- I. For each of the following lists of premises, derive the conclusion and supply the justification for it. There is only one possible answer for each problem.
- **★**(1) 1. G⊃F
 - 2. ~F 3. √G
 - (2) 1. S
 - 2. S⊃M 3. M
 - (3) 1. $R \supset D$ 2. E⊃R 3. E 3 D
- \bigstar (4) 1. $B \lor C$
 - **2.** ∼B
 - (5) 1. N
 - 2. $N \vee F$
 - 3. $N \supset K$ 1,3 MP
- (6) 1: $\simeq J \vee P$
- , in $2:\sim J$, in the second constitution of $4:F_{ar{ar{b}}}\sim L$ 3. $S \supset J$
 - 2,3 MT 4. ^ S
- \star (7) 1. $H \supset D$
 - 2. $F \supset T$
- 3. F⊃H 4. *F* ⊃ <u>b</u> 1,345
 - (8) 1. $S \supset W$
 - 2. ~S
 - 3. $S \vee N$
 - 4. N 2,3D5
 - (9) 1. $F \supset \sim A$

 - ZHMT 5. n-H

- \star (10) 1. $H \supset A$
 - 2. A
 - 3. $A \lor M$
 - 4.. G⊃H 5. G>A
 - (11) 1. W \(B
 - 2. W
- 3. B⊃T
 - $\mathbf{4.~W} \supset \mathbf{A}$
 - 5 A
 - - 2. ~R
 - 3. R ∨ S
 - 4. R ⊃ T
 - 2,3 ∂≤ 5. **S**
 - ★(13) 1. ~Cう~F
 - 2. $L \supset F$

 - 5.74 ~ C
 - (14) 1. N⊃~E
 - 2. ~~S
 - 3. $\sim E \lor \sim S$
 - 4. ~S∨N
 - (17.15.24 . 24 DS
 - (15) 1. ∼R⊃∼T 2. $\sim T \vee B$

 - 3. C⊃~R
- 2. N∋A *(16) 1. ≒K
 - - 2. ~K⊃~P
 - 3. $\sim K \vee G$
 - 4. G⊃ O
 - 2,MP

(18) 1. $(R \supset M) \supset D$

2. $M\supset C$

3. $D \supset (M \vee E)$

2(MVE) 1,3 HS

2. ~S 3. $\sim L$

4. S⊃(K⊃L) 5. ~(5vC) 1,3 MT

(20) 1. $(A \lor W) \supset (N \supset Q)$

2. $Q \supset G$

3. $\sim A$

 \bigstar (19) 1. $(S \lor C) \supset L$

4. $(Q \supset G) \supset (A \lor N)$

5. AVN 2,4 MP

II. The following symbolized arguments are missing a premise. Write the premise needed to derive the conclusion (last line), and supply the justification for the conclusion. Try to construct the simplest premise needed to derive the conclusion.

 \bigstar (1) 1. $B \vee K$ 2.~B

12 DS 3. K

(2) 1. $N \supset S$

2. N 3. S

(3) 1. $K \supset T$

2. _ ユT

3. ∼*K*

 \bigstar (4) 1. $C \supset H$

2. R>C

3. R ⊃ H _1,2 HS

(5) 1. $F \supset N$

2. $N \supset T$

3. ~P

4. ~F 13 MT

(6) 1. $W \vee T$

2. $A\supset W$

3. WOT

4. A⊃T <u>~3</u>H5

 \bigstar (7) 1. $M \supset B$

2. Q⊃M 3. <u>Q</u> 2,3 MP

(8) 1. $C \vee L$

2. $L\supset T$

3. <u>~C</u>

1,3 DS

(9) 1. $E \supset N$

2. $T \lor \sim E$

3. $S \supset E$

5. E

 \bigstar (10) 1. $H \supset A$

2. $S \supset H$

3. ~M∨H 4. ~M

しんれて

(11) 1. $T \supset N$

2. $G \supset T$

3. $H \lor T$

4. F>G

5. F⊃T 2,4 KS

(12) 1. $G \supset C$

2. $M \vee G$

3. *T*∨~*G* 4. ~*M*

2,405 5. G

 \bigstar (13) 1. $\sim S \supset \sim B$

2. $R \lor \sim B$

3. ~B⊃ ~S 4. ~~5 ₃

5. ~~B ⋅

(14) 1. $\sim R \supset D$

2. $\sim J \supset \sim R$

3. $N \lor \sim R$

4.~F>~J

5, ~F⊃~R <u>1,4</u>#\$

(15) 1. $\sim S \vee \sim$	P
2. ~K⊃	P
3. ~ <i>P</i> ⊃ <i>I</i>	
4.~~ S	
$\frac{1}{\sim P}$	1,4D5

*(16) 1.
$$J \supset E$$

2. $B \lor \sim J$
3. $\sim Z \supset J$
4. $\sim Z$
5. J 3, \mathcal{L} M P

(18) 1.
$$(S \supset M) \supset G$$

2. $S \supset (M \cdot G)$
3. $G \supset (R \supset \sim S)$
4. $\sim (M \cdot G)$
5. $\sim S$ $\sim S$

*(19) 1.
$$(W \lor \sim F) \supset H$$

2. $(H \lor G) \supset \sim F$
3. $T \supset (F \supset G)$
4. $\cancel{H} \lor \cancel{G}$
5. $\sim F$ $\cancel{2f(M)}$

(20) 1.
$$(H \cdot A) \vee T$$

2. $\sim S \supset (P \supset T)$
3. $(N \vee T) \supseteq P$
4. $\sim (H \cdot A)$
5. T

III. Use the first four rules of inference to derive the conclusions of the following symbolized arguments.

*(1) 1.
$$\sim C \supset (A \supset C)$$

2. $\sim C$ / $\sim A$

(3) 1.
$$(K \bullet B) \lor (L \supset E)$$

2. $\sim (K \bullet B)$
3. $\sim E$ / $\sim L$

(5) 1.
$$\sim W \supset [\sim W \supset (X \supset W)]$$

2. $\sim W$ / $\sim X$

(6) 1.
$$J \supset (K \supset L)$$

2. $L \lor J$
3. $\sim L$ / $\sim K$

*(7) 1.
$$\sim$$
S \supset D
2. \sim S \vee (\sim D \supset K)
3. \sim D / K

(8) 1.
$$A \supset (E \supset \sim F)$$

2. $H \lor (\sim F \supset M)$
3. A
4. $\sim H$ / $E \supset M$

(9) 1.
$$\sim G \supset (G \lor \sim A)$$

2. $\sim A \supset (C \supset A)$
3. $\sim G$ / $\sim C$

★(10) 1.
$$N \supset (J \supset P)$$

2. $(J \supset P) \supset (N \supset J)$
3. $N \neq P$

(11) 1.
$$G \supset [\sim O \supset (G \supset D)]$$

2. $O \lor G$
3. $\sim O$ / D

(12) 1.
$$\sim M \lor (B \lor \sim T)$$

2. $B \supset W$
3. $\sim \sim M$
4. $\sim W$ / $\sim T$

★(13) 1.
$$R \supseteq (G \lor \sim A)$$

2. $(G \lor \sim A) \supseteq \sim S$
3. $G \supseteq S$
4. $R \nearrow \sim A$

(14) 1.
$$(L \equiv N) \supset C$$

2. $(L \equiv N) \lor (P \supset \sim E)$
3. $\sim E \supset C$
4. $\sim C$ / $\sim P$

(15) 1.
$$\sim J \supset [\sim A \supset (D \supset A)]$$

2. $J \lor \sim A$
3. $\sim J / \sim D$

Strategy 7: If the conclusion is a conjunctive statement, consider obtaining it via conjunction by first obtaining the individual conjuncts:

- $I. A \supset C$
- 2. B
- 3. ~C
- 1 B . ~C
- 4. B ~C
- 2, 3, Conj

Strategy 8: If the conclusion is a disjunctive statement, consider obtaining it via constructive dilemma or addition:

- 1. $(A \supset B) \cdot (C \supset D)$
- 2. B⊃ C
- 3. $A \lor C$
- / B V D
- 4. B v D
- 1, 3, CD
- 1. $A \lor C$
- 2. B
- 3. C⊃D
- $I B \vee D$
- 4. B v D
- 2, Add

Strategy 9: If the conclusion contains a letter not found in the premises, addition must be used to introduce that letter.

Strategy 10: Conjunction can be used to set up constructive dilemma:

- I. $A \supset B$
- 2. C⊃D
- 3. $A \lor C$
- $I B \lor D$
- 4. $(A \supset B) \cdot (C \supset D)$
- I, 2, Conj
- 5. B ∨ D
- 3, 4, CD

1,3,cd

EXERCISE

I. For each of the following lists of premises, derive the indicated conclusion and complete the justification. In problems 4 and 8 you can add any statement you choose.

- \star (1) 1. $S \vee H$
 - 2. B E
 - 3. $R \supset G$
 - **2**, Simp 4. B
 - (2) 1. $(N \supset T) \cdot (F \supset Q)$
 - - 2. $(N \supset R) \lor (F \supset M)$
 - 3. $N \vee F$

- (3) 1. D
 - 3. D.W 1,2 Coni
- $\bigstar(4)$ 1. H
 - 2, **// //** , Add
 - (5) 1. $R \cdot (N \vee K)$
 - 2. $(G \bullet T) \vee S$
 - 3. $(Q \bullet C) \supset (J \bullet L)$

3. $(\sim R \supset A) \cdot (P \supset \sim N)$ 4. $(\sim R \supset A) \cdot (P \supset \sim N)$

 \bigstar (7) 1. $(Q \lor K) \bullet \sim B$

2. $(M \cdot R) \supset D$

3. $(W \cdot S) \vee (G \cdot F)$ _____, Simp 4. Qv K

2(E-6) VX 1, Add

(9) 1. $\sim B$ 2. $F \vee N$, Conj

 \bigstar (10) 1. $S \lor \sim C$

2. $(S \supset \sim L) \cdot (\sim C \supset M)$

3. $(\sim N \supset S) \cdot (F \supset \sim C)$ 4. $\sim L \vee M$

II. In the following symbolized arguments, derive the line needed to obtain the conclusion (last line), and supply the justification for both lines.

 \star (1) 1. $G \supset N$

2. G • K 3. <u>G</u> 4. $G \vee T$

(2) 1. $\sim A$

2. $A \vee E$ 1,205

4. $\sim A \cdot E$

(3) 1. $B \supset N$

2. $B \vee K$

4/B>H) (K2R) 1,3 Cons

5. NVR 2,4CD

 \bigstar (4) 1. T

2. $T \supset G$

3. $(T \lor U) \supset H$

4. TV U 3,4 MA

5. H

(5) 1. $S \supset E$

2. $E \vee (S \bullet P)$

2. $N \supset F$

3. $(N \supset A) \cdot (F \supset C)$ 4. $N \cdot F$

5. $A \lor C$

 \star (7) 1. J

2. $\sim L$

3. $F \supset L$

2,3 MT 4.~F

5. ~F.J 1,4 Conj

(8) 1. $(E \supset B) \cdot (Q \supset N)$

2. $K \supset E$

3. $B \supset K$

4. EPB

5. $E \supset K$

(9) 1. $G \vee N$

2. $\sim G$

3. $\sim G \supset (H \cdot R)$

 \star (10) 1. M

2. $(M \bullet E) \supset D$

Strategy 14: Distribution can be used in two ways to set up disjunctive syllogism:

- 1. $(A \lor B) \cdot (A \lor C)$
- 2. *∼A*
- 3. $A \vee (B \cdot C)$
- I, Dist 2, 3, DS
- 4. B C
- I. $A \cdot (B \vee C)$
- 2. $\sim (A \cdot B)$
- 3. $(A \cdot B) \vee (A \cdot C)$ I, Dist
- 4. A C
- 2, 3, DS

Strategy 15: Distribution can be used in two ways to set up simplification:

- 1. $A \vee (B \cdot C)$
- 2. $(A \lor B) \cdot (A \lor C)$
- I, Dist
- 3. $A \lor B$
- 2, Simp
- 1. $(A \cdot B) \vee (A \cdot C)$
- 2. $A \cdot (B \vee C)$
- I, Dist
- 3. A
- 2, Simp

Strategy 16: If inspection of the premises does not reveal how the conclusion should be derived, consider using the rules of replacement to deconstruct the conclusion. (See the final example in this section.)

RCISE 7.3

- 1. For each of the following lists of premises, derive the indicated conclusion and complete the justification. For double negation, avoid the occurrence of triple fildes. Exercise 6 has two possible answers.
- **★(1)** 1. ~(E⊃H)
 - 2. $\sim (N \vee G)$
 - 3. ~A∨D
 - 4.~/...6 DM. کے
 - (2) 1. $G \supset (N \supset K)$
 - 2. $R \lor (D \supset F)$
 - S (T ∨ U)
 - **3**_,Dist
 - (3) 1. $M \vee (G \vee T)$
 - 2. $P \cdot (S \supset N)$
 - 3. $D \cdot (R \vee K)$
 - 4/<u>M</u>VG|VI
 - Assoc
- $\bigstar(4)$ 1. $B\supset W$
 - 2. $G \equiv F$
 - 3. $S \cdot A$

 - 3 ,Com 4. <u>ኢ-</u> ፓ

```
(5) 1. \sim \sim R \vee T
         2. \sim N \vee \sim B
         3. \sim A \supset \sim H
         4. PVT (,DN
    (6) 1. (F \vee N) \supset (K \bullet D)
         2. (H \bullet Z) \lor (H \bullet W)
         3. (P \supset H) \lor (P \supset N)
                                       2 Dist
         4. H. (2 v W)
 \bigstar(7) 1. \sim(G • \sim0)
         2. \sim (K \equiv \sim B)
         3. \sim T \supset \sim F
         4.~Gv~~Q ( ,DM
    (8) 1. G \supset (\sim L \supset T)
         2. L \equiv (\sim R \supset \sim C)
         3. I \supset (S \lor \sim N)
         4.7> (N.S) 3, Com
    (9) 1. S \supset (M \supset D)
         2. (K \bullet G) \vee B
         3. (E \cdot H) \cdot Q
         4. E- (H-Q) 3, Assoc
\bigstar(10) 1. \sim R \vee \sim P
         2. \sim F \supset \sim W
         3. G \cdot \sim A
         4.~ (R.P)
                              , DM
  (11) 1. \sim B \vee E
         2. \sim E \cdot \sim A
         3. \sim C \supset \sim R
         4.1BVANE 1, DN
  (12) 1. \sim G \cdot (S \supset A)
         2. \sim S \supset (B \bullet K)
         3. \sim Q \vee (T \cdot R)
         MAQUT)-LAQUE
\bigstar(13) 1. F \supset (\sim S \lor M)
         2. H \supset (\sim L \cdot \sim D)
                                                           + + - (L v D)
         3. N \supset (\sim G \supset \sim C)
                                 <u>z</u>,DM tf
  (14) 1. F \supset (P \supset \sim E)
         2. C \vee (S \bullet \sim B)
                                                     (M.R). at
         3. M \cdot (R \cdot \sim T)
```

- II. In the following symbolized arguments, derive the line needed to obtain the conclusion (last line), and supply the justification for both lines.
- \bigstar (1) 1. $K \lor C$ 2. $\sim C$ 3. CVK 1 Com 2,3 A5 4. K
 - (2) 1. $G \supset (R \vee N)$ 2. ~R·~N 3.~(R·N) 2 DM 43 MT
 - (3) 1. $H \cdot T$ 2. Teff 1 Com 3. T 25,mp
- ★(4) 1. (L•S)•F 2. <u>L•(5•F</u>)
 - (5) 1. $\sim B \vee K$ 2. ~Brunk I. DN 3. ~(B • ~K) 2 DM
 - (6) 1. $C \supset \sim A$ 2. A 3, ~~ A 2 DN 4. ~C 43 MT
- \bigstar (7) 1. $(D \bullet M) \lor (D \bullet N)$ 1 Distr 2. D. (MVN)
 - (8) 1. $(U \lor T) \supset R$ 2. $T \vee U$

- (9) 1. $\sim L \vee M$ 2. L
- \bigstar (10) 1. $D \lor (N \bullet H)$ 3. $D \vee N$
 - (11) 1. $(K \vee E) \bullet (K \vee G)$ 2. ~K 3. Kv(E·G) 1 Dista 4. E·G >13 Dista
 - (12) 1. $(N \supset T) \cdot (F \supset Q)$ **2.** $F \vee N$ 3. NYF 4. $T \vee Q$
- \bigstar (13) 1. $(M \lor G) \lor T$ $2. \sim M$ 3. MV(GVT) 1 KOSOC 4. $G \vee T$
 - (14) 1. $(\sim A \supset T) \cdot (\sim S \supset K)$ 2. $\sim (A \cdot S)$ 3. ~Av~ 5 MG S 1,3 CD 4. $T \vee K$
 - (15) 1. $\sim R$ 2. ARVAT 3. $\sim (R \cdot T)$ 2 DM
- III. Use the first thirteen rules of inference to derive the conclusions of the following symbolized arguments:
 - \bigstar (1) 1. $(\sim M \supset P) \cdot (\sim N \supset Q)$ 2. $\sim (M \cdot N)$ $/P \lor O$
 - 2. T
 - (2) 1. $\sim S$
- $/\sim (F \cdot S)$

/ J

(5) 1. $H \supset \sim A$ 2. A

 $\bigstar(4)$ 1. $\sim (N \cdot T)$

- (3) 1. $J \vee (K \bullet L)$ 2. $\sim K$

 $/\sim (H\vee\sim A)$

 $/\sim N$

Strategy 22: Transposition can be used to set up constructive dilemma:

1.
$$(A \supset B) \cdot (C \supset D)$$

3.
$$(\sim B \supset \sim A) \cdot (C \supset D)$$

Strategy 23: Constructive dilemma can be used to set up tautology:

1.
$$(A \supset C) \cdot (B \supset C)$$

Strategy 24: Material implication can be used to set up tautology:

I.
$$A \supset \sim A$$

Strategy 25: Material implication can be used to set up distribution:

I.
$$A \supset (B \cdot C)$$

2.
$$\sim A \vee (B \cdot C)$$

3.
$$(\sim A \vee B) \cdot (\sim A \vee C)$$

EXERCISE 7.4

I. For each of the following lists of premises, derive the indicated conclusion and complete the justification. For tautology, derive a conclusion that is simpler than the premise.

$$\star$$
(1) 1. $H \vee F$

2.
$$N \lor \sim S$$

3.
$$\sim G \vee Q$$

(2) 1.
$$R \supset (S \supset N)$$

2.
$$T \supset (U \lor M)$$

3.
$$K \cdot (L \supset W)$$

(3) 1.
$$G = R$$

2.
$$H \supset P$$

3.
$$\sim F \vee T$$

$$\bigstar$$
(4) 1. $(B \supset N) \bullet (N \supset B)$

2.
$$(R \vee F) \cdot (F \vee R)$$

3.
$$(K \supset C) \lor (C \supset K)$$

__, Equiv

```
(5) 1. E \lor \sim E
         2. A \lor A
         3. G \cdot \sim G
         4. A 2 Taut
    (6) 1. S \vee \sim M
         2. \sim N \cdot \sim T
         3. \sim L \supset Q
                             3, Trans
         4. ~Q =~~/
 \bigstar(7) 1. \sim C \supset \sim F
         2. D \lor \sim P
         3. \sim R \cdot Q
                             2__, Impl
         4. D>~P
    (8) 1. E \supset (R \cdot Q)
         2. (G \cdot N) \supset Z
         3. (S \supset M) \supset P
         4. (+> (N>Z) 2, Exp
   (9) 1. (D \cdot H) \vee (\sim D \cdot \sim H)
         2. (F \supset J) \cdot (\sim F \supset \sim J)
         3. (N \lor T) \bullet (\sim N \lor \sim T)
                                   3, Equiv
         4. N=T
\bigstar(10) 1. L\supset (A\supset A)
         2. K \supset (R \lor \sim R)
         3. S \supset (G \cdot G)
                               3, Taut
         4.5>6
  (11) 1. K \cdot (S \vee B)
         2. \sim F \supset \sim J
         3. \sim E \vee \sim M
         (12) 1. H \supset (K \bullet J)
         2. (N \vee E) \supset B
         3. C \supset (H \supset A)
         4(C.H)=+
                                 <u>ڪ</u>, Exp
\star(13) 1. (A \supset \sim C) \cdot (C \supset \sim A)
         2. (W \supset \sim T) \cdot (\sim T \supset W)
         2. (W \supset \sim I) (\sim E) • (\sim M \supset E) 2. , Equiv
  (14) 1. (\sim K \lor M) \equiv S
         2. T \vee (F \bullet G)
         3. R \equiv (N \cdot \sim H)
         4. (K>M)=5 1, Impl
  (15) 1. (S \vee S) \supset D
         2. K \supset (T \cdot \sim T)
         3. (Q \supset Q) \supset M
                                  l _, Taut
```

4. <u>5 °</u> D

II. In the following symbolized arguments, derive the line needed to obtain the conclusion (last line), and supply the justification for both lines.

- \bigstar (1) 1. $\sim J \vee M$ 2. $M \supset B$ 3. J>M 2,3 HS **4.** $J \supset B$
 - (2) 1. $(J \bullet F) \supset N$ 3. T > (F>N) 1 Exp 4. F⊃N 2,3 MP
 - (3) 1. $C \supset A$ **2.** $A \supset C$ 3/ (x>+) . (x>C) 1/2 (-n) 3 Eguil
- \bigstar (4) 1. $(G \supset K) \bullet (T \supset K)$ **2.** $G \vee T$
 - (5) 1. $(G \supset B) \cdot (\sim C \supset \sim H)$ 2. $G \lor H$ 3. $G \ni B$). $(H \ni C)$ 1 Tans \star (13) 1. $(N \ni A) \cdot (-N \ni -A)$
 - (6) 1. $J \supset (M \supset Q)$
- **★**(7) 1. H⊃(~C∨R) 2. H3(c3R) 2 Trans 3. $(H \cdot C) \supset R$

- (8) 1. $\sim G \supset \sim T$ 2. $G \supset N$ 3. T>6 1, Trans 4. TON 23 HS
- (9) 1. $K\supset (A\supset F)$
- \bigstar (10) 1. $H \supset \sim H$ 2.2Hraft 1 Impl 3. ∼*H*
 - (11) 1. $\sim S$ 2. 25 × K 1 Achol. 3. 8 ⊃ K 2 I mp1
 - (12) 1. $M \supset (M \supset D)$ 2 $(M \cdot M) \supset D$ 3. $M \supset D$
 - 3. $N \equiv A$
 - 3. $E \equiv R$
- (15) 1. $Q \supset (\sim W \supset \sim G)$ 2. (2 > (6 > W) 3. $(Q \cdot G) \supset W$

III. Use the eighteen rules of inference to derive the conclusions of the following symbolized arguments.

- \bigstar (1) 1. $(S \bullet K) \supset R$ $/S \supset R$ 2. K
 - (2) 1. $T \supset (F \vee F)$ 2. $\sim (F \bullet F)$ $/\sim T$
 - (3) 1. $G \supset E$ 2. $H \supset \sim E$ $/G\supset \sim H$
- $\bigstar(4)$ 1. $S \equiv Q$ 2. ~S $/\sim Q$
 - (5) 1. $\sim N \vee P$ 2. $(N \supset P) \supset T$