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AN ASYMPTOTIC RELATION BETWEEN THE DISTRIBUTION OF PRIME NUMBERS AND THE COLLATZ FUNCTION

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To my wife, for all her support

Abstract. This work shows an asymptotic relationship between the distribution of prime integers and the cardinal growth of order isomorphic, disjoint, chains of integers, congruent to the Collatz function. The proof defines and utilizes a novel algebraic structure existing within the total order of sequences formed by the Collatz function.

Introduction

The Collatz conjecture states that from any positive integer, given two functions, a sequence always follow a trajectory ending at 1 such that:

$$f(n) = \begin{cases} n/2 & \text{if} \quad n \equiv [0]_2 \\ 3n+1 & \text{if} \quad n \equiv [1]_2 \end{cases}$$

Despite vast research and large computer based evidence, an accepted proof has yet confirmed that all integer trajectories eventually lead to 1. Commonly, research methods analyze an integer's trajectory directed towards 1. The following work rather analyzes groups of order isomorphic posets of integer chains directed away from a root minimum element. From this, a well ordered, dynamic algebraic structure is formed. This work uses common notations found in the context of Order Theory.

AN OVERVIEW OF THE PROOF'S LOGIC.

The Collatz function dictates that for all odd integers x, and even integers y, a directed mapping away from 1 such that:

$$1 \prec x \prec y/2^z \equiv \; [1]_3 \prec x$$

Therefore, to map the path of an odd integer, directed away from 1, requires it to be multiplied by some $2^z : z = 1 \; or \; 2$, until an even integer, $y \equiv [1]_3$ is reached. At which point, an even integer's path away from 1 is found simply by reversing the order of the Collatz function; subtracting 1 from such even integer then dividing it by 3. By continuing this process a chain of ordered elements is formed until reaching such poset's maximal element. Every poset has a defined minimal and maximal element. The following section provides the definition.

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1. Minimal and Maximal elements of a poset

1.1. **Maximal Element.** The reason for defining a poset's maximal element stems from an observation that, for all chains, a limit point exists such that nothing dynamic occurs past such point. The following is an example of an integers path away from 1, illustrating what is meant by the dynamic part of a chain.

Example 1.1.

$$88 \mapsto 29 \mapsto 58 \mapsto 19 \mapsto 76 \mapsto 25 \mapsto 100 \mapsto 33 \mapsto 66 \mapsto 132 \mapsto 264 \mapsto 528$$

The Collatz function dictates that all even integers y: $y \equiv [0]_3$, will never precede an odd integer on a path directed towards 1.

Definition 1.2. A poset's maximal element is given for all even integers y such that:

$$y \equiv [0]_6 \cdot 2^1$$

The dynamic part of an integers path away from 1 stops at this point because no further iteration of such an even number:

$$y \equiv [0]_6 \cdot 2^x$$

will ever be approachable by an odd integer. The final even integer reached, moving away from 1 by this process, denotes the maximal element of such poset.

1.2. **Minimal Element.** A poset's minimal element is the first element in the chain. As previously stated, an odd integer reaches an even integer, $y \equiv [1]_3$, when multiplied by either, 2^1 or 2^2.

Therefore, the mapping of an odd integer's path away from 1 never contacts an even integer containing a factor of 2 greater than 2. A poset's minimal element is defined:

Definition 1.3. For all posets, the minimal element is equal to an even integer y, raised to a power x such that: $y \not\equiv [0]_3$, and x≥ 3

2. Partitioning and indexing

A partitioning of the odd integers is performed in accordance to the following observation. The Collatz function dictates that an odd integer x: $x \equiv [1]_4$, falls to an integer less than x after three operations of the function.

Example 2.1. $5 \mapsto 16 \mapsto 8 \mapsto 4$

Example 2.2. $17 \mapsto 52 \mapsto 26 \mapsto 13$

The remaining set of odd integers:

$$x \equiv [3]_4$$

move about other integers greater than themselves for an unspecified, larger number of steps before reaching an integer less than themselves.

Example 2.3. $25 \mapsto 76 \mapsto 38 \mapsto 19 \mapsto 58 \mapsto 29 \mapsto 88 \mapsto 44 \mapsto 22$

Example 2.4. $27 \mapsto 82 \mapsto 41 \mapsto \ 124 \mapsto \ 62 \mapsto 31 \mapsto 94 \mapsto 47 \mapsto 142 \mapsto 71 \mapsto 214 \mapsto 112 \mapsto 56 \mapsto 23$

THE FIRST PARTITION

The two classes of odd integers, behaving distinctly within the Collatz function, are partitioned into two disjoint sets, κ and μ. We will name the set containing all positive odd integers θ:

$$\theta \cup x : x \equiv [1]_2, \forall x \in Z^+$$

We will partition θ into proper subsets κ and μ:

$$\theta = \kappa \cup \mu$$

$$\kappa_y \cup \theta : y \equiv [1]_4, \forall\, y \in Z^+$$

$$\mu_y \cup \theta : y \equiv [3]_4, \forall\, y \in Z^+$$

All odd integers are now respectively partitioned into two sets:

$$\kappa = 1, 5, 9, 13, 17...$$

$$\mu = 3, 7, 11, 15, 19...$$

3. Indexing sets κ and μ

All of the elements of κ and μ will be indexed within their respective set:
Let,

$$\kappa \cup \alpha :$$

$$\alpha_x = \{(y+3)/4\} | \kappa_y \equiv [1]_4 \cup Z^+ : \kappa_y = \{4x - 3 | x = 1, 2, 3...\}$$

Let,

$$\mu \cup \beta :$$

$$\beta_x = \{(y+1)/4\} | \mu_y \equiv [3]_4 \cup Z^+ : \beta_y = \{4x - 1 | x = 1, 2, 3...\}$$

Example 3.1.

$$\alpha_0 = \kappa_1, \quad \alpha_5 = \kappa_{21}, \quad \alpha_{370} = \kappa_{1481}$$

Example 3.2.

$$\beta_0 = \mu_3, \quad \beta_{19} = \mu_{75}, \quad \beta_{68} = \mu_{271}$$

4. Indexing Even Integers

Even integers will be indexed in accordance to two relations, all of which will be denoted as set C:

$$C \cup x \equiv [0]_2 : x \in Z^+$$

A new notation will be defined for the purposes of this work. First, the odd component of an even integer, being either a prime or composite odd number, is noted in the subscript of C. Second, the power to which 2 is raised in such even number, will be noted in the superscript of C.

Definition 4.1.

$$C_x^y = x, y : \tag{4.1}$$

for all even integers, x = The odd component, y = The power to which 2 is raised.

Example 4.2.

$$C_3^3 = 24, \quad 24 = 3 \cdot 2^3 \tag{4.2}$$

Example 4.3.

$$C_{35}^5 = 1120, \quad 1120 = 35 \cdot 2^5 \tag{4.3}$$

Functions, later to be defined, will only utilize "x" as an input, though the y component is necessary to define whether an even number exists as the generator of a sets sequence, as was previously discussed.

The next section will discuss the minimum element, being the only integer that maps directly to all other elements, along with a definition of a set's first and last member.

5. An infinite group of order isomorphic posets

The Collatz conjecture asks if every positive integer, given the rules, will eventually map to 1. If an integer ≥ 16 maps to 1 then that integer goes through 16. For this reason, 16 is denoted as the minimum element. The integers below 16 all map to 1 and will not be considered in this work.

6. Establishing the root

Therefore, the minimum element to and from which all elements map, for the purposes of this work is defined:

Definition 6.1. The Minimum Element:

$$C_1^4 = 16 \tag{6.1}$$

7. Bases, Origins and Leafs

Previous definitions have established a sets first and last element though some additional comments and examples will help clarify such conditions. In all cases, only the even elements of a set are labeled either as a Root, Origin or Leaf. The odd elements of a set, labeled a_x or b_y , will later undergo permutations and relabelling. Listed below are a few of the sequences from the Collatz function, and the matching sequence of numbers under the transformations described in this work. The definitions of a Base, Origin and Leaf will be applied to these examples. In all cases the trajectories of the following examples are directed away from 1, as is the focus.

Example 7.1.

$$\mathbf{16}, Root \mapsto \mathbf{5}, \alpha_1 \mapsto \mathbf{10}, Origin \mapsto \mathbf{3}, \beta_1 \mapsto \mathbf{6}, Leaf \tag{7.1}$$

$$C_1^4, Root \mapsto \alpha_1 \mapsto C_5^1, Origin \mapsto \beta_1 \mapsto C_3^1, Leaf \tag{7.2}$$

Example 7.2.

$\mathbf{160}, Base \mapsto \mathbf{53}, \alpha_{13} \mapsto \mathbf{106}, Origin \mapsto \mathbf{35}, \beta_9 \mapsto \mathbf{70}, Origin \mapsto \mathbf{23}, \beta_6 \mapsto \mathbf{46}, Origin \mapsto \mathbf{15}, \beta_4 \mapsto \mathbf{30}, Leaf$

(7.3)
$C_5^5, Base \mapsto \alpha_{13} \mapsto C_{53}^1, Origin \mapsto \beta_9 \mapsto C_{35}^1, Origin \mapsto \beta_6 \mapsto C_{23}^1, Origin \mapsto \beta_4 \mapsto C_{15}^1, Leaf$

Example 7.3.

$\mathbf{2752}$,Base$\mapsto \mathbf{917}, \alpha_{229} \mapsto \mathbf{1834}, Origin \mapsto \mathbf{611}, \beta_{153} \mapsto \mathbf{1222}, Origin \mapsto$

$\mathbf{407}, \beta_{102} \mapsto\mapsto \mathbf{814}, Origin \mapsto \mathbf{271}, \beta_{68} \mapsto \mathbf{1084}, Origin \mapsto \mathbf{361}, a_{90} \mapsto \mathbf{1444}, Origin \mapsto$

$\mathbf{481}, \alpha_{120} \mapsto\mapsto \mathbf{1924}, Origin \mapsto \mathbf{641}, \alpha_{160} \mapsto \mathbf{1282}, Origin \mapsto \mathbf{427}, \beta_{107} \mapsto \mathbf{1708}, Origin \mapsto$

$\mathbf{569} \alpha_{142} \mapsto \mathbf{1138}, Origin \mapsto \mathbf{379}, \beta_{95} \mapsto \mathbf{1516}, Origin \mapsto \mathbf{505}, \alpha_{126} \mapsto \mathbf{2020}, Origin \mapsto$

$\mathbf{673}, \alpha_{168} \mapsto \mathbf{2692}, Origin \mapsto \mathbf{897}, \beta_{224} \mapsto \mathbf{1794}, Leaf$

Example [5.2] displays the root,**16** and C_1^4, of all trajectories. In all examples, the final member of the set is labeled its leaf. A leaf, being equivalent to $[0]_3$, is never approachable by an odd integer, and therefore will only generate sequences of even numbers. A sets Base is the first element of the set.

8. Bases

In agreement with the earlier [Definition 0.2.], denoting a sets first element, additional remarks are added that further define a Base.

Definition 8.1. A sets first element, C_x^y is labeled as its Base. A Base is always an even integer containing a factor: $y = 2^{\geq 3}$. The odd component "x", is fixed for such even integers containing such prime factors along with their particular exponents.

Example 8.2. $C_{35}^1 = 70, \qquad C_{35}^2 = 140, \qquad C_{35}^7 = 4480,$

$C_{429}^3 = 3432, \qquad C_{429}^6 = 27456, \qquad C_{429}^{13} = 3,514,368$

The Base of a set is always followed by an element from $\alpha_x \cup \kappa_y$, as can be seen in the examples above.

All of the elements between a set's Base, or Root in a single case, and its Leaf are labeled Origins.

8.1. **The Origin of a Base.** The elements within a set labeled " Origins," are even integers, C_x^y, as such being contained between a Base and a Leaf.

Definition 8.3. An Origin C_o:

$$C_o \cap [C_x^y],\ for\ y \leq 2 \equiv [0]_2 \in Z \tag{8.1}$$

Every Origin is the parent of an infinite group of Base children raised to increasing powers of 2. Therefore every Base has a parent of Origin and Bases whose component x is equal: $C_x^3 = C_x^{17}$, have the same Origin parent.

Now that a set's elements are clearly defined, we will partition the elements of sets a and b into separate equivalence classes.

9. Partitioning α and β into equivalence classes

Every element in $\alpha$and β maps to and from another element in α or β according to the equivalence class in which it belongs. Before the individual elements of, $\alpha_x$or β_x, are indexed within their individual equivalence classes, we must partition α and β according the covering sets of equivalence classes they contain:

$$\alpha = [0]_6 \cup [1]_6 \cup [3]_6 \cup [4]_6 \cup [2]_3 : \forall \alpha_x \in Z^+$$

$$\beta = [0]_6 \cup [3]_6 \cup [1]_3 \cup [2]_3 : \forall \beta_x \in Z^+$$

Each equivalence class in union with α and β is named individually:

$$\alpha = \{a_x : x \equiv [1]_6 \cup b_x : x \equiv [3]_6 \cup c_x : x \equiv [4]_6 \cup d_x : x \equiv [0]_6 \cup i_x : x \equiv [2]_3\}$$

$$\beta = \{e_x : x \equiv [2]_3 \cup f_x : x \equiv [3]_6 \cup g_x : x \equiv [0]_6 \cup h_x : x \equiv [1]_3\}$$

10. Modular Mapping

The finite set of modular relations, shown in the table below, are the only ones needed to map each trajectory of every positive integer. The way of obtaining the indexed values of sets, $a, b, c, d, e, f, g, h,$and i, as shown in the table below, is given in the following chapter.

A MAPPING OF THE FINITE GROUP OF MODULAR RELATIONSHIPS EXISTING BETWEEN ALL ELEMENTS, AS A REPLACEMENT TO COLLATZ' FUNCTIONS

	1	2	3	4	5	6	7	8	9	$x \equiv [y]_z$
$a_k \subset \alpha_x$:	a_1	a_7	a_{13}	a_{19}	a_{25}	a_{31}	a_{37}	a_{43}	a_{49}	$a_k \equiv [1]_6$
	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$
h_x, e_x, f_x :	h_1	e_5	f_9	h_{13}	e_{17}	f_{21}	h_{25}	e_{29}	f_{33}	$h_x, e_x, f_x \equiv [1]_4$
$b_k \subset \alpha_x$:	b_3	b_9	b_{15}	b_{21}	b_{27}	b_{33}	b_{39}	b_{45}	b_{51}	$b_k \equiv [3]_6$
	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$
c_x, d_x, i_x:	c_4	d_{12}	i_{20}	c_{28}	d_{36}	i_{44}	c_{52}	d_{60}	i_{68}	$c_x, d_x, i_x \equiv [4]_8$
a_k:	c_4	c_{10}	c_{16}	c_{22}	c_{28}	c_{34}	c_{40}	c_{46}	c_{52}	$c_x \equiv [4]_6$
	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$
f_x, h_x, e_x :	f_3	h_7	e_{11}	f_{15}	h_{19}	e_{23}	f_{27}	h_{31}	e_{35}	$f_x, h_x, e_x \equiv [3]_4$
a_k :	d_6	d_{12}	d_{18}	d_{24}	d_{30}	d_{36}	d_{42}	d_{48}	d_{54}	$d_x \equiv [0]_6$
	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$
i_x, c_x, d_x:	i_8	c_{16}	d_{24}	i_{32}	c_{40}	d_{48}	i_{54}	c_{62}	d_{70}	$i_x, c_x, d_x \equiv [0]_8$
b_k :	e_2	e_5	e_8	b_{11}	e_{14}	e_{17}	e_{20}	e_{23}	e_{26}	$e_x \equiv [2]_3$
	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$
i_x, d_x, c_x:	i_2	d_6	c_{10}	i_{14}	d_{18}	c_{22}	i_{26}	d_{30}	c_{34}	$i_x, d_x, c_x \equiv [2]_4$
b_k :	f_3	f_9	f_{15}	f_{21}	f_{27}	f_{33}	f_{39}	f_{45}	f_{51}	$f_x \equiv [3]_6$
	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$
e_x, g_x, h_x:	e_2	g_6	h_{10}	e_{14}	g_{18}	h_{22}	e_{26}	g_{30}	h_{34}	$e_x, g_x, h_x \equiv [2]_4$
b_k :	g_6	g_{12}	g_{18}	g_{24}	g_{30}	g_{36}	g_{42}	g_{48}	g_{54}	$g_x \equiv [0]_6$
	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$
h_x, e_x, g_x:	h_4	e_8	g_{12}	h_{16}	e_{20}	g_{24}	h_{28}	e_{32}	g_{36}	$h_x, e_x, g_x \equiv [0]_4$
b_k :	h_1	h_4	h_7	h_{10}	h_{13}	h_{16}	h_{19}	h_{22}	h_{25}	$h_x \equiv [1]_3$
	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$
C^y_x:	C^1_3	C^1_{15}	C^1_{27}	C^1_{39}	C^1_{51}	C^1_{63}	C^1_{75}	C^1_{87}	C^1_{99}	$C^y_x \equiv [3]_{12}$
a_k :	i_2	i_5	i_8	i_{11}	i_{14}	i_{17}	i_{20}	i_{23}	i_{26}	$i_x \equiv [2]_3$
	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$
C^y_x:	C^1_9	C^1_{21}	C^1_{33}	C^1_{45}	C^1_{57}	C^1_{69}	C^1_{81}	C^1_{93}	C^1_{105}	$C^y_x \equiv [9]_{12}$

11. Indexing the Elements of sets, $a, b, c, d,\ e, f, g, h,\ i$

The recursive functions laid out in the next section utilize the indexed values of the elements in sets, $a, b, c, d,\ e, f, g, h,\ i$. The following list of functions are used to index the elements of each set:

Let,

(11.1) $$a_x \cup A_j :$$

$$A_j = (x+5)/6 \mid a_x, \forall x \equiv [1]_6 \cup Z^+ : a_x = \{6j - 5 \mid j = 1, 2, 3...\}$$

(11.2) $$b_x \cup B_j :$$

$$B_j = (x+3)/6 \mid b_x, \forall x \equiv [3]_6 \cup Z^+ : b_x = \{6j - 3 \mid j = 1, 2, 3...\}$$

$$c_x \cup C_j : \tag{11.3}$$
$$C_j = (x+2)/6 \mid c_x, \forall x \equiv [4]_6 \cup Z^+ : c_x = \{6j-2 \mid j = 1,2,3...\}$$

$$d_x \cup D_j : \tag{11.4}$$
$$D_j = x/6 \mid d_x, \forall x \equiv [0]_6 \cup Z^+ : d_x = \{6j \mid j = 1,2,3...\}$$

$$e_x \cup E_j : \tag{11.5}$$
$$E_j = (x+1)/3 \mid e_x, \forall x \equiv [2]_3 \cup Z^+ : e_x = \{3j-1 \mid j = 1,2,3...\}$$

$$f_x \cup F_j : \tag{11.6}$$
$$F_j = (x+3)/6 \mid f_x, \forall x \equiv [3]_6 \cup Z^+ : f_x = \{6j-3 \mid j = 1,2,3...\}$$

$$g_x \cup G_j : \tag{11.7}$$
$$G_j = x/6 \mid g_x, \forall x \equiv [0]_6 \cup Z^+ : g_x = \{6j \mid j = 1,2,3...\}$$

$$h_x \cup H_j : \tag{11.8}$$
$$H_j = (x+2)/3 \mid h_x, \forall x \equiv [1]_3 \cup Z^+ : h_x = \{3j-2 \mid j = 1,2,3...\}$$

$$i_x \cup I_j : \tag{11.9}$$
$$I_j = (x+1)/3 \mid i_x, \forall x \equiv [2]_3 \cup Z^+ : i_x = \{(3j-1 \mid j = 1,2,3...\}$$

12. A Set of Recursive functions which produce order isomorphic chains

The following functions map every possible trajectory of an odd integer's path from a Base to a Leaf. Every sequence begins from an element of $a_x$or b_x, while ending on either$h_x$or i_x.

$$T(a_x) = \begin{cases} f(2(\mathrm{A}_j)/3) & \text{if} \quad \mathrm{A}_j \equiv [0]_3 \\ h(3(\mathrm{A}_j)\text{-}1) & \text{if} \quad \mathrm{A}_j \equiv [1]_3 \\ e((4(\mathrm{A}_j)\text{-}2)/3) & \text{if} \quad \mathrm{A}_j \equiv [2]_3 \end{cases}$$

$$T(b_x) = \begin{cases} i((8B_j/3)-1) & \text{if} \quad b^j \equiv [0]_3 \\ c((4B_j-1)/3) & \text{if} \quad b^j \equiv [1]_3 \\ d((4B_j-2)/3) & \text{if} \quad b^j \equiv [2]_3 \end{cases}$$

$$T(c_x) = \begin{cases} e(4C_j/3) & \text{if} \quad c^j \equiv [0]_3 \\ f((2C_j+1)/3) & \text{if} \quad c^j \equiv [1]_3 \\ h(((C_j+1)/3)+c^j) & \text{if} \quad c^j \equiv [2]_3 \end{cases}$$

$$T(d_x) = \begin{cases} d((4D_j+1)/3) & \text{if} \quad d^j \equiv [0]_3 \\ i((8D_j+1)/3) & \text{if} \quad d^j \equiv [1]_3 \\ c((4D_j/3)+1) & \text{if} \quad d^j \equiv [2]_3 \end{cases}$$

$$T(e_x) = \begin{cases} c(2E_j/3) & \text{if} \quad e^j \equiv [0]_3 \\ i(4(E_j-1)/3) & \text{if} \quad e^j \equiv [1]_3 \\ d((2E_j-1)/3) & \text{if} \quad e^j \equiv [2]_3 \end{cases}$$

$$T(f_x) = \begin{cases} h(4F_j/3) & \text{if} \quad f^j \equiv [0]_3 \\ e((4F_j-1)/3) & \text{if} \quad f^j \equiv [1]_3 \\ g((2F_j-1)/3) & \text{if} \quad f^j \equiv [2]_3 \end{cases}$$

$$T(g_x) = \begin{cases} g(4G_j/6) & \text{if} \quad g^j \equiv [0]_3 \\ h((4G_j+2)/3) & \text{if} \quad g^j \equiv [1]_3 \\ e((4G_j+1)/3) & \text{if} \quad g^j \equiv [2]_3 \end{cases}$$

The recursive use of these functions produce the table of sets in the following section. Every set, beginning from some a_j or b_j, is a path's last dynamic segment, in accordance to the Collatz function. The algebraic structure, created thus far, is sufficient for the purpose of this proof. Now, with a defined algebraic structure, an exploration of the astounding number of numerical relationships existing within its field begins.

In section 8. a series of tables demonstrate the resulting sets produced by the functions above. The python program attached to this work is coded by the use of these functions and is of good use when analyzing large numbers.

This algebraic structure can also be viewed as a coordinate graph, giving it geometric structure. The aspects of such will be addressed in a later section.

13. Table Talk

The tables in this section are a select sample of the infinite collection of such tables. To provide examples, I have chosen correlating tables that demonstrate the recurring properties of numbers and sets within this algebraic system.

Graph 1

A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
a_1	a_7	a_{13}	a_{19}	a_{25}	a_{31}	a_{37}	a_{43}	a_{49}
h_1	e_5	f_9	h_{13}	e_{17}	f_{21}	h_{25}	e_{29}	f_{33}
	d_6	g_6		c_{22}	e_{14}		i_{38}	h_{22}
	i_8	h_4		f_{15}	d_{18}			
				h_{10}	d_{24}			
					i_{32}			

Graph 2

A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}	A_{16}	A_{17}	A_{18}
a_{55}	a_{61}	a_{67}	a_{73}	a_{79}	a_{85}	a_{91}	a_{97}	a_{103}
h_{37}	e_{41}	f_{45}	h_{49}	e_{53}	f_{57}	h_{61}	e_{65}	f_{69}
	d_{54}	g_{30}		c_{70}	e_{38}		i_{86}	h_{46}
	d_{72}	e_{20}		e_{47}	i_{50}			
	d_{96}	i_{26}		i_{62}				
	i_{128}							

Graph 3

A_{19}	A_{20}	A_{21}	A_{22}	A_{23}	A_{24}	A_{25}	A_{26}	A_{27}
a_{109}	a_{115}	a_{121}	a_{127}	a_{133}	a_{139}	a_{145}	a_{151}	a_{157}
h_{73}	e_{77}	f_{81}	h_{85}	e_{89}	f_{93}	h_{97}	e_{101}	f_{105}
	d_{102}	g_{54}		c_{118}	e_{62}		i_{134}	h_{70}
	c_{136}	g_{36}		h_{79}	c_{82}			
	h_{91}	g_{24}			h_{55}			
		h_{16}						

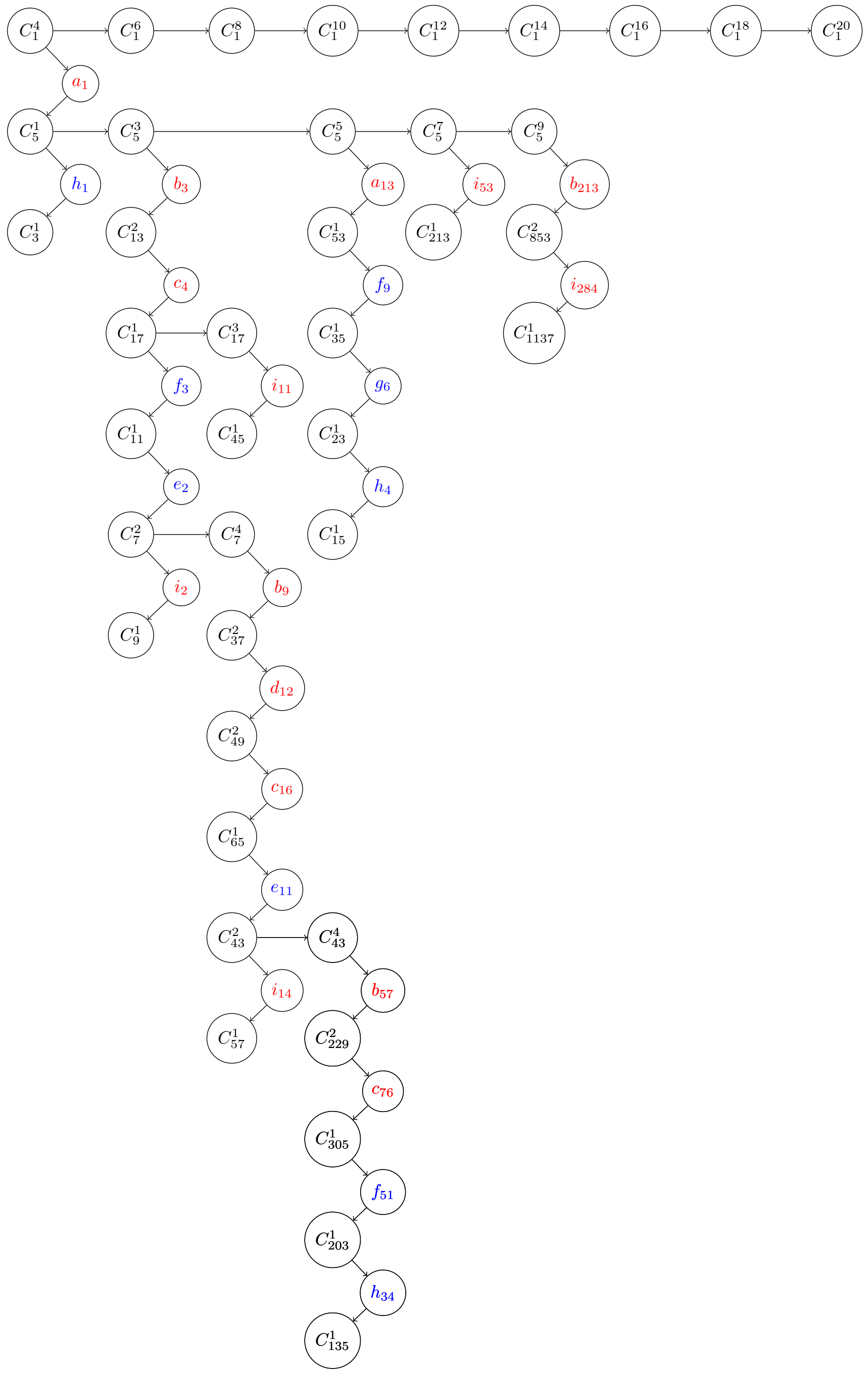
Graph 4

A_{28}	A_{29}	A_{30}	A_{31}	A_{32}	A_{33}	A_{34}	A_{35}	A_{36}
a_{163}	a_{169}	a_{175}	a_{181}	a_{187}	a_{193}	a_{199}	a_{205}	a_{211}
h_{109}	e_{113}	f_{117}	h_{121}	e_{125}	f_{129}	h_{133}	e_{137}	f_{141}
	d_{113}	f_{117}		c_{166}	e_{86}		i_{182}	h_{94}
	i_{200}	h_{52}		f_{111}	d_{114}			
				e_{74}	i_{52}			
				i_{98}				

Graph 5

A_{37}	A_{38}	A_{39}	A_{40}	A_{41}	A_{42}	A_{43}	A_{44}	A_{45}
a_{217}	a_{223}	a_{229}	a_{235}	a_{241}	a_{247}	a_{253}	a_{259}	a_{265}
h_{145}	e_{149}	f_{153}	h_{169}	e_{161}	f_{165}	h_{169}	e_{173}	f_{177}
	d_{198}	g_{102}		c_{214}	e_{110}		i_{230}	h_{118}
	d_{264}	e_{68}		e_{143}	i_{146}			
	c_{352}	d_{90}		c_{190}				
	h_{211}	d_{120}		h_{127}				
		c_{160}						
		e_{107}						
		c_{142}						
		e_{95}						
		d_{126}						
		d_{168}						
		i_{223}						

An astounding number of relationships exist between integers and sets of integers.



C_1^4
C_1^6
C_1^8
C_1^{10}
C_1^{12}
C_1^{14}
C_1^{16}
C_1^{18}
C_1^{20}
a_1
C_5^1
C_5^3
C_5^5
C_5^7
C_5^9
h_1
b_3
a_{13}
i_{53}
b_{213}
C_3^1
C_{13}^2
C_{53}^1
C_{213}^1
C_{853}^2
c_4
f_9
i_{284}
C_{17}^1
C_{17}^3
C_{35}^1
C_{1137}^1
f_3
i_{11}
g_6
C_{11}^1
C_{45}^1
C_{23}^1
e_2
h_4
C_7^2
C_7^4
C_{15}^1
i_2
b_9
C_9^1
C_{37}^2
d_{12}
C_{49}^2
c_{16}
C_{65}^1
e_{11}
C_{43}^2
C_{43}^4
i_{14}
b_{57}
C_{57}^1
C_{229}^2
c_{76}
C_{305}^1
f_{51}
C_{203}^1
h_{34}
C_{135}^1

14. THE COMMON DIFFERENCE BETWEEN ELEMENTS OF ORDER ISOMORPHIC POSETS

The relation that defines the order isomorphism between poset's is their common ordering of letters. A poset's cardinality is used to define the common difference of its elements in arithmetic progression. Due to the many permutations and indexations thus far, it is important to know which set of numbers are being used when defining the common difference of a poset's elements in arithmetic progression. Three definitions are given for the common difference, d of a chain,C:

$$\text{Given an arithmetic progression}: \; a_n = a_1 + (n-1)d, \tag{14.1}$$

Given a poset's cardinality,C, the common difference, D_x,D_y,and D_z, between minimal elements is such:

Definition 14.1. D_x= The real odd integers: 5,7,9,11....

$$D_x = 2^3 \cdot 3^{|C|} \tag{14.2}$$

Definition 14.2. D_y = The real odd integers after the first permutation and indexing: $a_1, b_1, a_2, b_2, a_3, b_3 ...,$

$$D_y = 2 \cdot 3^{|C|} \tag{14.3}$$

Definition 14.3. D_z = The real odd integers after the second indexing that followed the first permutation and indexing:

$$D_z = 3^{|C-1|} \tag{14.4}$$

The following section defines the relationship between the common elements of isomorphic sets.

15. Ascending and descending values of, $2^x \cdot 3^y$, between elements of isomorphic sets

If two chains are order isomorphic then a bijective relationship exists between the elements of the poset. For all order isomorphic chains in this algebraic system, the common difference between each element is illustrated by the tables below.

A Poset's Equivalence class in respect to their Common Difference, D_x,D_y,and D_z

$C^y_x:$	C^3_{11}	$\mapsto$	C^4_{127}	$\mapsto$	C^3_{497}	$=$	116	$=$		
$D_x:$	29	$\mapsto$	677	$\mapsto$	1325	$=$	648	$=$	$2^3 \cdot 3^4$	$A \equiv [29]_{648}$
$D_y:$	a_7	$\mapsto$	a_{169}	$\mapsto$	a_{331}	$=$	162	$=$	$2^1 \cdot 3^4$	$a_x \equiv [7]_{162}$
$D_Y:$	e_5	$\mapsto$	e_{113}	$\mapsto$	e_{221}	$=$	108	$=$	$2^2 \cdot 3^3$	$e_x \equiv [5]_{108}$
$D_y:$	d_6	$\mapsto$	d_{150}	$\mapsto$	d_{294}	$=$	144	$=$	$2^4 \cdot 3^2$	$d_x \equiv [6]_{144}$
$D_y:$	i_8	$\mapsto$	i_{200}	$\mapsto$	i_{392}	$=$	192	$=$	$2^6 \cdot 3^1$	$i_x \equiv [8]_{192}$
$D_z:$	2	$\mapsto$	29	$\mapsto$	56	$=$	27	$=$	$2^0 \cdot 3^3$	$a_x \equiv [2]_{27}$

$D_x :$	125	$\mapsto$	5957	$\mapsto$	11,789	$=$	5,832	$=$	$2^3 \cdot 3^6$	$A \equiv [125]_{5832}$
$D_x :$	125	$\mapsto$	5957	$\mapsto$	11,789	$=$	5,832	$=$	$2^3 \cdot 3^6$	$A \equiv [125]_{5832}$
$D_y :$	a_{31}	$\mapsto$	$a_{1,489}$	$\mapsto$	$a_{2,947}$	$=$	1,458	$=$	$2^1 \cdot 3^6$	$a_x \equiv [31]_{1,458}$
$D_y :$	f_{21}	$\mapsto$	f_{993}	$\mapsto$	$f_{1,965}$	$=$	972	$=$	$2^2 \cdot 3^5$	$f_x \equiv [21]_{972}$
$D_y :$	e_{14}	$\mapsto$	e_{662}	$\mapsto$	$e_{1,310}$	$=$	648	$=$	$2^3 \cdot 3^4$	$e_x \equiv [14]_{648}$
$D_y :$	d_{18}	$\mapsto$	d_{882}	$\mapsto$	d_{1746}	$=$	864	$=$	$2^5 \cdot 3^3$	$d_x \equiv [18]_{864}$
$D_y :$	d_{24}	$\mapsto$	$d_{1,176}$	$\mapsto$	$d_{2,328}$	$=$	1,152	$=$	$2^7 \cdot 3^2$	$d_x \equiv [24]_{1,152}$
$D_y :$	i_{32}	$\mapsto$	$i_{1,568}$	$\mapsto$	$i_{3,104}$	$=$	1,536	$=$	$2^9 \cdot 3^1$	$i_x \equiv [32]_{1,536}$
$D_z :$	6	$\mapsto$	249	$\mapsto$	492	$=$	243	$=$	$2^0 \cdot 3^5$	$a_x \equiv [6]_{243}$

16. Finding the source element

This section addresses a relation that exists between the first elements of onto, isomorphic, groups of chains in relation to the series of odd integers reached, when directed towards 1, using the Collatz functions. I'll do my best to explain with examples. I'm trying to say that a relation exists when looking at the modular relation of the cross products of the series. This may not of been worded properly so I'll give examples. Here's a series of pairs from the work.

Graph 1

Poset C^3_{11}		a_7, e_5, d_6, i_8	$aRb \mid bRc \mid cRd \mid aRd$
a:11	$\longmapsto$	b:127	$11 \cdot 677 = 7,447$
$\Updownarrow$		$\Updownarrow$	
c:29	$\longmapsto$	d:677	$29 \cdot 127 = 3,683$
			$7,477 \equiv [81]_{3,683}$
127	$\longmapsto$	497	$127 \cdot 1,325 = 336,469$
$\Updownarrow$		$\Updownarrow$	
677	$\longmapsto$	1,325	$677 \cdot 497 = 168,275$
			$81 \equiv [336,469]_{168,194}$
185	$\longmapsto$	983	$185 \cdot 2,621 = 484,885$
$\Updownarrow$		$\Updownarrow$	
1,973	$\longmapsto$	2,621	$1,973 \cdot 983 = 1,939,459$
			$1,939,459 \equiv [81]_{484,885}$
983	$\longmapsto$	613	$983 \cdot 3,269 = 3,213,427$
$\Updownarrow$		$\Updownarrow$	
2,621	$\longmapsto$	3,269	$2,621 \cdot 613 = 1,606,673$
			$81 \equiv [3,213,427]_{1,606,592}$
613	$\longmapsto$	1,469	$613 \cdot 3,917 = 4,802,161$
$\Updownarrow$		$\Updownarrow$	
3,269	$\longmapsto$	3,917	$3,269 \cdot 1,469 = 2,401,121$
			$4,802,161 \equiv [81]_{2,401,121}$
1,469	$\longmapsto$	107	$1,469 \cdot 4,565 = 6,707,985$
$\Updownarrow$		$\Updownarrow$	
3,917	$\longmapsto$	4,565	$3,917 \cdot 107 = 419,119$
			$81 \equiv [6,707,985]_{419,038}$

Poset C^4_{19}		$a_{25}, b_{17}, c_{22}, f_{15}, h_{10}$	$aRb\|bRc\|cRd\|aRd$
a:19	$\longmapsto$	d:767	$19 \cdot 2,045 = 38,855$
$\Updownarrow$		$\Updownarrow$	
b:101	$\longmapsto$	c:2045	$101 \cdot 767 = 77,467$
			$243 \equiv [77,467]_{38,612}$
767	$\longmapsto$	187	$767 \cdot 3,989 = 3,059,563$
$\Updownarrow$		$\Updownarrow$	
2,045	$\longmapsto$	3,989	$2,045 \cdot 187 = 382,415$
			$3,059,563 \equiv [243]_{382,415}$
187	$\longmapsto$	2,225	$187 \cdot 5,933 = 1,109,471$
$\Updownarrow$		$\Updownarrow$	
3,989	$\longmapsto$	5,933	$3,989 \cdot 2,225 = 8,875,525$
			$243 \equiv [8,875,525]_{1,109,228}$
2,225	$\longmapsto$	1,477	$2,225 \cdot 7,877 = 17,526,325$
$\Updownarrow$		$\Updownarrow$	
5,933	$\longmapsto$	7,877	$5,933 \cdot 1,477 = 8,763,041$
			$17,526,325 \equiv [243]_{8,762,798}$
1,477	$\longmapsto$	3,683	$1,477 \cdot 9,821 = 14,505,617$
$\Updownarrow$		$\Updownarrow$	
7,877	$\longmapsto$	9,821	$7,877 \cdot 3,683 = 29,010,991$
			$243 \equiv [29,010,991]_{14,505,374}$
3,683	$\longmapsto$	1,103	$3,683 \cdot 11,765 = 43,330,495$
$\Updownarrow$		$\Updownarrow$	
9,821	$\longmapsto$	11,765	$9,821 \cdot 1,103 = 10,832,563$
			$43,330,495 \equiv [243]_{10,832,563}$

17. Matrices: The limit of sets given a certain cardinality

$$\lambda_{\{4,4\}} = \begin{pmatrix} 2_A, 7_a, 29_\lambda & 3_A, 13_a, 53_\lambda & 15_A, 73_a, 341_\lambda & 23_A, 133_a, 533_\lambda \\ 2_E, 5_e, 19_\mu & 2_F, 9_f, 35_\mu & 10_F, 57_f, 227_\mu & 30_E, 89_e, 355_\mu \\ 1_D, 6_d, 25_\lambda & 1_G, 6_g, 23_\mu & 13_E, 38_e, 153_\mu & 20_C, 118_c, 473_\lambda \\ 3_I, 8_i, 33_\lambda & 2_H, 4_h, 15_\mu & 17_I, 50_i, 201_\lambda & 27_H, 79_h, 315_\mu \end{pmatrix}$$

$$\lambda^{-1}_{\{4,4\}} = \begin{pmatrix} 25_A, -155_a, 619_\lambda & 24_A, -149_a, 595_\lambda & -12_A, -89_a, -307_\lambda & -4_A, -29_a, -74_\lambda \\ -36_E, -103_e, 19_\mu & 2_F, -99_f, 35_\mu & -18_F, -51_f, 227_\mu & 30_E, -19_e, 355_\mu \\ 1_D, 6_d, 25_\lambda & 1_G, 6_g, 23_\mu & 13_E, 38_e, 153_\mu & 20_C, 118_c, 473_\lambda \\ 3_I, 8_i, 33_\lambda & 2_H, 4_h, 15_\mu & 17_I, 50_i, 201_\lambda & 27_H, 79_h, 315_\mu \end{pmatrix}$$

18. New Rewrite

For any $k \in Z$ we'll denote the equivalence class of a by $\bar{a}$. This is called the residue class of a mod n and consists of the integers which differ from a by an integral multiple of n such that,

$$\begin{aligned} \bar{a} &= \{a + kn | k \in Z\} \\ &= \{a, a \pm n, a \pm 2n, a \pm 3n...\} \end{aligned}$$

If R defines an equivalence relation on A, then the equivalence class of $a \in A$ is defined to be

$$\{x \in A | xRa\}$$

An equivalence relation is similarly defined such that:

$$R : aRb \iff n|b - a$$

If R is an equivalence relation on a set A, then the equivalence classes of R partition A. Given a set A:

$$A = Z - \{\emptyset\}$$

If R defines an equivalence relation on A then the set of equivalence classes of R form a partition of A. Let the equivalence classes containing all if the odd and even integers partition A into subsets o and e:

$$\begin{gathered} o \equiv [1]_2 \subset A \\ e \equiv [0]_2 \subset A \\ A = o \cup e \end{gathered}$$

Next, the subset o in A is parti

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