1)

i) It is considered false.

According to the gale-Shapley algorithm, all men and women should find one stable match and no one should be left alone.

According to the statement given, if they choose based on their self-interest, it could generate more chaos and not self–enforcing. Hence, we couldn't find a stable matching

For example:

Men's Preference Profile

Men	1st	2nd
Robert	Taylor	Emma
Ross	Emma	Taylor

Women's Preference Profile

Women	1st	2nd
Taylor	Ross	Robert
Emma	Robert	Ross

According to the men's preference:

1) Let us consider, Robert prefers Taylor to Emma and Ross prefers Emma to Taylor. So, (Robert, Taylor) and (Ross, Emma) is a pair

According to women's preference:

2) Taylor prefers Ross to Robert and Emma prefers Robert to Ross. Here, (Taylor, Ross) and (Emma, Robert) is a pair

Considering men's preference, the stable matching of (Ross, Emma) and (Robert, Taylor), the women's preference list is not with the first preference so we can't pair them.

ii) The answer is true.

There are two existing pairs (m, w') and (m', w) in S but then m prefers w to w', and w prefers

m to m'. In this case, m and w prefer each other to their

current partners to become a match.

We'll say that such a pair (m, w) is an instability, considered as a contradiction, m and w prefers the other to their partner in S, so (m,w) belongs to S

2.

(a)

Yes, there will be a solution for indifference in stable Matching Problem with no strong instability.

For example, women w is indifferent between m and m', we can still assume that w may prefer m to m' and run the GS Algo on the problem. Any solid shakiness in the outcome concerning the first indifference would need to compare to a flimsiness in the consequence of the Gale-Shapley calculation. Since this is unthinkable, the arrangement from the Gale-Shapley calculation should establish a strong instability-free solution to the problem with indifferences

The algorithm for the stable matching problem is:

Let us consider n men M and n women W

Algorithm

```
While(there is any unassigned man):
  m1 prefers the most preferred woman w1 in his preference list
  if(w1 is free and not assigned to any man)
      assign w1 to m1
        Elseif (the woman w1 is not free)
      assign w1 to m2
        check preference list of w1
          if (man m1 is preferred over w1 to w2)
            assign m1 to w1
          else //
           woman w1 prefers m2.
```

In the Stable Marriage algorithm, every man will propose every woman only once, that is the reason we are deleting the woman id from the preference list of man, in the algorithm.

2. b. The answer is No.

For example:

Men	1st	2nd
M	W	W'
M’	W	W’

Women	1st	2nd
W	M’	M
W’	M	M’

The two possible perfect matchings are (M, W); (M', W') and (M, W'); (M', W).

Considering the first case, (M’,W) considered as a weak instability; in the second case, (M, W) considered as a weak instability.

3)

For each ship's schedule we need to find the ship's stopping point. These stopping points would be the truncations

Let us find the stopping point of the ship’s which is known as truncations.

Let us take ships s1,s2,s3….sn, the ports p1,p2,p3...pn. Let us assign the ships to ports.

1) Here, s1 is assigned to p1

2) s2 is assigned to p2

3) s1 can only go to p2 port after arriving at p1 and after s2 leaves.

Each ship prefers the ports in ascending order. But, Each port prefers ships in descending order and find the stable matching using GS Algo

Ship s1 visits p2 but there is another ship over there, but s1 ranked p2 higher and p2 ranks s1 higher than the ship which is already in the port. It contradicts but there was a stable matching

<u>Pseudocode:</u>

Initially all ships and ports are free

While there is a port p1 is free and hasn’t occupied by any ship

  Choose such a port p1

  If p1 is free and no ships arrived there then

    s1 is assigned to p1

  Else p1 is already occupied with some ship sk

```
        If p1 prefers s1 to sk then
            sk remains free
        Else p1 prefers sk to s1
           p1 is assigned to sk
           s1 remains free
        Endif
     Endif
Endwhile
```

4.

Yes. Switching the preference lists could improve the partner of a women.

For example:

Men's Preference	1st	2nd	3rd
Peter	Nicole	Kate	Emma
Mark	Kate	Nicole	Emma
Joe	Nicole	Kate	Emma

Women's Preference	1st	2nd	3rd
Kate	Peter	Mark	Joe
Emma	Peter	Mark	Joe
Nicole	Mark	Peter	Joe

1) Peter chooses Nicole and they become a pair

2) Mark chooses Kate and they become a pair

3) Joe chooses Nicole but Peter already asked her, there is a clash. It's Nicole's turn to make a decision based on her preference list. According to her, she prefers Peter to Joe and rejects Joe's proposal.

4) Now Joe chooses Kate but Mark already asked her, there is a clash. It’s Kate's turn to make a decision based on her preference list. According to her, she prefers Mark to Joe and rejects Joe’s proposal.

5) Joe chooses his least preference Emma and no one asked Emma, she is free so they become a pair

Therefore, as per the Gale-Shapley algorithm, (Peter, Nicole), (Mark, Kate) and (Joe, Nicole) becomes a perfect match

According to the statement, if someone lies take an example of Nicole lied and change her preference list as follows:

Nicole prefers Mark>Joe>Peter then the algorithm will be executed as:

1) Peter chooses Nicole and they become a pair

2) Mark chooses Kate and they become a pair

3) Joe chooses Nicole and peter is less preference to her compared to Joe so they become a pair and leave Peter

4) Now, Peter will choose Kate and she prefers Peter more than Mark so they become a pair and leave Mark

5) Mark will choose Nicole and Mark is more preference to her compared to Joe so they become a pair and leave Joe

6) Finally, Joe will choose Emma and they become a pair

So, the final pair is as follows:

(Mark, Nicole) (Joe, Emma) and (Peter, kate)

So here, Nicole ended up with her favorite preference when she lied