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Do polls create momentum in political competition?☆

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ABSTRACT

We explore how public opinion polls affect candidates' campaign spending in political competition. Generally, polls lead to (more) asymmetric behavior. Under a majority rule, there always exists an equilibrium in which the initially more popular candidate invests more in the campaign and thereby increases her lead in expectation: polls create momentum. When campaigning is very effective and the race is very close, a second type of equilibrium may exist: The trailing candidate outspends and overtakes his opponent. Regardless of the type of equilibrium, polls have a tendency to decrease expected total campaigning expenditures by amplifying ex-ante asymmetries between candidates and thus defusing competition. When candidates care also for their vote share in addition to having the majority, candidates' incentives crucially depend on the distribution of voters' candidate preferences.

1. Introduction

While an informed electorate is generally considered essential for a well-functioning democracy, one exception concerns polls of candidates' relative standing. Critics claim that polls undermine both the incentive to vote as well as the vote itself, thus distorting voting decisions. As a consequence, the preferences of the populace are warped by the echo chamber of opinion polls. The effects of polls on voters' decisions at the ballot have been studied extensively, whereas the other side of the political arena, the politicians, has received less attention. However, candidates' behavior during political campaigns is considered an important determinant of electoral success. In this paper, we study the effect of opinion polls on candidates' incentives to invest in their campaign. These investments in turn influence the voters' ballot choice on election day and thus the final election outcome.

Polls are ubiquitous. In addition to traditional providers like Gallup and Rasmussen Reports, newspapers and TV stations conduct their own polls. Poll aggregator websites, such as realclearpolitics.com, collect the plethora of poll results and offer a structured overview to the public. In addition, prediction markets such as politicalbetting.com and oddschecker.com offer alternative sources of voters' approval (Berg et al., 2008). Given the relevance of polls, numerous studies have analyzed the various channels through which information about candidates' relative standing might affect voters' behavior. A prominent hypothesis is the existence of a bandwagon effect, which posits that voters increasingly cast their vote for candidates doing well in the polls. This suggests that polls create momentum in the sense that the front-runner improves her position over time. However, there are also studies predicting anti-momentum, for example, due to a mobilization effect, with the consequence that the front-runner in the polls experiences a loss of support in the election.

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0047-2727/

While scholars show a strong interest in the analysis of voters' reaction to polls, the other side of the political market—the politicians and parties—has been virtually neglected. To increase their chances of election, political candidates invest aggressively into their campaigns: total campaign spending during the 2012 presidential campaign in the United States amounted to more than USD 2 billion.[1] Because the fraction of partisan voters has been shrinking in many countries (e.g. Dalton and Wattenberg (2001)), there are more swing voters to be swayed in a campaign, making campaigning increasingly important in determining the election outcome. Advances in social media and communication technology offer a plethora of new channels to target specific voter groups more effectively and thus increase the impact of a campaigning dollar.

Voter preferences and voting intentions are not directly observable to the political candidates and typically change over the course of time due to taste or information shocks. The possibility of reliable up-to-date information about candidates' relative standing is thus at the heart of our analysis. Modeling campaigning as a dynamic contest, we analyze how this information influences the incentives of candidates to spend in the course of their political campaign. In particular, we are interested in how candidates' *relative* incentives to compete are affected and whether polls affect the likelihood of an incumbent winning the election. We construct a model in which candidates for political office may spend resources early and late in the campaign. Relative spending then determines the popularity of both candidates with the decisive voter and thus the likelihood of success on election day. If there is no poll, candidates a priori have a common belief about the decisive voter's candidate ranking and preference intensity and cannot update their beliefs as the election day comes closer. If there is a poll, candidates know the decisive voter's candidate ranking and preference intensity when making their investment decisions. Our main results are as follows:

- If candidates are mainly concerned with winning the election, polls give the front-runner an incentive to campaign harder than her opponent and thereby create momentum. Thus, there always exists an equilibrium where the front-runner increases her lead in expectation.
- In an environment where candidates' campaign expenditures are highly effective in influencing voters, polls may also create anti-momentum. The trailing candidate may run a more costly campaign than his adversary, but only if candidates have relatively similar popularity. Equilibria with momentum and anti-momentum co-exist.
- Polls have a tendency to decrease expected total campaigning expenditures by amplifying ex-ante asymmetries between candidates and thus defusing competition. Concave marginal costs of campaigning act as a mitigating factor.
- When candidates are sufficiently interested in their vote shares and not only in gaining a majority, unique equilibria exhibiting anti-momentum may occur in a polarized electorate.

To illustrate the intuition behind our results, consider two ex-ante equally popular candidates. They campaign over a certain period of time, and during this time, random (unobservable) shocks to their popularity occur. Without a poll, candidates never learn about their current standing with the voter and thus at any point in time incentives are completely symmetric. With polls on the other hand, candidates receive updates about their current relative standing. This alters their incentives in the following way. A candidate who receives the information that she is ahead, now has an additional incentive to invest. The reason is that any additional investment now affords her an even greater lead in expectation in the next poll. This in turn decreases the expected intensity and thus expected costs of future competition. A trailing candidate on the other hand has a weaker incentive to invest. Any additional unit of investment now brings him closer in expectation to his opponent in the next poll and thus it makes future competition more fierce and costly in expectation. Consequently, polls drive a wedge between the investment incentives of the trailing and leading candidate and thereby create momentum.

When campaigning is relatively effective and the candidates are relatively close, this intuition is valid for the trailing candidate as well. A large investment helps him overtake his opponent and at the same time defuse future competition in expectation. In these situations, both candidates have an incentive to preempt the other with the objective to save costs in the future, and the game resembles a possibly asymmetric hawk–dove game. In terms of aggregate expenditures of the candidates, note that initial asymmetries are amplified through a poll. This tends to defuse competition and thus lowers investments into the campaigns. In addition, though, the candidates also face a less noisy decision environment, which has a differential effect on expenditures. The overall effect of polls on aggregate campaign spending is thus ambiguous and depends on the shape of the cost of campaigning function.

The paper is organized as follows. In the remainder of this section, we review the relevant literature. Section 2 sets up and explains the basic model. In Section 3, we explore the effect of providing information through polls in a system of pure majority voting. We discuss candidates' expected spending profiles in Section 4.1 and the effect of polls on the intensity of political competition in Section 4.2. Section 5 contains a discussion of possible extensions of the basic model. Section 6 concludes. We relegate all formal proofs to the appendix.

1.1. Related literature

Other scholars have directed their attention to the effect of polls on election outcomes. The incentives to conduct polls about voters' policy preferences to inform campaign positioning choices are analyzed for example in Ledyard (1989), Bernhardt et al. (2009) and Jacobs and Shapiro (1994). In contrast, we focus on polls' strategic effects on campaign spending choices when platform choices are already determined.

To be able to study polls and the associated repercussions for electoral outcomes, we study candidate incentives in a dynamic campaigning model. The dynamic nature of the model is essential, since the effects we are interested in can only emerge if candidates can learn over time. The only other paper we are aware of studying dynamics in a contest model of campaign spending is Klumpp and Polborn (2006). While the authors study incentives in sequential electoral contests during the primaries, we consider spending dynamics and the role of informational feedback within a single contest.

Also, the effect of polls on voters' decisions has been investigated. McKelvey and Ordeshook (1985) and Hong and Konrad (1998), among others, show how polls can distort voters' decisions at the ballot in favor of the more popular/leading candidate. Other authors, e.g. Goeree and Großer (2007), argue in the opposite direction, claiming that leading in the polls might actually be harmful, due to a negative mobilization effect.

A closely related literature considers momentum effects in sequential elections such as the U.S. primaries, see, for example, the studies due to Bikhchandani et al. (1992), Callander (2007), Knight and Schiff (2010), Ali and Kartik (2012), or Morton et al. (2013). Voters in later elections observe results from earlier elections or from exit polls of earlier elections and use this information to update their priors about candidates. Hence, a focus of this literature is on social learning. Our paper also addresses the effect of polls on voting decisions and election outcomes, but through a different, indirect channel: the strategies of political candidates. Candidates vie for voters by spending time and money on their campaign. We study how candidates' relative incentives to engage in such persuasive efforts are affected by the presence of polls.

We also add to the literature studying the allocation of campaign funds and candidates' time during a campaign contest, for example, Brams and Davis (1973, 1974), Snyder (1989), Stromberg (2008), Iaryczower and Mattozzi (2013), Denter (2013), and Meirowitz

[1] Source: http://www.opensecrets.org/pres12. Accessed: September 10, 2013.

(2008). We add to this literature by studying the strategic repercussions of information feedback through polls.

The momentum effect we identify is closely related to the "discouragement effect" found in dynamic sequential contests. Konrad (2012) illustrates this effect for models of the "tug-o-war" and the "race" class from the industrial organization literature (see also Harris and Vickers (1985, 1987)). We show that this effect is also relevant in a different class of models tailored to study political economy questions and where competition takes place over a fixed time horizon.

Finally, a literature that is related because of the class of models employed analyzes workers' incentives in labor market tournaments. Aoyagi (2010), Ederer (2010), Gershkov and Perry (2009), as well as Klein and Schmutzler (2013) study the optimal feedback policy of a principal in a dynamic promotion tournament. We add to this literature in two ways. First, in order to study momentum in a campaign, we depart from the modeling practice of this literature by looking at multiple feedback stages and ex-ante asymmetric candidates. Second, we study the effect of two different "reward schemes": promotions (concern for winning) and sharing of a bonus (concerns for vote share).

2. The model

Two candidates, $i\in\{F,T\}$ compete for voters' approval over two stages, $t=1$ and $t=2$.[2] We assume that policy platforms or ideologies were chosen and committed to before $t=1$. Campaigning in $t=1$ and $t=2$ is purely persuasive, and candidates sway voters by choosing investments $x_i^t\geq 0$, $i=F,T$, $t=1,2$, which can, for example, be thought of as TV advertisements or public appearances. Throughout the campaign, candidates experience random shocks to their popularity. Polls inform candidates about their current popularity. In stage three, $t=3$, the election takes place. Fig. 1 summarizes the timing of the game.

We assume that a simple majority rule is used ("first past the post"-system, FPTP). This implies that candidates vie for the voter that will just afford them the majority of votes. We refer to this voter as the "decisive voter" in the following. Voters vote for the candidate who affords them the highest expected utility. Formally, we define the (perceived) difference in utilities between candidate F and T from the perspective of voter v on election day ($t=3$) as

$$d_v^3=a_v+\sum_{t=1}^{2}\left(x_F^t-x_T^t\right)-\sum_{t=1}^{2}e^t. \tag{1}$$

For the decisive voter, we drop the index v, such that $d^3>0$ means the decisive voter prefers F and thus the election is decided in F's favor. The first term, a_v, respectively, a for the decisive voter, represents a voter's realized utility difference at the outset of the campaign before any campaign effort was chosen and can be thought of as an amalgam of valence, platforms, ideologies etc. In case the election were held just before period 1, a would solely determine the election outcome. From the candidates' perspective, a is the realization of a random variable α, which is normally distributed with mean μ_α and variance σ_α^2. The second term in (1) represents the difference in campaigning efforts between F and T in period 1 and 2. Thus, if F outspends T, she will, ceteris paribus, increase her popularity with the decisive voter. Finally, e^t is the realization of a (macro)-shock after the investment decision in period $t=1$ and $t=2$.[3] An example for such a shock is a blunder in a publicly broadcasted speech. For example, in the midst of economic turmoil 2008, Republican U.S. presidential nominee John McCain claimed that "the

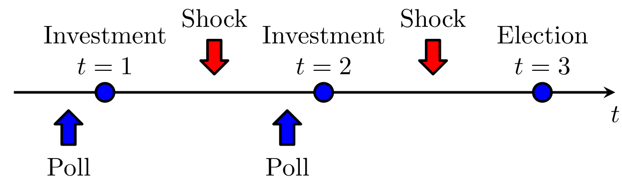

Fig. 1. Timing.

fundamentals are strong", making him look extremely out of touch with reality.[4] Another example is (unexpectedly) backfiring campaign ads. We denote the respective random variables corresponding to realizations e^1 and e^2 as ϵ^1 and ϵ^2 and assume that they are independently drawn from a normal distribution with zero mean and variance σ_ϵ^2.[5] On election day, the decisive voter votes for candidate F if $d^3>0$, while he votes for candidate T if $d^3<0$. If the decisive voter is indifferent ($d^3=0$), so is the electorate as a whole, and both candidates receive a vote share of 50 percent. Denote by δ^3 the random variable belonging to d^3. d^2 and δ^2 are defined analogously as the (realized) difference in perceived utility in $t=2$. Since $a=d^1$, we will use only the former. Throughout the paper, we shall use the symbols ϕ and Φ to denote the PDF and CDF of the standard normal distribution where needed.

The candidates' objective functions are equal to

$$E\left[\pi_F^1\right]=Pr\left(\delta^3>0\right)-\frac{c}{2}\left(x_F^1\right)^2-\frac{c}{2}\left(x_F^2\right)^2,$$
$$E\left[\pi_T^1\right]=Pr\left(\delta^3<0\right)-\frac{c}{2}\left(x_T^1\right)^2-\frac{c}{2}\left(x_T^2\right)^2.$$

We normalize the utility from holding office to one. By Eq. (1) we see that expending campaigning effort x_i^t, $t=1,2$ increases the chances of winning the decisive voter's vote and thus the election. Furthermore, two types of factors not under the control of the candidates are relevant for the probability of capturing the decisive voter on election day. On the one hand, there is uncertainty about the exogenous parameter α. Thus, one candidate may start the campaign in $t=1$ with a popularity advantage. Without loss of generality, we focus on situations where $\mu_\alpha>0$ and thus we refer to candidate F as the "expected front-runner."[6] Similarly, when analyzing the candidates' decisions with polls, we focus on cases where $a>0$ and $d^2>0$ such that we refer to candidate F as the current "front-runner." Furthermore, the realizations of the two macro shocks ϵ^1 and ϵ^2 add uncertainty in each stage. Investments come at quadratic costs $C(x_i^t)=\frac{c}{2}\left(x_i^t\right)^2$, $c>0$. The convexity of costs may reflect the fact

[2] We will in the following refer to F as 'she' and to T as 'he'.

[3] Assuming the shock is common to the whole populace implies that the identity of the decisive voter does not change over time. The assumption of such a "macro" shock (together with the assumption that campaign spending affects all voters equally) is not necessary but significantly simplifies the exposition. In general, the shocks could as well have any other form that allows inferring the expected ranking of candidates in the next stage. In that case, the identity of the decisive voter changes over time.

[4] http://www.nytimes.com/2008/09/17/world/americas/17iht-mccain.4.16251777.html.

[5] There are many other possible interpretations. For example, Gassebner et al. (2008) show the influence of terrorist attacks on politicians' popularity. While this may still be related to policy or qualification, other studies show that pure random events—from the point of view of a politician—influence election outcomes. For example, Healy et al. (2010) study the impact of local college football games just before an election takes place. If the local team wins, the incumbent's chances to win improve significantly. Similarly, natural disasters may have a direct influence of voters' candidate ranking. A recent example for such a shock is the disaster caused by the damaged Fukushima Daiichi nuclear power plant in Japan that followed the earthquake and the thereby caused tsunami of March 11, 2011. In the aftermath, in many countries around the world, a shift of voters' preferences in favor of green or anti-nuclear movements was accounted for. In the U.S., support for nuclear energy dropped to a historical low, with numbers even beneath those immediately after the Three Mile Island incident in 1979 (Cooper and Sussman, 2011). In Germany, the Green Party's support surged in the disaster's aftermath and helped them to win the state election in one of Germany's most influential states, Baden-Württemberg, which until then was a traditional stronghold of the conservative party CDU of Angela Merkel (Dempsey, 2011). Similar shifts in public opinion happened in other countries, such as France (Buffery, 2011), Switzerland (Kanter, 2011), and India (Gupta, 2011). Apart from shocks to candidate ranking, ϵ may as well be interpreted as a random shock muting candidates' campaign efforts in a given stage, so that effective effort is not x_i^t but $x_i^t-\epsilon_i^t$. This is a standard assumption in the literature on labor tournaments that has been pioneered by Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983). Finally, it may be interpreted as a poll's error (see the Supplementary Appendix).

[6] F can also be interpreted as the incumbent and μ_α as the expected incumbency advantage.

that the marginal costs of fundraising go up with every dollar already raised or that each additional unit of investment becomes less effective at a given point in time.[7] For example, if we interpret spending x_i^t as a measure of how many voters a candidate reaches at a given time, convex costs would imply it gets harder and harder for the candidates to reach yet another voter with their messages. Note that despite convex costs, the candidates' objective functions need not be quasi-concave in investments. Thus, throughout most of the paper, we assume c and σ_ϵ are sufficiently large such that interior pure strategy equilibria exist.[8] As a solution concept to determine spending, we will use subgame perfect equilibrium.

3. The effect of polls

We now start our analysis of the campaign by studying candidate behavior in a FPTP electoral system. A candidate's goal is to win the simple majority of votes, and thus, as long as a majority is achieved, the candidate is indifferent whether he wins by a small or by a large margin. We discuss a situation where candidates also care about vote share in Section 5.

In order to identify the effect of polls, we consider two polar cases: a situation without any poll, and thus no information about current relative standing, and a situation where polls perfectly reveal current popularity before each investment decision is taken by the candidates. Thus, in these benchmark cases, we use "poll" as a metaphor for new and precise information. First consider as a benchmark a situation without any poll.

3.1. A campaign without a poll

In both $t = 1$ and $t = 2$, the candidates choose campaign expenditures. Absent a poll they do not receive a signal as to their current popularity with the electorate in either stage and thus they base their decisions solely on their common prior beliefs $\Phi(\frac{\alpha}{\sigma_\alpha})$. Note that we implicitly assume that investments are unobservable to the candidates. Thus candidates receive no information whatsoever between stage 1 and 2. The problem is thus strategically equivalent to a game where first and second period investments are chosen simultaneously at the beginning of stage 1.[9] The next proposition summarizes our first result:

Proposition 1. *The equilibrium of the campaigning game without polls is unique. Both candidates choose identical investments in each stage, and investments are also identical across stages. Consequently, absent a poll the ex-ante expected popular vote of a candidate in $t = 3$ is identical to his popularity in $t = 1$.*

Proof. See appendix. □

We find that investments are completely identical, even if the frontrunner F has an expected popularity advantage. To see the intuition for the result, it is instructive to look at candidates' marginal incentives. For a realized decisive voter position a, note that the marginal increase in probability to win of F is exactly the density function of the macro shock evaluated at the current popularity level. Of course, this also holds for T. Taking into account uncertainty about α, this must hold also in expectation. Moreover, since candidates have identical costs of investment, they also have identical marginal products and marginal costs of spending. Consequently, for any prior distribution $\Phi(\frac{\alpha}{\sigma_\alpha})$, both candidates have identical marginal incentives, and hence the equilibrium must be symmetric. Furthermore, due to the convexity of costs, candidates have an incentive to spread their campaigning expenditures evenly across both periods, implying $x_i^1 = x_i^2$, $i = F, T$. In Supplementary Appendix A.3, we discuss the precision of polls and that the benchmark we just established can also be interpreted as arising when polls are very imprecise, such that they do not convey information.

A direct implication of the proposition is that the campaigning stage does not have *any* effect on the expected winner of the election, absent polls. Candidates choose identical investments, the net effect is zero, and the winning probability will be unaffected in equilibrium. Here we clearly see how political campaigns, or contests in general, are similar to the prisoner's dilemma: Both candidates would be better off by reducing spending by some $k > 0$, or by spending nothing at all. However, individually, they have an incentive to try to outspend their opponent.

3.2. A campaign with polls

In most modern democracies, polls are a pervasive element of the political landscape. Information about relative popularity is accessible in abundance. Apart from polls, politicians may learn more about their popularity from prediction markets on the Internet. Thus, candidates can react to changes in their popularity, and at the same time, even if popularity does not change over time (e.g. $a = d^2$), this gives additional information to the candidates. After all, learning about the popularity in $t = 2$ is confounded by less noise than it was in $t = 1$, and for sure by less than absent any feedback. In this section, we introduce polls into the political campaign. Before their investment decisions in $t = 1,2$, both candidates learn their relative popularity with the decisive voter $d^1 = a$ and $d^2 = a + x_F^1 - x_T^1 - e^1$.[10]

3.2.1. Stage 2

We start with the candidates' investment decisions in stage 2. After learning d^2 through the poll, both candidates decide on x_i^2. This situation is strategically similar to the situation in the absence of polls, with one distinction: the amount of uncertainty until the election in $t = 3$ is reduced. But we have seen from Proposition 1 that the amount of uncertainty was not decisive for *relative* incentives. Hence, although investment levels differ from the situation without poll, candidates in stage 2 also choose identical investments.

[7] One might argue that candidates in a campaign are budget constrained. They cannot spend more money than is donated by their supporters. While this is true in the short run, over the course of the campaign, candidates can invest in fundraising and open up new sources of funding. Thus, we believe it is reasonable for our purposes to assume that spending has some opportunity costs which increase with the amount of campaigning effort expended in a given period. Also empirically positive opportunity costs seem appropriate. For example, John McCain did not use a significant fraction of his campaigning purse (some USD 26 m) during the 2008 presidential campaign (see http://www.fec.gov/finance/disclosure/candcmte_info.shtml). This suggests that there are some costs to using campaign funds.

[8] The details can be found in the respective proofs. Our results generalize beyond the normal-quadratic case, to which we confine our analysis for expositional simplicity. In Supplementary Appendix A.1, we show robustness of the current framework to changes in functional assumptions. In particular, we study general convex cost functions and arbitrary symmetric and quasi-concave noise distributions. Furthermore, our results also apply to situations where only mixed strategy equilibria exist, as we show for the case of $\sigma_\epsilon \to 0$, again in Supplementary Appendix A.1.

[9] This assumption is used in the literature on feedback in dynamic tournaments; see, for example, Ederer (2010). A discussion of the robustness of our results with respect to noisy polls and observable effort can be found in Supplementary Appendix A.3. Note that since there are no proper subgames in this game, the set of subgame perfect equilibria coincides with the set of Nash equilibria.

[10] Of course, real-world polls do not show the preference intensity difference between candidates for the decisive voter, but vote shares in the population. Assume that voters can be ordered by their preference intensity in favor of one of the candidates and all voters are equally affected by campaigning. The difference in voters' preferences between candidates can then be represented by a distribution function. With the family of the distribution of voter preferences known and just the location unknown—e.g. it is known that the CDF is normal with variance σ^2 but the mean is unknown—a candidate can then, if the PDF of preferences is strictly positive on its support and each voter receives a positive vote share, easily infer from her vote share the position of the voters that are just indifferent between her and her opponent and use this to determine the distribution's location. It is then straightforward to determine the preference distance of the decisive voter. To save on notation, we assume the poll immediately reports these distances.

Proposition 2. *There is a unique equilibrium in stage 2. Both candidates choose identical investments $x_F^2(d^2) = x_T^2(d^2)$, no matter what their popularity advantage in $t = 2$.*

Proof. See appendix. □

The intuition for the symmetry of investments is identical to before: By marginally increasing spending, the additional probability to win equals exactly the opponent's loss in winning probability; the marginal benefit of investment is identical for both candidates independent of their popularity. Because marginal costs are also identical, both must choose identical spending. Hence, we can conclude that the expected popularity of the front-runner in $t = 3$, $E[\delta^3]$, equals the popularity in stage 2, d^2.

What are the properties of stage 2 investments? Intuitively, higher marginal costs c decrease spending. The effect of increasing the noisiness of the macro shock σ_ϵ is ambiguous. Increasing σ_ϵ implies ϕ goes up at the tails and goes down in the center, overall becoming flatter. Therefore, in case competition is lopsided with one candidate enjoying a popularity advantage, increasing the variance of the shock increases equilibrium spending. Intuitively, if the variance is large, the trailing candidate has a realistic chance to catch up without spending overly in the campaign. This also gives him an incentive to increase spending, since, as we have discussed before, ϕ determines the marginal product of investment. To the contrary, if σ_ϵ is relatively low, catching up due to luck is relatively unlikely, and hence the trailing candidate has no incentive to compete. As a result, both candidates' investments decrease. In case candidates are similarly popular, an increase in variance always decreases the incentives to invest, as the marginal product of investment goes down. Finally, consider the effect of the popularity advantage d_2. This comparative static is of special importance for the candidates as they are able to influence their popularity through investing in stage 1. Since the second stage equilibrium is symmetric, the difference in spending, $x_F^2 - x_T^2$, is zero, independent of d^2. Thus, the marginal increase in the winning probability *in equilibrium* is equal to the density of the shock, evaluated at d^2, $\frac{1}{\sigma_\epsilon}\phi(\frac{d_2}{\sigma_\epsilon})$. The normal density is strictly quasi-concave and symmetric at zero, and thus $\phi(\frac{d_2}{\sigma_\epsilon})$ is strictly decreasing in $|d_2|$. This implies spending decreases as $|d^2|$ gets larger. Thus, the intensity of competition in $t = 2$ increases as the popularity difference vanishes.

3.2.2. Stage 1

Next turn to stage 1. How does the existence of a poll in period 2 affect candidates' incentives? In order to answer this question, we focus on situations where pure strategy equilibria exist. This depends on the cost parameter c and the variance of the macro shock σ_ϵ^2, or more specifically, on $\rho := c\sigma_\epsilon^2$. We can interpret ρ as a measure of the relative importance of campaigning and randomness. Obviously, higher marginal costs c makes it more expensive to gain a certain advantage relative to the opponent through campaigning and thus shocks become relatively more important in determining the outcome. A high variance σ_ε^2 implies that the density in the center of the shock distribution is low and thus that the expected absolute value of the shock is greater. Thus, at the margin, campaign spending in a close race has only a small impact on the probability to win the campaign when ρ is large.[11] First we consider values of ρ such that a *unique* pure strategy equilibrium exists. Specifically, we require ρ to be sufficiently large or $\rho > \left(3^{\frac{3}{4}}\sqrt{\pi}\right)^{-1} = \overline{\rho}$.[12]

Consider the decision each candidate has to make in $t = 1$. There are three different channels through which his decision influences a candidate's expected utility. First, campaigning effort increases the probability to win the election. This effect is, as in the previous section, identical for both, and hence cannot be a cause of differences in campaign disbursements. Second, spending has immediate costs. But candidates have identical cost functions, and hence marginal costs are also identical. Therefore, this cannot be a reason for different behavior, either. There is, however, a third channel. By increasing spending in $t = 1$, a candidate changes the expected state of the campaign in $t = 2$ in her favor, because $E[\delta^2] = a + x_F^1 - x_T^1$; the marginal impact of investment in $t = 1$ on $E[\delta^2]$ is equal to one. Changing $E[\delta^2]$ has consequences for expected future spending, and hence for expected future costs. From the discussion of the comparative statics in $t = 2$, we know that costs are highest when the race is tied, $d^2 = 0$, and that costs decrease monotonically if we let $|d^2|$ grow, because equilibrium spending goes down. This implies that the leading candidate, by exerting campaign effort in stage 1, increases $|d^2|$ in expectation and hence decreases her expected costs in stage 2, while the opposite holds for the trailing candidate. In more technical terms, we find that the leading candidate's efforts in stage 1 and 2 are strategic substitutes, while the trailing candidate's efforts are strategic complements (Bulow et al., 1985). Thus, the leader has lower (expected) marginal costs of investment than her opponent. As a consequence, she spends more in equilibrium and thereby in expectation increases the difference to her opponent. We have a situation in which the leader acts tough, while her opponent takes a softer stance. We summarize this result in the following proposition:

Proposition 3. *Let $\rho > \overline{\rho}$. In the unique equilibrium, the candidate leading in the poll chooses greater investment in stage 1 than her opponent. If the race is tied, both candidates choose identical spending.*

Proof. See appendix. □

Together, Propositions 2 and 3 characterize the candidates' spending decisions over the course of the campaign. While in stage 1 spending is typically asymmetric, with the front-runner outspending the trailing candidate, expenditures are symmetric just before the election in stage 2. Overall, with polls, the front-runner enjoys *momentum* in expectation, i.e. an initial lead grows in expectation over time. In Proposition 1, we characterized spending decisions absent polls. We found that behavior is completely symmetric and thus an initial advantage translates into an equal expected advantage. With these observations, we can state our main result:

Corollary 1. *The effect of polls: If campaigning is not very effective, respectively, the macro shock is sufficiently important, $\rho > \overline{\rho}$, polls create momentum in favor of the front-runner. Thus, the introduction of polls increases the likelihood that the front-runner wins the election.*

Proof. This follows immediately from Propositions 1, 2, and 3. □

We find that the availability of information about relative popularity crucially changes the incentives of the candidates. In fact, a candidate starting into the campaign with an incumbency advantage will make this advantage grow over time in expectation as the campaign progresses.

So far, we have assumed that ρ is sufficiently large. To the contrary, when ρ is small, campaigning is effective and cheap, and hence influential. Thus, influencing future popularity through campaigning is attractive. Furthermore, because of the competitive environment, expected costs in period 2 are relatively high,[13] making it attractive to use period

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11 We can use ρ to illustrate the importance of shocks vs. campaign spending in symmetric races. To see this, note that campaign spending in the second stage with $d^2 = 0$ equals $\frac{1}{\sqrt{2\pi}\,c\,\sigma_\epsilon}$. The expected absolute value of the shock per period is $\sqrt{\frac{2}{\pi}}\sigma_\epsilon$. Hence, shocks and campaign spending have a comparable magnitude when $\frac{1}{\sqrt{2\pi}\,c\,\sigma_\epsilon} = \sqrt{\frac{2}{\pi}}\sigma_\varepsilon \Leftrightarrow \rho := c\sigma_\epsilon^2 = 1/2$. When ρ increases, the shocks get relatively more important.

12 As we show in the proof of Proposition 3, $\overline{\rho}$ is the smallest value of ρ for which the system of first order conditions has a unique solution. Moreover, we show in the proof of Proposition 6 that at $\rho = \overline{\rho}$, the slope of the individual best responses for a $= 0$ and on the main diagonal, i.e. where $x_F^1 = x_T^1$, is -1. The absolute value of the slope at this point is strictly decreasing in ρ. Whether this slope is larger or smaller than -1 is sufficient to determine whether or not multiple equilibria exist.

13 c affects actual stage 2 costs in two ways. First, a greater c implies greater marginal costs, which decreases equilibrium spending. Lower equilibrium spending, however, implies lower costs, and this effect dominates. Hence, with lower marginal costs c, equilibrium costs in stage 2 are actually greater and thus there is greater potential to use diffused competition to save on cost.

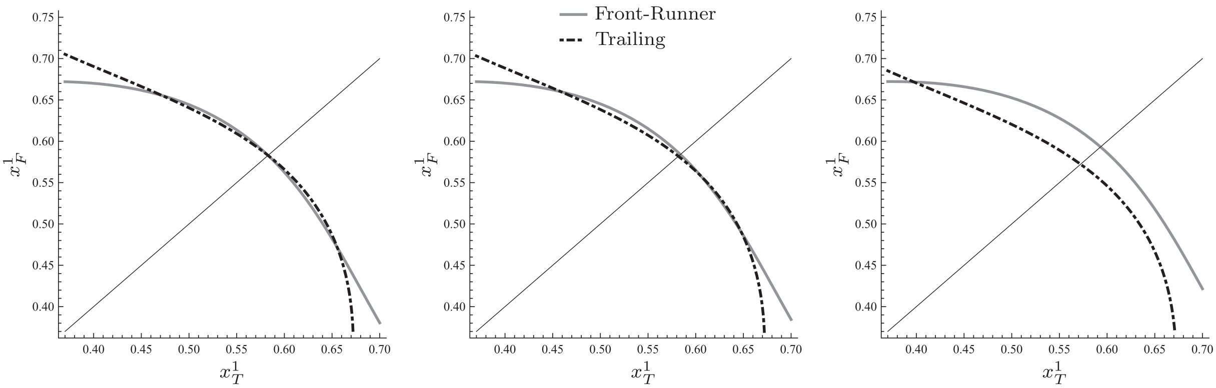

Fig. 2. Reaction functions for F (solid) and T (dashed) and stage 1 equilibria for $\rho = 0.242$ and $a \in \{0, 0.04, 0.1\}$. The diagonal is the 45 degree line, depicting symmetric spending.

1 spending to defuse competition and save on future costs. In such a situation, "fighting for resurrection" turns out to be a possible equilibrium. However, the popularity difference between candidates must not be too large. To see why this is the case, recall the aforementioned logic of strategic substitutes and complements. A candidate's investments are strategic substitutes if and only if he is in expectation more popular than his opponent in stage 2, given equilibrium efforts. If this is not the case, a marginal increase in spending decreases the expected popularity gap and hence increases expected cost. Consequently, the trailing candidate may spend more in equilibrium, but if he does this, he needs to spend enough to turn the state in his favor in expectation. Of course, turning the state becomes increasingly expensive as the difference between candidates increases, and there exists a threshold gap determining the maximum lead that the trailing candidate may try to turn by investing heavily in campaigning.[14]

Obviously, if the trailing candidate can take charge and act aggressively in the campaign in equilibrium, this is also possible for the front-runner. After all, it is cheaper for her to stay in the lead than for her opponent to turn the electorate's sentiment. Consequently, when $\rho \leq \overline{\rho}$, there exist multiple equilibria in close-enough races.

It remains to discuss what happens when the race is tied. As before, to conjecture that there exists a symmetric equilibrium is appealing. But we have also seen that there are multiple equilibria if no candidate dominates. Those asymmetric equilibria also exist in the tied race. Hence, there exist three equilibria for $a = 0$, and more generally, when a is small. The symmetric equilibrium at $a = 0$ is asymptotically unstable, while the asymmetric ones are stable. We note in passing that this has the interesting implication that the stage 1 subgame with low ρ resembles a (possibly asymmetric) hawk–dove game (e.g. Rapoport and Chammah (1966) or Baliga and Sjöström (2012)).

In the next proposition, we summarize the results formally:

Proposition 4. *If ρ is relatively small, $\rho \leq \overline{\rho}$, there exist multiple equilibria in close races. If we confine ourselves to asymptotically stable equilibria, there exists no symmetric equilibrium, even when $a = 0$. In close races, either candidate may spend more in stage 1 in equilibrium, while if one candidate has a sufficiently large advantage, this candidate outspends her opponent in stage 1 and the equilibrium is unique.*

Proof. See appendix. □

Even though the two types of equilibria may look very different at first sight, they follow in fact the same underlying logic. Candidates find it worthwhile to defuse competition in period 2 by investing asymmetrically in period 1. One candidate takes a soft stance and suffers a lower probability of success, but this will be more than compensated through the saved costs. The other candidate takes a tough stance and thus incurs high costs in period 1. He is more than compensated through the increased probability of success and the cost savings in period 2.

What is the underlying cause in the difference in results between Propositions 3 and 4? When ρ is large, the trailing candidate cannot effectively close the gap to the front-runner at a reasonable cost, because the relative influence of campaigning is limited. Moreover, the benefits of diffusing competition are low because spending tends to be low in such a situation. Thus, there needs to be momentum in such a situation. As ρ decreases, campaigning becomes more and more important. Whenever the trailing candidate outspends the front-runner by more than a, he turns the expected state in his favor, and as randomness becomes less important, the probability that he keeps this newly gained advantage increases. This way, when ρ is low, by spending excessively in the campaign, T can put F in exactly the same situation in which he used to be himself: By marginally increasing campaign investments in stage 1, the former front-runner F increases the fierceness of competition in stage 2, which is to her own detriment, and which results in her being content with not fighting back. Moreover, because ρ is low, T indeed has sufficient control over the campaign to be able to achieve this at a reasonable cost. Hence, as the cost-effectiveness of campaigning increases, there might be anti-momentum. Of course, this cannot be the case for all possible a, because turning the state becomes increasingly expensive as the difference between candidates increases, and there exists a threshold gap determining the maximum lead that the trailing candidate may try to turn by investing heavily in campaigning. When F has a lead greater than this threshold, anti-momentum disappears and only momentum remains possible in equilibrium.

In order to illustrate the multiplicity of equilibria in stage 1 graphically, we plot the best response functions of the candidates for $\rho = 0.242 < \overline{\rho}$ and $a \in \{0, 0.04, 0.1\}$ in Fig. 2. In the left panel, the symmetric case is depicted. The best response functions cross three times. The symmetric equilibrium is not asymptotically stable; a slight deviation results in best response dynamics away from this equilibrium, because the slope of best responses at this point is less than -1.[15] In the panel in the middle, F has a small advantage. Best response functions cross three times as well and thus multiple equilibria exist, the two more asymmetric ones being asymptotically stable similar to the symmetric case with $a = 0$. In the panel on the right, F's advantage is sufficiently large such that best response functions cross only once. Thus there is a unique equilibrium exhibiting momentum.

[14] $\overline{x} = \sqrt{\frac{2}{c}}$ is as an upper bound for this threshold, because this is the spending level which costs exactly 1, which is the value of winning the election. Hence, a sufficient condition for momentum is $a > \overline{x}$.

[15] We show that the slope of best responses on the symmetric intersection decreases in ρ and that at $\rho = \overline{\rho}$, the slope is exactly -1. This implies that for $\rho < \overline{\rho}$, the symmetric equilibrium cannot be stable.

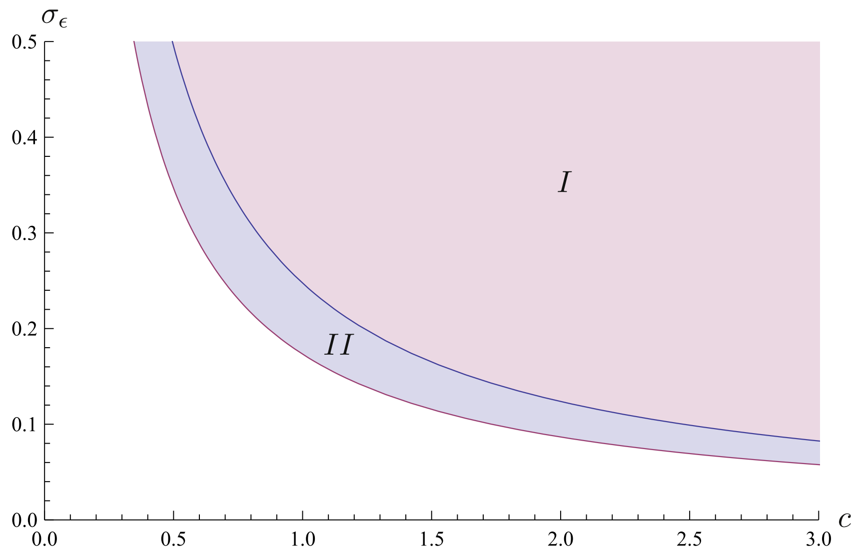

Fig. 3. Number of pure strategy equilibria in stage 1 depending on c and σ_ϵ. In Region *I*, there is a unique equilibrium for all possible realizations of a. In Region *II*, there are two stable equilibria when a is small and a unique equilibrium when a is large. In the region below *II*, for some a, the second order conditions are violated and hence existence of a pure strategy equilibrium cannot be guaranteed for all a.

Next, consider the effect of polls on the election outcome. For the case of $\rho \leq \overline{\rho}$, we need to amend Corollary 1 slightly.

Corollary 2. *The effect of polls: If the macro shock is relatively unimportant, $\rho < \overline{\rho}$, polls create momentum if there is a clear front-runner. Thus, polls increase the probability that the front-runner is elected. In close races, momentum and anti-momentum may occur, and consequently, the position of the front-runner may be both strengthened or weakened through the introduction of polls.*

Proof. This follows immediately from Propositions 1, 2, and 4. □

A final comment concerning the equilibria when $\rho \leq \overline{\rho}$ is in order. A low value of ρ implies that either marginal costs are low or the campaign is–to borrow from the theory of contests–relatively discriminating (low σ_ϵ), or both. Either one makes it unlikely that an interior pure strategy equilibrium exists, since the second order conditions are then likely to be violated. Typically there exists only a 'small' range of values for $\rho \leq \overline{\rho}$ for which the second order conditions hold. Fig. 3 illustrates combinations of c and σ_ϵ for which pure strategy equilibria as discussed above exist. However, it can be shown that our results are qualitatively identical to the ones described in Proposition 4 when $\sigma_\epsilon \to 0$.[16] In this case, the game becomes a dynamic All-pay auction with equilibria in mixed strategies in stage 2 and pure strategy equilibria in stage 1. Also here, momentum and anti-momentum equilibria co-exist in close races, while in lopsided races only momentum occurs.

To sum up, there always exists an equilibrium in which the front-runner outspends the trailing candidate and thus there is momentum. Furthermore, in close races when campaigning is relatively effective, also the trailing candidate may outspend the front-runner. While in this section we compared relative spending, in the next section we are interested in the shape of the candidates' spending profiles.

4. Polls and candidate spending

4.1. Polls and candidates' spending profiles

In this section we take a closer look at the spending profiles of the candidates. Without polls the result is straightforward, spending decreases as one candidate becomes more and more popular (in expectation). In this section we focus on spending profiles given that polls are conducted. Because comparative statics are only meaningful in settings with unique equilibria, we restrict ourselves to this case. We compare our results with empirical findings and show that the model performs relatively well in predicting spending profiles.

We know already from Section 3 that (given a unique equilibrium) polls always create momentum and the more popular candidate invests more heavily in the campaign. We are now interested in how the candidates' spending varies in absolute terms as we increase the realized popularity advantage $|a|$ of the front-runner from zero. In stage 2, we observed that spending decreases monotonically in $|d^2|$, i.e. when one candidate becomes more and more popular with the electorate. Also in the studies of Snyder (1989), Erikson and Palfrey (2000), or Klumpp and Polborn (2006) intensity of competition decreases as candidates become increasingly asymmetric. However, in stage 1, we find a non-monotonic spending profile for the front-runner. The reason is that compared to a symmetric race, the front-runner has an additional incentive to invest in stage 1 in order to reduce future competition. This also explains why the trailing candidate's spending profile is monotonically decreasing. Thus, we find that introducing dynamics may change results significantly. The following proposition states the result:

Proposition 5. *Assume $\rho > \overline{\rho}$. Candidate F's expected total investment increases in a when $a = 0$, while candidate T's expected total investment decreases.*

Proof. See the supplementary appendix. □

The trailing candidate's campaign spending decreases monotonically as $|a|$ grows larger. His more popular adversary, however, has an incentive to first increase spending when her lead grows larger, before she also cuts down on spending when she becomes more and more advantaged in the campaign. We can now describe the two candidates spending profiles completely:

Corollary 3. *As $|a|$ increases from zero, the more popular candidate first increases spending in the campaign and her spending declines only after she reaches a certain popularity advantage. The trailing candidate monotonically decreases spending and spends less than his opponent for all $|a| > 0$.*

[16] We show this in Supplementary Appendix A.1.

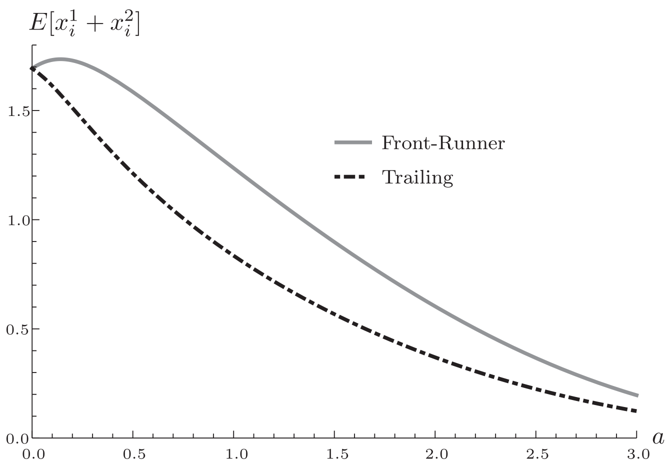

Fig. 4. Total expected spending per candidate as a function of initial popularity for $\sigma_\epsilon = 1$ and $c = \frac{1}{3}$. The patterns closely resemble those of Fig. 3 of Erikson and Palfrey (2000).

Proof. See the supplementary appendix. □

How well does the model perform in predicting candidate spending in a campaign? In Fig. 4, we plotted expected total campaign spending for both candidates for $\sigma_\epsilon = 1$ and $c = \frac{1}{3}$. The spending patterns as a function of initial popularity are quite similar to those discovered by Erikson and Palfrey (2000), see Fig. 3 of their paper.[17] Hence, despite its simplicity, our model seems to predict spending patterns well.

4.2. Polls and the intensity of political competition

Campaign expenditures can be quite substantial. For example, expenditures during both the 2008 and 2012 U.S. presidential campaign exceeded one billion U.S. dollars. Thus, the total of campaign disbursements is also an important aspect to study and relate to the existence of polls. The following proposition shows the result:

Proposition 6. Expected aggregate campaign expenditures are lower in the presence of polls.

Proof. See the supplementary appendix. □

Because polls create momentum (or anti-momentum), initial asymmetries are amplified. This decreases the expected intensity of competition in subsequent rounds, and hence expected aggregate expenditures decrease, too.

We derive results on aggregate spending also for more general convex cost functions $C(x)$ in Supplementary Appendix A.1. We find that when $C'''(x) \geq 0 \ \forall \ x$, aggregate campaign expenditures decrease in expectation with the introduction of polls, strictly so when $|\mu_\alpha| > 0$. For $C'''(x) < 0 \ \forall \ x$, aggregate expenditures may increase. This is in line with earlier findings by Aoyagi (2010), Ederer (2010), as well as Gershkov and Perry (2009). Intuitively, there are two effects at work. Initial asymmetries are amplified through a poll which tends to defuse competition and thus lower investments into the campaigns. In addition though, the candidates also face a less noisy decision environment, which has a differential effect on expenditures. When the race is revealed to be close (lopsided), competition becomes more (less) fierce. The overall effect of polls on aggregate campaign spending is thus ambiguous and depends on the shape of the marginal cost function.

5. Discussion

In this section, we offer a short discussion of possible extensions of the baseline model and discuss our results in comparison to Klumpp and Polborn (2006).

5.1. Literature

At this point, it is illustrative to link our results to those of Klumpp and Polborn (2006), who also study a dynamic electoral competition model. First, we clarify the differences in set-up. While our study focuses on the dynamics in spending during a single contest as time evolves and information is revealed, they look at a grand contest and focus on behavior in each single, static component contest. The candidate winning the majority of the component contests is elected. A notable difference to our set-up is that instead of accumulating a lead equal to the spending difference (plus noise) over time, the candidates accumulate wins over time and each win changes the difference between candidates by a fixed amount.

The set-up of Klumpp and Polborn allows them to explain the so-called New Hampshire effect in U.S. presidential primaries: On average, candidates front-load their electoral campaigns even though early states—like New Hampshire—are small and hence seemingly negligible for outcomes. The intuition is that achieving an early lead creates an asymmetry in the candidates' values of winning future component contests and thereby creates momentum for the early winner. As an early loser faces lower continuation values, he is discouraged to invest relative to the early winner. This strategic interdependency of intertemporal investments is often referred to as the discouragement effect and is well documented in other scenarios, see Konrad (2012) for a discussion.

We uncover another variant of this discouragement effect. Initial asymmetries encourage the front-runner to invest more heavily in her campaign early on as she can benefit from reduced competition in the future. The exact manifestation of the discouragement effect differs from the setting of Klumpp and Polborn. While in both settings, the discouragement effect produces asymmetries and thus results in an overall decrease in competitiveness, in our setting the front-runner often becomes more aggressive compared to the symmetric situation. Moreover, in contrast to Klumpp and Polborn, we identify the possibility of multiple equilibria: The discouragement effect may work against the stronger candidate and lead to anti-momentum when campaigning is very effective and no candidate has a large advantage.[18]

5.2. Concerns for vote share

In many situations, candidates do not only care about winning the election but also about the vote share they receive. For example, a government that has only slightly more than 50 percent of the vote share might have difficulties in passing laws because some delegates might refuse to toe the party line. Similarly, in proportional representation electoral systems, the vote share is important because it determines relative power of different parties in the parliament. We can generalize the baseline model to account for this mix of motives. Payoffs are a continuously increasing function of the obtained vote share, and there is a discrete jump once the vote share hits the 50 percent mark due to a plurality premium. Hence, our modeling approach follows the one of

[17] In their paper, the incumbent's spending increases in popularity until it peaks at some 55 percent and decreases thereafter until the predicted incumbent vote reaches 80–85 percent. Then, somewhat surprisingly, spending goes up again. One reason for this upwards turn for a dominating front-runner might lie in her ability to attract funding. In our model, costs do not depend on popularity. When increased popularity makes fundraising easier and thus costs of investment lower, an additional benefit of being a front-runner arises. The challenger's spending decreases in the incumbent's popularity advantage and remains below the incumbent's spending at all time.

[18] This difference is not driven by the difference in investment effectiveness between the two settings. Konrad and Kovenock (2009) study a framework à la Klumpp and Polborn (2006) where each component contest is modeled as an all-pay auction and thus only investments matter for success. Also in their setting, anti-momentum does not occur.

Iaryczower and Mattozzi (2013). For details, see Supplementary Appendix A.2.

How does this change incentives? Intuitively, when vote shares become important, instead of just focusing on the decisive voter, candidates will consider the whole distribution of voter preferences. Thus, the shape of the distribution of voter preferences will be crucial for relative campaigning incentives. In the analysis so far, that is if there is only a plurality premium and candidates do not care about vote shares but only about winning the majority, competition will be fiercest in stage 2 when candidates hold equal vote shares. This need not be the case anymore. For example, if candidates only care about the vote share, competition is fiercest when the mode of the preference distribution is just indifferent, because at this point, the vote share obtained by raising campaign expenditures marginally is greatest. In the simplest case of a quasi-concave distribution of voter preferences, this is the case when the distribution peaks at d^2. As before, a candidate's incentives to invest in stage 1 depend on whether this will bring him closer in expectation to a position of fierce future competition. If the electorate is polarized, i.e., there is a large mass of voters at the tails of the preference distribution, while only a small mass is in the center, competition is fiercest when candidates are not equally popular. Thus, in these cases, the trailing candidate may have an incentive to expend more in stage 1 than the front-runner as this will enable him to defuse competition in the future.

Two results emerge from this extended set-up. First, as long as the weight put on vote share is relatively small, the results from the baseline model are qualitatively robust. On the flip-side, if candidates care sufficiently about vote share and the electorate is sufficiently polarized, there exist unique anti-momentum equilibria in close races. What can we conclude from this? In FPTP systems, it is likely that candidates value obtaining the majority much more than an increase in the vote share. In this case, the results from the baseline model remain valid and momentum is likely to occur. However, in other electoral systems, results may differ. For example, in proportional electoral systems, the vote share is more important than the plurality premium, especially when there are multiple parties of similar strength such that it is quite unlikely for any to win the majority. If in such a system, the electorate is polarized, anti-momentum might be the unique outcome. Hence, with respect to momentum in political campaigns, we find significant differences between electoral systems.

5.3. The precision of polls

We assumed that without a poll, candidates receive no feedback about their relative standing with the populace, while in the presence of polls, they learn current popularity for sure. These were interesting benchmark cases and serve well to illustrate the relevant intuitions. We extend our baseline model to analyze imprecise polls to show that our results hold more generally, see Supplementary Appendix A.3 for more details. Here we assume that instead of fully revealing current popularity, polls offer a noisy, though unbiased, public signal.

Also in this generalized set-up we find that the first-period front-runner always outspends the trailing candidate (given a unique equilibrium exists). Furthermore, when the first-period advantage of the front-runner is sufficiently small, an increase in poll precision increases the spending gap between front-runner and trailing candidate. If the front-runner's advantage is quite large, on the other hand, increasing the precision of the poll decreases momentum as the spending differential goes to zero.

Intuitively, being front-runner in the poll at $t = 1$, one can be more sure that one is indeed preferred by the decisive voter, ceteris paribus, when the precision of polls increases. This has two opposing effects. On the one hand, she will be more likely to benefit from reduced expenditures in the future, giving her an extra incentive to invest. On the other hand, she can be more sure to be actually in the lead, thereby depressing incentives to invest in the campaign. In close races, the former effect dominates, while in races with a very strong candidate, the latter effect dominates and both candidates' spending levels decrease with poll precision. This may also suppress the spending differential, such that greater poll precision decreases momentum.

5.4. Costs of campaigning

We assumed that costs of campaigning effort are additive across periods. This implies that the candidates' expected costs of spending in the future do not (directly) depend on current expenditures and vice versa. Thinking about costs of fund raising, one can easily imagine, though, that high expenditures in early periods make it harder to raise money later (ceteris paribus). This would be the case if first the most willing donors are targeted for donations. Then, in later periods, new, less willing donors have to be convinced to contribute. We expect this to have a mitigating effect on the magnitude of momentum. When thinking about whether to increase spending, the front-runner will trade off a decrease in future competition with higher expected costs of raising funds.

On the other hand, it may also seem plausible that early success lowers (marginal) costs of raising funds in the future. This would be the case when donors condition their donation on expected success in the election, maybe because they are expecting the implementation of favorable policies from the winner or because they simply like to support the winner. Then investing early has an additional advantage in that it improves popularity in expectation and thus lowers the cost of fundraising. In such a case, momentum will be amplified. Which effect is more relevant is an empirical question.

Finally, we assumed that candidates are similar in their costs of raising funds. Thus, we abstract away from any differences between candidates not related to their relative popularity. Adding asymmetries, for example, in ability or costs, will naturally lead to more asymmetric campaigning behavior in some situations and might mitigate the effects described in the current paper in other settings. However, the general intuitions we developed should be preserved also in such a more general setting. The benefit of our modeling approach is being able to disentangle different effects that play a role in creating momentum more clearly.

6. Conclusion

In this paper, we explore the effect of opinion polls on candidates' incentives to campaign and the final election outcome. We find that when candidates mostly care about winning the election, polls generally cause momentum for the front-runner. The reason is that the front-runner has an additional incentive to invest in her campaign. By investing early, it becomes more likely that she comes out ahead in the poll in the future. This in turn defuses competition and thus she can save on campaigning expenditures closer to the election. In close campaigns, when campaigning is very effective, also the trailing candidate may adopt a tough stance and outspend the leader.

We also show that polls tend to decrease campaign expenditures. The reason is that candidates tend to spend most when the campaign race is close, but since polls create momentum, they induce more lopsided campaigns and hence depress aggregate spending. A sufficient condition for this to hold more generally is a non-negative third derivative of the cost of campaigning function.

As discussed, we show that momentum arises quite generally when candidates care mostly about winning the election. In case candidates are mainly interested in their vote share and the electorate is relatively polarized, the trailing candidate may have stronger incentives to campaign in relatively close elections. Anti-momentum may be the unique equilibrium outcome. Thus, we expect the effect of polls to differ between countries with a FPTP system and countries with proportional representation. In the latter, no general statement about momentum can be made as the voter preference distribution becomes important.

An implication of the model is that given an incumbency advantage at the outset of the campaign, polls are likely to foster this advantage and thus decrease the rate of turnover in political offices. Indeed, the spending profile predicted by our model fits well with the estimated profiles in Erikson and Palfrey (2000), who show that incumbents with an early popularity edge tend to improve their chances by spending more than their opponents in the competition.

Our model shows that a ban on the publication of opinion polls, as many countries impose in the pre-election period, will likely not eliminate all effects on voters. In particular, we have shown that there exists an indirect effect of polls on candidates' campaigning investments, which in turn influence voters' decisions at the ballot. This effect is still in place as long as candidates are allowed to commission polls in that period. Thus, a ban on the publication of polling results needs to be complemented with a ban or a cap on campaigning if potential causes of momentum are to be eliminated.

In our model, voters react only to expenditures of the candidates, not to their relative standing. Other papers have argued that voters' preferences may be influenced also by relative standing (e.g. Hong and Konrad (1998) or Callander (2007)). Voters receive additional utility by voting for a candidate who is ahead in the polls. Thus, for given popularity levels and equal campaigning effort, the advantage of the front-runner grows. In terms of our model, this can be interpreted as an additional popularity dependent shift of the voter distribution in favor of the front-runner. Under this interpretation of the “bandwagon” effect, momentum still results. Thus, accounting for the bandwagon effect only introduces further stickiness into our model and potentially intensifies momentum.

For future research, it is interesting to empirically validate the results proposed by the theory. In contrast to theories that focus on the direct effect of polls on voters, our theory has predictions regarding candidates' campaign expenditures. Another interesting direction for future research is to study how polls influence donors' decisions to contribute to the candidates' campaigns. As Fig. 4 shows, clear front-runners have a significant spending advantage over trailing candidates which is not completely captured by our model, but which we believe to be related to donors' behavior.

Appendix A

A. Proof of Proposition 1

Proof. If there is no poll, candidates do not observe the realization of ϵ^1 nor investments and hence their information does not change between periods 1 and 2. They maximize

$$\max_{(x_F^1,x_F^2)\in\mathbb{R}_+^2} \int_{-\infty}^{\infty} \Phi\left(\frac{a+x_F^1+x_F^2-x_T^1-x_T^2}{\sqrt{2}\sigma_\epsilon}\right)\frac{1}{\sigma_\alpha}\phi\left(\frac{a-\mu_\alpha}{\sigma_\alpha}\right)d\,a-\frac{c}{2}\left(x_F^1\right)^2-\frac{c}{2}\left(x_F^2\right)^2,$$

$$\max_{(x_T^1,x_F^2)\in\mathbb{R}_+^2} 1-\int_{-\infty}^{\infty} \Phi\left(\frac{a+x_F^1+x_F^2-x_T^1-x_T^2}{\sqrt{2}\sigma_\epsilon}\right)\frac{1}{\sigma_\alpha}\phi\left(\frac{\alpha-\mu_\alpha}{\sigma_\alpha}\right)d\alpha-\frac{c}{2}\left(x_T^1\right)^2-\frac{c}{2}\left(x_T^2\right)^2,$$

where $\Phi(\frac{\cdot}{\sqrt{2}\sigma_\varepsilon})$ is the CDF of the convolution of two Gaussian distributions with zero mean and variance σ_ϵ^2. The system of first order conditions is

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2}\sigma_\epsilon}\phi\left(\frac{a+x_F^1+x_F^2-x_T^1-x_T^2}{\sqrt{2}\sigma_\epsilon}\right)\frac{1}{\sigma_\alpha}\phi\left(\frac{a-\mu_\alpha}{\sigma_\alpha}\right)d\,a-c\cdot x_i^t=0,$$

$i=F,T$ and $t=1,2$. It is easily observed that in pure strategy equilibrium, it must hold that $x_F^1=x_F^2=x_T^1=x_T^2=x^*$. In particular,

$$x^*=\frac{1}{c}\int_{-\infty}^{\infty}\frac{1}{\sqrt{2}\sigma_\epsilon}\phi\left(\frac{a}{\sqrt{2}\sigma_\epsilon}\right)\frac{1}{\sigma_\alpha}\phi\left(\frac{a-\mu_\alpha}{\sigma_\alpha}\right)d\,a=\frac{e^{-\frac{\mu_\alpha^2}{2\sigma_\alpha^2+4\sigma_\epsilon^2}}}{\sqrt{2\pi}c\sqrt{\sigma_\alpha^2+2\sigma_\epsilon^2}}.$$

Note that corner solutions with $x_i^t=0$ cannot be an equilibrium since the Gaussian density functions are unbounded and marginal cost at $x_i^t=0$ are zero. The second order conditions evaluated at x^* are

$$\begin{aligned} F:&\quad \int_{-\infty}^{\infty}\frac{1}{2\sigma_\epsilon^2}\phi'\left(\frac{a}{\sqrt{2}\sigma_\epsilon}\right)\frac{1}{\sigma_\alpha}\phi\left(\frac{a-\mu_\alpha}{\sigma_\alpha}\right)d\,a-c<0,\\ T:&\quad -\int_{-\infty}^{\infty}\frac{1}{2\sigma_\epsilon^2}\phi'\left(\frac{a}{\sqrt{2}\sigma_\epsilon}\right)\frac{1}{\sigma_\alpha}\phi\left(\frac{a-\mu_\alpha}{\sigma_\alpha}\right)d\,a-c<0. \end{aligned}$$

If

$$c>\mathrm{Abs}\left[\int_{-\infty}^{\infty}\frac{1}{2\sigma_\epsilon^2}\phi'\left(\frac{a}{\sqrt{2}\sigma_\epsilon}\right)\frac{1}{\sigma_\alpha}\phi\left(\frac{a-\mu_\alpha}{\sigma_\alpha}\right)d\,a\right]=\mathrm{Abs}\left[\frac{\mu_\alpha e^{-\frac{\mu_\alpha^2}{2\left(\sigma_\alpha^2+2\sigma_\epsilon^2\right)}}}{\sqrt{2\pi}\left(\sigma_\alpha^2+2\sigma_\epsilon^2\right)^{3/2}}\right]$$

the second order conditions hold for both. Maximizing this condition with respect to μ_α, we find that this is most critical for $\mathrm{Abs}[\mu_\alpha]=\sqrt{2\sigma_\epsilon^2+\sigma_\alpha^2}$. Thus, when $c>\frac{1}{\sqrt{2e\pi}(\sigma_\alpha^2+2\sigma_\epsilon^2)}$, the second order conditions generally hold. For example, when $\sigma_\epsilon=\sigma_\alpha=1$, we need $c>0.1141$ to hold. For the remainder of the paper, we assume this to be the case. □

B. Proof of Proposition 2

Proof. Consider the subgame in stage 2 when there is a poll. In this case, both candidates know the decisive voter's exact position in stage 2, d^2. Candidates maximize

$$\max_{x_F^2\in\mathbb{R}_+}\Phi\left(\frac{d^2+x_F^2-x_T^2}{\sigma_\epsilon}\right)-\frac{c}{2}\left(x_F^2\right)^2 \quad\text{and}\quad \max_{x_T^2\in\mathbb{R}_+}1-\Phi\left(\frac{d^2+x_F^2-x_T^2}{\sigma_\epsilon}\right)-\frac{c}{2}\left(x_T^2\right)^2$$

First order conditions read

$$\frac{1}{\sigma_\epsilon}\phi\left(\frac{d^2+x_F^2-x_T^2}{\sigma_\epsilon}\right)-c\,x_i^2=0,$$

where $i=F,T$. It is immediately observed that—as without a poll—both candidates choose identical spending, $x_F^2=x_T^2$. In particular, $x_F^2=x_T^2=x^{**}(d^2)=\frac{1}{c}\frac{1}{\sigma_\epsilon}\phi(\frac{d^2}{\sigma_\epsilon})$.

The second order condition for F is $\frac{1}{\sigma_\epsilon^2}\phi'(\frac{d^2}{\sigma_\epsilon})-c<0$. Maximizing this condition with respect to d^2 shows that it attains a maximum when $d^2=-\sigma_\epsilon$. The condition is generally fulfilled when

$$c>\frac{1}{\sqrt{2e\pi}\sigma_\epsilon^2} \tag{B.1}$$

which we assume to be the case henceforth. If $\sigma_\epsilon=1$, it must hold that c > 0.2419. □

C. Proofs of Propositions 3 and 4

Proof. The expected utilities of F and T, conditional on being in state d^2 in the second stage, are

$$E\left[\pi_F^2\left(d^2\right)\right]=\Phi\left(\frac{d^2}{\sigma_\epsilon}\right)-\frac{c}{2}\left(\frac{\phi\left(\frac{d^2}{\sigma_\epsilon}\right)}{c\,\sigma_\epsilon}\right)^2 \text{ and } E\left[\pi_T^2\left(d^2\right)\right]=1-\Phi\left(\frac{d^2}{\sigma_\epsilon}\right)-\frac{c}{2}\left(\frac{\phi\left(\frac{d^2}{\sigma_\epsilon}\right)}{c\,\sigma_\epsilon}\right)^2.$$

Note that $d^2=a+x_F^1-x_T^1-e^1$. Then we can write the optimization problem of the candidates as:

$$\max_{x_F^1\geq 0}\int_{-\infty}^{\infty}\left(\Phi\left(\frac{a+x_F^1-x_T^1-e^1}{\sigma_\epsilon}\right)-\frac{\left(\phi\left(\frac{a+x_F^1-x_T^1-e^1}{\sigma_\epsilon}\right)\right)^2}{2c\,\sigma_\epsilon^2}\right)\frac{\phi\left(\frac{e^1}{\sigma_\epsilon}\right)}{\sigma_\epsilon}de^1-\frac{c}{2}\left(x_F^1\right)^2,$$

$$\max_{x_T^1\geq 0}\int_{-\infty}^{\infty}\left(1-\Phi\left(\frac{a+x_F^1-x_T^1-e^1}{\sigma_\epsilon}\right)-\frac{\left(\phi\left(\frac{a+x_F^1-x_T^1-e^1}{\sigma_\epsilon}\right)\right)^2}{2c\,\sigma_\epsilon^2}\right)\frac{\phi\left(\frac{e^1}{\sigma_\epsilon}\right)}{\sigma_\epsilon}de^1-\frac{c}{2}\left(x_T^1\right)^2.$$

Before we analyze the equilibrium in detail, we now need to show that an equilibrium exists.

Lemma 1. *If c is sufficiently large, $c>\overline{c}$, a pure strategy equilibrium exists for all realizations a.*

Proof. The second derivative of F's payoff function with respect to x_F^1 is

$$Q=\int_{-\infty}^{\infty}\left(\frac{\phi'\left(\frac{\kappa-e^1}{\sigma_\epsilon}\right)}{\sigma_\epsilon^2}-\frac{\left(\phi'\left(\frac{\kappa-e^1}{\sigma_\epsilon}\right)\right)^2+\phi\left(\frac{\kappa-e^1}{\sigma_\epsilon}\right)\phi''\left(\frac{\kappa-e^1}{\sigma_\epsilon}\right)}{c\,\sigma_\epsilon^4}\right)\frac{\phi\left(\frac{e^1}{\sigma_\epsilon}\right)}{\sigma_\epsilon}de^1-c \tag{C.1}$$

$$=\frac{e^{-\frac{\kappa^2}{3\sigma_\epsilon^2}}\left(2\sqrt{3}\left(3\sigma_\epsilon^2-2\kappa^2\right)-27\sqrt{\pi}\kappa\sigma_\epsilon^3e^{\frac{\kappa^2}{12\sigma_\epsilon^2}}\right)}{108\pi c\sigma_\epsilon^6}-c \tag{C.2}$$

where we defined $\kappa:=a+x_F^1-x_T^1$. Note that $\lim_{|\kappa|\to\infty}Q=-c$ and that Q is continuous in κ. Hence, its range is compact. The absolute value of the first part of Q is strictly decreasing in c. Hence, there always exists a smallest c, $\overline{c}=\Gamma(g)$, such that $Q\leq 0$, implying the individual decision problem is strictly concave. Moreover, candidate T's second derivative is identical to F's if we substitute $-\kappa$ for κ. Consequently, an interior pure strategy equilibrium exists by the Debreu, Fan, and Glicksberg Theorem (see for example Theorem 1.2 in Fudenberg and Tirole (1991)). □

Now let us go more into the details of the first order conditions to determine the properties an equilibrium must have. Knowing now that a pure strategy equilibrium exists, we focus on interior equilibria henceforth, but we show in the supplementary appendix that results also hold when no such equilibrium exists. First order conditions in an interior equilibrium are

$$\int_{-\infty}^{\infty}\left(\frac{\phi\left(\frac{\kappa-e^1}{\sigma_\epsilon}\right)}{\sigma_\epsilon}-\frac{\phi\left(\frac{\kappa-e^1}{\sigma_\epsilon}\right)\phi'\left(\frac{\kappa-e^1}{\sigma_\epsilon}\right)}{c\,\sigma_\epsilon^3}\right)\frac{\phi\left(\frac{e^1}{\sigma_\epsilon}\right)}{\sigma_\epsilon}de^1-c\,x_F^1=0\Leftrightarrow\frac{9\sqrt{\pi}c\sigma_\epsilon^3 e^{-\frac{\kappa^2}{4\sigma_\epsilon^2}}+\sqrt{3}\kappa e^{-\frac{\kappa^2}{3\sigma_\epsilon^2}}}{18\pi c\sigma_\epsilon^4}-c\,x_F^1=0,$$

$$\int_{-\infty}^{\infty}\left(\frac{\phi\left(\frac{\kappa-e^1}{\sigma_\epsilon}\right)}{\sigma_\epsilon}+\frac{\phi\left(\frac{\kappa-e^1}{\sigma_\epsilon}\right)\phi'\left(\frac{\kappa-e^1}{\sigma_\epsilon}\right)}{c\,\sigma_\epsilon^3}\right)\frac{\phi\left(\frac{e^1}{\sigma_\epsilon}\right)}{\sigma_\epsilon}de^1-c\,x_T^1=0\Leftrightarrow\frac{9\sqrt{\pi}c\sigma_\epsilon^3 e^{-\frac{\kappa^2}{4\sigma_\epsilon^2}}-\sqrt{3}\kappa e^{-\frac{\kappa^2}{3\sigma_\epsilon^2}}}{18\pi c\sigma_\epsilon^4}-c\,x_T^1=0.$$

Using $\Delta^1=x_F^1-x_T^1$, it follows that in equilibrium, it must hold that $\Delta^1=\Sigma(\Delta^1+a)$, where

$$\Sigma\left(\Delta^1+a\right):=-\frac{2}{c}\int_{-\infty}^{\infty}\left(\frac{\phi\left(\frac{\kappa-e^1}{\sigma_\epsilon}\right)\phi'\left(\frac{\kappa-e^1}{\sigma_\epsilon}\right)}{c\,\sigma_\epsilon^3}\right)\frac{\phi\left(\frac{e^1}{\sigma_\epsilon}\right)}{\sigma_\epsilon}de^1=\frac{\left(a+\Delta^1\right)e^{-\frac{\left(a+\Delta^1\right)^2}{3\sigma_\epsilon^2}}}{3\sqrt{3\pi}\,c^2\sigma_\epsilon^4}. \tag{C.3}$$

The shape of this function is now important to determine equilibrium behavior. It will be useful to define

$$\xi(\kappa)=-\frac{\phi\left(\frac{\kappa-e^1}{\sigma_\epsilon}\right)\phi'\left(\frac{\kappa-e^1}{\sigma_\epsilon}\right)}{c\,\sigma_\epsilon^3}=-\frac{\kappa e^{-\frac{\kappa^2}{3\sigma\epsilon^2}}}{6\sqrt{3\pi}c\sigma\epsilon^4},$$

which represents the effect of stage 1 spending on stage 2 payoffs for candidate F. Obviously, Sign$[\xi(\kappa)]=-$ Sign$[\kappa]$ and $\xi(0)=0$. We now establish a few more lemmata that help us to characterize equilibria.

Lemma 2. *Sign*$[\Sigma(\Delta^1+a)]=$ *Sign*$[\Delta^1+a]$. *Moreover,* $\Sigma(\Delta^1+a)$ *is continuous, bounded, point symmetric at* $-a$ *in* Δ^1, *and* $\lim_{|\Delta^1|\to\infty}\Sigma(\Delta^1+a)=0$.

Proof. It is readily observed that $\Sigma(0)=0$, $\Sigma(+)=(+)$, and $\Sigma(-)=(-)$. For the symmetry properties, look at $a=0$ first. It is immediately observed that $\Sigma(\kappa)=-\Sigma(-\kappa)$, proving the symmetry property.
Also, note that by adding an arbitrary constant—for example a—to the argument of a function, the function is shifted horizontally by $-a$. Hence, Σ must be point symmetric at $-a$. Continuity follows from differentiability and boundedness follows from continuity and $\lim_{|\kappa|\to\infty}\Sigma(\kappa)=0$. □

Lemma 3. *Assume* $a\neq0$ *and let the investment of the more popular candidate in stage 1 be* x_F^1 *and the investment of his opponent be* x_T^1. *In any equilibrium, it holds that* $x_T^1\notin(x_F^1,x_F^1+|a|)$.

Proof. This follows immediately from Lemma 2. To see this, look at the first order conditions. Without loss of generality, assume $a>0$ and also assume $x_T^1\in(x_F^1,x_F^1+a)$. This implies $\kappa>0$ and thus $\xi(\kappa)<0$. Hence, T's investments in stage 1 and 2 are strategic complements, and F's investments are strategic substitutes. If F's first order condition holds, T's must be strictly negative and he hence would like to decrease investment. If T's first order condition holds, F's must be strictly positive and he would like to increase investment. Hence, this cannot be an equilibrium. □

Lemma 4. *Assume* $a\neq0$ *and let the investment of the more popular candidate in stage 1 be* x_F^1 *and the investment of his opponent be* x_T^1. *There exists* $\overline{d}\geq0$ *such that if* $a>\overline{d}$, *the equilibrium in stage 1 is unique and* $x_F^1>x_T^1$.

Proof. This follows from Lemma 3 and the fact that any investment greater than $\overline{x}:=\sqrt{\frac{2}{c}}$ is strictly dominated. Thus, when a becomes larger and larger, outspending the leading candidate becomes too expensive and cannot be part of an equilibrium. □

The proposition we want to prove states that in close games, there might be both equilibria in which the leading candidate spends more and some in which the trailing candidate spends more, depending on the variance σ_ϵ^2 and on c. If one candidate has a sufficiently large advantage, in all equilibria, this candidate will spend weakly more. If the equilibrium is unique for all a, in this equilibrium, the leading candidate will always spend weakly more. A necessary and sufficient condition for a unique equilibrium for all a is that $\Sigma'(\Delta^1)<1$ for all Δ^1. To see this, note that if there are to be multiple equilibria, that is $\Delta^1=\Sigma(\Delta^1)$ intersect more than once, then Σ must be steeper than Δ^1 somewhere. Starting from an intersection of the two functions, if the slope is strictly smaller than 1 to the right of the intersection Σ is strictly smaller than Δ^1, and to the left strictly larger, and hence there cannot be another equilibrium. If, however, there is some region in which the slope is larger than 1, there exists an a shifting Σ in a way such that there are multiple equilibria. Hence, if and only if $\frac{\partial\Sigma}{\partial\Delta^1}<1$ for all Δ^1, there is a unique equilibrium. Note that the absolute value of the slope is strictly decreasing in c (evaluated at the equilibrium). If c is sufficiently large, the equilibrium is unique for all a. For $a=0$, the equilibrium is symmetric and $\Delta^1=0$. If we now increase a, we thereby shift Σ to the left, which, because of the fact that $\Sigma(+)=(+)$ (see Lemma 2), implies that the intersection is now where $\Delta^1>0$. This remains true for all $a>0$, and the opposite is similarly true for $a<0$. If

$$\frac{\partial\Sigma}{\partial\Delta^1}>1 \Leftrightarrow \frac{e^{-\frac{\left(\Delta^1\right)^2}{3\sigma_\epsilon^2}}\left(3\sigma_\epsilon^2-2\left(\Delta^1\right)^2\right)}{9\sqrt{3}\pi c^2\sigma_\epsilon^6}>1$$

for some Δ^1, there are multiple equilibria for some a. This follows from the discussion above. $\frac{\partial\Sigma}{\partial\Delta^1}$ is strictly positive for $|\Delta^1|<\sqrt{\frac{3}{2}}\sigma_\epsilon$, negative for $|\Delta^1|>\sqrt{\frac{3}{2}}\sigma_\epsilon$, and zero for $|\Delta^1|=\sqrt{\frac{3}{2}}\sigma_\epsilon$. The maximum of $\overline{\Sigma}=\frac{1}{3\sqrt{2e}\pi c^2\sigma_\epsilon^3}$ is attained at $\Delta^1=\sqrt{\frac{3}{2}}\sigma_\epsilon$ and the minimum of $\underline{\Sigma}=-\frac{1}{3\sqrt{2e}\pi c^2\sigma_\epsilon^3}$ is attained at $\Delta^1=-\sqrt{\frac{3}{2}}\sigma_\epsilon$. Now look at the second derivative of Σ,

$$\frac{\partial^2\Sigma}{\partial\left(\Delta^1\right)^2}=\frac{2\Delta^1\left(2\left(\Delta^1\right)^2-9\sigma_\epsilon^2\right)e^{-\frac{\left(\Delta^1\right)^2}{3\sigma_\epsilon^2}}}{27\sqrt{3}\pi c^2\sigma_\epsilon^8}.$$

This is strictly negative on $[-\infty,-\frac{3}{\sqrt{2}}\sigma_\epsilon)\cup(0,\frac{3}{\sqrt{2}}\sigma_\epsilon)$, and hence the function is strictly concave in this region, which also must include (and does) the maximum. Hence, Σ is strictly concave between zero and the maximum, and decreases monotonically thereafter. Therefore, if $\Sigma'(0)\le 1$, the slope is strictly smaller than 1 (the slope of Δ^1) for all $\Delta^1>0$. Therefore, there exists a unique Δ^1 fulfilling $\Delta^1=\Sigma(\Delta^1)$ not only for $a=0$, but for all $a\in\mathbb{R}$. Using the condition $\Sigma'(0)\le 1$, we can derive the lower bound on ρ for which a unique equilibrium exists in the benchmark model, $\rho>\overline{\rho}=\frac{1}{3^{3/4}\sqrt{\pi}}$. Thus, for given σ_ϵ, the derivative becomes arbitrarily small at $\Delta^1=0$ if we increase c, and hence there exists $\tilde{c}$ such that for all $c>\tilde{c}$, the slope is less than 1, and larger than 1 else. If the derivative at $\Delta^1=0$ is larger than 1, there exist multiple equilibria. Because Σ is strictly concave on $[0,\sqrt{\frac{3}{2}}\sigma_\epsilon]$, and the derivative is zero at the end of this interval, it must become equal to 1 at some $\Delta^1\in[0,\sqrt{\frac{3}{2}}\sigma_\epsilon]$. Denote this by $\tilde{\Delta}$. If we increase a now from zero, we thereby shift Σ to the left by a. Hence, the two outer intersections of Δ^1 and Σ move to the right (Δ^1 increases), while the inner intersection moves to the left. Hence, there are two intersections converging to each other, the ones where $\Delta^1<0$. At $\tilde{d}$, they converge to $\tilde{\Delta}$, and hence, there are only two equilibria left. If we increase a now further, this equilibrium vanishes and only one equilibrium remains, in which $\Delta^1>0$.

To complete the proof, we now show by example that the second order condition can hold in both stages when there are multiple equilibria in stage 1. The second order condition in stage 2 holds when (B.1) holds for all d^2, that is when $c>\frac{1}{\sqrt{2\,e\,\pi\sigma_\epsilon^2}}$. Hence, let $c=\frac{t}{\sqrt{2\,e\,\pi\sigma_\epsilon^2}}$ for some $t>1$. Moreover, let $\kappa=k\sigma_\epsilon$, which will allow us to get rid of one variable. Then

$$\left.\frac{\partial^2 E\left[\pi_F^1\right]}{\partial\left(x_F^1\right)^2}\right|_{\kappa=k\,\sigma_\epsilon,c=\frac{t}{\sqrt{2\,e\,\pi\sigma_\epsilon^2}}}=-\frac{e^{-\frac{k^2}{4}}k}{4\sqrt{\pi}\sigma_\epsilon^2}+\frac{e^{\frac{1}{2}-\frac{k^2}{3}}\left(3-2k^2\right)}{9\sqrt{6\pi}\sigma_\epsilon^2 t}-\frac{t}{\sqrt{2e\pi}\sigma_\epsilon^2}$$

This is strictly decreasing in t for all $k>0$ and hence we let $t=1$. Then, after a few manipulations,

$$\left.\frac{\partial^2 E\left[\pi_F^1\right]}{\partial\left(x_F^1\right)^2}\right|_{\kappa=k\,\sigma_\epsilon,c=\frac{t}{\sqrt{2\,e\,\pi\sigma_\epsilon^2}},t=1}=-\frac{e^{-\frac{k^2}{3}-\frac{1}{2}}\left(27e^{\frac{1}{12}\left(k^2+6\right)}k+54\sqrt{2}e^{\frac{k^2}{3}}+2\sqrt{6}e\left(2k^2-3\right)\right)}{108\sqrt{\pi}\sigma_\epsilon^2}.$$

The sign of this derivative is independent of σ_ε and depends on the sign of the expression in parentheses. This expression has a minimum of $26.7568>0$ when $k=-0.4339$, implying the expression in parentheses is strictly positive, which in turn implies the problem of candidate F is strictly concave in both stages also for all $t>1$. Because T's optimization problem is identical to that of F when $-\kappa$ is substituted for κ, the proof is complete.

What is left to prove Proposition 4 is to show that no stable symmetric equilibrium exists when $\rho<\overline{\rho}$. In the following, we show that the slope of each candidate's best response function for $a=0$, evaluated at the symmetric intersection, is less than -1. Each candidate's best response is implicitly defined by

$$BR_F^1(x_T^1)=\max\left\{\left\{x_F^1:\int_{-\infty}^{\infty}\left(\frac{1}{\sigma_\epsilon}\phi\left(\frac{x_F^1-x_T^1-e^1}{\sigma_\epsilon}\right)-\frac{\phi\left(\frac{x_F^1-x_T^1-e^1}{\sigma_\epsilon}\right)\phi'\left(\frac{x_F^1-x_T^1-e^1}{\sigma_\epsilon}\right)}{c\,\sigma_\epsilon^3}\right)\frac{1}{\sigma_\epsilon}\phi\left(\frac{e^1}{\sigma_\epsilon}\right)de^1-cx_F^1=0\right\},0\right\},$$

$$BR_T^1(x_F^1)=\max\left\{\left\{x_T^1:\int_{-\infty}^{\infty}\left(\frac{1}{\sigma_\epsilon}\phi\left(\frac{x_F^1-x_T^1-e^1}{\sigma_\epsilon}\right)+\frac{\phi\left(\frac{x_F^1-x_T^1-e^1}{\sigma_\epsilon}\right)\phi'\left(\frac{x_F^1-x_T^1-e^1}{\sigma_\epsilon}\right)}{c\,\sigma_\epsilon^3}\right)\frac{1}{\sigma_\epsilon}\phi\left(\frac{e^1}{\sigma_\epsilon}\right)de^1-cx_T^1=0\right\},0\right\}.$$

Assuming a symmetric equilibrium with $x_F^1 = x_T^1$, it follows from $\xi(0) = 0$ that the indirect effect is zero for both and hence $x_F^1 = x_T^1 = \frac{1}{2\sqrt{\pi}c\sigma_\epsilon}$ is an equilibrium. From the implicit function theorem, it follows that the slope of the best responses is

$$\left.\frac{\partial BR_i^1\left(x_j^1\right)}{\partial x_j^1}\right|_{a=0,x_F^1=x_T^1} = \frac{E_{\epsilon^1}\left[\frac{1}{\sigma_\epsilon^4}\left(\phi'\left(\frac{\epsilon^1}{\sigma_\epsilon}\right)\right)^2 + \frac{1}{\sigma_\epsilon^4}\phi\left(\frac{\epsilon^1}{\sigma_\epsilon}\right)\phi''\left(\frac{\epsilon^1}{\sigma_\epsilon}\right)\right]}{E_{\epsilon^1}\left[\frac{1}{\sigma_\epsilon^4}\left(\phi'\left(\frac{\epsilon^1}{\sigma_\epsilon}\right)\right)^2 + \frac{1}{\sigma_\epsilon^4}\phi\left(\frac{\epsilon^1}{\sigma_\epsilon}\right)\phi''\left(\frac{\epsilon^1}{\sigma_\epsilon}\right)\right] + c^2}.$$

If this is smaller than minus 1, the equilibrium is unstable. Note that the denominator must be positive, because it is the negative of the second derivative in equilibrium, and this has to be negative in equilibrium. The derivative is smaller than minus 1 whenever

$$-2E_{\epsilon^1}\left[\frac{1}{\sigma_\epsilon^4}\left(\phi'\left(\frac{\epsilon^1}{\sigma_\epsilon}\right)\right)^2 + \frac{1}{\sigma_\epsilon^4}\phi\left(\frac{\epsilon^1}{\sigma_\epsilon}\right)\phi''\left(\frac{\epsilon^1}{\sigma_\epsilon}\right)\right] > c^2,$$

which further simplifies to $\rho = c\sigma_\epsilon^2 < \left(3^{3/4}\sqrt{\pi}\right)^{-1} = \overline{\rho}$. This is the condition for the existence of multiple equilibria. Hence, if there are multiple equilibria, there is no stable symmetric equilibrium. □

Appendix B. Supplementary material

Supplementary data to this article can be found online at http://dx.doi.org/10.1016/j.jpubeco.2015.07.003.

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