Model Predictive Control with Prioritised Actuators

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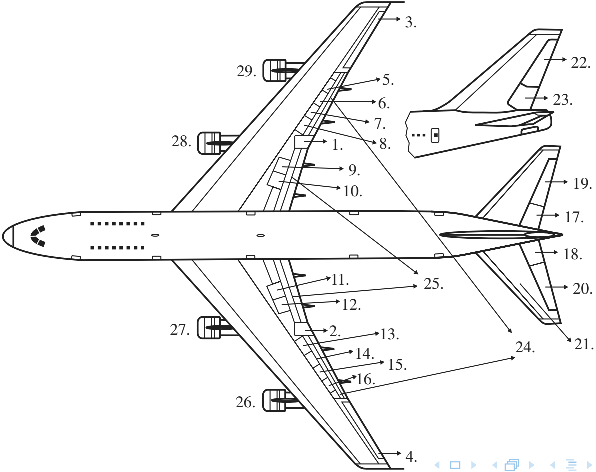
Multiply-actuated systems

Aircraft example: 16 states, nearly 30 actuators



LASSO Model Predictive Control (ℓ_{asso}-MPC)

Control of multiply-actuated systems – e.g. Aircraft $\approx$ 30 inputs

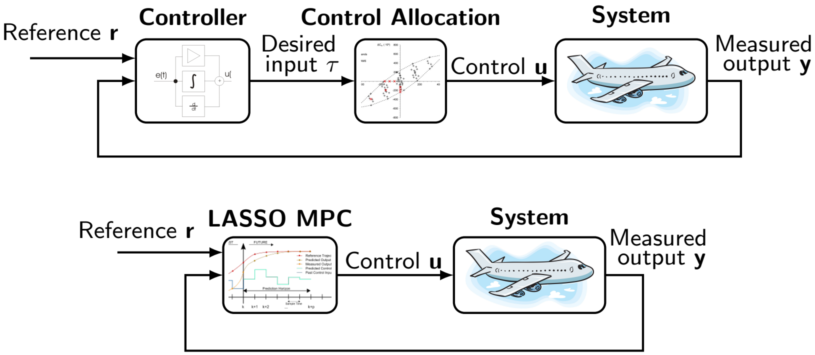


Figure: Integration of multiple control layers through LASSO MPC.

LASSO Model Predictive Control (ℓ_{asso}-MPC)

Preferred actuators

Cost function: $\ell(x_j, u_j) = x_j^T Q x_j + u_j^T R u_j + \|S u_j\|_1$

- Standard MPC moves all the actuators all of the time
- We may have *preferred* actuators:
    - Aircraft: Ailerons normally, spoilers only if necessary.
- We want *sparse control allocation*.
- LASSO: ℓ_1-norm regularisation.
- Quadratic cost on state $\implies$ smooth plant behaviour.

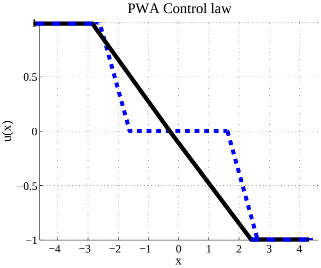
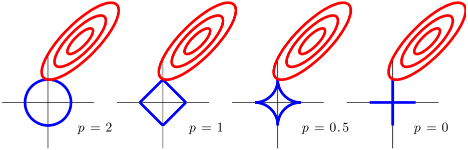


Figure: Primary (solid) and secondary actuator (dashed).

Who inspired our work ?

Authors who have previously investigated regularised MPC

- H. Ohlsson, F. Gustafsson, L. Ljung, and S. Boyd, 2010. "Trajectory generation using sum-of-norms regularization", IEEE CDC.
  - MPC with sparse Δu. Reduce actuator's wear and tear.
- M. Nagahara and D.E. Quevedo, 2011. "Sparse representations for packetized predictive networked control", Proc. IFAC World Con.
  - Remote control. Compress control message. Send future N moves. Unconstrained boundedness and robustness to communication losses.
- T. Hastie *et al.* 2011. "The elements of statistical learning", Springer. LASSO for Machine learning. Solution path.



ℓ_{asso}-MPC (or ℓ_1-regularised MPC)

Let's start from a standard quadratic MPC

Control the constrained discrete-time LTI system

$$x(k+1) = Ax(k) + Bu(k) + B_w w(k),$$

$$x(k) \in \mathbb{X},\ \ u(k) \in \mathbb{U},\ \ \|w(k)\|_\infty \leq \mu,\ \forall k \geq 0,$$

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$$x(k+1) = Ax(k) + Bu(k),$$

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by applying the first move of the optimal sequence

$$\underline{u}^\star = \arg\min_{\underline{u}} \hat{x}_N^T P \hat{x}_N \qquad + \sum_{j=0}^{N-1} \hat{x}_j^T Q \hat{x}_j + \hat{u}_j^T R \hat{u}_j \qquad \text{s.t. constraints.}$$

ℓ_{asso}-MPC (or ℓ_1-regularised MPC)

Version 1: Add a 1-norm penalty to obtain sparse u (or Δu)

Control the constrained discrete-time LTI system

$$x(k+1) = Ax(k) + Bu(k),$$

$$x(k) \in \mathbb{X},\ u(k) \in \mathbb{U},\ \forall k \geq 0,$$

by applying the first move of the optimal sequence

$$\underline{u}^\star = \arg\min_{\underline{u}} \hat{x}_N^T P \hat{x}_N + \sum_{j=0}^{N-1} \hat{x}_j^T Q \hat{x}_j + \hat{u}_j^T R \hat{u}_j + \|S\hat{u}_j\|_1 \quad \text{s.t. constraints.}$$

ℓ_{asso}-MPC (or ℓ_1-regularised MPC)

Version 2: Add a second 1-norm penalty for stability (difficult conditions)

Control the constrained discrete-time LTI system

$$x(k+1) = Ax(k) + Bu(k),$$

$$x(k) \in \mathbb{X},\ \ u(k) \in \mathbb{U},\ \ \forall k \geq 0,$$

by applying the first move of the optimal sequence[1]

$$\underline{\mathrm{u}}^{\star} = \arg\min_{\underline{\mathrm{u}}} \hat{x}_N^T P \hat{x}_N + \|Z\hat{x}_N\|_1 + \sum_{j=0}^{N-1} \hat{x}_j^T Q \hat{x}_j + \hat{u}_j^T R \hat{u}_j + \|S\hat{u}_j\|_1 \quad \text{s.t. constraints.}$$

[1] M. Gallieri and J. M. Maciejowski, *Stabilising Terminal Cost and Terminal Controller for lasso-MPC: Enhanced Optimality and Region of Attraction*, ECC 2013

Apply the first move of the optimal sequence

$$\underline{\mathrm{u}}^\star = \arg\min_{\underline{\mathrm{u}}} F(\hat{x}_N) + \sum_{j=0}^{N-1} \hat{x}_j^T Q \hat{x}_j + \hat{u}_j^T R \hat{u}_j + \|S \hat{u}_j\|_1$$

$$\text{s.t. } \hat{x}_j \in \mathbb{X},\ \hat{u}_j \in \mathbb{U},\ \forall j = 0,\ 1, \ldots, N-1,\ \hat{x}_N \in \mathbb{X}_f.$$

MPC stability assumption (Mayne et al., 2000)

- $F((A + BK)\hat{x}_N) - F(\hat{x}_N) \leq -\ell(\hat{x}_N, K\hat{x}_N)$, for some K, $\forall \hat{x}_N \in \mathbb{X}_f$.
- $\mathbb{X}_f$ is *positively invariant* under $u(k) = Kx(k)$.

ℓ_{asso}-MPC (or ℓ_1-regularised MPC)

Asymptotic Stability: How to choose the terminal cost?

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Two possible strategies

- *Partial regularisation*: $S = [0 \ \bar{S}]$, $F(x) = x^T P x$. Prioritised actuators (first actuators stabilise the plant).
- *New terminal cost*: Multiple levels of priority. Using Minkowski functions for stability[2]

[2]Gallieri M., Maciejowski J. M., *Soft constrained LASSO-MPC for robust LTI tracking: Enlarged feasible region and an ISS gain estimate*, CDC 2013.

Objectives:

- Use preferred actuators ($u^{(\mathrm{i})}$) to stabilise system
- If control error large $\Rightarrow$ Use also auxiliary actuators ($u^{(\mathrm{ii})}$)

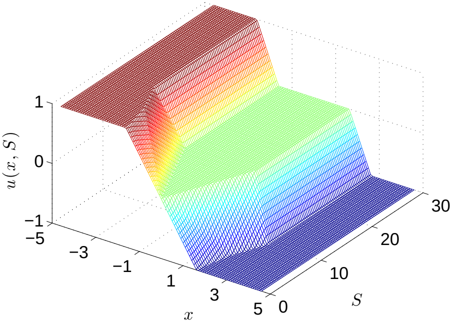
Is this possible?

- ℓ_1-regularisation as exact penalty function $\Rightarrow$ $u^{(\mathrm{ii})} = 0$ when x close to the origin
- For LTI systems, ℓ_{asso}-MPC is strongly convex
- Explicit solution is piecewise affine in x and S (assume S diagonal or change coordinates).

ℓ_{asso}-MPC (or ℓ_1-regularised MPC)

SISO Example: Solution path for varying S

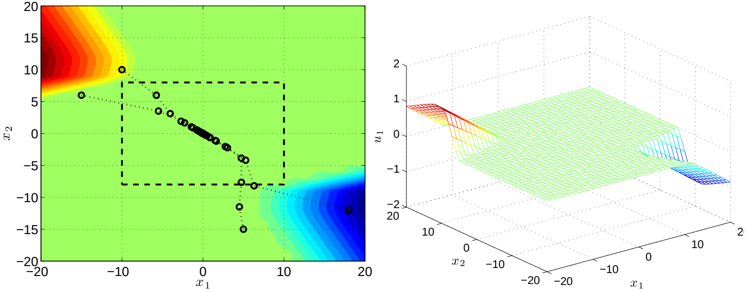
- We can use explicit MPC to compute the PWA solution path and the regularisation parameter (upper bound)
- Example: $A = B = Q = R = 1$, $N = 3$, $|x| \leq 5$, $|u| \leq 1$, $S \in (0,\ 30]$.



Theorem

The maximal feasible set for which it is possible to have $u^{(\mathrm{ii})} = 0$ *is*

$$\tilde{\mathbb{X}}_N^{(\mathrm{i})} = \mathcal{Q}^{(\mathrm{i})}(\mathcal{K}_{N-1}(\mathbb{X}, \mathbb{X}_f)) \cap \mathbb{X}. \quad (1)$$



ℓ_{asso}-MPC (or ℓ_1-regularised MPC)

Tuning procedure for regularisation parameter

1. Choose $\mathbb{X}_{\text{nom}}$ in $\mathbb{X}$ in which to enforce $u^{(\text{ii})} = 0$.
2. Find MPQP solution to modified MPC problem with auxiliary constraints:
   - $u^{(\text{ii})} = u_+^{(\text{ii})} - u_-^{(\text{ii})}$, s.t. $u_+^{(\text{ii})} = 0$, $u_-^{(\text{ii})} = 0$.
3. Use MPQP solution to find $\gamma^\star =$ maximum Lagrange multiplier for auxiliary constraints.
4. Set regularisation parameter $\bar{S} = \lambda I$, $\lambda \geq \gamma^\star \Rightarrow$ Cost of regularisation $>$ cost reduction for original problem.

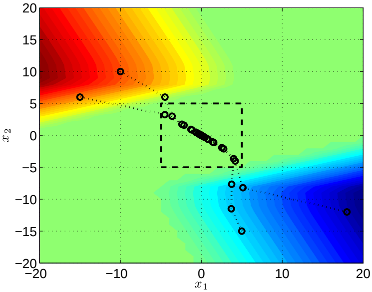
ℓ_{asso}-MPC (or ℓ_1-regularised MPC)

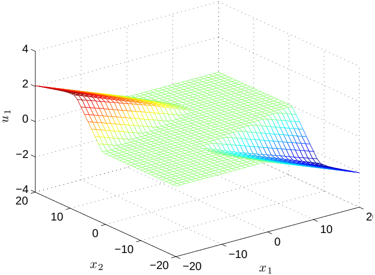
MIMO example: Prioritised actuators

$$A = \begin{bmatrix} 0.2 & 0.1 \\ 0 & 1.1 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \; Q = I, \; R = I, \quad N = 3,$$

$$\mathbb{X} = \{x \mid \|x\|_\infty \leq 20\}, \; \mathbb{U} = \{u \mid \|u\|_\infty \leq 5\},$$

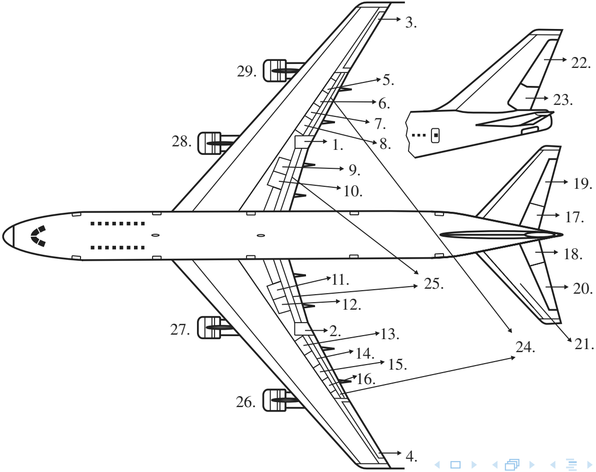
$$\mathbb{X}_{\mathrm{nom}} = \{x : |\|x\| + \infty \leq 5\}, \; \gamma^\star = 13.07.$$





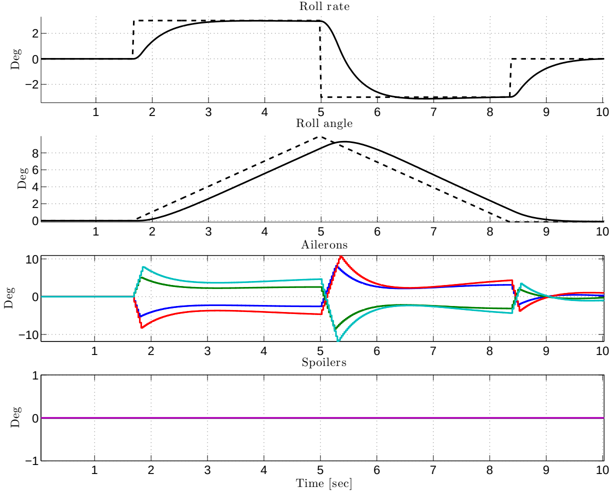
Case study: Aircraft roll control

16 states, nearly 30 actuators



Aircraft roll control

Case 1: Moderate roll rate command (3 deg/sec)



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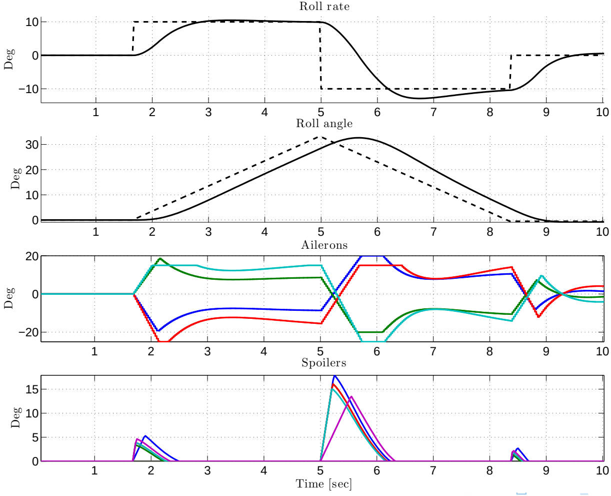
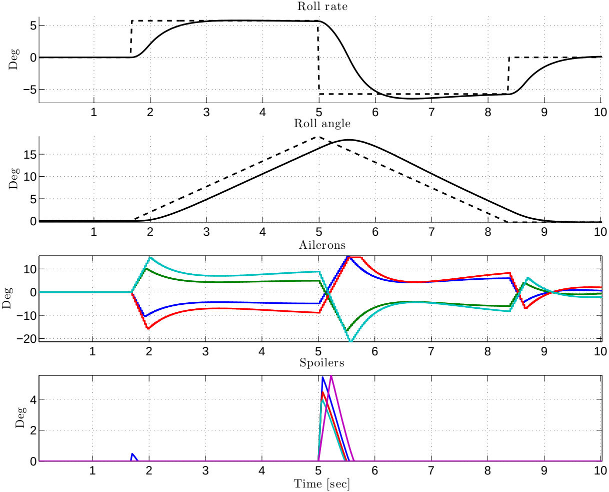
Aircraft roll control

Case 2: Large roll rate command (5 deg/sec)



Aircraft roll control

Case 3: Very large roll rate command (10 deg/sec)



Why 'ℓ_{asso}-MPC'?

- A new concept — Perform only most relevant input actions (moves).
- When to use ℓ_{asso}-MPC:
  1. Use less input – Sparse in time signals (e.g. save fuel).
  2. Move actuators less – Reduce wear and tear. ($\|\hat{u}_{j+1} - \hat{u}_j\|_1$).
  3. Prioritised actuators – Sparsity in channels.

Thank you for the attention!
Any questions?