Appendix B: Network structure comparisons and remarks for *Social influence network simulation design affects behavior of system-level entropy*

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1. Appendix B: Network structure comparisons and remarks

We use three network structure models in our experimental design: Erdős-Rényi random, small-world β, and preferential attachment. We also use an assortment of parameters for these models, detailed below. In total, this provides ten levels in the experimental design for the network structure factor.

Each level is represented as one of the following:

- *erdos_renyi_random(N)* produces an Erdős-Rényi random graph with density $\frac{\ln N}{N}$. This density equation is chosen as it is percolation threshold above which the probability of the network being connected approaches one.
- *small_world(N, p, k)* uses the β-model of Watts and Strogatz [1], where p is the probability of rewiring each edge in a ring lattice network that initially has each node connected to its k nearest neighbors.
- *scale_free(N, m)* produces a graph using the Barabasi-Albert preferential attachment process [2], where nodes are added to the network one at a time using m edges.

The ten network structure levels in the experimental design are:

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a. *erdos_renyi_random(N)*,

b. *small_world(N, 0.0, 3)*,

c. *small_world(N, 0.0, 10)*,

d. *small_world(N, 0.33, 3)*,

e. *small_world(N, 0.33, 10)*,

f. *small_world(N, 0.66, 3)*,

g. *small_world(N, 0.66, 10)*,

h. *scale_free(N, 1)*,

i. *scale_free(N, 3)*, and

j. *scale_free(N, 5)*.

N is the population size of the network. In our design, $N \in \{100, 1000, 10{,}000\}$.

Figure 1 shows an instance of each of the the network structures for $N = 100$. Based on the shapes of the networks, three structural patterns are apparent: tree-like, ring lattices, and denser networks.

Each trial replication uses a different random number seed to create the network structure, so the structures differ slightly across the replications of a single trial. However, the seeds are controlled such that for every trial, replications use the same sequence of seeds. Therefore, the first replication for all trials with a given setting for N and network structure model have the same network structure, and so on for each replication. Figure 1 diagrams the network structures for $N = 100$ using the random number seed for the first replication; diagrams for structures created using the seed for the 100th replication (not pictured) show the same three patterns while differing in exact details.

These three groups are preserved when increasing the population size. Figure 2 shows the results for $N = 1000$. Unfortunately, the networks for $N = 10{,}000$ exceeded the rendering software's capabilities, but we predict the same three patterns to be present.

Degree distributions (Figure 3) did not exhibit the expected similarity between distribution of the same visual pattern, nor did average degree connectivity or shortest paths. However, network density proved to be the most

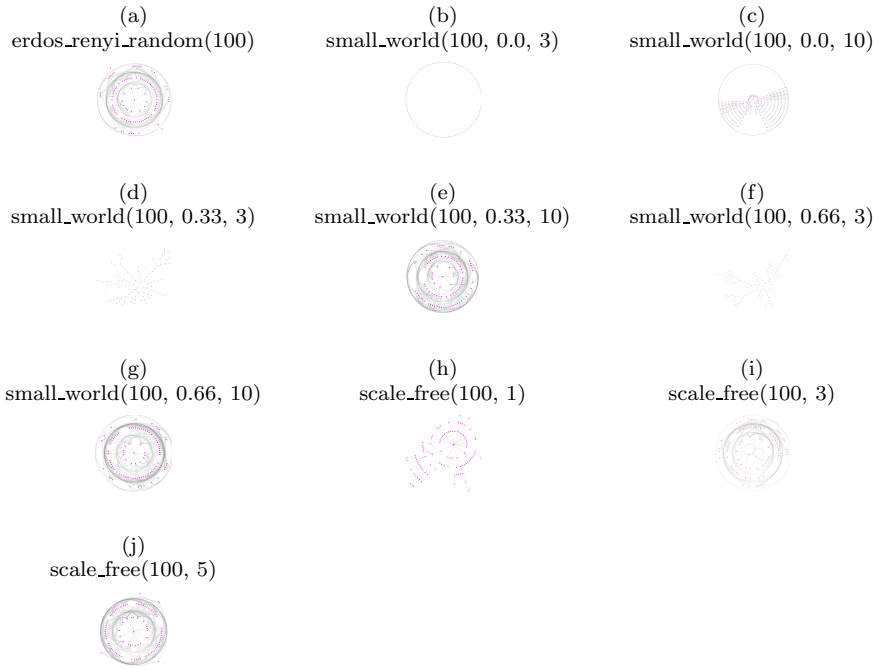

Figure 1: Network structures for the first replication of $N = 100$ trials. Based on the visual layout, the structures exhibit three patterns. Structures d, f, and h are tree-like; structure b and c are ring lattices; and the remaining structures (a, e, g, i, and j) appear as denser networks. The graphics are created using the yFiles radial layout in Cytoscape.

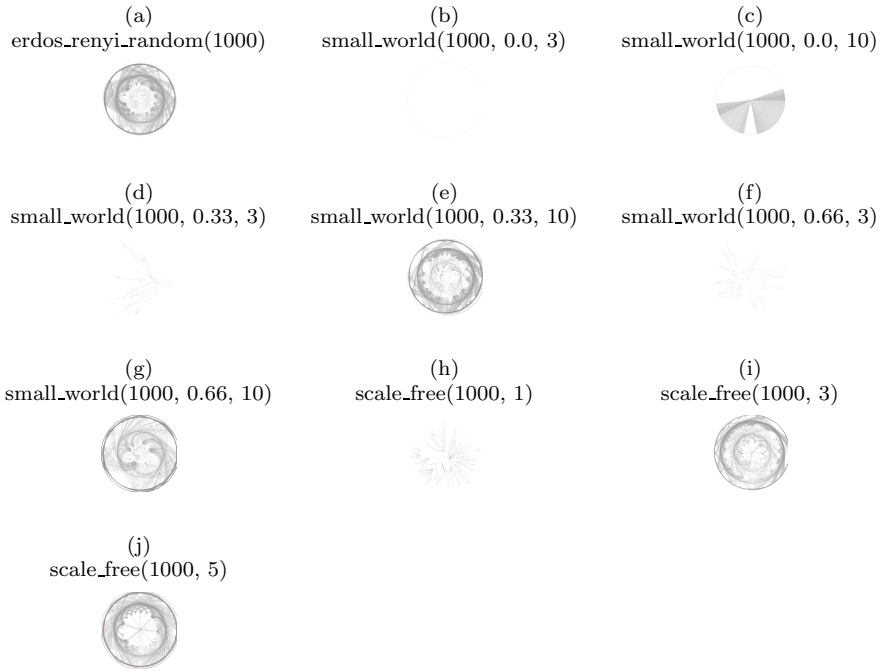

Figure 2: Network structures for the first replication of $N = 1000$ trials. Despite the larger network population, these visual layouts fall into the same three categories as with $N = 100$ (Figure 1) and are a qualitative match with their counterparts that have the smaller population.

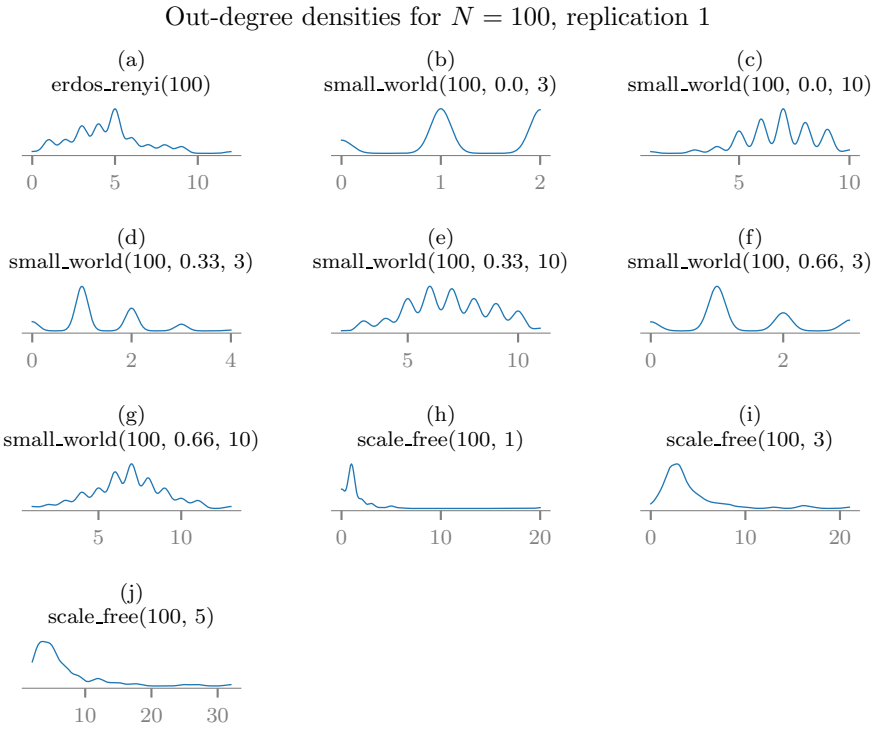

Figure 3: Out-degree density distributions of the network structures in Figure 1 do not fully align with the three visual patterns (tree-like, ring lattice, and denser networks), where we would expect more similarity between structures h and f, as well as between j and g.

compelling metric for understanding (some of) the similarities and differences in simulation response variable distributions across each network structure (Figure 4).

References

[1] D. J. Watts, S. H. Strogatz, Collective dynamics of 'small-world' networks, Nature 393 (1998) 440–442.

[2] A.-L. Barabasi, R. Albert, Emergence of scaling in random networks, Science (New York, N.Y.) 286 (1999) 509–12.

Figure 4: The groupings of network densities more closely align with the response variable behavior, so density may have an effect on the entropies measured in this research.