AE 6355 Homework # 1

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Problem 1: Comparison of Unmanned Entry Vehicles

In this comparison we will look at the Galileo entry probe and the Mars Perseverance Entry vehicle. The Galileo entry probe had a payload mass of 30 kg, with an overall mass of 339 kg. Perserverance had a 575 kg aeroshell descent vehicle, a 440 kg heat shield, a 1070 kg descent stage, with a 1025 kg payload. The Mars entry vehicle was far larger than the Galileo vehicle, and likely had a far larger ballistic coefficient. Galileo was a blended cone aeroshell, using a parachute pack for descent. Mars Perservence also had a conic aeroshell, but included a parachute phase, a powered descent phase, and a sky-crane to safely deliver the physics package to Mars.

Galileo entered Jupiter's atmosphere at 48 km/s, at a flight path angle of -8.4 degrees, and experiences peak temperatures surface temperatures of the heat shield equal to 16000 C, which far exceeded expected values. Mars Perseverance entered Martian atmosphere at a lower 5.327 km/s, and a lower flight path angle of -15.48 degrees, and experienced peak surface temperatures of the heat shield around 1600 C. The thermal protection systems of both vehicles are similar in theory, though Mars Perseverance is far more advanced in materials. The Galileo probe utilized chopped-molded carbon phenolic for a 1.26 meter diameter heatshield. Mars Perseverance utilized a newer material, Phenolic Impregnated Carbon Ablator, on a larger 4.5 meter diameter heatshield. The atmospheric density during entry for Mars Perseverance was underestimated by approximately 60 percent, but the heatshield still was able to handle the conditions. For Galileo, the peak heating was far underestimated, resulting in more ablative loss of the heat shield than designed for.

For Galileo, the initial cost was slated to be $410 million dollars, but after 10 years of delays, the cost grew to $1.6 billion. The project was high risk due to increases in the size of the vehicle and the novelty of the mission design. Mars Perseverance cost $2.8 billion dollars, but was able to utilize technology from the Mars Science Laboratory, thereby reducing risk and cost.

Problem 2: Changes in properties across a normal shock

The upstream conditions for the problem are:

$$
\begin{aligned}
u_1 &= 10000.72 \text{ m/s} \\
T_1 &= 208.4 \text{ K} \\
\rho_1 &= 3.99109\text{e-5 kg/m}^3 \\
p_1 &= 2.3875 \text{ N/m}^2 \\
\beta &= 90^\circ \text{ degrees; this is the shock angle}
\end{aligned}
$$

a) Assume the air is a calorically perfect gas

In this case we can assume that the ratio of specific heats for air is $\gamma_{air} = 1.4$, and we can compute the gas constant for air and subsequently the Mach number.

$$
\begin{aligned}
R &= 8.3145\text{e}3 \ \frac{\text{N m}}{\text{K kmol}} \\
m_{air} &= 28.97 \ \frac{\text{kg}}{\text{kmol}} \\
R_{air} = \frac{R}{m_{air}} &= \frac{8.3145\text{e}3}{28.97} = 287.004 \ \frac{\text{N m}}{\text{K kg}} \\
M_1 &= u_1 / \sqrt{\gamma_{air} R_{air} T_1} \\
&= \frac{10000.72 \text{ m/s}}{\sqrt{1.4287.004 \ \frac{\text{N m}}{\text{K kg}} 208.4 \text{ K}}} = 37.046
\end{aligned}
$$

The ratios are

$$
\begin{aligned}
\frac{p_2}{p_1} &= 1 + \frac{2\gamma_{air}}{\gamma_{air}+1}(M_1^2 \sin(\beta)^2 - 1) \\
&= 1 + \frac{21.4}{1.4+1}(37.046^2 \sin(90)^2 - 1) = 1600.95 \\
\frac{\rho_2}{\rho_1} &= \frac{(\gamma+1)M_1^2 \sin(\beta)^2}{(\gamma-1)M_1^2 \sin(\beta)^2 + 2} \\
&= \frac{1.4+1)37.046^2 \sin(90)^2}{(1.4-1)37.046^2 \sin(90)^2 + 2} = 5.978 \\
\frac{T_2}{T_1} &= \frac{p_2}{p_1}\frac{\rho_1}{\rho_2} \\
&= 1600.95/5.978 = 267.797
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
\frac{p_2}{p_1} &= 1600.95 && (1) \\
\frac{\rho_2}{\rho_1} &= 5.978 && (2) \\
\frac{T_2}{T_1} &= 267.797 && (3)
\end{aligned}
}
$$

b) The air is approximated as pure diatomic nitrogen

The gas is in local thermal equilibrium and we are given the function for enthalpy $h(T)$. We can compute the upstream enthalpy h_1 assuming that it is a perfect gas upstream.

$$
\begin{aligned}
cp_{N_2,200K} &= 1.039 \\
m_{N2} &= 14.01 \ \frac{\text{kg}}{\text{kmol}} \\
R_{N2} &= 593.47 \ \frac{\text{N m}}{\text{K kg}} \\
h_1 &= cp_{N_2,200K} T_1 \\
&= (1.039)(208.4) = 216.527 \text{ kJ / kg}
\end{aligned}
$$

We will use an iterative procudure with the following steps. First we write the inner loop function.

```
def inner_loop(T, h_2_est):  # Input Temp (K) and Enthalpy (kJ / mol)
       return h_2_est - enthalpy_N2(T)  # Returns enthalpy difference
                                              in (kJ / mol)
```

We use a root finding algorithm to find the appropriate value of temperature. Next we describe the outer loop.

```
def outer_loop(e0, p_1, rho_1, h_1, u_1, T_1):
    # p_1 (N m^2), rho_1 (kg/m^3), h_1 (kJ / kg), u_1 (m/s), T_1 (K)
    p_2_est = p_1 + rho_1 * u_1 ** 2 * (1 - e0)
    h_2_est = h_1 + u_1 ** 2 / 2 * (1 - e0 ** 2)
    T0 = T_1   # Random guess using T_1
    T_converged = sp.optimize.newton(inner_loop, T0, args=(h_2_est))
    rho_2 = p_2_est / (R_N2 * T_converged)
    eps_check = rho_1 / rho_2
    return e0 - eps_check
```

We can use a root finding algorithm to find the appropriate epsilon. For the conditions listed above, $\varepsilon = 0.12736$

```
epsilon = sp.optimize.newton(outer_loop, epsilon_est, args=(p_1, rho_1
                                   , h_1, u_1, T_1))
```

The appropriate ratios are then found using the conservation equations and the equation of state. The equation of state.

$$
\begin{aligned}
p_2 &= p_1 + \rho_1 \frac{u_1^2}{2}(1-\varepsilon) \\
&= 2.3875 \text{ N/m}^2 + 3.99109\text{e-}5 \text{ kg/m}^3 \frac{(10000.72 \text{ m/s})^2}{2}(1-0.12736) \\
&= 4004.76 \text{ N/m}^2 \\
\rightarrow \frac{p_2}{p_1} &= \frac{4004.76 \text{ N/m}^2}{2.3875 \text{ N/m}^2} = 1677.387 \\
\rho_2 &= \frac{\rho_1}{\varepsilon} \\
&= \frac{3.99109\text{e-}5 \text{ kg/m}^3}{0.12736} = 3.133781\text{e-}4 \text{ kg/m}^3 \\
\rightarrow \frac{\rho_2}{\rho_1} &= \frac{3.133781\text{e-}4 \text{ kg/m}^3}{3.99109\text{e-}5 \text{ kg/m}^3} = 7.8519 \\
T_2 &= \frac{p_2}{\rho_2 R_{N2}} \\
&= \frac{4004.76 \text{ N/m}^2}{3.133781\text{e-}4 \text{ kg/m}^3 \; 593.47 \; \frac{\text{N m}}{\text{K kg}}} = 21533.27 \text{ K} \\
\rightarrow \frac{T_2}{T_1} &= \frac{21533.27 \text{ K}}{208.4 \text{ K}} = 103.327
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
\varepsilon &= 0.12736 && (4) \\
\frac{p_2}{p_1} &= 1677.387 && (5) \\
\frac{\rho_2}{\rho_1} &= 7.8519 && (6) \\
\frac{T_2}{T_1} &= 103.327 && (7)
\end{aligned}
}
$$

c) Utilize curve fits for both enthalpy and temperature

This routine is similar to the first in setup. We will calculate h_1 with air and the perfect gas assumption and then setup our outer loop.

```
def outer_loop_curve_fit(epsilon_c):
    # Given the estimated density ratio, compute the estimated density
                                        downstream of the shock
    rho_2_c_est = rho_1 / epsilon_c

    # Compute the estimated pressure downstream of the shock
    p_2_c_est = p_1 + rho_1 * u_1 ** 2 * (1 - epsilon_c)

    # Compute the estimated enthalpy downstream of the shock
    h_2_c_est = h_1 + u_1 ** 2 / 2 * (1 - epsilon_c ** 2)

    # Perform the curve fits on gamma tilde
```

```
    gamma_tilde_est = gamma_curve.fit(p_2_c_est, rho_2_c_est)

    # Compute the enthalpy from the curve fit
    h_2_c_check = compute_curve_fit_enthaply(p_2_c_est, rho_2_c_est,
                                              gamma_tilde_est[0])

    return h_2_c_check - h_2_c_est
```

This function is then set to zero with a root finding algorithm. We can then compute our pressure and density, then use the temperature curve fit. For the conditions listed above, $\varepsilon = 0.05886$

```
epsilon_c = sp.optimize.newton(outer_loop_curve_fit, epsilon_c_est)

p_2_c = p_1 + rho_1 * u_1 ** 2 * (1 - epsilon_c)
rho_2_c = rho_1 / epsilon_c
T_2_c = temp_curve.fit(p_2_c, rho_2_c)[0]
```

The full code is given via a zip file, while the results are tabulated here.

$$
\begin{aligned}
p_2 &= p_1 + \rho_1 \frac{u_1^2}{2}(1-\varepsilon) \\
&= 2.3875 \text{ N/m}^2 + 3.99109\text{e-}5 \text{ kg/m}^3 \frac{(10000.72 \text{ m/s})^2}{2}(1-0.05886) \\
&= 4318.908 \text{ N/m}^2 \\
\rightarrow \frac{p_2}{p_1} &= \frac{4318.907 \text{ N/m}^2}{2.3875 \text{ N/m}^2} = 1808.967 \\
\rho_2 &= \frac{\rho_1}{\varepsilon} \\
&= \frac{3.99109\text{e-}5 \text{ kg/m}^3}{0.05886} = 6.780266\text{e-}4 \text{ kg/m}^3 \\
\rightarrow \frac{\rho_2}{\rho_1} &= \frac{6.780266\text{e-}4 \text{ kg/m}^3}{3.99109\text{e-}5 \text{ kg/m}^3} = 16.988 \\
T_2 &= 9919.981 \text{ K} \\
\rightarrow \frac{T_2}{T_1} &= \frac{9919.981 \text{ K}}{208.4 \text{ K}} = 47.60
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
\varepsilon &= 0.05886 && (8) \\
\frac{p_2}{p_1} &= 1808.967 && (9) \\
\frac{\rho_2}{\rho_1} &= 16.988 && (10) \\
\frac{T_2}{T_1} &= 47.60 && (11)
\end{aligned}
}
$$

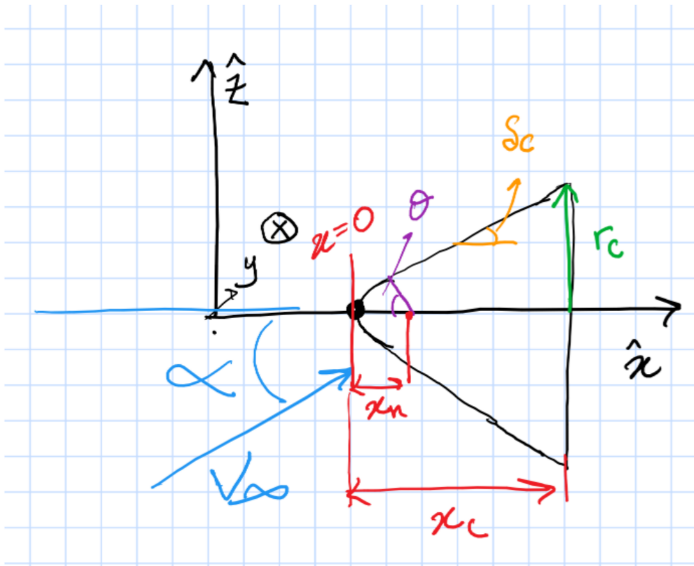

Figure 1: Blunted Cone Variables and Axes

Problem 3: Coefficient of Moment for Sphere-Cone

The majority of this work was done in Mathematica, and the full notebook is attached at the end of this pdf. The code is attached in a zip.

$$\boxed{\begin{aligned} M = \frac{1}{6}(\cos^2\delta_c)\Big(&2\xi^3\sin(\alpha)\cos(\alpha)(\cos\delta_c)(3\cos\delta_c - 4) \\ &- 4\sin(2\alpha)(\cot\delta_c)\left(\xi^3\cos^3\delta_c - 1\right)\Big)\end{aligned}} \tag{12}$$

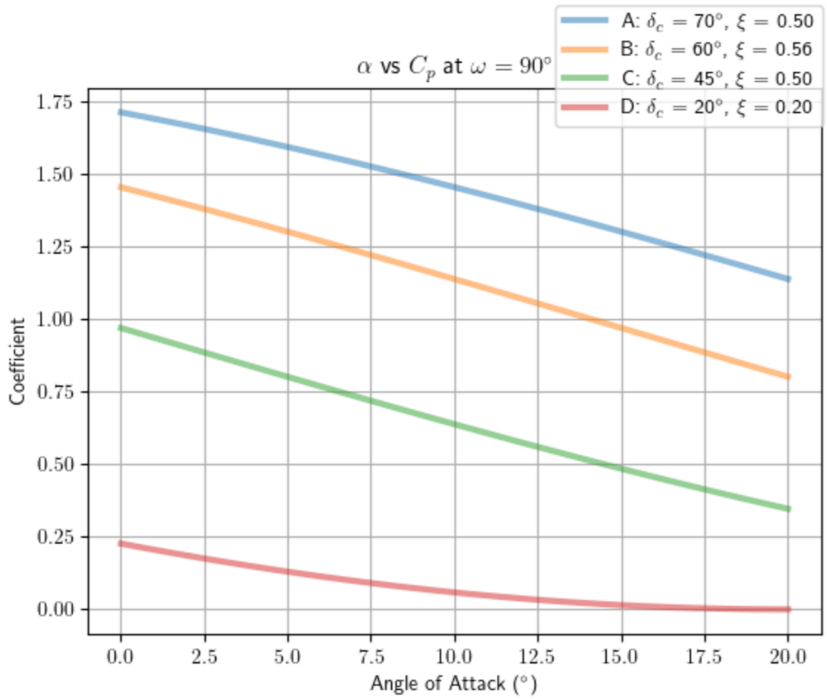

Figure 2: C_p graph along the centerline of the body of a blunted cone.

Problem 4: Graph C_p, C_l, and C_d for a general blunted cone.

e) Comments on accuracy and order of magnitude

We assumed a modified Newtonian flow, which corrects for Cpmax using the density ratio epsilon. Newtonian flow is valid when the wave angle of the shock is close to the cone angle of the vehicle, such the flow is traversing along the cone. As the angle of attack increases, the ratio of the cone angle and the wave angle should move away from 1, making estimates less accurate. ince we neglected skin friction and parasite drag, the accuracy of the solution should decrease as it becomes closer to a flat plate. This means as the cone angle and bluntness parameter increase, the solution becomes less accurate.

In terms of the order of magnitude, at large cone angles, the bluntness parameter does not matter as much for Cd. If the cone angle becomes smaller, the bluntess parameter impacts Cd much heavier. For example at a 20 degree cone angle, increasing the bluntness parameter from 0.2 - 0.56 increase Cd from .281 to .489 at 5 degrees alpha.

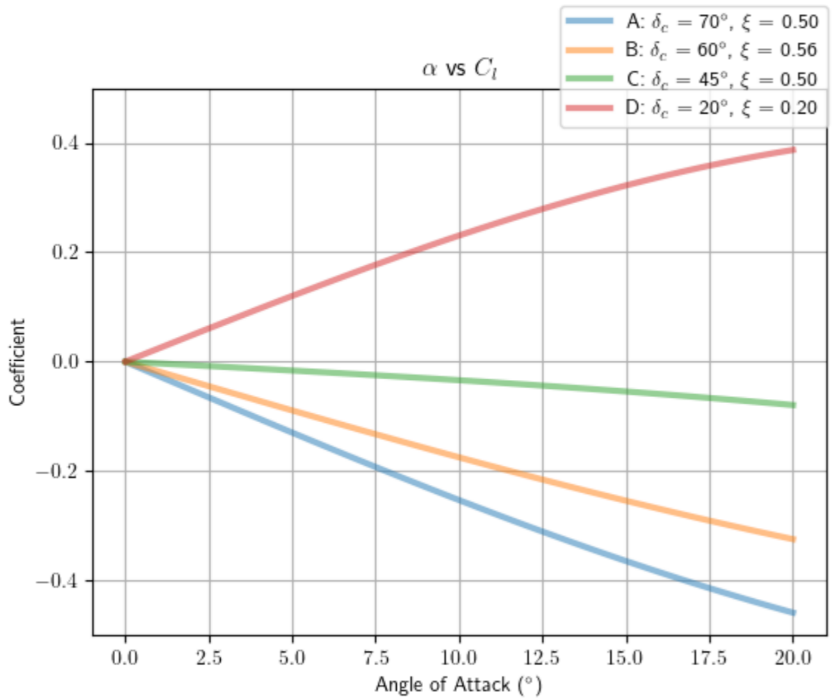

Figure 3: C_l graph for the body of a blunted cone.

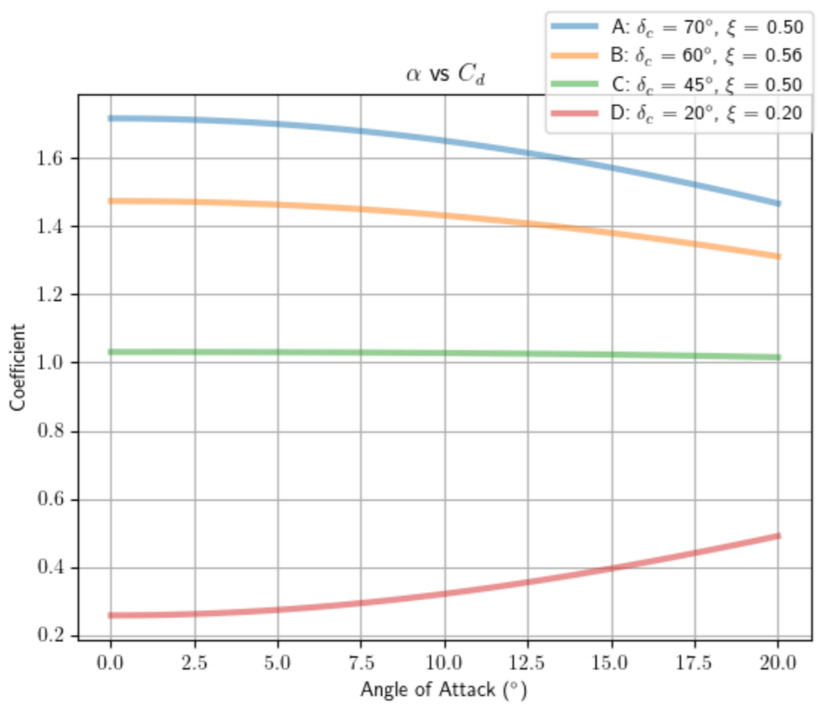

Figure 4: C_d graph for the body of a blunted cone.

Problem 5: AFE Aerodynamics

The upstream conditions for the problem are:

$$\begin{aligned}
u_1 &= 9750 \text{ m/s} \\
T_1 &= 189 \text{ K} \\
\rho_1 &= 2.862e - 5\text{e-5 kg/m}^3 \\
p_1 &= 1.556 \text{ N/m}^2 \\
cp_{air,200K} &= 1.0025 \text{ kJ /(kg K)} \\
h_1 &= cp_{air,200K} * T_1 \\
&= 189.4725 \text{ kJ / kg}
\end{aligned}$$

a) Downstream conditions behind normal shock

Utilizing the same procedure as in problem 2c, we compute an $\varepsilon = 0.05616$

$$\begin{aligned}
p_2 &= p_1 + \rho_1 \frac{u_1^2}{2}(1-\varepsilon) \\
&= 1.556 \text{ N/m}^2 + 2.862\text{e-5 kg/m}^3 \frac{(9750 \text{ m/s})^2}{2}(1-0.05616) \\
&= 2569.442 \text{ N/m}^2 \\
\rho_2 &= \frac{\rho_1}{\varepsilon} \\
&= \frac{2.862\text{e-5 kg/m}^3}{0.05616} = 5.09586\text{e-4 kg/m}^3 \\
h_2 &= h_1 + \frac{u_1^2}{2}(1-\varepsilon^2) \\
&= 189.4725 \text{ kJ / kg} + \frac{(9750 \text{ m/s})^2}{2}(1-0.05616^2) \\
&= 4.738\text{e7 kJ / kg} \\
T_2 &= 8420.88 \text{ K} \\
u_2 &= u_1\varepsilon \\
&= 9750 \text{ m/s}(0.05616) = 547.591 \text{m/s}
\end{aligned}$$

$$\boxed{\begin{aligned}
\varepsilon &= 0.05616 && (13) \\
T_2 &= 8420.88 \text{ K} && (14) \\
p_2 &= 2569.442 \text{ N/m}^2 && (15) \\
\rho_2 &= 5.09586\text{e-4 kg/m}^3 && (16) \\
u_2 &= 547.591 \text{m/s} && (17) \\
h_2 &= 4.738\text{e7 kJ / kg} && (18)
\end{aligned}}$$

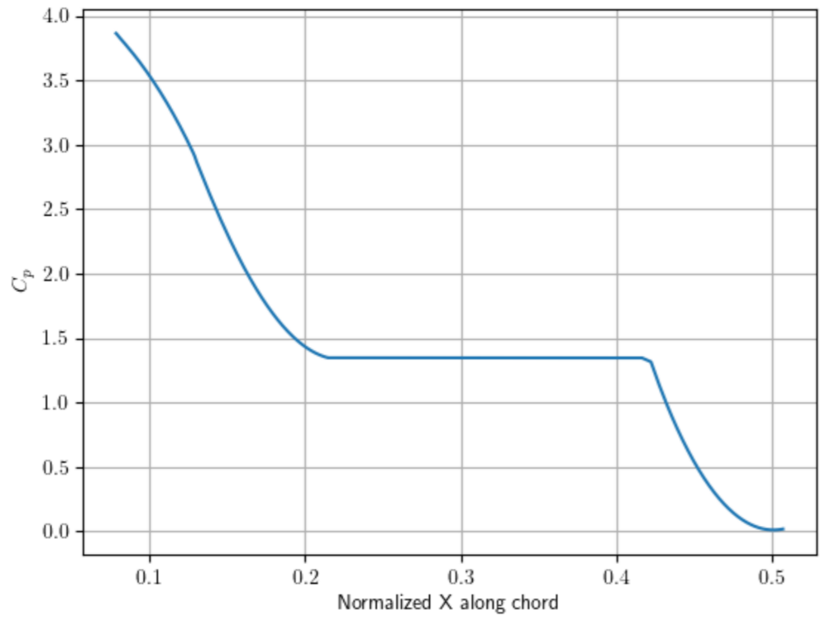

Figure 5: C_p along the AFE symmetry axis at $\alpha = 4°$

b) Modified Newtonian pressure coefficient along the symmetry plane of the AFE

In this case, we have assumed trim angle of attack is 4 degrees, from Ref 2, Figure 3, where $\frac{C_m}{C_{m,ref}} \approx 0$ for air

Physically this distribution should have a higher contribution of pressure from the bottom, thus being positive. Additionally since the nose of the vehicle close to normal to the flow, it should experience the largest pressure, and we will see a drop as the continue down the chord. The pressure should remain constant along the straight part of the cone, since the upper part of the body does not experience any flow (under the modified newtonian assumptions), and the pressure decreases as the angle between the flow and normal vector decrease.

c) AFE lift and drag coefficients

Since in this vehicle, we have computed $C_p(x)$, and $\theta(x)$ (the body angle of the vehicle with respect to the x-axis) as a function of the chord x_c. Additionally $\theta(x)$ differs between the top and bottom of the vehicle, so both sides have to be integrated separately and then summed. We can use the following two equations for C_l and C_d

$$
\begin{aligned}
C_l &= \frac{1}{c}\int_{x_1}^{x_c} C_p(x)\cos\theta(x)\mathrm{d}x \\
&= \frac{1}{c}\int_{x_1}^{x_c} C_p(x)\cos\theta_u(x)\mathrm{d}x + \frac{1}{c}\int_{x_1}^{x_c} C_p(x)\cos\theta_l(x)\mathrm{d}x \\
C_d &= \frac{1}{c}\int_{x_1}^{x_c} C_p(x)\sin\theta\mathrm{d}x \\
&= \frac{1}{c}\int_{x_1}^{x_c} C_p(x)\sin\theta_u(x)\mathrm{d}x + \frac{1}{c}\int_{x_1}^{x_c} C_p(x)\sin\theta_l(x)\mathrm{d}x
\end{aligned}
$$

These are approximated via sums over infinitesimal segments to result in $C_d = 1.94383$ and $C_l = -0.33621$.

$$
\boxed{\begin{aligned} C_{l,AFE} &= -0.33621 \qquad (19) \\ C_{d,AFE} &= 1.94383 \qquad (20) \end{aligned}}
$$

d) Transition between free-molecular and continuum flow for coefficients

From class, we have been given a bridging function for the Knudsen numbers $0.001 < Kn < 10$:

$$
c_{continuum} + (c_{free} - c_{continuum})\sin(\phi)^2
$$
$$
\phi = \pi(3/8 + 1/8 * \log_{10}(Kn))
$$

Applying this with the given endpoints, we get the following figures:

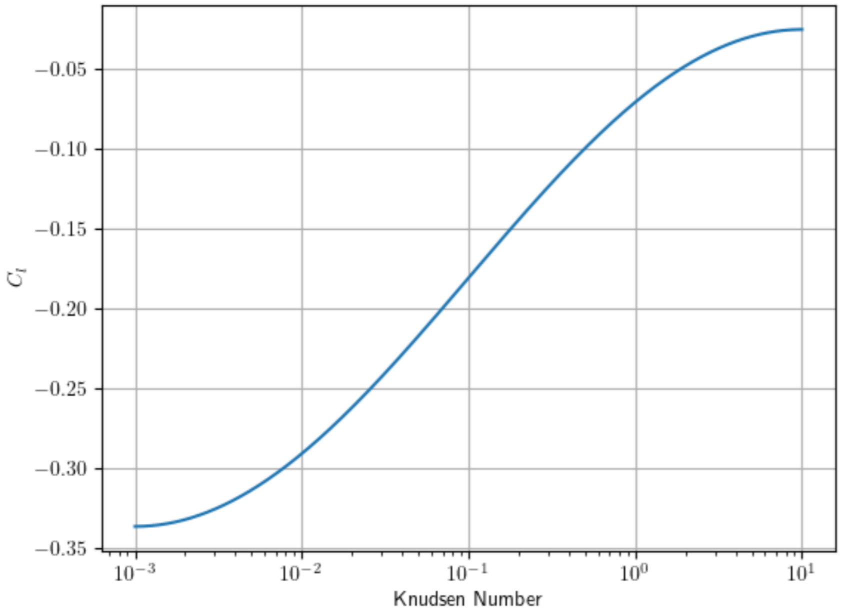

Figure 6: C_l as a function of Knudsen number Kn

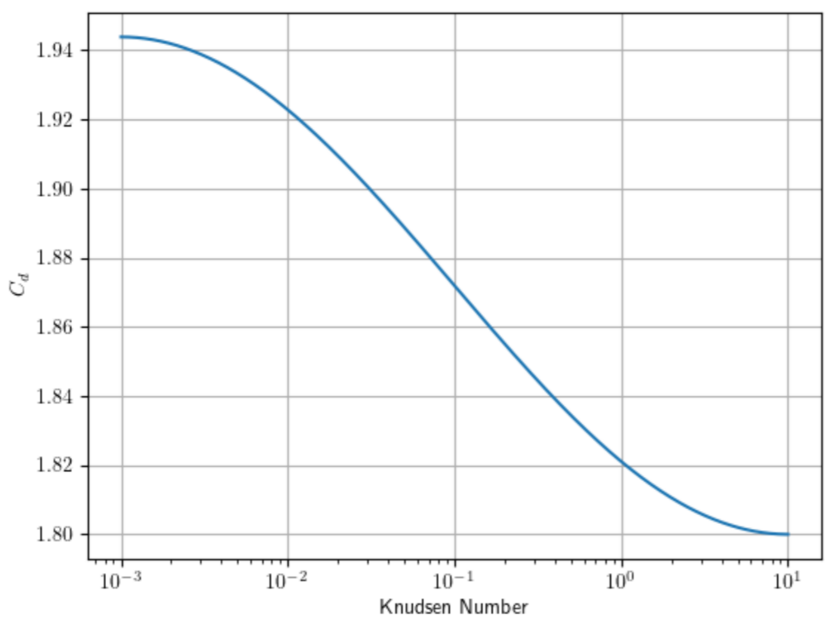

Figure 7: C_d as a function of Knudsen number Kn