Assignment 10. Regression

Your name

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Solve the following two problems – each problem on a separate page – and submit to Gradescope for grading.

Clearly present every step of your solution.

1 Least squares [9 points]

1.1 Explain [3 points]

Explain the method of least squares in your own words (Short, no chatGPT please).

1.2 Compute [3 points]

Compute the equation of the line y=ax+b which best fits the following data points: $(0,6),(1,0),(2,0)$.
Show all intermediate steps.

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} 1 \end{pmatrix}^{-1} \begin{pmatrix} \sum_{i=1}^{n} x_i y_i \\ \sum_{i=1}^{n} y_i \end{pmatrix}$$
$$= \begin{pmatrix} - & - \\ - & - \end{pmatrix}^{-1} \begin{pmatrix} - \\ - \end{pmatrix} =$$

1.3 Read and learn [3 points]

What is the SSR error of your model?
Learn about the *coefficient of determination* here:
https://en.wikipedia.org/wiki/Coefficient_of_determination

Compute the value of the coefficient of determination r^2 for your model.
Based on the resulting value – how well do you think the variance in x describes the variance in y?

2 Gradient descent [6 points]

2.1 Watch and learn [3 points]

Watch the following step-by-step explanation of the gradient descent algorithm: https://www.youtube.com/watch?v=sDv4f4s2SB8.

Explain how you understand the idea of gradient descent in your own words (one short paragraph, and no chatGPT please).

2.2 First step for intercept [3 points]

Let's use the same data points as before: $(0, 6), (1, 0), (2, 0)$

Let our original random guess for a slope be $a = -1$. Keeping a constant, we look for an optimal value of b starting with $b = 0$. Using learning rate $\eta = 0.1$ what would be the value of b after one step of the gradient descent algorithm?

Show all the steps of your computation, including partial derivatives