

# RELAX, TENSORS ARE HERE...WITH EXOGENOUS COVARIATES

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# model specification

Dependent variables:  $\text{Log}(\text{Exports})$  and  $\text{Stdzed}(\text{Material Conflict})$ .

Direct ( $i$ ), reciprocal ( $ji$ ) and transitive ( $ijk$ ) 1 month lags of these included as IVs.

Exogenous Covariates:

Number of Preferential Trade Agreements (PTA) between  $i$  and  $j$  (this is an undirected, yearly level variable). Direct and transitive version of this variable included as covariates.

Presence of a defensive alliance relationship between  $i$  and  $j$  (undirected, yearly level). Direct and transitive versions.

Centroid distance between  $i$  and  $j$  (directed). Direct version.

Polity, monthly level variable. Polity of sender included.

$\text{Log}(\text{GDP})$ , yearly level variable but imputed at the monthly level. GDP of sender.

$\text{Log}(\text{Population})$ , yearly level variable but imputed at the monthly level.

Population of sender.

$\text{Log}(\text{Total Exports to any country})$ , monthly level variable. Exports of sender.

Our sample is comprised of 161 countries over the period of March 2001 to December 2014

Data sources:

Exports: [IMF Direction of Trade Statistics](#)

Material Conflict: ICEWS

PTA: [Design of Trade Agreements Database](#)

Alliance: [Correlates of War](#)

Distance: [cshapes](#)

Polity: [Polity IV Project](#)

GDP, Population: [IMF World Economic Outlook Database](#)

Multilinear tensor regression framework

MCMC run for 1300 iterations with first 600 used as burn-in<sup>1</sup>

The model has the following form:

$$\mathbf{Y} = \mathbf{X} \times \{\beta_1, \beta_2, \beta_3\} + \mathbf{E}$$

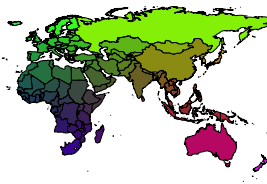
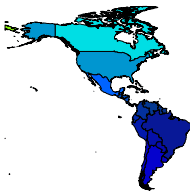
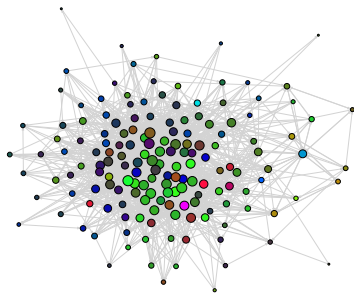
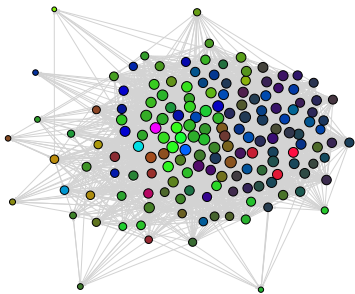
$\mathbf{Y}$  is a  $161 \times 161 \times 2 \times 165$  array

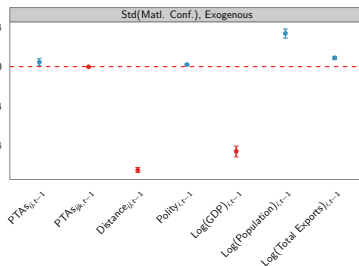
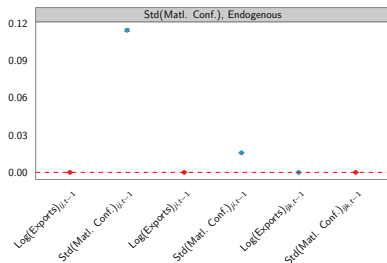
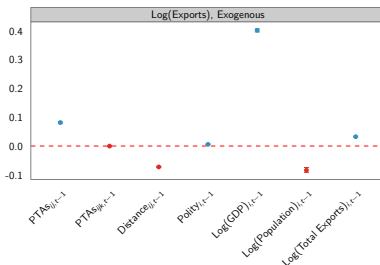
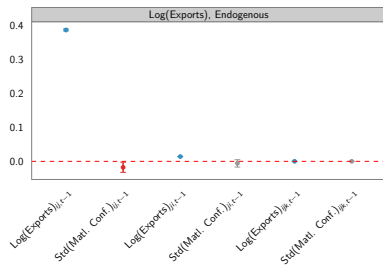
$\mathbf{X}$  is a  $161 \times 161 \times 13 \times 165$  array, where each of the 13 variables is lagged by one month

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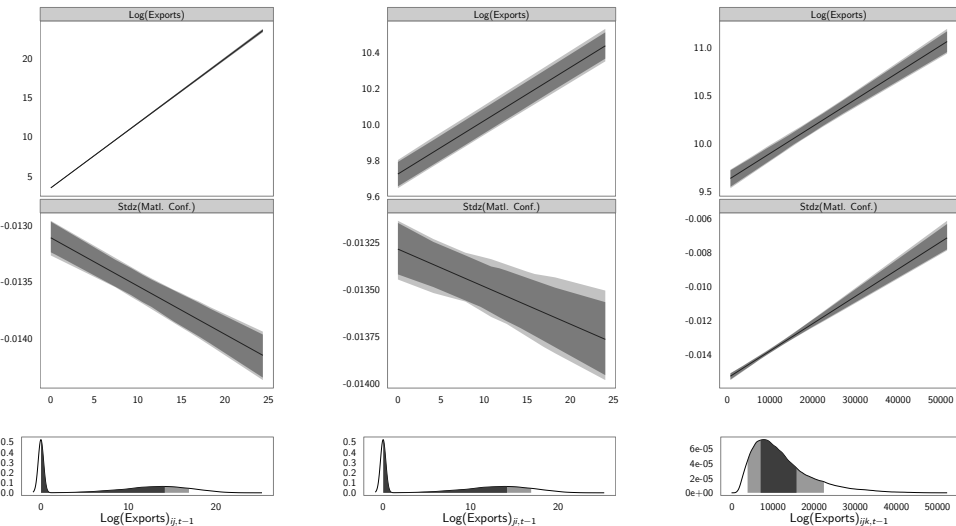
<sup>1</sup>Using this many datapoints takes time the MCMC will keep running for another 3700 iterations so these results are preliminary, but trace plots at the end of this pdf look stable after 600 iterations

$\beta_1$  &  $\beta_2$ , sig. + shown,  $\alpha = 0.01$

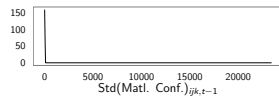
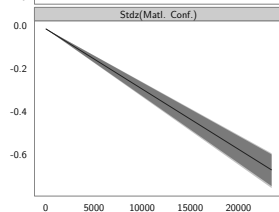
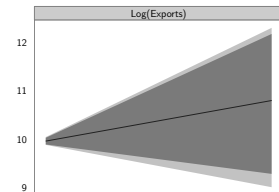
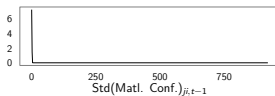
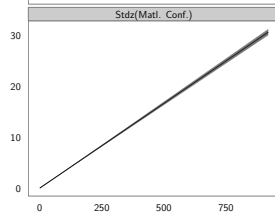
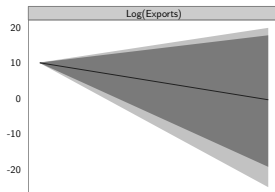
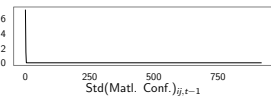
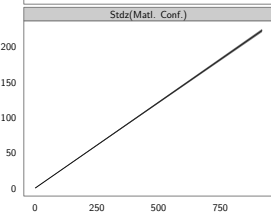
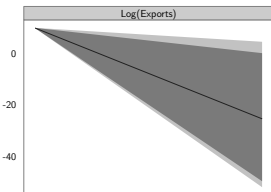




# endog. effects of $\log(\text{exports})$

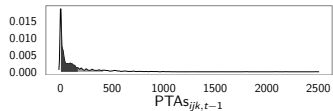
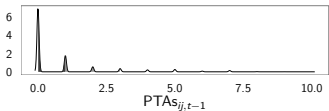
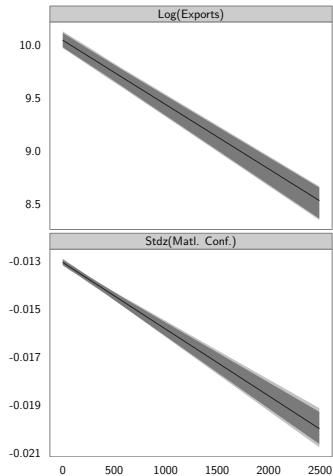
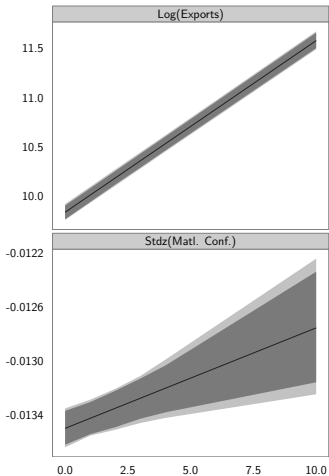


# endog. effects of std(matl. conf.)



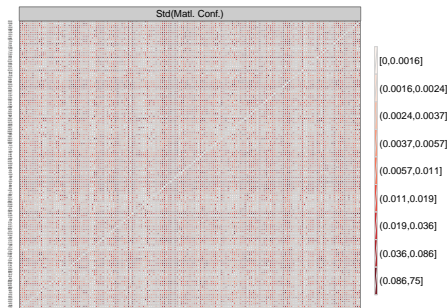
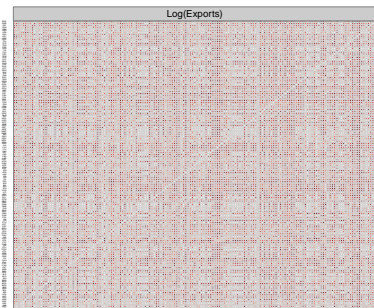


# effect of ptas: direct and transitive

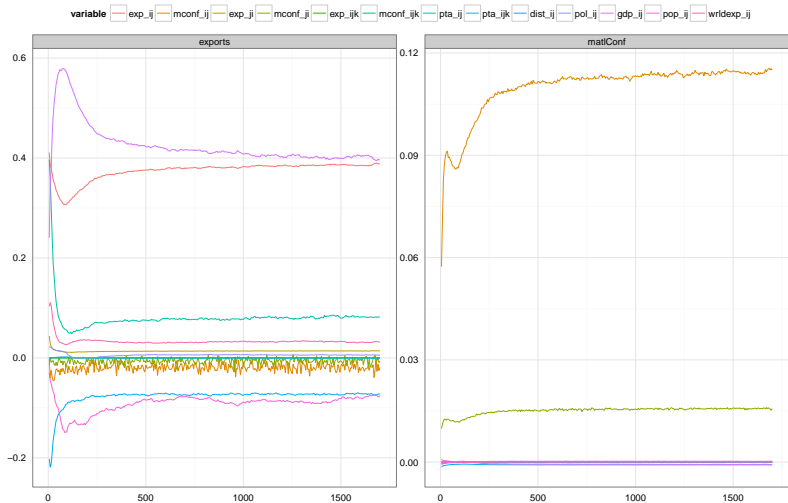


# aggregate performance & rmse by i-j

	RMSE	$R^2$
Log(Exports)	2.32	0.95
Std(Mat. Conf.)	0.85	0.28



# trace plots for $\beta_3$



## comparison with directed dyadic model

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Here I run a similar analysis using the standard directed dyadic (dd) framework

The covariates for both models are the same

Instead of taking a vector autoregression approach, I just run two separate directed dyadic linear regressions

dd coefficient results, std. errors in (), \* sig. at  $p < 0.05$

	Log(Exports)	Std(Matl. Conf.)
(Intercept)	0.20*	0.22*
	(0.02)	(0.01)
exp_ij	0.82*	-0.00*
	(0.00)	(0.00)
mconf_ij	-0.00	0.49*
	(0.00)	(0.00)
exp_ji	0.06*	-0.00
	(0.00)	(0.00)
mconf_ji	0.00	0.03*
	(0.00)	(0.00)
exp_ijk	0.00*	0.00*
	(0.00)	(0.00)
mconf_ijk	-0.00*	0.00*
	(0.00)	(0.00)
pta_ij	0.10*	-0.00
	(0.00)	(0.00)
pta_ijk	-0.00*	-0.00*
	(0.00)	(0.00)
dist_ij	-0.09*	-0.01*
	(0.00)	(0.00)
pol_ij	0.01*	-0.00*
	(0.00)	(0.00)
gdp_ij	0.13*	0.01*
	(0.00)	(0.00)
pop_ij	-0.02*	0.00*
	(0.00)	(0.00)
wrldexp_ij	0.01*	-0.01*
	(0.00)	(0.00)
N	4250400	4250400

# parameter estimate comparisons: mltr & dd

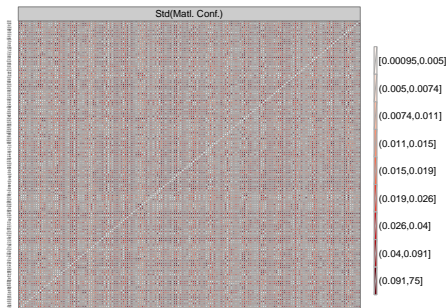
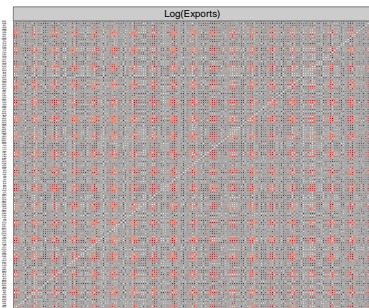
+ = sig at 95% interval and positive

- = sig at 95% interval and negative

	Log(Exports)		Std(Matl. Conf.)	
	MLTR	Dyadic	MLTR	Dyadic
Log(Exports) $_{ij,t-1}$	+	+	-	-
Std(Matl. Conf.) $_{ij,t-1}$	-		+	+
Log(Exports) $_{ji,t-1}$	+	+	-	
Std(Matl. Conf.) $_{ji,t-1}$			+	+
Log(Exports) $_{ijk,t-1}$	+	+	+	+
Std(Matl. Conf.) $_{ijk,t-1}$		-	+	-
PTAs $_{ij,t-1}$	+	+	+	
PTAs $_{ijk,t-1}$	-	-	-	-
Distance $_{ij,t-1}$	-	-	-	-
Polity $_{i,t-1}$	+	+	+	-
Log(GDP) $_{i,t-1}$	+	+	-	+
Log(Population) $_{i,t-1}$	-	-	+	+
Log(Total Exports) $_{i,t-1}$	+	+	+	-

# dd aggregate performance & rmse by i-j

	RMSE	$R^2$
Log(Exports)	2.37	0.89
Std(Matl. Conf.)	0.86	0.26



## performance comparisons: mltr & dd

Across all cases the  $R^2$  is higher using the MLTR approach for both exports (95% v. 89%) and matl. conf. (28% v. 26%)

MLTR has a lower RMSE in  $\approx 57\%$  of cases for Log(Exports)

MLTR has a lower RMSE in  $\approx 80\%$  of cases for Std(Matl. Conf.)

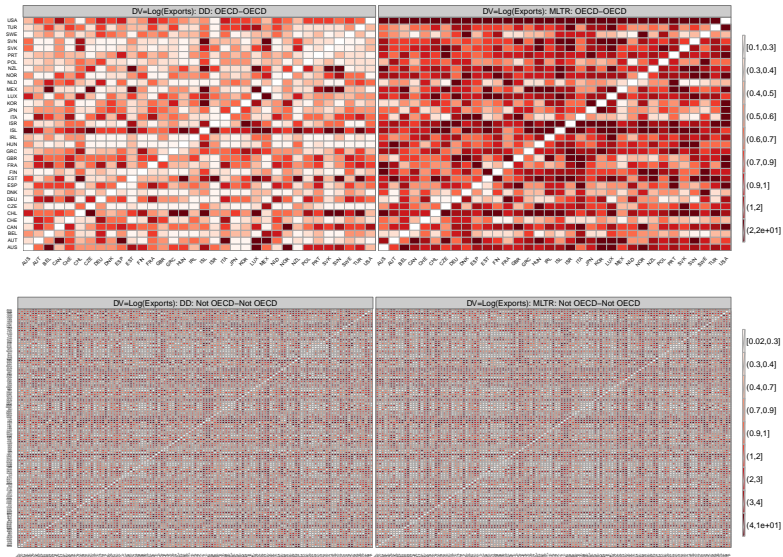
Across all cases the RMSE is lower using the MLTR approach for both exports (2.32 v. 2.37) and matl. conf. (0.85 v. 0.86)

However, as shown by the aggregate RMSE statistics right above, the differences in performance are small

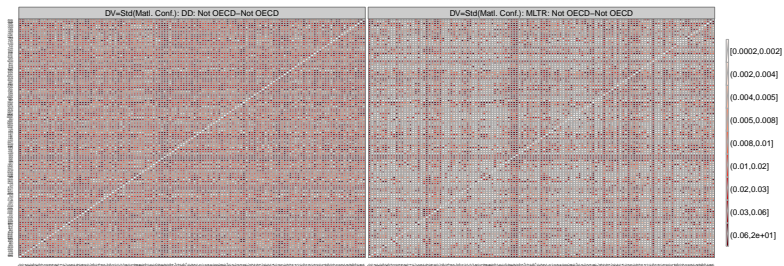
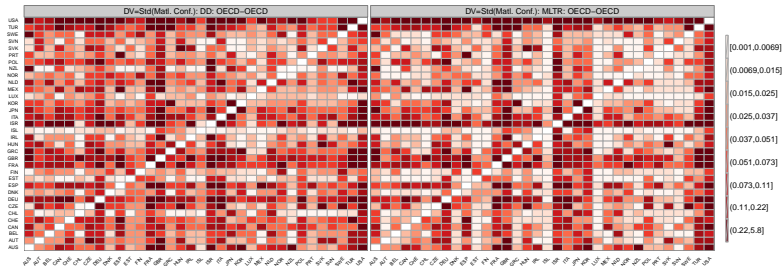
Additionally, in the next two slides I break out the performance, in terms of RMSE, by showing the results for OECD–OECD and Not OECD–Not OECD countries



# performance on oecd–oecd countries: $\log(\text{exports})$



# performance on oecd-oecd countries: std(matl. conf.)



NEXT STEPS?