Common constraint mistakes

SUPPLY CHAIN ANALYTICS IN PYTHON

Aaren Stubberfield
Supply Chain Analytics Mgr.

Dependent demand constraint

Context

- Production Plan
- Planning for 2 products *(A, and B)*
- Planning for production over 3 months *(Jan - Mar)*
- Product A is used as an input for production of product B

Constraint Problem

- For each unit of B, we must also have at least 3 units of A

Dependent demand constraint

For each unit of B, we must also have at least 3 units of A

- $3B \leq A$
- $3(2) \leq A$
- $6 \leq A$

Common Mistake:

- $B \leq 3A$
- $3B = A$

Code example

```python
from pulp import *
demand = {'A':[0,0,0],'B':[8,7,6]}
costs = {'A':[20,17,18],'B':[15,16,15]}

# Initialize Model
model = LpProblem("Aggregate Production Planning",
                   LpMinimize)

# Define Variables
time = [0, 1, 2]
prod = ['A', 'B']
X = LpVariable.dicts(
     "prod", [(p, t) for p in prod for t in time],
      lowBound=0, cat="Integer")
```

```python
# Define Objective
model += lpSum([costs[p][t] * X[(p, t)]
                for p in prod for t in time])

# Define Constraint So Production is >= Demand
for p in prod:
    for t in time:
        model += X[(p, t)] >= demand[p][t]
```

Code example continued

```
for t in time:
    model += 3*X[('B',t)] <= X[('A',t)]
```

Extended constraint

For each unit of B, we must also have at least 3 units of A and ***account for direct to customer sells of A***.

- $3B + \text{Demand}_{A} \leq A$

Combination constraint

Context

- Warehouse distribution plan
- 2 warehouses *(WH1, and WH2)*
- We ship 2 products *(A, and B)* from each warehouse
- Warehouse WH1 is small and can either ship 12 A products per a week or 15 B products per a week

Constraint Problem

- What combinations of A, or B can be shipped in 4 weeks?

- 1 week only: $(1/12)A + (1/15)B \leq 1$

Correct Form

- $(1/12)A + (1/15)B$? $\leq$
- $(1/12)(32) + (1/15)(20) \leq 4$
- $(32/12) + (20/15) \leq 4$
- $4 \leq 4$

Common Mistakes

- $12A + 15B \leq 4$
- $(1/12)A + (1/15)B = 4$

```
from pulp import *
import pandas as pd
demand = pd.read_csv("Warehouse_Constraint_Demand.csv", index_col=['Product'])
costs = pd.read_csv("Warehouse_Constraint_Cost.csv", index_col=['WH','Product'])

# Initialize Model
model = LpProblem("Distribution Planning", LpMinimize)

# Define Variables
wh = ['W1','W2']
prod = ['A', 'B']
cust = ['C1', 'C2', 'C3', 'C4']
X = LpVariable.dicts("ship", [(w, p, c) for c in cust for p in prod for w in wh],
                      lowBound=0, cat="Integer")
```

Code example continued

```
# Define Objective
model += lpSum([X[(w, p, c)]*costs.loc[(w, p), c]
               for c in cust for p in prod for w in wh])

# Define Constraint So Demand Equals Total Shipments
for c in cust:
    for p in prod:
        model += lpSum([X[(w, p, c)] for w in wh]) == demand.loc[p, c]
```

Code example continued

Constraint

```
model += ((1/12) * lpSum([X['W1', 'A', c] for c in cust])
          + (1/15) * lpSum([X['W1', 'B', c] for c in cust])) <= 4
```

Extend constraint

Warehouse WH1 is small and either ship 12 A products per a week, 15 B products per a week, ***or 5 C products per a week***. What combinations of A, B, or C can be shipped in 4 weeks?

- $(1/12)A + (1/15)B + (1/5)C \leq 4$

Summary

- Common Mistakes
  - Dependent constraint
  - Combination selection constraint
- How to extend constraints
- Check constraint by plugging in a value

Let's practice

SUPPLY CHAIN ANALYTICS IN PYTHON

Capacitated plant location - case study P2

SUPPLY CHAIN ANALYTICS IN PYTHON

Aaren Stubberfield
SuSupply Chain Analytics Mgr.

Capacitated plant location model

Modeling

- Production at regional facilities
  - Two plant sizes (low / high)
- Exporting production to other regions
- Production facilities open / close

Decision variables

What we can control:

- x_{ij} = quantity produced at location **_i_** and shipped to **_j_**
- y_{is} = 1 if the plant at location **_i_** of capacity **_s_** is open, 0 if closed
  - s = ***low*** or ***high*** capacity plant

Constraints

- Total Production = Total Demand
  - $\sum_{i=1}^{n}$ x_{ij} = D_j for $j = 1, ..., m$
  - n = number of production facilities
  - m = number of markets or regional demand points

Constraints

- Total Production ? Total Production Capacity
  - $\sum_{j=1}^{m} x_{ij}$? $\sum_{s=1} K_{is} y_{is}$
  - K_{is} = potential production capacity of plant **_i_** of size **_s_**

```python
from pulp import *

# Initialize Class
model = LpProblem("Capacitated Plant Location Model", LpMinimize)

# Define Decision Variables
loc = ['A', 'B', 'C', 'D', 'E']
size = ['Low_Cap','High_Cap']
x = LpVariable.dicts("production_", [(i,j) for i in loc for j in loc],
                     lowBound=0, upBound=None, cat='Continuous')
y = LpVariable.dicts("plant_", [(i,s) for s in size for i in loc], cat='Binary')

# Define Objective Function
model += (lpSum([fix_cost.loc[i,s]*y[(i,s)] for s in size for i in loc])
          + lpSum([var_cost.loc[i,j]*x[(i,j)] for i in loc for j in loc]))
```

Code example continued

```
# Define the Constraints
for j in loc:
    model += lpSum([x[(i, j)] for i in loc]) == demand.loc[j,'Dmd']
for i in loc:
    model += lpSum([x[(i, j)] for j in loc]) <= lpSum([cap.loc[i,s]*y[(i,s)]
                                                       for s in size])
```

Summary

Capacitated Plant Location Model:

- Constraints
  - Total Production = Total Demand
  - Total Production ≤ Total Production Capacity

Review time

SUPPLY CHAIN ANALYTICS IN PYTHON

Solve the PuLP model

SUPPLY CHAIN ANALYTICS IN PYTHON

Aaren Stubberfield
Supply Chain Analytics Mgr.

Common modeling process for PuLP

1. ~~Initialize Model~~
2. ~~Define Decision Variables~~
3. ~~Define the Objective Function~~
4. ~~Define the Constraints~~
5. Solve Model
   - call the `solve()` method
   - check the status of the solution
   - print optimized decision variables
   - print optimized objective function

Solve model - solve method

```
.solve(solver=None)
```

- `solver` = Optional: the specific solver to be used, defaults to the default solver.

```python
# Initialize, Define Decision Vars., Objective Function, and Constraints
from pulp import *
import pandas as pd
model = LpProblem("Minimize Transportation Costs", LpMinimize)
cust = ['A','B','C']
warehouse = ['W1','W2']
demand = {'A': 1500, 'B': 900, 'C': 800}
costs = {('W1','A'): 232, ('W1','B'): 255, ('W1','C'): 264,
         ('W2','A'): 255, ('W2','B'): 233, ('W2','C'): 250}
ship = LpVariable.dicts("s_", [(w,c) for w in warehouse for c in cust],
                        lowBound=0, cat='Integer')
model += lpSum([costs[(w, c)] * ship[(w, c)] for w in warehouse for c in cust])
for c in cust: model += lpSum([ship[(w, c)] for w in warehouse]) == demand[c]

# Solve Model
model.solve()
```

Solve model - status of the solution

```
LpStatus[model.status]
```

- **Not Solved:** The status prior to solving the problem.
- **Optimal:** An optimal solution has been found.
- **Infeasible:** There are no feasible solutions (e.g. if you set the constraints x ≤ 1 and x ≥ 2).
- **Unbounded:** The object function is not bounded, maximizing or minimizing the objective will tend towards infinity (e.g. if the only constraint was x ≥ 3).
- **Undefined:** The optimal solution may exist but may not have been found.

[1] Keen, Ben Alex. "Linear Programming with Python and PuLP [2] Part 2." _Ben Alex Keen_, 1 Apr. 2016, benalexkeen.com/linear-programming-with-python-and-pulp-part-2/._{{5}}

```python
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from pulp import *
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         ('W2','A'): 255, ('W2','B'): 233, ('W2','C'): 250}
ship = LpVariable.dicts("s_", [(w,c) for w in warehouse for c in cust], lowBound=0, cat='Integer')
model += lpSum([costs[(w, c)] * ship[(w, c)] for w in warehouse for c in cust])
for c in cust: model += lpSum([ship[(w, c)] for w in warehouse]) == demand[c]
# Solve Model
model.solve()
print("Status:", LpStatus[model.status])
```

```
Status: Optimal
```

Print variables to standard output:

```
for v in model.variables():
    print(v.name, "=", v.varValue)
```

Pandas data structure:

```
o = [{A:ship[(w,'A')].varValue, B:ship[(w,'B')].varValue, C:ship[(w,'C')].varValue}
     for w in warehouse]
print(pd.DataFrame(o, index=warehouse))
```

- loop model variables
- store values in a pandas DataFrame

```python
# Solve Model
model.solve()
print(LpStatus[model.status])
o = [{A:ship[(w,'A')].varValue, B:ship[(w,'B')].varValue, C:ship[(w,'C')].varValue}
     for w in warehouse]
print(pd.DataFrame(o, index=warehouse))
```

Output:

```
Status: Optimal
|       |A      |B      |C      |
|:------|:------|:------|:------|
|W1     |1500.0 |0.0    |0.0    |
|W2     |0.0    |900.0  |800.0  |
```

Solve model - optimized objective function

Print the value of optimized objective function:

```
print("Objective = ", value(model.objective))
```

```python
# Solve Model
model.solve()
print(LpStatus[model.status])
output = []
for w in warehouse: t = [ship[(w,c)].varValue for c in cust] output.append(t)
opd = pd.DataFrame.from_records(output, index=warehouse, columns=cust)
print(opd)
print("Objective = ", value(model.objective))
```

```
Status: Optimal
|       |A      |B      |C      |
|:------|:------|:------|:------|
|W1     |1500.0 |0.0    |0.0    |
|W2     |0.0    |900.0  |800.0  |
Objective = 757700.0
```

Summary

Solve Model

- Call the `solve()` method
- Check the status of the solution
- Print values of decision variables
- Print value of objective function

Let's practice!

Sanity checking the solution

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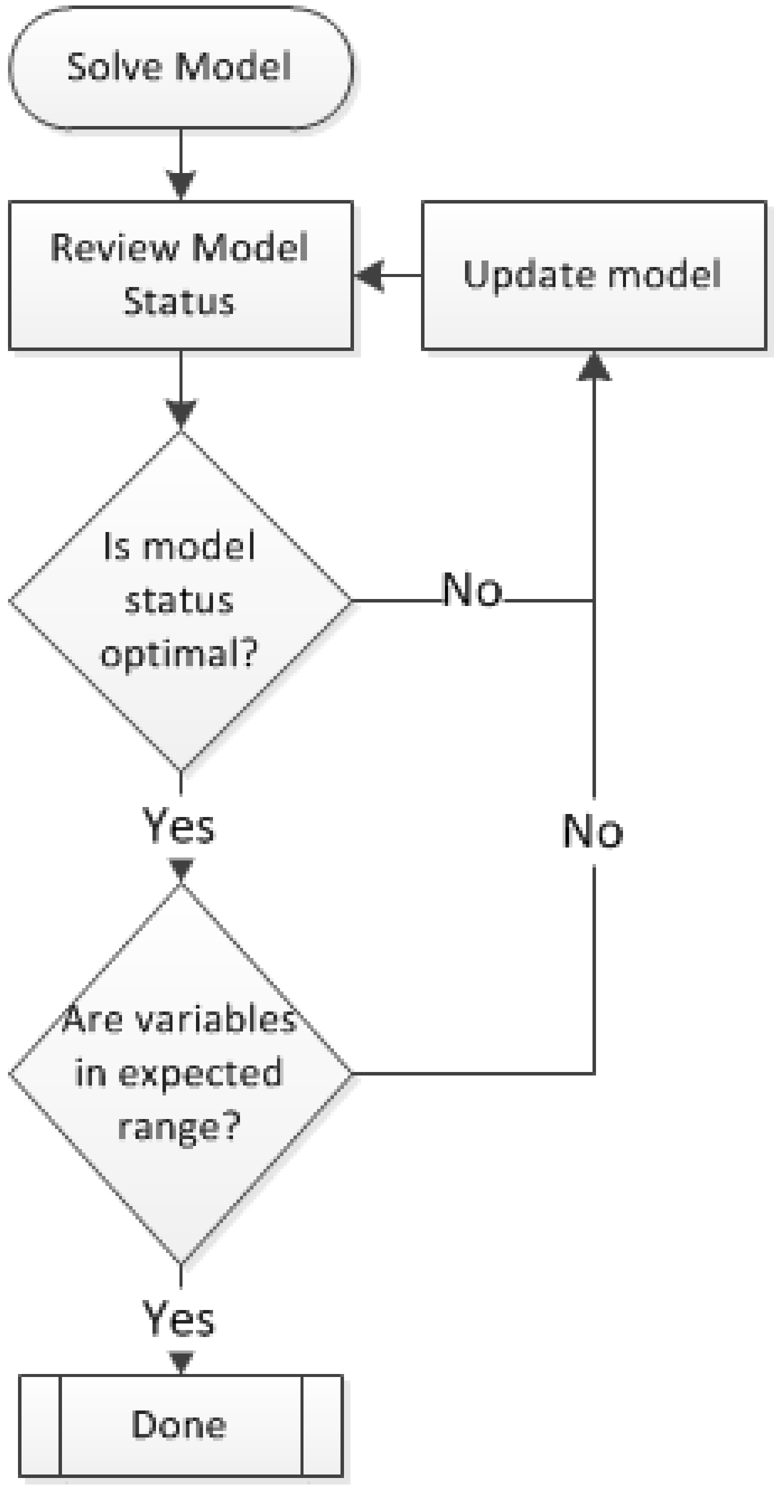
Supply Chain Analytics Mgr.


Solve Model
Review Model Status
Update model
Is model status optimal?
No
Yes
No
Are variables in expected range?
Yes
Done

Check the model status

- **Infeasible:** There are no feasible solutions.
    - Review the constraints
- **Unbounded:** The object function is not bounded, maximizing or minimizing the objective will tend towards infinity.
    - Review the objective function
- **Undefined:** The optimal solution may exist but may not have been found.
    - Maybe the best available solution
    - Review how you are modeling the problem

Check if results are within expectations

Are the **decision variables** and value of **objective** within expected range?

- Based on knowledge / understanding of problem
- If "Yes", then you have a valid solution
- If "No", then review:
  - Python code
  - Data
  - Write the LP File

Write LP

```
writeLP(filename)
```

- `filename` = The name of the file to be created

Shows:

- Name of problem
- Objective function and if minimizing or maximizing
- Constraints, including constraints on Decision Variables called Bounds
- Decision variables

Code example

```
\* Aggregate Production Planning *\
Minimize
OBJ: 20 prod_('A',_0) + 17 prod_('A',_1)
    + 18 prod_('A',_2) + 15 prod_('B',_0)
    + 16 prod_('B',_1) + 15 prod_('B',_2)
Subject To
_C1: prod_('A',_0) >= 0
_C2: prod_('A',_1) >= 0
_C3: prod_('A',_2) >= 0
_C4: prod_('B',_0) >= 8
_C5: prod_('B',_1) >= 7
_C6: prod_('B',_2) >= 6
```

```
Bounds
0 <= prod_('A',_0)
0 <= prod_('A',_1)
0 <= prod_('A',_2)
0 <= prod_('B',_0)
0 <= prod_('B',_1)
0 <= prod_('B',_2)
Generals
prod_('A',_0)
prod_('A',_1)
prod_('A',_2)
prod_('B',_0)
prod_('B',_1)
prod_('B',_2)
```

Summary

Strategy for Sanity Checking

- Check the model status
- Check decision variables and objective inside expected range
- Use `writeLP()` if needed

Practice time!