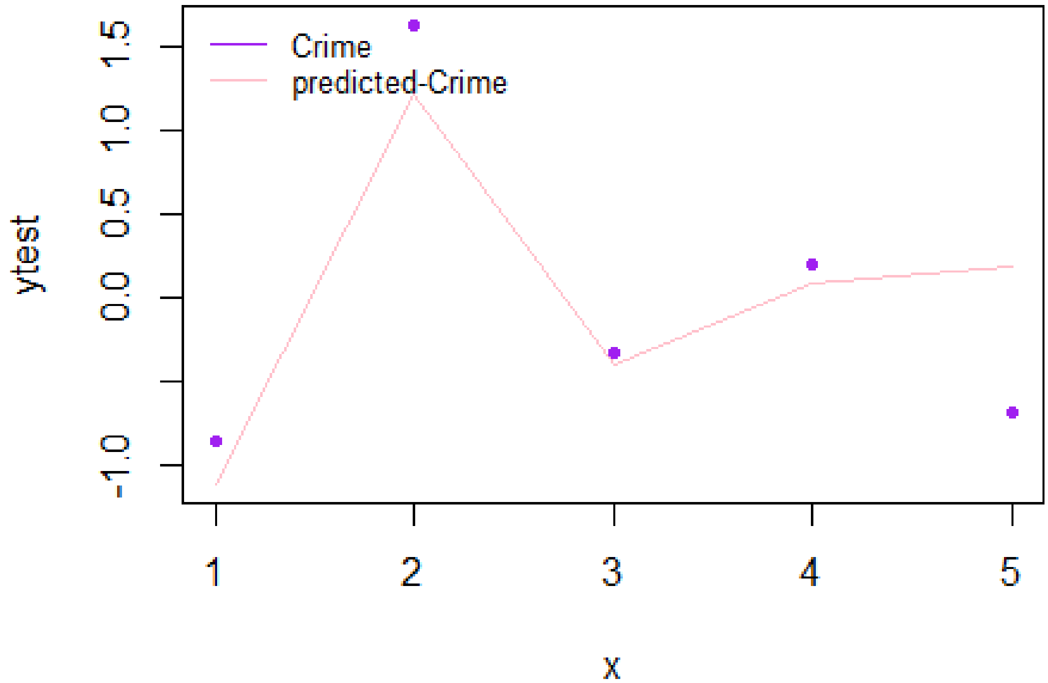
Prediction and evaluations (LASSO)

```r
# Compute R^2 from true and predicted values
eval_results <- function(true, predicted, crime) {
  SSE <- sum((predicted - true)^2)
  SST <- sum((true - mean(true))^2)
  R_square <- 1 - SSE / SST
```

```r
  RMSE = sqrt(SSE/nrow(crime))


  # Model performance metrics
data.frame(
  RMSE = RMSE,
  Rsquare = R_square
)

}

# Prediction and evaluation on train data
yhat.train = predict(lasso_mod, xtrain)
eval_results(ytrain, yhat.train, traindata)

##        RMSE   Rsquare
## 1 0.4634677 0.7799586

# Prediction and evaluation on test data
yhat.test = predict(lasso_mod, xtest)
eval_results(ytest, yhat.test, testdata)

##        RMSE  Rsquare
## 1 0.4510459 0.745697
```

Plot of lasso prediction

```r
x = 1:length(ytest)
plot(x, ytest, ylim=c(min(yhat.test), max(ytest)), pch=20, col="purple")
lines(x, yhat.test, lwd="1", col="pink")
legend("topleft", legend=c("Crime", "predicted-Crime"),
       col=c("purple", "pink"), lty=1,cex = 0.8, lwd=1, bty='n')
```



Observation #2:

The optimal lambda was 0.02221254 from the plot

RSME and R squares were fairly similar for both training and testing data sets and very comparable to linear regression

Because Stepwise was done w/o scaling, the RSME metric cannot be compared with Lasso as its scaled. So we will be using R's as the metric of comparison going forward. As expected Lasso's R-square saw improvement

Elastic Net:

```
# Set training control
train_cont <- trainControl(method = "repeatedcv",
                           number = 10,
                           repeats = 5,
                           search = "random",
                           verboseIter = F)


# Train the model
elastic_reg <- train(Crime ~ .,data = as.matrix(scale(traindata)),method =
```

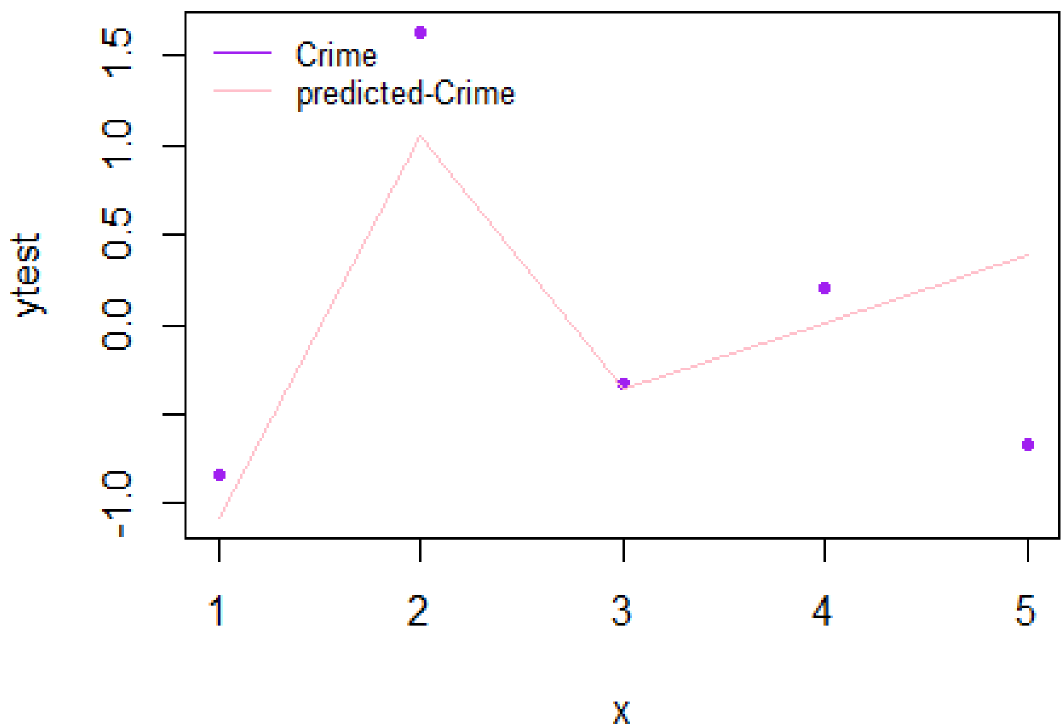
```
"glmnet",preProcess = c("center", "scale"),
                           tuneLength = 10,#10 different combinations of
values for alpha and lambda are to be tested
                           trControl = train_cont)

## Warning in nominalTrainWorkflow(x = x, y = y, wts = weights, info =
trainInfo,
## : There were missing values in resampled performance measures.

# Best tuning parameter
elastic_reg$bestTune

##        alpha     lambda
## 1 0.00381962 0.09804711
```

Prediction and evaluation (elastic net)

```
# Make predictions on training set
predictions_train <- predict(elastic_reg, xtrain)
eval_results(ytrain, predictions_train, as.matrix(traindata))

##       RMSE  Rsquare
## 1 0.477752 0.766186

# Make predictions on test set
predictions_test <- predict(elastic_reg, xtest)
eval_results(ytest, predictions_test, as.matrix(testdata))

##        RMSE   Rsquare
## 1 0.5595539 0.6086243
```

Plot of elastic Net predition

```
x = 1:length(ytest)
plot(x, ytest, ylim=c(min(predictions_test), max(ytest)), pch=20,
col="purple")
lines(x, predictions_test, lwd="1", col="pink")
legend("topleft", legend=c("Crime", "predicted-Crime"),
       col=c("purple", "pink"), lty=1,cex = 0.8, lwd=1, bty='n')
```



Observation 3:

Since there's no definite alpha for Elastic net, using the argument tuneLength specifies that 10 different combinations of values for alpha and lambda are to be tested

Based on the above iterations and output, best tuned alpha & best tuned lambda were listed above

Note: Potentially better results (or worst) would've been gotten if I had tried out different tune length

From a quality perspective, regularized R-square dropped slightly from training to test set like it should

Overall, all the models performed well with decent R-squared and stable RMSE values. Strangely enough for this data set, witnessed improvements going from traditional Linear Regression to regularization models in terms of R squares