NAVAL POSTGRADUATE SCHOOL

A Salvo Model of Warships in Missile Combat Used to Evaluate Their Staying Power

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Agenda

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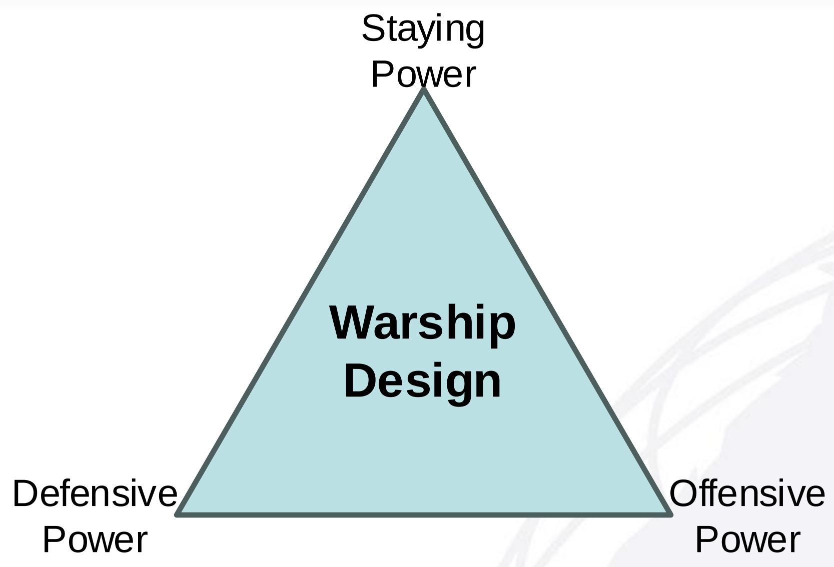
Balanced Warship Design



- Key Attributes:
  - <u>Offensive Power</u> – a combination of firepower, fighting power, striking power, and/or combat power
  - <u>Defensive Power</u> – a composite of all defensive actions to reduce susceptibility to hits by the enemy
  - <u>Staying Power</u> – the ability of a ship to absorb hits and continue fighting
- Balancing these key attributes is challenging due to trade-offs in weight, cost, and space.

"You cannot have everything. If you attempt it, you will lose everything . . . On a given tonnage . . . There cannot be the highest speed, *and* the heaviest battery, *and* the thickest armor, *and* the longest coal endurance."

- Alfred Thayer Mahan, 1911

Introduction to Salvo Models

- The salvo model was originally developed by CAPT Wayne Hughes in 1986 to provide insight into modern naval missile engagements, where traditional continuous-fire models were inadequate.
- The model provides a way to compare naval forces by analyzing offensive, defensive, and staying power during naval combat.
- The salvo model analyzes combat as sequential volleys instead of continuous fire, which captures the impact of high-damage, high-risk exchanges.
- The Fractional Exchange Ratio (FER) is the key MoE that compares how much of each force is destroyed after an exchange of salvos.
- Salvo models aid in designing ships and strategies by assessing the balance of offensive, defensive, and staying power in naval engagements.

Basic Model Assumptions and Limitations

Assumptions

- Striking power of the attacker is the number of accurate ASCMs launched.
- Accurate ASCM shots are spread equally over all targets.
- Counterfire from area and point defense systems of the targeted force eliminates all accurate shots until the force's defenses are saturated.
- Staying power is the number of hits required to achieve a firepower kill (not to sink it).
- Hits on a target force will diminish its whole fighting strength linearly and proportionate to the remaining hits the target force can take.
- Weapon range is sufficient on both sides and neither side has an advantage of detecting, tracking, and targeting the other while remaining safely outside of the enemy's weapon range.
- Losses are measured in warships put out of action.

Limitations

- Undersea warfare, including use of submarines and torpedoes, is not represented in the model.
- The model assumes that all ships in the fleet have similar missile capabilities.
- The model is deterministic in nature.

The Basic Model: Force-on-Force Dynamics

The Basic Model:

$$\Delta B = \frac{\boxed{\alpha A - b_3 B}}{b_1} \qquad \Delta A = \frac{\boxed{\beta B - a_3 A}}{a_1} \qquad (1)$$

Calculates combat work achieved by a single salvo at any time step (ships out of action)

Combat Power: P_a or P_b measured in hits that damage target force.

A = # of units in force A

B = # of units in force B

α = # of well-aimed missiles fired by each A unit

β = # of well-aimed missiles fired by each B unit

a_1 = # of hits by B's missiles needed to put one A out of action

b_1 = # of hits by A's missiles needed to put one B out of action

a_3 = # of well-aimed missiles destroyed by each A

b_3 = # of well-aimed missiles destroyed by each B

ΔA = # of units in force A out of action from B's salvo

ΔB = # of units in force B out of action from A's salvo

Measure of Effectiveness: Fractional Exchange Ratio

Exchange Ratio

$$\frac{\Delta B}{\Delta A}$$

vs

Fractional Exchange Ratio (FER)

$$\frac{\Delta B / B}{\Delta A / A}$$

Ratio of fractional losses after A & B exchange salvos

FER > 1 → A is winning

FER < 1 → B is winning

| "winning" meaning that side will have forces with combat power remaining when the enemy is "impotent" |

A higher FER indicates that one side achieves more damage relative to its losses.

Useful for evaluating comparative mission effectiveness without detailed battle information

Fighting Strengths:

"Military worth of a force" – F.W. Lanchester

Under his square law conditions, strength of A units each with individual hitting rates α is equal to αA^2.

If fighting strength of A is greater than that of B, then A will have survivors when B is annihilated

Salvo equations necessitate a defender capability (defensive power) be included. To determine the winner of a single salvo exchange:

If $a_1 \alpha A^2 - a_1 A b_3 B > b_1 \beta B^2 - b_1 B a_3 A$ then A wins salvo exchange

If $b_1 \beta B^2 - b_1 B a_3 A > a_1 \alpha A^2 - a_1 A b_3 B$ then B wins salvo exchange (2)

When second term is larger than first, defense is too strong and zero (not a negative) loss results.

Model Based Conclusions:

1. Missile combat is force-on-force, must examine the fractional force put out of action:

$$\frac{\Delta B}{B} = \frac{\alpha A \; - \; b_3 B}{b_1 B} \quad \frac{\Delta A}{A} = \frac{\beta B \; - \; a_3 A}{a_1 A} \qquad (3)$$

FER > 1 means A will have forces remaining when B is out of action and vice versa.

2. Comparative fighting strengths evident with FER by dividing one equation by the other:

$$\text{FER} = \frac{\Delta B / B}{\Delta A / A} = \frac{(\alpha A - b_3 B)(a_1 A)}{(\beta B - a_3 A)(b_1 B)} \qquad (4)$$

3. Overkill (excessive offensive or defensive power) now has significant effect on results. (Caution with FER)

4. Eq. 4 implies when each unit of A has 2 x striking power, 2 x defensive power, and 2 x staying power of each B, then B can still achieve parity if force is twice as numerous as A.

5. Other linear combinations of A's attributes won't achieve FER = 1.

Note separation of terms in fractions.

Salvo Equation Example

<u>Consider two forces, typical of modern American combatants:</u>

A has great striking and defensive power, but weak staying power.

B's forces are more numerous but less capable, focusing on only striking power

Number of units	A = 2	B = 6
Staying power	$a_1 = 2$	$b_1 = 1$
Defensive Power	$a_3 = 16$	$b_3 = 1$
Striking power	$\alpha = 24$	$\beta = 6$

$$\text{FER} = \frac{\Delta B / B}{\Delta A / A} = \frac{(\alpha A - b_3 B)(a_1 A)}{(\beta B - a_3 A)(b_1 B)}$$

$$= \frac{[(24 * 2) - (1 * 6)](2 * 2)}{[(6 * 6) - (16 * 2)](1 * 6)} = \frac{168}{24} = 7$$

Side A clearly has a huge advantage, and one would think A walks away victorious… but what about "overkill?"

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<u>Considering overkill</u>:

$$\frac{\Delta B}{B} = \frac{\alpha A - b_3 B}{b_1 B} = \frac{48 - 6}{6} = 7.0 \qquad \frac{\Delta A}{A} = \frac{\beta B - a_3 A}{a_1 A} = \frac{36 - 32}{4} = 1.0$$

A can cause total firepower kills across all B seven times over with **<u>one</u>** salvo.

However... B possesses enough combat power to put all of A out in one salvo as well (exactly one).

<u>The Big "So What"</u>: Despite great advantage in offensive and defensive power, A's force cannot survive an exchange (in the model's terms so far). Meanwhile B's forces, though a suicidal mission, can take out a foe with far superior fighting power.

Model Insights: Instability

Previous calculation highlights the concept of instability for weak staying power vs strong combat power.

$$\Delta B = \frac{\alpha A - b_3 B}{\boxed{b_1}} \qquad \Delta A = \frac{\beta B - a_3 A}{\boxed{a_1}} \qquad (1)$$

If staying power cannot be easily and affordably added, then instability is only restored through adding more units (A's and B's of affordable attributes)

This type of instability heavily favors circumstances for unanswered strikes and calls for superior scouting for the first effective attack (embellished model).

"Fire effectively first."
– The Great Wayne P. Hughes Jr.

A simple salvo model demonstrating such instability implies limited value of specific and detailed studies **<u>until</u>** we better understand the ***general*** nature of warship characteristics and modern combat.

Focus on combat power alone increases risk of investing too much capability into a single package. Must consider factors of greater numbers and staying power.

The Embellished Model:

$$\Delta B = \frac{\boxed{(\alpha' A - b_3' B) b_4}}{b_1} \qquad \Delta A = \frac{\boxed{(\beta' B - a'_3 A) a_4}}{a_1}$$

where

α' = fighting power in hits of an attacking unit of side A modified for scouting and training deficiencies and the effect of defender B's distraction chaff

β' = fighting power in hits of an attacking unit of side B modified for scouting and training deficiencies and the effect of defender A's distraction chaff

$a_3{}'$ = hits denied to A by defender counterfire of B, degraded for defender alertness and training deficiencies

$b_3{}'$ = hits denied to B by defender counterfire of A, degraded for defender alertness and training deficiencies

a_4 = A's effectiveness in employing seduction chaff to cause otherwise accurate well aimed B missiles to miss after counterfire has failed

b_4 = B's effectiveness in employing seduction chaff to cause otherwise accurate well aimed A missiles to miss after counterfire has failed

Combat Power P_a or P_b are now based on partial offensive and defensive effectiveness

This model incorporates values for combat features that are difficult to capture quantitatively

Modified Combat Effectiveness:

$$\alpha' = \sigma_a \tau_a \rho_b \alpha$$

$$b_3' = \delta_b \tau_b b_3$$

$$\beta' = \sigma_b \tau_b \rho_a \beta$$

$$a_3' = \delta_a \tau_a a_3$$

τ is present in both offensive and defensive effectiveness for both sides

These additional factors can only degrade combat effectiveness, not enhance

where

σ = *Scouting effectiveness.* Takes values between 0 and 1 that measure the extent to which striking power is diminished due to less than perfect targeting and distribution of fire against the target force

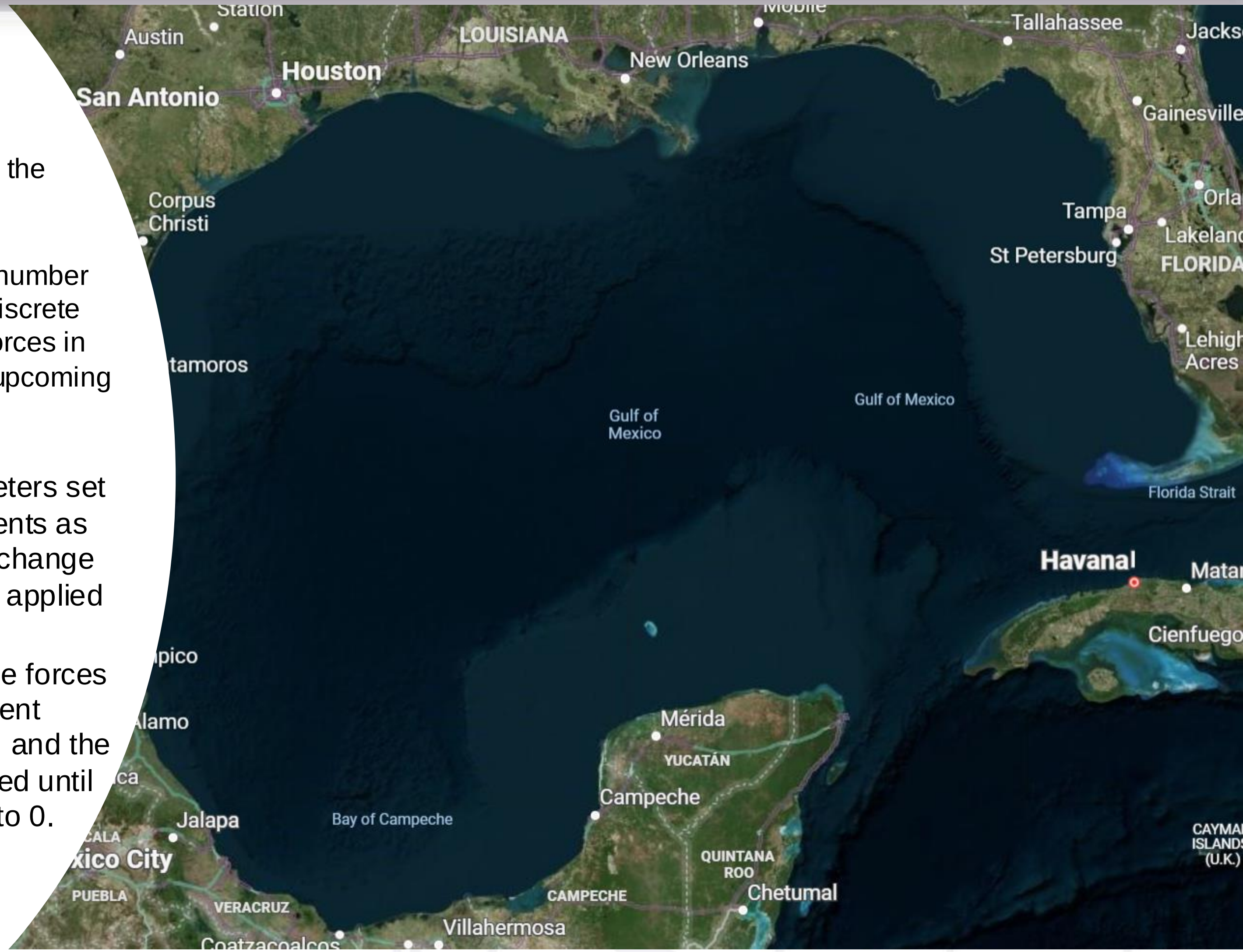
δ = *Defender alertness.* Takes values between 0 and 1 that measure the extent to which counterfire is diminished due to less than perfect readiness or fire control designation to destroy the missiles of an enemy attack

ρ = *Distraction Chaff.* Multiplier between 0 and 1 that reduces the number of accurate shots that must be destroyed by counterfire. Distracts each enemy shot with a fixed probability.

τ = *Skillfulness/Training multiplier.* Multiplier between 0 and 1, the degree to which a firing or target unit fails to achieve its full combat potential due to inadequate training, organization, or motivation.

Simulation Concept

- A_0 and B_0 set at the start of the simulation.
- Half of each side are in the engagement area and the number of forces is selected by a discrete random uniform (1 – # of forces in engagement area) for the upcoming exchange.
- Exchange dynamics are dependent on the parameters set by the design of experiments as the inputs to the salvo exchange equations and losses are applied to the number of forces.
- Reserves are added to the forces available in the engagement area, forces are selected, and the salvo exchange is repeated until one of the sides is equal to 0.



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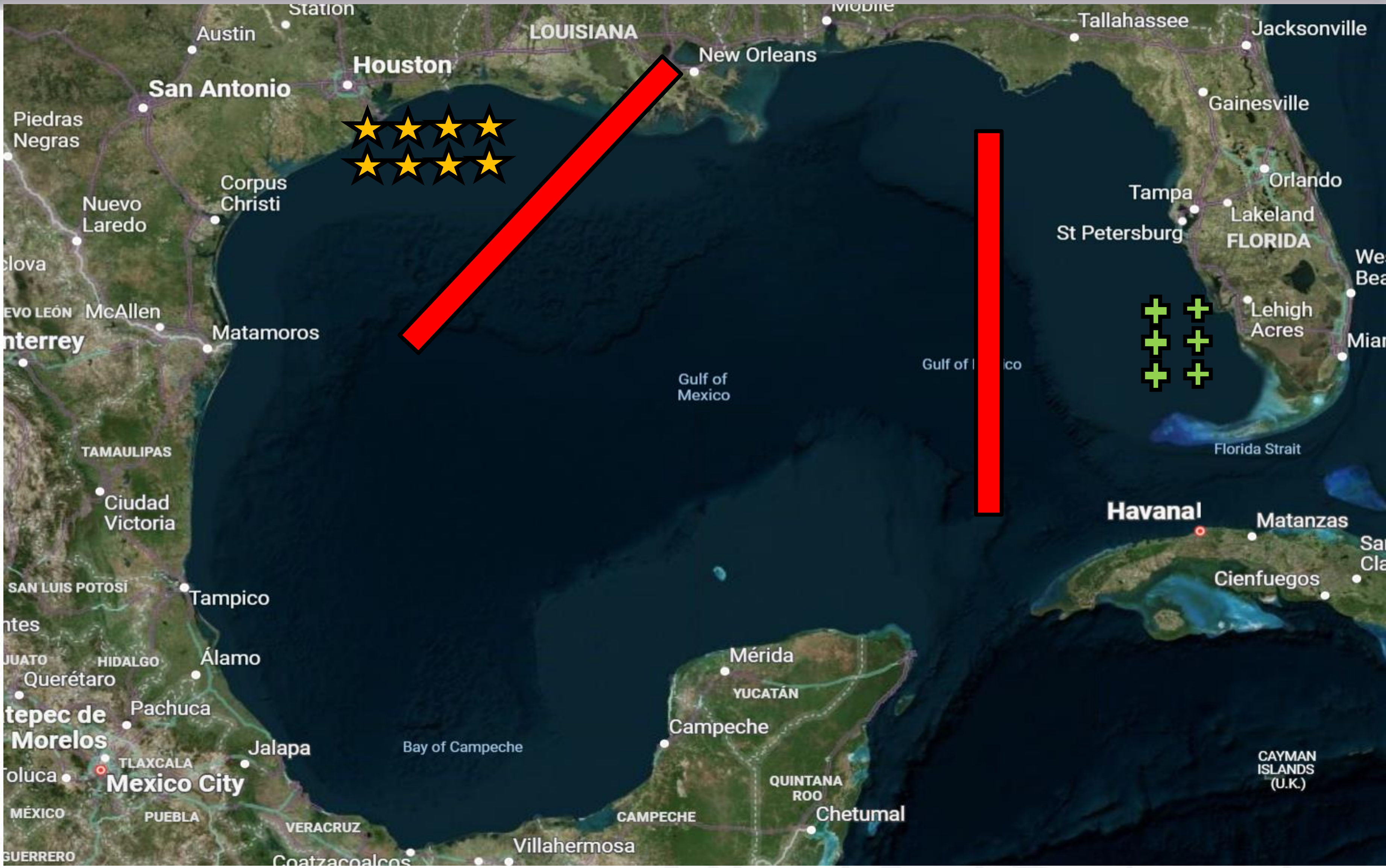
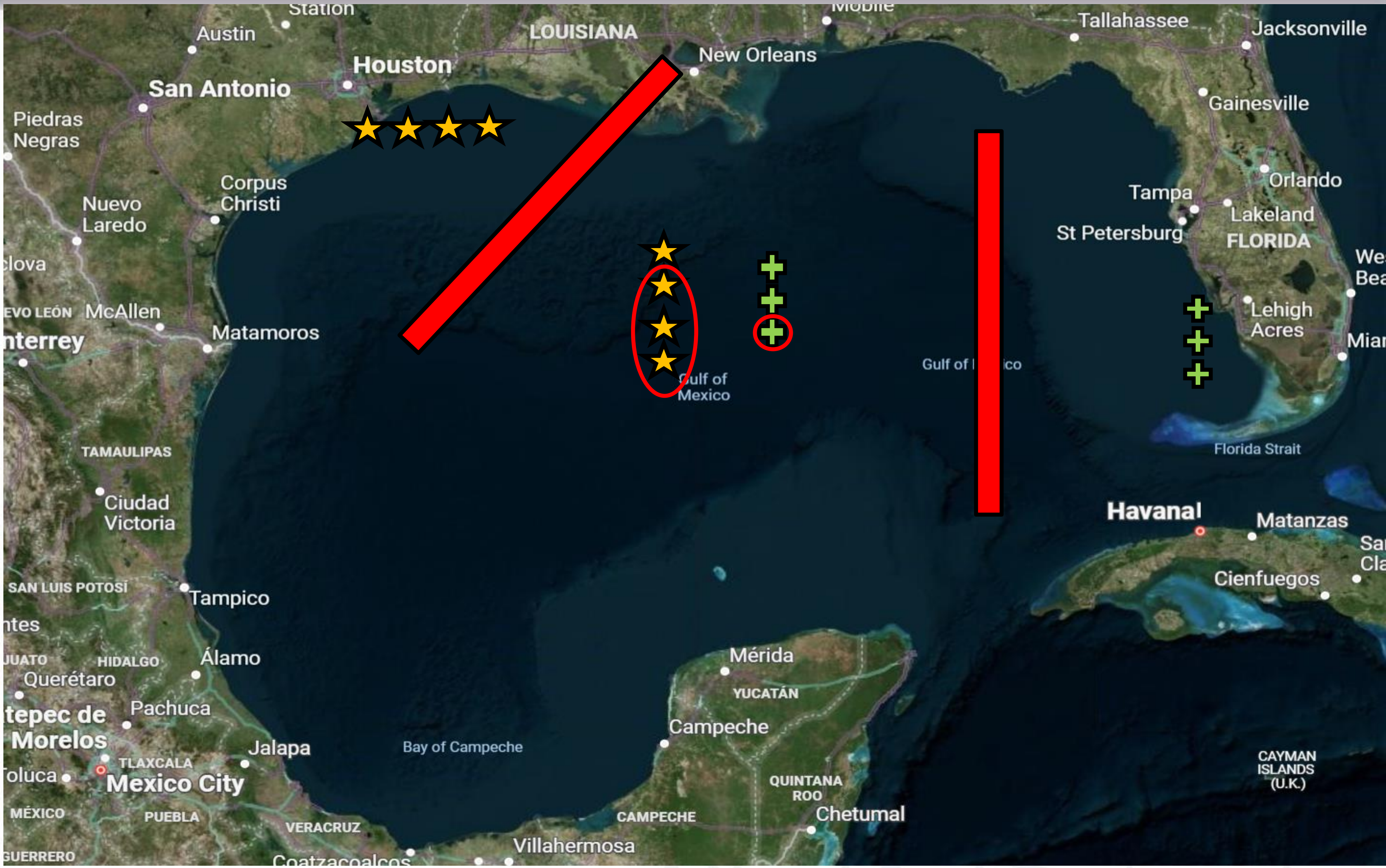
Half of the Total Force is Available for each Engagement

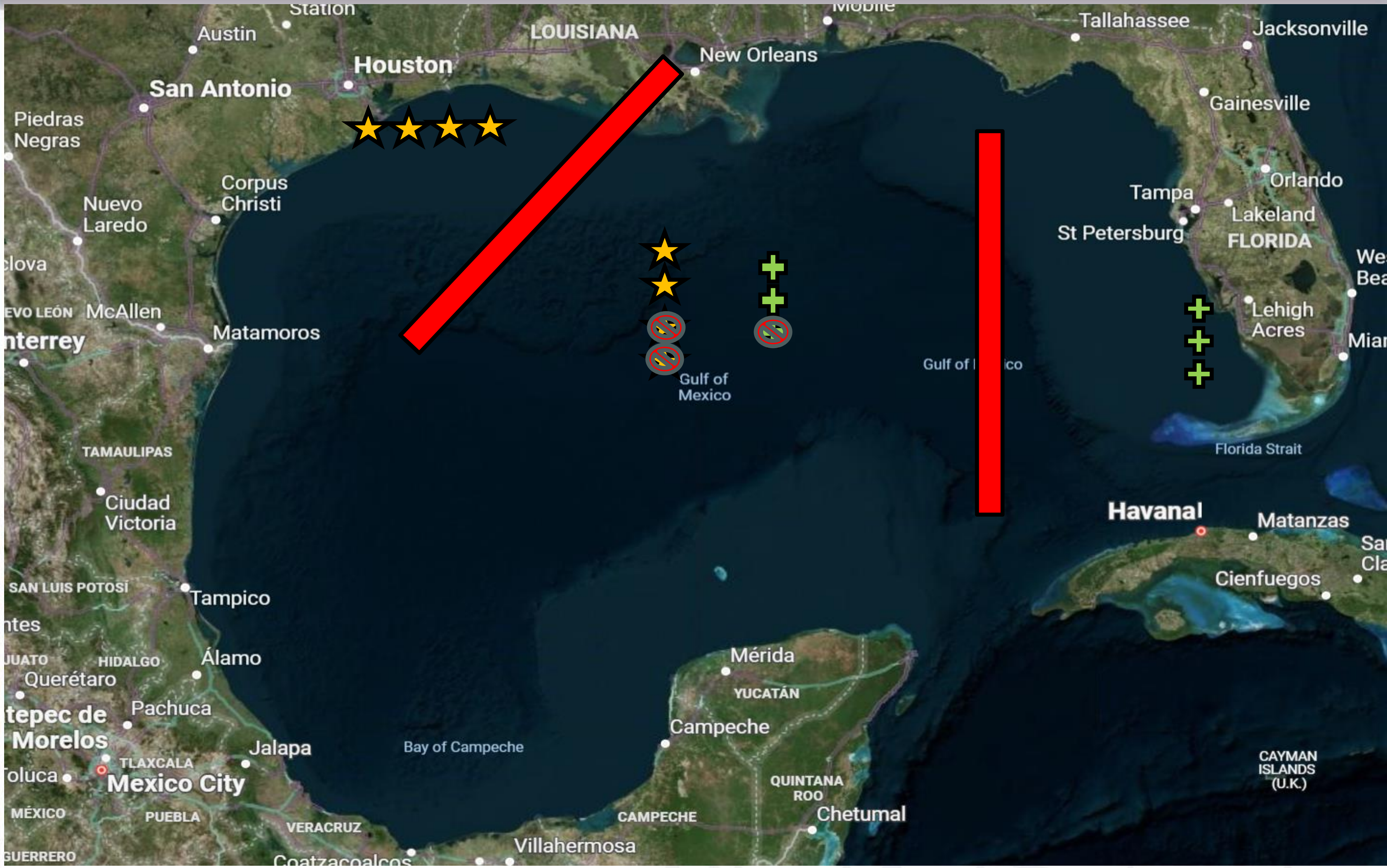


Forces are Randomly Selected for the Exchange



Conduct Salvo Exchange Based on Design Point Parameters

Forces are Replaced and Half of the Total Force is Available for Next Exchange



Parameters (Basic Equation)

Basic Random Selection Model

- **A**: initial number of units in force A
- **B**: initial number of units in force B
- **α** : number of **well aimed** missiles fired by each A unit.
- **β** : number of **well aimed** missiles fired by each B unit.
- $\mathbf{a_1}$: number of hits by B's missiles needed to put one A unit out of action.
- $\mathbf{b_1}$: number of hits by A's missiles needed to put one B unit out of action.
- $\mathbf{a_3}$: number of **well aimed** missiles **destroyed** by each A.
- $\mathbf{b_3}$: number of **well aimed** missiles **destroyed** by each B.

Stochastic Basic Model

- **A**: initial number of units in force A
- **B**: initial number of units in force B
- **α** : number of missiles fired by each A unit.
- **β** : number of missiles fired by each B unit.
- $\mathbf{a_1}$: number of hits by B's missiles needed to put one A unit out of action.
- $\mathbf{b_1}$: number of hits by A's missiles needed to put one B unit out of action.
- $\mathbf{a_3}$: number of missiles that can be destroyed by each A.
- $\mathbf{b_3}$: number of missiles that can be destroyed by each B.
- $\mathbf{P(A_{\text{Well-Aimed Missile}})}$: probability that an A unit fires a missile that can hit a B unit
- $\mathbf{P(B_{\text{Well-Aimed Missile}})}$: probability that a B unit fires a missile that can hit an A unit
- $\mathbf{P(A_{\text{Destroy Well-Aimed Missile}})}$: probability that an A unit can destroy a missile that can hit an A unit
- $\mathbf{P(B_{\text{Destroy Well-Aimed Missile}})}$: probability that a B unit can destroy a missile that can hit a B unit

Parameters (Modified Equation)

Modified Random Selection Model

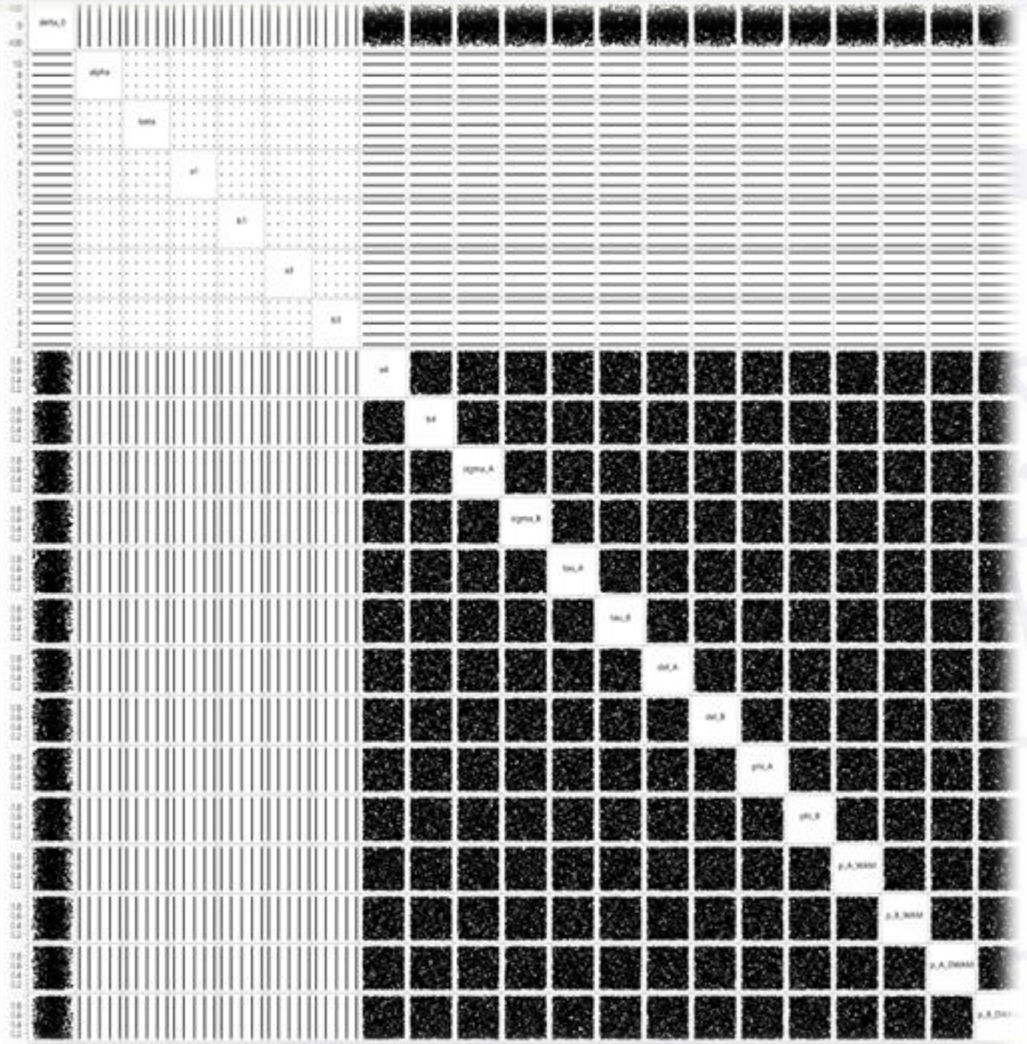
- **A**: initial number of units in force A
- **B**: initial number of units in force B
- **α** : number of **well aimed** missiles fired by each A unit.
- **β** : number of **well aimed** missiles fired by each B unit.
- **a_1** : number of hits by B's missiles needed to put one A unit out of action.
- **b_1** : number of hits by A's missiles needed to put one B unit out of action.
- **a_3** : **number** of **well aimed** missiles **destroyed** by each A.
- **b_3** : **number** of **well aimed** missiles **destroyed** by each B.
- **a_4** : probability that accurate shots miss an A unit after their counterfire has failed.
- **b_4** : probability that accurate shots miss a B unit after their counterfire has failed.
- **σ_A** : scouting effectiveness due to less than perfect targeting of A units.
- **σ_B**: scouting effectiveness due to less than perfect targeting of B units.
- **τ_A** : training effectiveness of A units.
- **τ_B** : training effectiveness of B units.
- **δ_A** : alertness of unit A's defense that is degraded by less than perfect readiness or fire control.
- **δ_B** : alertness of unit B's defense that is degraded by less than perfect readiness or fire control.
- **ρ_A** : effectiveness of unit A chaff that draws off shots before its counterfire against B's missiles.
- **ρ_B** : effectiveness of unit B chaff that draws off shots before its counterfire against A's missiles.

Stochastic Modified Model

- **A:** initial number of units in force A
- **B:** initial number of units in force B
- **α** : number of missiles fired by each A unit.
- **β** : number of missiles fired by each B unit.
- **a_1** : number of hits by B's missiles needed to put one A unit out of action.
- **b_1** : number of hits by A's missiles needed to put one B unit out of action.
- **a_3** : number of missiles that can be destroyed by each A.
- **b_3** : number of missiles that can be destroyed by each B.
- **a_4** : probability that accurate shots miss an A unit after their counterfire has failed.
- **b_4** : probability that accurate shots miss a B unit after their counterfire has failed.
- **σ_A** : scouting effectiveness due to less than perfect targeting of A units.
- **σ_B**: scouting effectiveness due to less than perfect targeting of B units.
- **τ_A** : training effectiveness of A units.
- **τ_B** : training effectiveness of B units.
- **δ_A** : alertness of unit A's defense that is degraded by less than perfect readiness or fire control.
- **δ_B** : alertness of unit B's defense that is degraded by less than perfect readiness or fire control.
- **ρ_A** : effectiveness of unit A chaff that draws off shots before its counterfire against B's missiles.
- **ρ_B** : effectiveness of unit B chaff that draws off shots before its counterfire against A's missiles.
- **$P(A_{\text{Well-Aimed Missile}})$** : probability that an A unit fires a missile that can hit a B unit
- **$P(B_{\text{Well-Aimed Missile}})$** : probability that a B unit fires a missile that can hit an A unit
- **$P(A_{\text{Destroy Well-Aimed Missile}})$** : probability that an A unit can destroy a missile that can hit an A unit
- **$P(B_{\text{Destroy Well-Aimed Missile}})$** : probability that a B unit can destroy a missile that can hit a B unit

Design of Experiments

- (8) Discrete Variables
  - A_0, B_0, α, β, a_1, b_1, a_3, b_3
- (14) Continuous Variables
  - a_4, b_4, σ_A, σ_B, τ_A, τ_B, δ_A, δ_B, ρ_A, ρ_B, $P(A_{\text{Well-Aimed Missile}})$, $P(B_{\text{Well-Aimed Missile}})$, $P(A_{\text{Destroy Well-Aimed Missile}})$,$P(B_{\text{Destroy Well-Aimed Missile}})$
- (2000) Design Points and (500) Iterations of each Design Point [1,000,000 Simulations]
  - Random seed set to the iteration number
- NOAB Design (Max Pairwise Correlation Magnitude = 0.032)
- A_0 and B_0 were used as input parameters for the simulations but the difference, Δ_0, was calculated for the analysis.

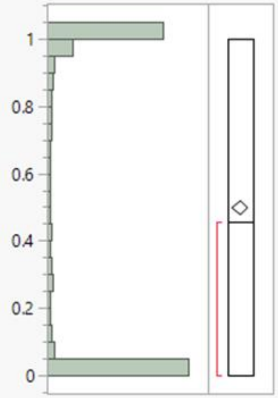


Model Comparisons

Proportion of Wins for Force A in the Models

Distributions

prop_A_wins_overall_basic



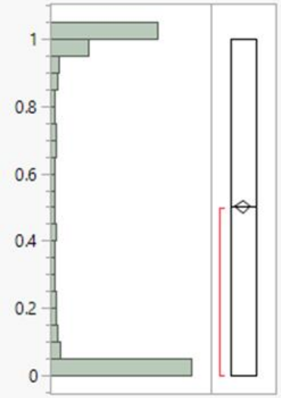
Quantiles

100.0%	maximum	1
99.5%		1
97.5%		1
90.0%		1
75.0%	quartile	1
50.0%	median	0.458
25.0%	quartile	0
10.0%		0
2.5%		0
0.5%		0
0.0%	minimum	0

Summary Statistics

Mean	0.499046
Std Dev	0.4622549
Std Err Mean	0.0103363
Upper 95% Mean	0.5193171
Lower 95% Mean	0.4787749
N	2000
N Missing	0

prop_A_wins_overall_basic_stoch



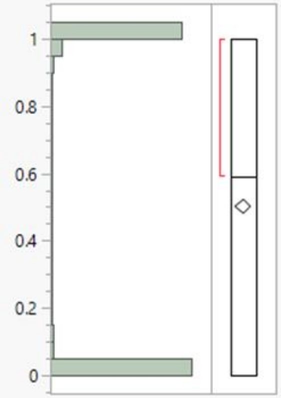
Quantiles

100.0%	maximum	1
99.5%		1
97.5%		1
90.0%		1
75.0%	quartile	1
50.0%	median	0.5
25.0%	quartile	0
10.0%		0
2.5%		0
0.5%		0
0.0%	minimum	0

Summary Statistics

Mean	0.500995
Std Dev	0.4516597
Std Err Mean	0.0100994
Upper 95% Mean	0.5208015
Lower 95% Mean	0.4811885
N	2000
N Missing	0

prop_A_wins_overall_mod



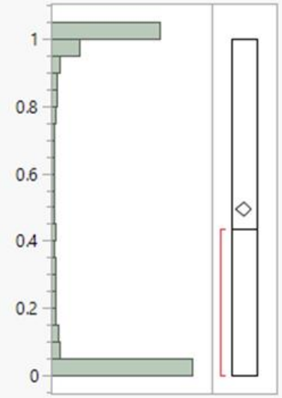
Quantiles

100.0%	maximum	1
99.5%		1
97.5%		1
90.0%		1
75.0%	quartile	1
50.0%	median	0.591
25.0%	quartile	0
10.0%		0
2.5%		0
0.5%		0
0.0%	minimum	0

Summary Statistics

Mean	0.5037
Std Dev	0.4850534
Std Err Mean	0.0108461
Upper 95% Mean	0.5249709
Lower 95% Mean	0.4824291
N	2000
N Missing	0

prop_A_wins_overall_mod_stoch



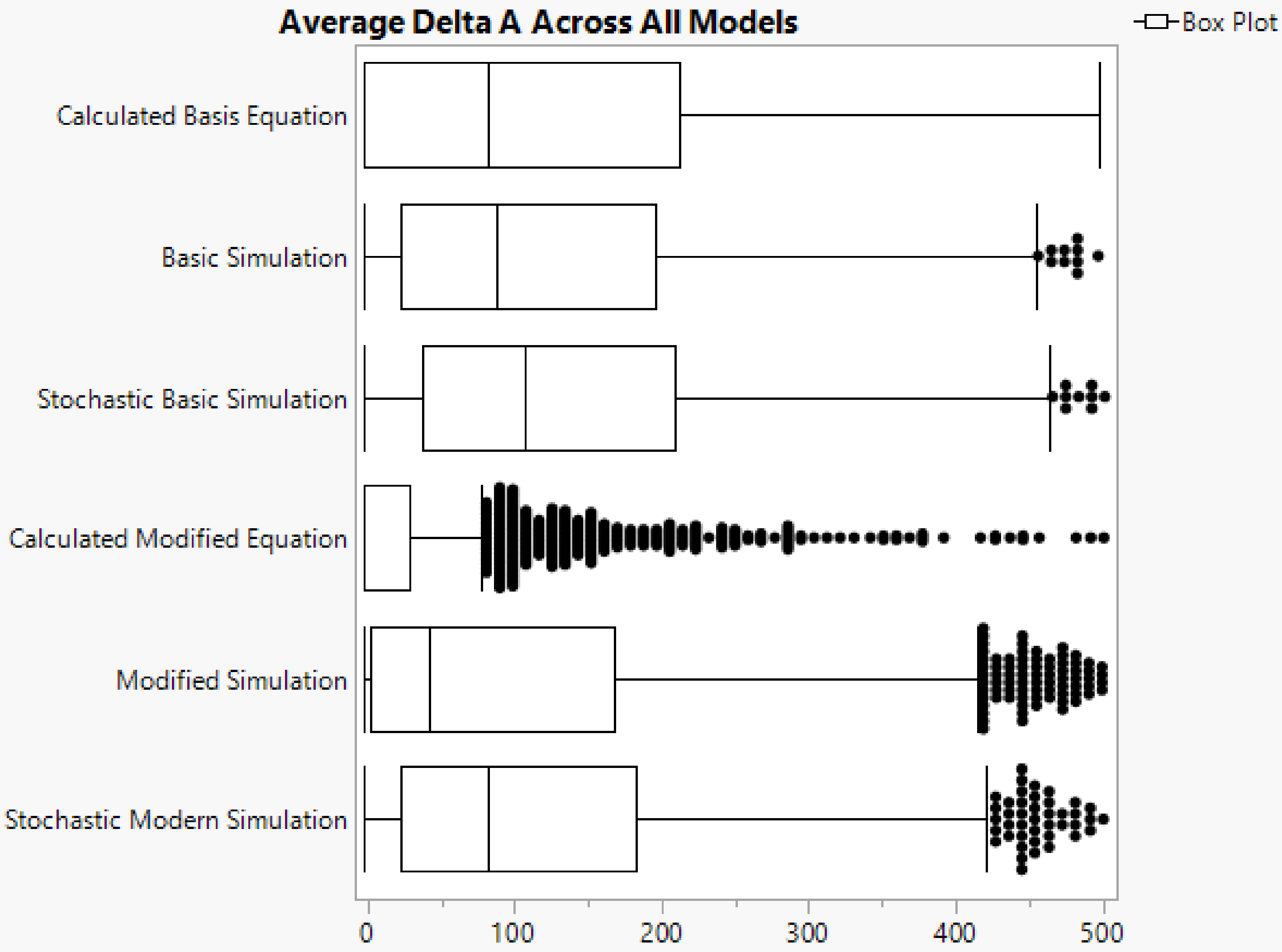
Quantiles

100.0%	maximum	1
99.5%		1
97.5%		1
90.0%		1
75.0%	quartile	1
50.0%	median	0.434
25.0%	quartile	0
10.0%		0
2.5%		0
0.5%		0
0.0%	minimum	0

Summary Statistics

Mean	0.493555
Std Dev	0.4589657
Std Err Mean	0.0102628
Upper 95% Mean	0.5136819
Lower 95% Mean	0.4734281
N	2000
N Missing	0

Average Units Out of Action after Simulation of each Design Point



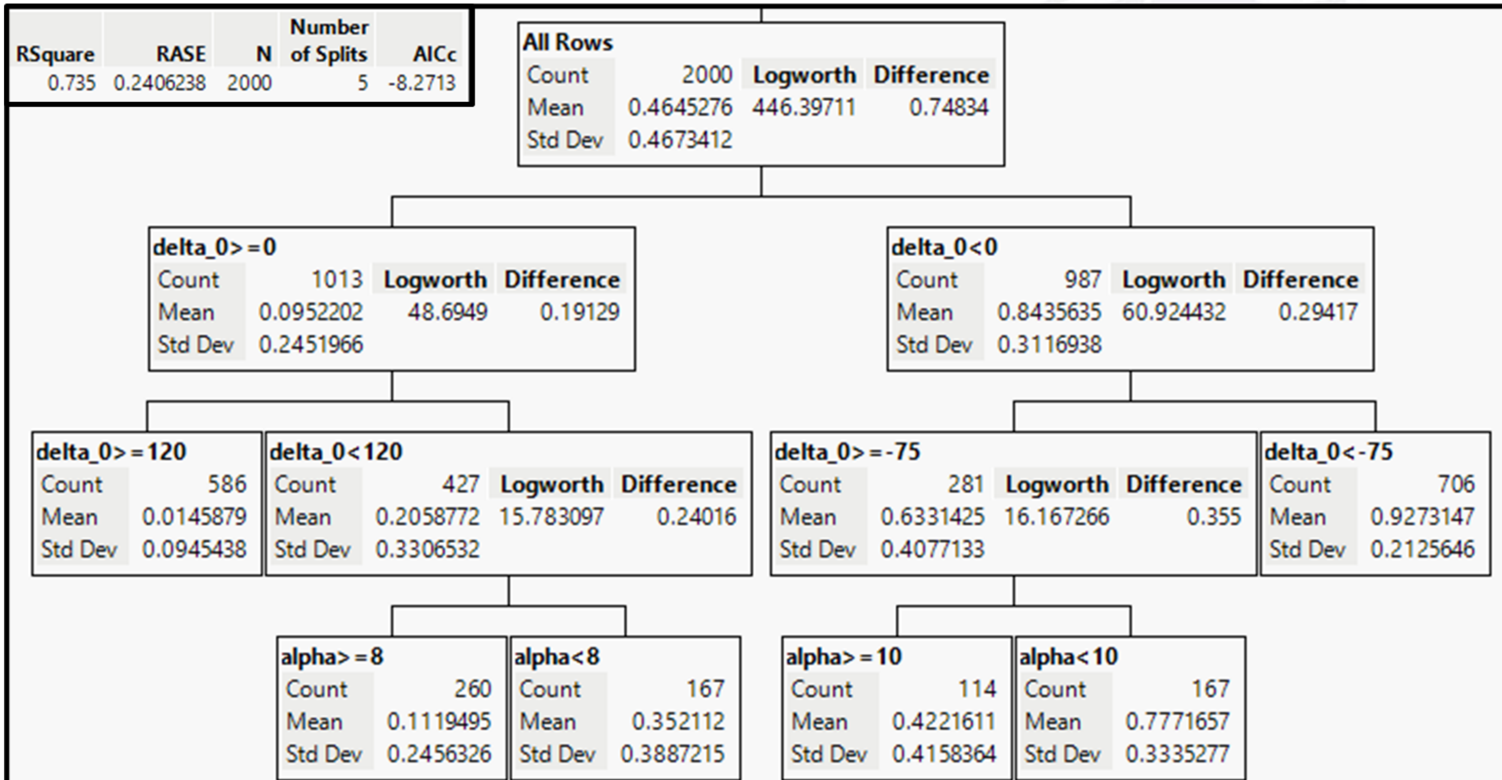
Results

Expected Loss Function Applied to the Proportion of Wins by Force A

$E[Loss] = (P_{Won\_under\_DP\_Parameters} - 1)^{(2)}$

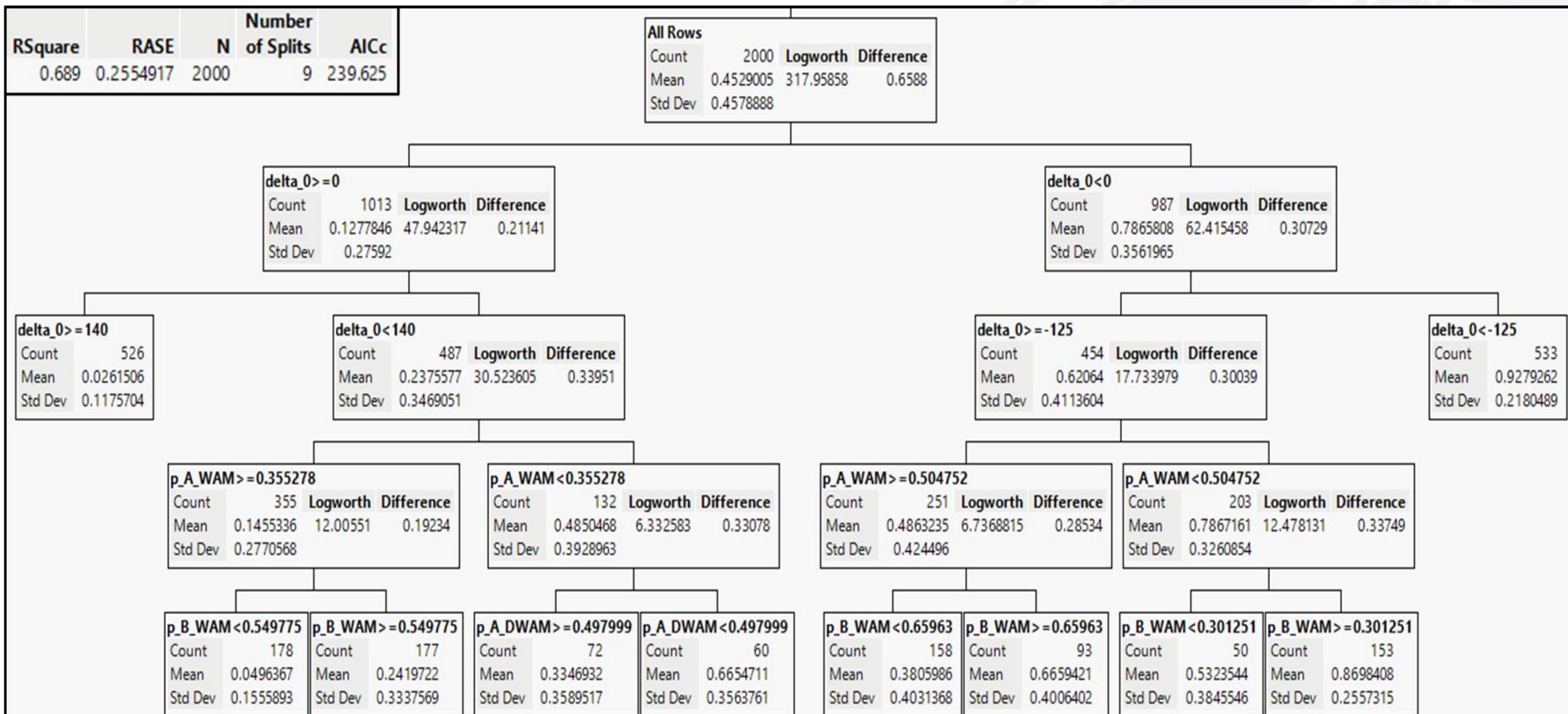
Partition Tree of the Basic Model

- Top Factors:
  - Initial Difference between forces
  - Number of “well aimed” missiles fired by a Force A unit



Partition Tree of the Stochastic Basic Model

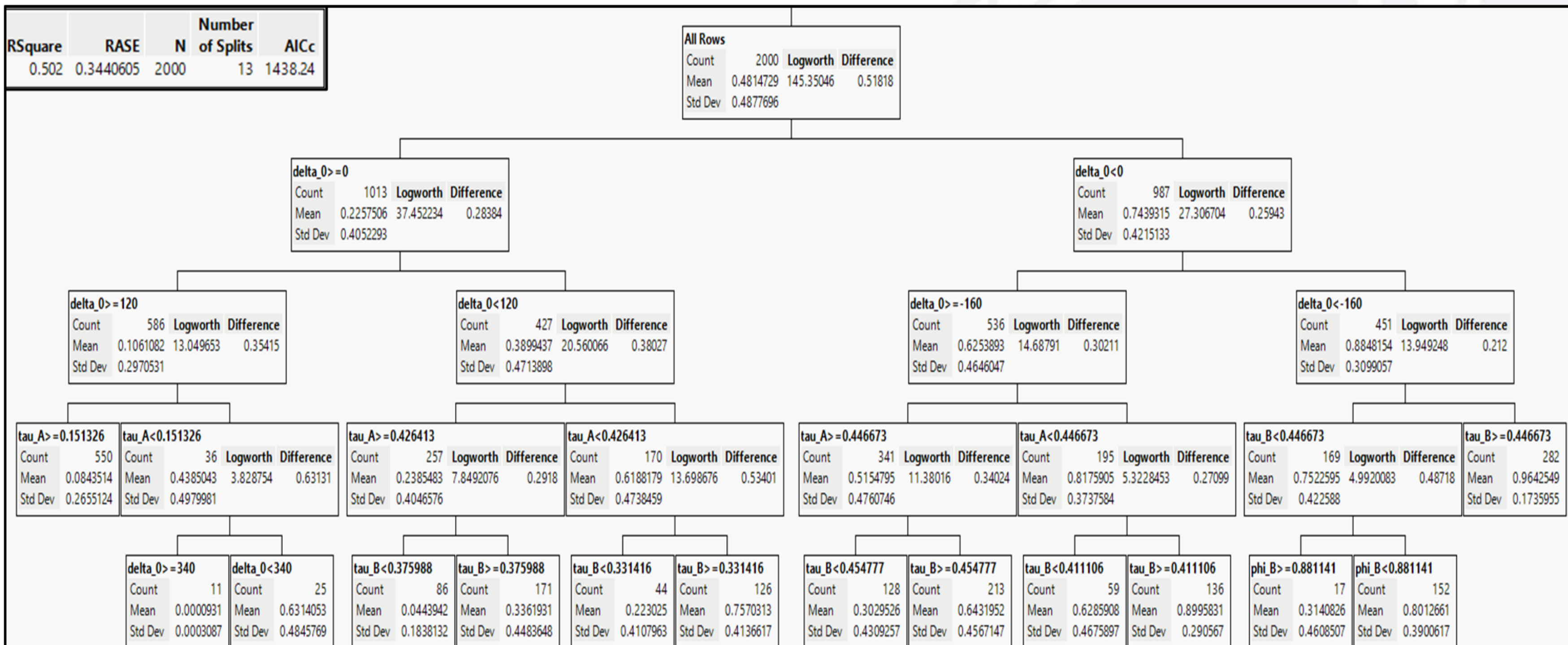
- Top Factors:
  - Initial Difference between forces
  - Probability of a “well-aimed” missile for both forces
  - Probability of a “well-aimed” missile destroyed by a Force A unit



Partition Tree of the Modified Model

- Top Factors:
  - Initial Difference between forces
  - Training effectiveness of both forces
  - Effectiveness of Force B unit’s chaff that draws off shots before its counterfire against A's missiles

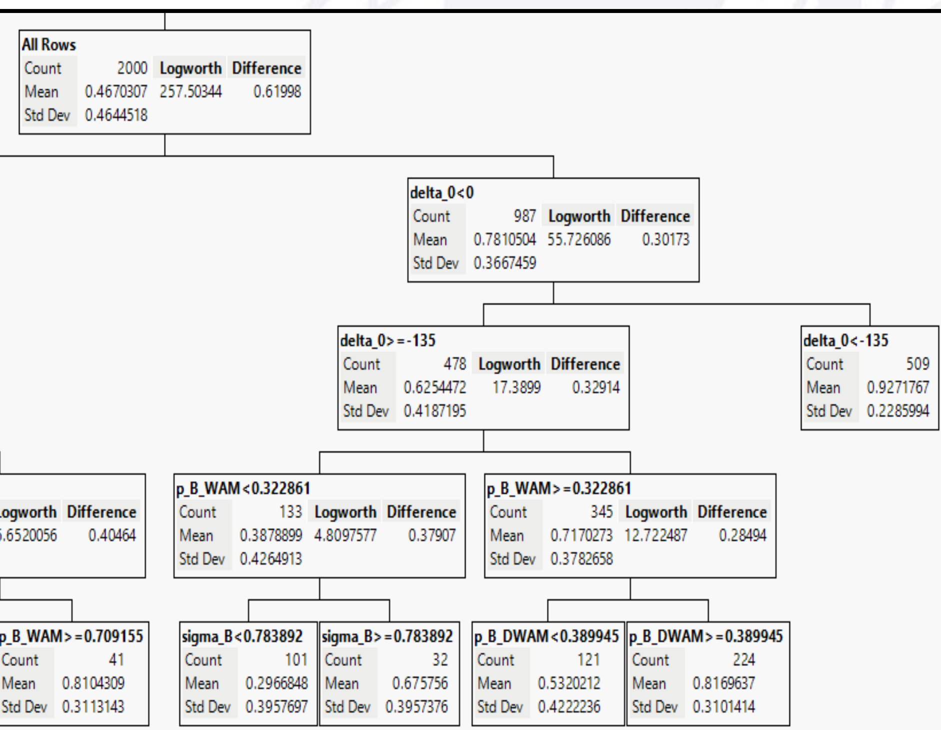
RSquare	RASE	N	Number of Splits	AICc
0.502	0.3440605	2000	13	1438.24



Partition Tree of the Stochastic Modified Model

- Top Factors:
  - Initial Difference between forces
  - Probability of a “well-aimed” missile for both forces
  - Probability of a “well-aimed” missile destroyed by Force B unit
  - Scouting effectiveness of Force B units

RSquare	RASE	N	Number of Splits	AICc
0.616	0.2875627	2000	9	712.63

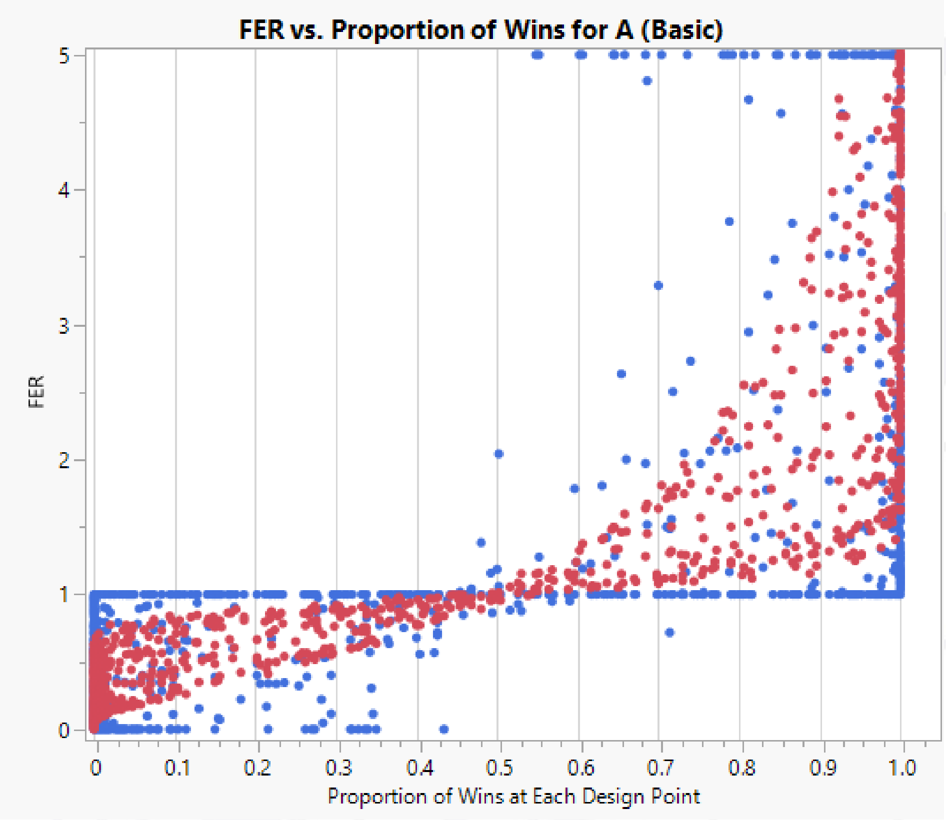


Results

Is the calculated FER a good measure of performance?

FER Comparison Overview

- Each observation corresponds to a design point and the x-axis is the proportion of wins for Force A in the 500 iterations of that design point.
- The blue corresponds to the calculated FER from the equation without the simulation.
  - For instances that resulted in 0 / 0, this was substituted with a 1, representing a tie.
  - For instances that resulted in a number divided by 0, this was substituted with a 5 to represent infinity on the plot.
- The red corresponds to the FER that was observed over the 500 iterations of each design point.

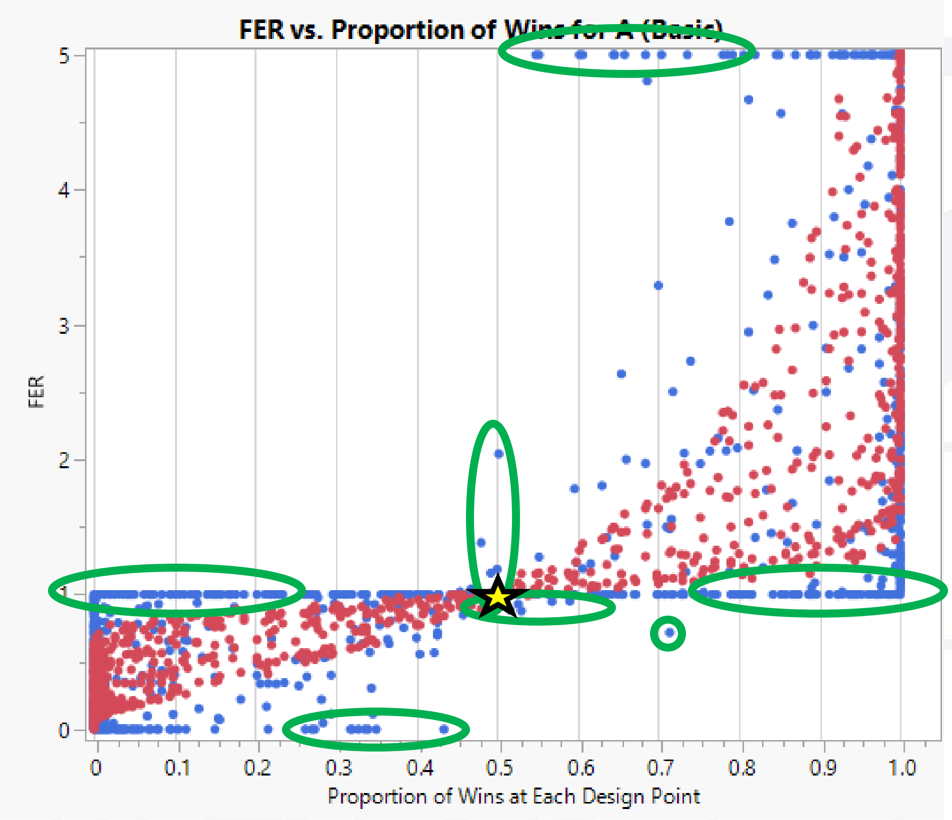


FER Comparison for Basic Model

- Overall Observations:
  - Both models have similar distributions with ranges of [0, 1] left of 0.5, and most observations to the greater than 0.5 are within [1, 5] with a few exceptions.
  - Both have an inflection point at approximately (0.5, 1), which is conditions of a tie.

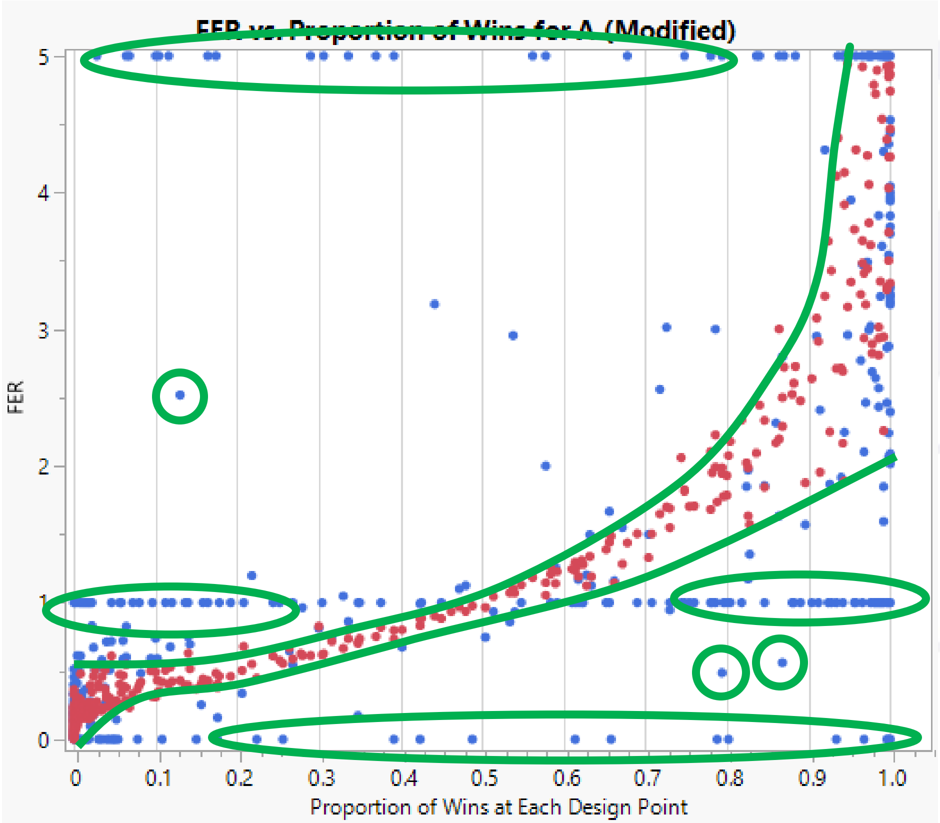
- Calculated FER from Initial Conditions (Blue):
  - Several scenarios calculated to end in a tie had very small and very high proportions of wins.
  - Exaggerated advantage for scenarios that have equivalent forces.
  - Instances that had proportions close to 0.5 were calculated to have a FER of 0 or infinity which can be misleading.

- Calculated FER from All Iterations in DP (Red):
  - All observations are consistent with the range of [0, 1] left of 0.5, and greater than 0.5 are within [1, 5].



FER Comparison for Modified Model

- Calculated FER from Initial Conditions (Blue):
  - Increased variance across all observations
  - Calculated FER does not reflect simulated proportion of wins
  - Several scenarios calculated to end in a tie had very small and very high proportions of wins.
- Calculated FER from All Iterations in DP (Red):
  - All observations are consistent with the range of [0, 1] left of 0.5, and greater than 0.5 are within [1, 5].
  - Smaller variance of FER values, especially around 0.5. High density of FER values closer to 0.



Conclusions

- Key attributes that bear on success are ***initial difference of units between forces and the number of well-aimed missiles fired by a force (higher probability of a "well aimed" missile)***
  - These conclusions are not drawn from Hughes' model, however his salvo equations led to insights about the relationships between attributes
- The FER is limited in its application and can lead to spurious results if applied to the initial conditions. The overall FER aligned well on the Basic Model with the proportion of wins over all iterations, but the several design points that were classified as a tie had either very high or very low probabilities of win based on the simulation.
- **Staying power** is inherently robust and should be treated with greater respect in determining warship design
- The advantage of numbers holds true in Hughes' salvo model. (i.e a force twice as numerous can achieve parity with a force twice as lethal)
- As combat power grows relative to staying power, the risk of **unstable circumstances** grows with it
- Scouting is the key to success and survival. This applies to delivering weapons at range and disrupting the enemies sensing and C2
- Maximum fighting strength (regardless of how it is achieved) is the proper warship design goal. Further research is needed comparing the relative worth of offensive and defensive power with the number of warships and staying power

- (T/F) The force with the greater combat power will always win in Hughes' Basic Salvo model.
    - False! : Combat power can be overcome with numbers and higher staying power of the less effective force. Fractional Exchange Ratio needs to be calculated.

- (T/F) Effective unit training and defensive readiness can **increase** offensive combat power and defensive readiness.
    - False! : None of the attributes in Hughes' embellished salvo model can increase offensive or defensive effectiveness. The best outcome is that neither is degraded.

- (T/F) When overkill is present in one of the force's attributes, FER must be used with caution.
    - True! : Overkill has the potential to show one force as having an overwhelming advantage when, in reality, both forces could have sufficient combat power to destroy each other.

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Questions?