Comprehensive Analysis on Least-Squares Lateration for Indoor Positioning Systems

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***Abstract*—In pursuit of the accomplishment of certain position estimations of targets in outdoor places, finding the locations of the targets in indoor environments has been a significant topic. Exact position estimations of the objects for indoor places have potentials for the enhancement of several emerging Internet-of-Things (IoT) applications, such as smart manufacturing, smart home, public security, social networks, transportation, traveling, marketing applications, and information services lead to a huge demand on the designing of low-cost and high-accuracy localization and navigation solutions. On the other hand, the global positioning system (GPS) technology designed for outdoor positioning applications, is not suitable to indoor positioning systems. Making exact position detection with GPS is a compelling problem for indoor positioning methods. In this study, received signal strength (RSS)-based least-squares triangulation approach that utilizes existing infrastructure, is proposed. By increasing the number of access points (APs) and using line fitting algorithms to the RSS values, the triangulation method improves the certainty of location estimation. The utilization of the existing infrastructure turns the proposed approach into cheaper when compared to existing localization methods which require expensive components. The proposed least-squares lateration algorithm is compared with pure lateration (PL) in terms of accuracy error under different Gaussian noise parameters for varying number of APs and varying dimensions of the measurement area. Usage of the least-square algorithm with line fitting approaches provides significant performance improvements for all cases when it compared with PL.**

***Index Terms*—Indoor positioning, least-squares methods, triangulation.**

I. Introduction

NOWADAYS, the position information of a device has become a significant requirement in various applications. In the late 1960s, the U.S. Department of Defense starts to develop a satellite-based positioning technique for military objectives that ultimately evolves into the global positioning system (GPS) in 1990. GPS becomes fully operational in 1995. The GPS is a space-based satellite navigation system that provides location and time information in all weather conditions, anywhere on or near the Earth where there is an unobstructed line of sight (LOS) to three or more GPS satellites [1]. The GPS includes the triangulation method to find out physical locations. It communicates with three satellites in sight (utilizing radio signal that travels at the speed of light) and then computes the distance between those satellites and the target by using the travel time of the signals. If the distance to those satellites is known, all possible positions are located on the surface of three spheres whose radius corresponds to the calculated distance. The position is the intersection point of three spheres. The traditional solution of improving accuracy of GPS devices is to seek data from four or more satellites. GPS becomes an enviable method for finding the locations in outdoor environments, however, it is not prospering for exactly specifying positions in the buildings [2]. The concrete walls cause attenuation of the signals transmitted from satellites and thus the signals do not penetrate walls. Therefore, it becomes nearly impossible to determine the location of an object for indoor environments. Considering the implementations in the GPS technology, sensitive GPS chip can occasionally take signals from satellites when it is situated in indoor building, but there are still considerable accuracy errors [3].

Latterly, due to the advancements in the technology, people spend most of their time in indoor places thus the necessity for indoor localization arises. The unsuccessful performance of the GPS for determining the location of targets inside the buildings has eventuated to develop new technologies. In these places, utilizer can use an indoor positioning system (IPS) just like a GPS, being led to definite destinations. For instance, they can use it to reach gates in the airports, clinic in a hospital, or a product in a supermarket. With accomplished implementations of the IPSs, several kinds of application can be performed for indoor places [4]. Supplier in a shopping center will take advantage of IPS for the sale.

Location-based message services can provide firms to transmit their messages and deliver personalized offers to their purchasers at the right place and the right time by using indoor positioning technologies. IPS can also ensure innovative and accurate ways to determine the locations of the people inside buildings in the case of emergency. By accurate determination of the position of a person inside the building, emergency situation operation can be considerably enhanced when an event happens. For instance, during the fire, having information about how many people stay inside the building, and determining their accurate positions in the affected place, gives an opportunity to gain some time to rescue the injured ones. Since people's life matters, gaining some time during this sort of occurrence is so crucial [5].

In recent years, indoor positioning is a significant research topic thus many studies have been done in this field. Several

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IPS techniques are designed by researchers [6]. Infrared (IR) positioning system utilizes IR light pulses to determine signals in buildings. IR receivers are placed in every room and when the IR tags pulse, they are read by the IR receivers. There are three different types of ultrasonic positioning systems which are active bats, crickets, and dolphin. In these technologies, the location can be determined by measuring distances between transmitters and receivers by applying trilateration [7]. In the received signal strength (RSS)-based localization systems [8] the position of the objects can be obtained by computing the distance of the object from the transmitter using triangulation or trilateration techniques.

Localization techniques can be classified according to the signal measurement or techniques that employed [4]. The common techniques used for signal measurement are: angle of arrival (AOA), time of arrival (TOA), time difference of arrival (TDOA), and RSS indicator (RSSI). AOA [9] uses the calculation of the angles at which the signals arrive from the un-located device to the anchor nodes. The computation of the angle and the distance provides to detect the location of the sender, and the information is used for tracing. This technique needs only two measuring units for 2-D positioning and it does not need synchronization between the measuring units. AOA can be used successfully when the LOS exists, however, the accuracy decreases in the multipath environment. TOA uses the distance between the transmitting node and the receiving node. It uses the transmission time delay and the corresponding speed of the signal to determine the position of the object [10], [11]. TOA ensures high accuracy, on the other hand, it brings extra costs because of the high hardware complexity. TDOA systems also use distance-based measurements to detect the location of the objects [12]. They determine the relative positions of the transmitters based on the difference of the TOA of the propagation of the signals of the transmitters and multisensors. When the signal is arrived at two reference points, the difference in arrival time can be utilized to compute the difference of the distances between the target and the two reference points. It is less complex than TOA and it performs remarkable accuracy performance.

The values of RSSI are the measures of the power levels of RSS. It is measured by decibel-milliwatts (dBm) which correspond to negative numbers. If the received signal is strong, the values become closer to zero. In order to locate an object with RSSI, the RSSI values between the sensor attached to an object and surrounding access points (APs) with fixed locations should be measured [13]. The combination of these multiple RSSI values can be used to calculate the approximate position of the object. Typically, at least three APs are required for the determination of the position of the object. The positions of the objects are obtained by computing the distances of the objects from senders by the help of triangulation or trilateration methods. RSSI-based positioning is easy to deploy in comparison to the methods which utilize AOA and TDOA [14]. For the RSSI method, specialized equipment both at the cellphone and the wireless interface card is not needed to be used [15].

Triangulation utilizes the geometric characteristics of triangles to estimate the locations of the targets by calculating angle measurements according to two recognized reference points. With triangulation, the position of the object can be found by the intersection of several pairs of angle direction lines [2]. The trilateration-based positioning scheme uses three fixed reference nodes to calculate the physical position of a target node [1]. The position of the target object is determined via TOA to measure the time taken by a signal to arrive at a receiver from a transmitter. The proximity algorithm is used to determine the position of the device utilizing the proximity relationship between the device and APs. A grid of APs with recognized position information is utilized to find the location. When a device is mobile, the nearest AP to that device is used to calculate its position. On the other hand, if mobile equipment is determined by a number of APs, the AP which has the most powerful signal is used for the determination of the position.

The fundamental basis of fingerprinting is building a fingerprint database. The fingerprinting scheme contains two stages: 1) sampling (offline) and 2) matching (online) [16]. In the sampling stage, the fingerprint database is formed. In this formation stage, the place is previously measured, the settlement of reference points is specified and RSS samples are collected according to each reference point. While the matching stage, a positioning method uses the available obtained signal powers and prior information to determine the location.

RSS-based techniques are realized easily by using a Wi-Fi integrated device, such as smartphone, PC, tablet, or laptop. Trilateration and fingerprinting are the main two Wi-Fi-based indoor localization techniques in the literature as mentioned before in this article. The fingerprinting techniques are also divided into two groups which are deterministic fingerprinting (D-FP) and probabilistic fingerprinting (P-FP). D-FP techniques determine the location of a target by regarding deterministic RSSs measurements solely. Probabilistic-based fingerprinting techniques compute the locations of the objects by using the RSSs measurements as a part of a stochastic process. Bisio *et al.* [17] aimed to propose energy efficient and less complex P-FD-based localization technique that can be easily operated over smartphone to obtain time and energy savings than the existing studies. In the proposed approach, at the positioning phase, a smartphone obtains the RSS measurements and computes an observation vector. Existing P-FD techniques make computations of the probabilities to perform an observation vector at a fixed reference point in the course of online step. Bisio *et al.* [17] presented a smart computation technique by exploiting the lower number of operations to provide time and energy savings. The proposed scheme is realized and tested on an Android smartphone practically. In the proposed smart P-FD scheme, the authors aim to use an algebraic factorization of the equations of the existing method to compute and store required variables in the training step for avoiding their calculations during the online localization stage. The authors make comprehensive computational cost analyzes of proposed smart P-FP and traditional P-FP schemes. They demonstrated that the proposed scheme provides significant complexity improvements when compared with traditional one. According to the performance evaluation analyzes and results of the proposed technique, the proposed scheme does not require any approximations and the accuracy performance is not changed provided by the traditional P-FP when the smart P-FP is used. The most significant performance enhancement is attained in energy efficiency. According to the results, the proposed scheme provides greater than 90% energy savings when compared with the traditional scheme for the case that the number of APs in the measurement area is greater than 3. Finally, the proposed scheme provides approximately 88% FLOPs savings when compared with traditional one. Existing studies related to the fingerprinting method aim to make enhancements in the measurement stage. In addition, some of the studies aim to estimate the locations associated with fingerprints. But, according to Baala *et al.* [18], the effect of the building architecture, the radio propagation, and the impact of spacing of the grids where location fingerprints are collected are not investigated. Therefore, in this study, the authors propose a WLAN-based positioning system (WPS) to present its performance. Moreover, the authors present two new indicators: 1) refined specific error ratio (RSER) and 2) refined global error ratio (RGER) to confirm the optimum configuration that provides lower accuracy error. According to analyzes and simulation results, the proposed statistical scheme determines the optimum configuration while providing smaller errors for several scenarios. The most extensive fingerprinting techniques are weighted K-nearest neighbors (WKNN) schemes, which compute K-nearest neighboring points to mobile devices. On the other hand, traditional WKNN schemes cannot solve the issues effectively. There are differences in observed AP sets during the offline and the online phases. In addition, not all the K neighbors are physically close to the users. Therefore, Hu *et al.* [19] aimed to use the similarity coefficient to obtain measurements of the similarities of AP sets, which are then combined with radio signal strength values to compute the fingerprint distances. They present and discuss two main common issues of existing fingerprinting approaches which are: APs mismatch and clustering inefficiency. To solve these two issues, they propose the combination of AP sets similarity and improved the semisupervised affinity propagation clustering scheme in aggregation with determination of isolated points, respectively. Real-time implementations are conducted on a university campus and the results of the experiments demonstrate that the proposed scheme outperforms existing techniques.

Industry 4.0, which is one of the most popular topics in many industries and academia world today, is seen as a technological experiment when it first appeared, and today it has become a necessity to protect and increase the competitiveness of industries. Industry 4.0 is defined as the sum of innovations produced and applied in a value chain to address the trends of digitalization, transparency, mobility, network collaboration, and socialization in information and communication technologies, and production systems of products, and processes. There are many components of Industry 4.0 technologies, such as cloud computing, big data analytics, and Internet of Things (IoT).

The concept of IoT was first used in a presentation by Kevin Ashton for a company called "Procter & Gamble" in 1999. The reason for the development of IoT in the recent years, not in the early years, is that other technologies that will contribute to the improvement of IoT have just begun to develop. For example, providing wireless access from anywhere regardless of time and space; low costs in data usage; the processors are stronger and cheaper; reduction of sensor costs; increasing number of smart devices; increased "big data" analyzes and capacities; the use of IPv6 which greatly increases the IP addressing capacity, allow for desired developments in the field of IoT. The IoT technology is mostly focused on the information technology. The vision of this technology is to connect the physical world and the virtual world in order to establish communication between all connected objects. IoT refers to the infrastructure of physical and digital objects equipped with (radio based) identification technologies, such as radio RFID, near-field communication (NFC), or beacon chip. These objects, on mobile or smart devices, use the IoT technology to exchange and process information to perform common tasks. In brief, the IoT refers to the technological innovation that connects devices over the Internet. Data transfer between various physical items and objects is carried out over a network and over the Internet. IoT consists of a series of digital technologies that can integrate physical systems with the digital world and provide continuous and easily accessible information about these systems. In today's world phones, tablets, computers, and smartwatches can be connected to the Internet. According to a study by the World Economic Forum, it is estimated that a trillion sensor will connect to the Internet by 2025. The advancement of the technology in this direction reveals the IoT technology. The IoT has the potential to be used in almost all industries, logistics activities, and research fields, such as healthcare, transportation, wireless communications, vehicular communications, mobile communications, smart factories, smart shops, IPSs, and remote patient monitoring.

In recent years, IPS plays a significant role in the scenarios of the IoT which require indoor location context. Especially, IoT-based IPS studies have increased in the last few years [20]–[26]. The authors proposed a novel clustering method for the election of reference points according to the virtual locations of APs for indoor positioning in the measurement field without using linear restrictions in [20]. This usage provides the robustness of techniques between the offline stage clustering and the online stage localization. To determine locations, the authors presented a weighted approach based on the physical distances. By utilizing angle velocity measurements, the weighted scheme is specifically adapted for mobile scenarios. In this study, the authors improved the traditional clustering schemes to obtain better accuracy results in positioning. According to the results of the analyzes and experiments, the proposed approach provides significant performance enhancements when it is compared with traditional K-nearest neighbor, Euclidean weighted K-nearest neighbor (EWKNN), Manhattan-WKNN, and GPS in terms of localization precision. In addition, the authors also prove that reference points in indoor places without linear restrictions can be clustered automatically by using the proposed scheme in this study. Du *et al.* [21] investigated the Wi-Fi signals under contemporary Wi-Fi infrastructure, signal models between APs, and the correlations between signals and indoor pathway maps to solve the issues of incoherent Wi-Fi signal measurements. The authors utilize spatial signal patterns to place the estimated location into a restricted field through signal coverage constraint (SCC). A localization technique that uses sibling signal patterns (SSPs) and SCC is proposed in this study. The results of the evaluations demonstrate that this technique provides significant localization certainty. The proposed approach in this article provides easy deployment to real-world environments and scenarios. Li *et al.* [22] proposed a dead reckoning-based localization technique where the radio map is automatically created without the need for any field survey. In the newly created robust radio map, there is no need to have prior information about building plans. By implementing three phase trace matching techniques, the robust radio map is constructed. This matching technique strengthens the gates of the buildings and Wi-Fi fingerprints as boundary marks to blend the noisy observations and exactly build the walking tracks of mobile users. Furthermore, an improved particle filter for fusing pedestrian dead reckoning (PDR), GPS, and fingerprints is used to perform accurate localization. The results of the evaluations show that the proposed scheme detects successfully the tracks of the mobile users. WKNN fingerprinting techniques that determine the locations of the objects based on K-nearest RPs measured previously are widely used in IPSs. The main problem of these schemes is to determine the optimum K value to provide accurate localization. Hu *et al.* [23] proposed a self-adaptive WKNN (SAWKNN) method which has an adaptive K. Based on the measured RSS, the value of K is arranging to provide localization precision enhancements. When the simulations and analyzes are examined, it is seen that the best value of K is 1 to apply this method to real-time scenarios. Feng *et al.* [24] proposed an integrated IPS which combines inertial measurement unit (IMU) and ultra wide band (UWB) systems together with extended Kalman Filter and unscented Kalman Filter to increase the accuracy of the existing IPSs. In this study, the geometric distributions of the APs are discussed. The results of the analyzes and simulations demonstrate that prior knowledge supplied by IMU can decrease the measurement errors of UWB considerably. Moreover, the performance of IPS highly depends on the UWB observations. The authors also show that the proposed method provides less computational complexity and thus it can be easily deployed to real-time applications. Radio-frequency (RF)-based wireless communications cannot assure QoS necessities of users, due to the restricted bandwidth (BW) and multipath interferences. Therefore, Yang *et al.* [25] presented a novel visible light positioning network (VLCP) for IoT to obtain fast communication performance and low localization accuracy errors together. By optimization of AP determination, BW allocation, adaptive modulations, and power efficiency, the authors aim to fulfill QoS requirements of indoor users while increasing communication speed significantly in the network which includes VLC APs. An iterative scheme that has less complexity is performed to enhance the existing resource management (RM) optimization problem. Moreover, PDR-based VLCP approach is implemented to obtain better accuracy performance under LOS blockages. The simulation results show that the proposed scheme outperforms other existing schemes in terms of data rate, localization precision, and QoS performance. With the wide usage of IoT systems, various crowdsourced data is convenient. Zhao *et al.* [26] aimed to use the crowdsourced Wi-Fi RSS data to implement indoor localization. For several cases, there is a huge amount of crowdsourced RSS data and the related RSS fingerprints are not available. Thus, constructing the radio map becomes a challenging problem. To solve this problem, in this article, the geometrical method is applied to transform the RSS measurements into dual distances between the fingerprints. To construct a radio map, multidimensional scaling (MDS) is used to calculate the locations of all the fingerprints. Furthermore, K-means clustering-based area partitioning scheme is also proposed. By applying parallel programming to the segmented data, computational complexity of the proposed MDS is decreased significantly when compared with existing optimization algorithms.

Industy 4.0 influences numerous sectors, such as manufacture, tourism, healthcare, transportation, and travel. Industry 4.0 evolution provides D2D communications and several IoT solutions to the users. A significant study in [27] is proposed to present the usage of ambient intelligence in IoT for smart products and services in Industry 4.0. To establish the practical effects of the proposed study, the authors present three successful implementations for smart factories, smart homes, and healthcare. In this study, the authors also present some significant challenges and their smart solutions in detail to provide foresight to researchers in the near future. Environmental sensing, power consumption, heterogeneity of Industry 4.0 schemes, and full connectivity are the major challenges investigated in this study. The efficiency of the manufacturing chain can be dramatically influenced, because searching of lost tools and entities is time-consuming process. IoT-based technologies provide helpful smart solutions for this issue. The authors propose an asset tracking framework that uses the Bluetooth low energy (BLE) technology that used in indoor positioning applications to solve the above-mentioned issue. According to the results, the authors obtain approximately full detection probability ratios for several cases. In addition, in smart home application part, the authors propose a successful speaker recognition scheme for security requirements of IoT applications. According to the results, 95% recognition performance is obtained at 1-m distance. Finally, IoT provides significant solutions for healthcare applications in home environments. Therefore, the authors aim to propose a wearable prototype post-stroke therapy. The designed prototype includes multiple sensors in wearable structures to provide smart sensing for guiding the patients. In addition, this system enables medical staff to make remote connections to the patients and checking whether the patients perform their exercises.

In this part, novel and up-to-date studies about indoor positioning are presented and discussed. Chen *et al.* [28] aimed to perform EntLoc which is the CSI-based P-FP approach utilizing commercial off-the-shelf Wi-Fi equipments. Autoregressive model-based entropy of channel state information is exploited as location fingerprint and this utilization provides simplicity. According to the results of the implementations, the proposed method provides accuracy performance enhancements by using the single signal transmitter when compared with the existing approaches.

Photodiodes-based visible light localization (VLL) has begun to become a popular research field nowadays. Thus, in [29], indoor real-time 3-D VLL method which uses fingerprinting and extreme learning machines is designed to acquire high localization precision, less interference, and well-adopted real-time behavior. When it is compared with existing KNN or SVM-based localization methods, the proposed scheme provides lower localization errors and higher localization speed. In addition, to decrease the dimensions of the fingerprint database and learning time significantly, VLL kernel is implemented. Simulation results demonstrate that the proposed method provides low interference in 3-D localization.

In [30], vision-based indoor localization method is proposed. Vision-based localization methods require only camera to accomplish indoor localization tasks. To enhance the robustness and certainty of the existing vision-based positioning methods, this study aims to present a pixel threshold-based eight points scheme and enhanced epipolar restraint approach. The existing eight point techniques in the literature exploit Euclidean distances as election indicators for feature points. In these existing methods, some feature points become discomposed when the localization scene varies and thus this can cause mismatches. The designed pixel threshold limitation in this study provides improvements in the quality of feature points. Moreover, the epipolar scheme is designed by using a new cost function to enhance the precision of essential matrix computation, so the localization accuracy errors are decreased. The results of the evaluations indicate that the proposed scheme enhance the indoor localization accuracy.

Simões *et al.* [31] aimed to design a hybrid localization method which combines linear weighted policy learner (mapping scheme), iterative PDR (navigation algorithm), and the obstacle detection approach to improve the performance metrics of existing methods. According to the results of the proposed hybrid model, it provides acceptable accuracy errors and computational costs for today's conditions. Another hybrid solution is proposed in [32]. It combines artificial neural networks and swarm intelligence method to solve the problems of indoor localizations and time-consuming tasks in the arrangement of the parameters of existing methods, respectively. The results of the experiments show that the precision errors of the proposed hybrid solution are approximately same with the existing methods, however, hybrid method decreases search times significantly when compared with the existing algorithms.

The remainder of this article is organized as follows. In Section II, lateration studies used for IPSs are presented as related works. In Section III, existing pure lateration (PL) algorithm is described. Section IV presents the proposed least-squares lateration (LSL) method, its complexity, and its cost analysis. In Section V, the simulation environment and the parameters are described. In Section VI, the performance evaluations of the LSL method through some performance metrics, such as varying number of APs and test points, different Gaussian noise values, and varying field dimensions are presented. Finally, the article concludes with conclusions in Section VII.

II. Related Works

When the lateration-based studies are investigated in the literature, the studies which are discussed below come to the forefront. Note that, the studies below use only classical lateration to implement indoor positioning. They can be classified into three groups: 1) classical lateration studies; 2) comparisons of traditional lateration with the other kinds of positioning algorithms; and 3) hybrid solutions which include PL and another kind of positioning algorithms.

The study in [33] provides a robust indoor localization by using real-time RSSI values. It makes investigations about path-loss exponent and provides path-loss exponent estimations from RSSI values. The obtained accuracy errors are still high (it is approximately 5 m) for indoor localizations. In addition, it does not provide line fitting solutions and investigations about the performance metrics, such as the number of APs, varying step sizes of test points, and varying dimensions of the measurement area. Using wireless LAN technologies for location estimations provides alternate means to enable position-based solutions without using any WSN infrastructure and special hardware. However, the main issues of wireless LAN-based location systems are calibrations of RSSs. Gwon and Jain [34] analyzed empirical error characteristics of PL in several spatial densities of calibrations. They propose triangular interpolation and extrapolation (TIX) schemes which are calibration-free positioning algorithms. According to their simulation results, they obtain mean distance error within 5.4 m which is very high for an indoor environment. Shchekotov [35] proposed to use indoor positioning based on the Wi-Fi PL approach. They obtain simulation results related to distance estimations. In this study, they cannot determine the locations of the users, they only find distances of them. In addition, common significant performance metrics that are analyzed in detail in this article are not considered in this study. Pathak *et al.* [36] aimed to provide a traditional lateration study for indoor localizations. They make some estimations about locations and they find accuracy errors in approximately 1.5–2 m. They do not consider line fitting solutions to enhance the accuracy performance of the proposed approach. This study only provides simulations about the PL. The significant performance metrics are not also considered and analyzed in this article. Rusli *et al.* [37] provided an improved traditional lateration technique to solve the signal blocking issue due to the obstacles are situated inside the building. This problem is resolved by enhanced RSSI measurements. This study just focuses on enhancements in RSSI measurements. This study can be used to improve the performance of the proposed method in this article by providing improved RSS measurements to the proposed line fitting-based lateration approach presented in this article.

Machado *et al.* [38] aimed to compare the performance of PL, KNN, and artificial neural networks techniques for real indoor positioning in WSNs. They use classical lateration algorithm in comparisons. An up-to-date study in [39] aims to design the beacon position detection-based fingerprinting approach. In simulations and analyzes, proposed approach is compared with traditional PL which does not include any novel enhancements. Yang *et al.* [40] proposed a bilateral greed iteration positioning method based on the greedy algorithm in order to utilize all of the effective anchor points. They compare their proposed method with traditional lateration, fingerprint, and maximum-likelihood method. El Ashry and Sheta [41] made real-time implementations of indoor localization using PL and fingerprinting. They obtain several kinds of comparison results for common existing two algorithms.

Herrera *et al.* [42] provided comparative simulation results about fingerprinting, traditional lateration, and hybrid approach include fingerprinting and PL. According to the results, hybrid solution provides better accuracy performance in indoor positioning. Indoor localization-based BLE beacons has attracted significant attention after the release of the BLE protocol. Numerous studies have been proposed to enhance the performance of BLE-based indoor localization. Huang *et al.* [43] focused on the issues caused by the dense Bluetooth environments. To solve the issues of the dense Bluetooth environments, the authors propose a hybrid method that combines sliding-window filtering, traditional lateration, dead reckoning and the Kalman filtering method to increase the accuracy performance of the BLE indoor localization. Retscher [44] aimed to combine fingerprinting and PL approaches to determine locations of the targets. They made some practical experiments and obtained interesting results about positioning accuracies.

The above-mentioned studies related about lateration which do not include line fitting modifications and comprehensive analyzes in terms of different performance metrics, such as varying number of access and test points, different Gaussian noise parameters, and varying dimensions of the measurement area.

III. Pure Lateration

Position determination by the means of the distance measurement using signal strengths is called lateration. This algorithm is used in indoor positioning due to its accuracy and low cost. The lateration method is based on location information of the reference points and the distances to them. If we have three APs which are placed in a room with coordinate points of $\mathrm{AP}_1(x_1, y_1)$, $\mathrm{AP}_2(x_2, y_2)$, and $\mathrm{AP}_3(x_3, y_3)$. The distance from each APs to a specific point at coordinate (x, y) in the room can be calculated by using the following equations:

$$d_1 = \sqrt{(x_1 - x)^2 + (y_1 - y)^2} \tag{1}$$

$$d_2 = \sqrt{(x_2 - x)^2 + (y_2 - y)^2} \tag{2}$$

$$d_3 = \sqrt{(x_3 - x)^2 + (y_3 - y)^2}. \tag{3}$$

In here, d_1, d_2, and d_3 correspond to the distances to AP_1, AP_2, and AP_3, respectively. We can use (4) and (5) to determine the location of point (x, y)

$$d_1^2 - d_2^2 = -2x_1x - 2y_1y - x_2^2 - y_2^2 + 2x_2x + 2y_2y + x_1^2 + y_1^2 \tag{4}$$

$$d_1^2 - d_3^2 = -2x_1x - 2y_1y - x_3^2 - y_3^2 + 2x_3x + 2y_3y + x_1^2 + y_1^2. \tag{5}$$

Equation (4) and (5) can be represented in a matrix form as

$$\begin{bmatrix} d_1^2 - d_2^2 + x_2^2 + y_2^2 - x_1^2 - y_1^2 \\ d_1^2 - d_3^2 + x_3^2 + y_3^2 - x_1^2 - y_1^2 \end{bmatrix} = \begin{bmatrix} 2(x_2 - x_1) & 2(y_2 - y_1) \\ 2(x_3 - x_1) & 2(y_3 - y_1) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}. \tag{6}$$

When the number of APs are increased to N APs for generalization to obtain better accuracy, the above equation can be expanded for N APs as

$$\begin{bmatrix} d_1^2 - d_2^2 + x_2^2 + y_2^2 - x_1^2 - y_1^2 \\ d_1^2 - d_3^2 + x_3^2 + y_3^2 - x_1^2 - y_1^2 \\ \vdots \\ d_1^2 - d_N^2 + x_N^2 + y_N^2 - x_1^2 - y_1^2 \end{bmatrix} = \begin{bmatrix} 2(x_2 - x_1) & 2(y_2 - y_1) \\ 2(x_3 - x_1) & 2(y_3 - y_1) \\ \vdots & \vdots \\ 2(x_N - x_1) & 2(y_N - y_1) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}. \tag{7}$$

From the above equation

$$A = \begin{bmatrix} 2(x_2 - x_1) & 2(y_2 - y_1) \\ 2(x_3 - x_1) & 2(y_3 - y_1) \\ \vdots & \vdots \\ 2(x_N - x_1) & 2(y_N - y_1) \end{bmatrix} \tag{8}$$

and

$$B = \begin{bmatrix} d_1^2 - d_2^2 + x_2^2 + y_2^2 - x_1^2 - y_1^2 \\ d_1^2 - d_3^2 + x_3^2 + y_3^2 - x_1^2 - y_1^2 \\ \vdots \\ d_1^2 - d_N^2 + x_N^2 + y_N^2 - x_1^2 - y_1^2 \end{bmatrix}. \tag{9}$$

Then (7) can rewritten as

$$B = A \begin{bmatrix} x \\ y \end{bmatrix}. \tag{10}$$

The above equation can be derived and a unique solution for this system can be obtained with (11) to determine the coordinates of (x,y)

$$\begin{bmatrix} x \\ y \end{bmatrix} = \left(A^T A\right)^{-1} A^T B. \tag{11}$$

RSSI is defined by the IEEE 802.11 Standard [5]. It is a measurement of the RF energy. Mobile client can get the RSSI from AP on the WLAN. In indoor environments where it is difficult to obtain line of sight (LOS), the RSSI and positioning may be affected by multipath fading and shadowing, hence the accuracy is reduced. The RSSI is decreased exponentially as the distance from AP increased, and this can be expressed by log-path-loss model [45]–[48]. A log-normal path-loss model for the estimation of distance between receiver and transmitter can be expressed as follows:

$$\mathrm{RSSI(dBm)} = -10n\log(r) + A. \tag{12}$$

TABLE I
PATH-LOSS EXPONENT FOR DIFFERENT ENVIRONMENTS

Environment	Path Loss Exponent
Free Space	2
Urban area cellular radio	2.7 to 3.5
Shadowed urban cellular radio	3 to 5
In building line-of-sight	1.6 to 1.8
Obstructed in buildings	4 to 6
Obstructed in factories	2 to 3

Here, RSSI shows received signal power in dBm, r corresponds to exact distance from the sender to the receiver in meters, n is the path-loss exponent, and A shows the RSS value in dBm at 1-m distance from the sender. The distance of an object from the transmitter can be calculated by using the following equation:

$$r = 10^{\frac{A-\mathrm{RSSI}}{10n}} \tag{13}$$

here, n is the significant parameter that corresponds to the power loss of the signals affected by environmental factors.

Path loss is highly associated with the environment where the transmitters and receivers are located. Path loss directly affects the link quality of the network and its applications, like distance-based localizations, when the power of received signal is an important factor [49]. Path-loss models are developed by using a combination of numerical methods and empirical approximations of measured data collected in channel sounding experiments. It depends on the frequency, antenna orientation, penetration losses through walls, the multipath propagation, and the interference from other signals. In various environments, the path loss and standard deviation change because of the environmental conditions. In Table I, the path-loss exponent for different environments are presented [45].

To calculate the path-loss exponent, the following process can be made. For instance, 196 reference points which has 0.4-m distance between each other can be determined for a room that has the area of 36 m^2. At each point, the RSS values can be measured with the sampling frequency of 10 Hz during 60-s interval. Then the measurements can be averaged for each AP at each reference point. Finally, the matrix in (14) can be obtained as

$$P_d = \begin{bmatrix} P_d(\mathrm{AP}_1)_1 & P_d(\mathrm{AP}_2)_1 & P_d(\mathrm{AP}_3)_1 & P_d(\mathrm{AP}_4)_1 \\ P_d(\mathrm{AP}_1)_2 & P_d(\mathrm{AP}_2)_2 & P_d(\mathrm{AP}_3)_2 & P_d(\mathrm{AP}_4)_2 \\ \vdots & \vdots & \vdots & \vdots \\ P_d(\mathrm{AP}_1)_{196} & P_d(\mathrm{AP}_2)_{196} & P_d(\mathrm{AP}_3)_{196} & P_d(\mathrm{AP}_4)_{196} \end{bmatrix} \tag{14}$$

where $P_d(\mathrm{AP}_i)_j$ is the averaged RSS value from AP i measured at reference point j. The distance $d_{j,i}$ computed with (15) where $d_{j,i}$ represents the distance of reference point j with (x_j, y_j) to AP_i at coordinates (x_i, y_i)

$$d_{j,i} = \sqrt{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2}. \tag{15}$$

TABLE II
SIGNAL STRENGTH ACCEPTANCES

RSSI (dBm)	Signal Strength
> -50	Excellent
-50 to -60	Good
-60 to -70	Fair
< -70	Weak

The above equation can be extended with a distance matrix as

$$D = \begin{bmatrix} d_{1,1} & d_{1,2} & d_{1,3} & d_{1,4} \\ d_{2,1} & d_{2,2} & d_{2,3} & d_{2,4} \\ \vdots & \vdots & \vdots & \vdots \\ d_{196,1} & d_{196,2} & d_{196,3} & d_{196,4} \end{bmatrix}. \tag{16}$$

Then the n value measured at reference point j for AP_i can be computed by the help of the following equation:

$$n = \frac{A - P_d(\mathrm{AP}_i)_j}{10\log_{10}\left(d_{j,i}\right)}. \tag{17}$$

Since path loss is a constant in the measurement field, the computed path-loss exponent values from (17) are averaged to approximate the n value using the following equation:

$$n \approx \frac{1}{kN}\sum_{j=1}^{k}\sum_{i=1}^{N}\frac{A - P_d(\mathrm{AP}_i)_j}{10\log_{10}\left(d_{j,i}\right)} \tag{18}$$

here, k shows the number of points in the field. N corresponds to the number of APs that are located. In this study, the n values are estimated using synthetic data.

IV. LEAST-SQUARE ESTIMATION FOR THE RSS VALUES

In this section, the proposed LSL method is presented.

A. Proposed Least-Squares Lateration Method

Least-square estimation (LSE) is generally used for obtaining a unique group of values for a group of unknowable parameters from a redundant set of observables from a known mathematical model. In particular, the line where x_i are the values at which y_i is measured and i denotes an individual observation that minimizes the sum of the squared distances (deviations) from the line to each observation is used to approximate a linear relationship

$$y_i = b + mx_i. \tag{19}$$

RSSI is generally stated in dB corresponding to a miliwatt (dBm) between zero and -120 dBm. The signal becomes stronger when the RSSI values approximate to 0. RSSI lower than -80 dBm is admitted that it is not useful for practical applications because of noise [50]. Table II denotes the signal strength acceptances according to the RSSI values.

In least-square estimation approach, the main purpose is to obtain m and b for minimizing the root MSE in the approximations of the data by using a first degree line equation (line fitting) according to (19). The error values of each point $\{(x_i, y_i)\}$ can be obtained by

$$E = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[b + mx_i - y_i\right]^2}. \tag{20}$$

Minimization of E is equal to minimize the sum in the above equation. So, to calculate the minimum value of m and b $G(b, m)$, the following equations can be used:

$$\frac{dG(b,m)}{db} = \sum_{i=1}^{n} 2\left[b + mx_i - y_i\right] = 0 \tag{21}$$

$$\frac{dG(b,m)}{dm} = \sum_{i=1}^{n} 2\left[b + mx_i - y_i\right]x_i = 0. \tag{22}$$

The solution of (21) and (22) can be expressed by the following equations:

$$m = \frac{\sum_{i=1}^{n} x_i y_i - \frac{1}{n}\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2} \tag{23}$$

$$b = \frac{1}{n}\sum_{i=1}^{n} y_i - \frac{m}{n}\sum_{i=1}^{n} x_i = \bar{y} - m\bar{x}. \tag{24}$$

Let

$$S_{xy} = \sum_{i=1}^{n} x_i y_i - \frac{1}{n}\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i \tag{25}$$

and

$$S_{xx} = \sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2. \tag{26}$$

Finally, m and b can be derived by the help of (25) and (26) as

$$m = \frac{S_{xy}}{S_{xx}} \tag{27}$$

$$b = \bar{y} - \frac{S_{xy}}{S_{xx}}\bar{x}. \tag{28}$$

This method is used frequently to estimate the locations of the user because of its simplicity and less complexity.

B. Computational Complexity and Hardware Costs of the LSL

In the analysis below, N_{AP} shows the number of APs and N_{TP} corresponds to the number of test points in the measurement area. For calculating the complexity, the number of APs value is determined as 4 (the optimum value).

$$O\left(21N_{\text{AP}}N_{\text{TP}} + 18N_{\text{TP}} + \sqrt{N_{\text{TP}}} + 41\right) \tag{29}$$

$$O\left(102N_{\text{TP}} + \sqrt{N_{\text{TP}}} + 41\right). \tag{30}$$

Considering, $N_{\text{TP}} \gg \sqrt{N_{\text{TP}}}$ and $N_{\text{TP}} \gg 41$ because $N_{\text{TP}} = 196$ for optimum situations.

Then complexity becomes as follows:

$$O(102N_{\text{TP}}) = O(N_{\text{TP}}) = O(N). \tag{31}$$

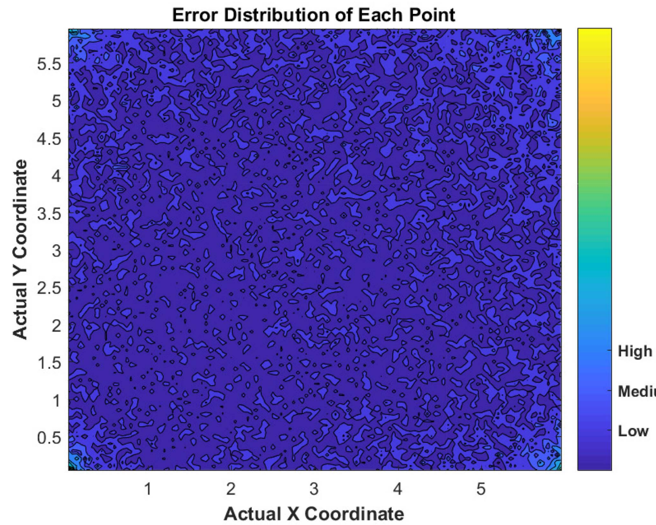

Fig. 1. Error measurements of each point.

According to the analysis, the computational complexity of the proposed LSL method, it only depends on the number of the test points in the measurement place.

According to Bisio *et al.* [17], when RSS-based techniques (lateration and fingerprinting) are compared with other IPS algorithms (TOA, TDOA, AOA, or ultrawide band), RSS-based methods can be applied only with a Wi-Fi integrated device (smartphone, tablet, and PC) without any redundant hardware. This remark shows that lateration and fingerprinting-based localization algorithms are cheap and easy to deploy in real-time applications. Moreover, RSSI-based lateration methods are robust in dynamic situations as well as having low complexity because they use only real-time RSSI measurements without requiring any change to the WLAN IEEE 802.11 network.

V. Simulation Environment and Parameters

The main purpose of this study is to compare the performance of lateration and LSL under different simulation environments, such as different grid sizes, varying Gaussian noise, and different number of APs in terms of mean and maximum accuracy errors. The simulations are conducted on MATLAB platform. To obtain scalable and robust results, the simulations are supported with 100 independent iterations.

VI. Results of the Simulations

A. Investigating Accuracy Error Performance of Each Measurement Point

Here, as shown in Fig. 1, by using four APs with 0.05-m step size, 141 61 test points are generated. The Gaussian noise that added to RSSI measurements has zero mean and standard deviation of 2 (low noise). This figure shows the measurement accuracy performance of each point. For every point, it is seen that low error is obtained almost all parts of the measurement area. The accuracy error increases on the points which are very close to APs which are located corners of the measurement area.

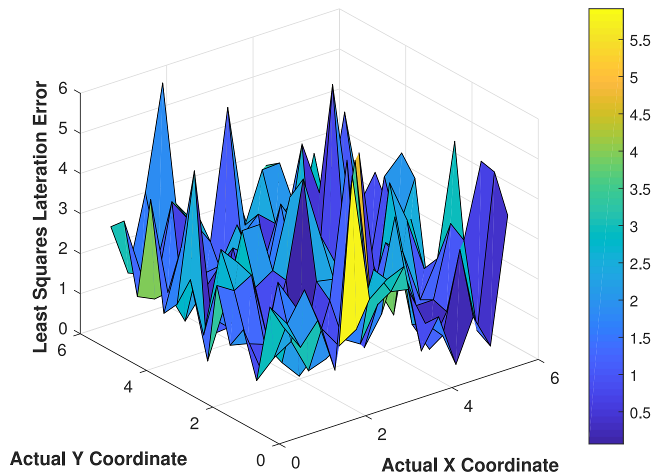

Fig. 2. 3-D least-square lateration error of four APs under high noise.

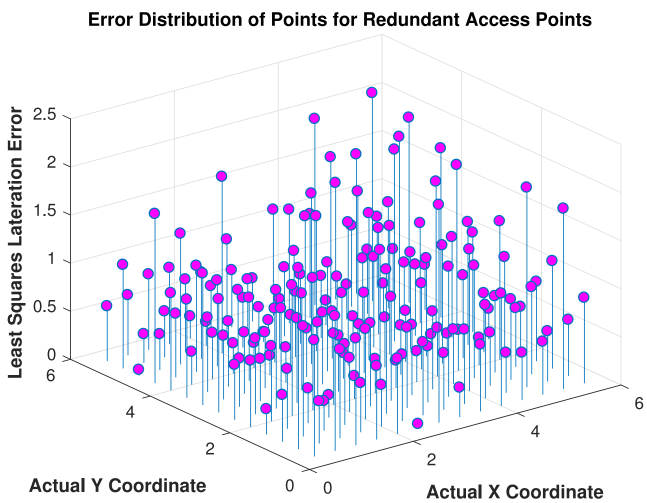

Fig. 3. Least-square lateration error performance of each point for six APs under low noise.

Fig. 2 illustrates the 3-D error distribution of each points in the measurement area for four APs and 196 measurements points under the Gaussian noise which has zero mean and standard deviation of 5 (high noise). According to this situation, average error is obtained as 2.31 m via least-square lateration and 5.45 m via PL.

In Fig. 3, the accuracy error of each point for the redundant number of APs (six APs) and 196 measurement points of the LSL method is shown. Under low Gaussian noise, the obtained average error performance of this method is approximately 0.9 m.

To investigate the effect of Gaussian error on each point of the LSL method in the measurement area, Fig. 4 is obtained. When the Gaussian error has standard deviation of 5, the number of black points that have the error value greater than 2 m increases in the measurement area.

When we decrease the Gaussian error to the standard deviation of 2, the number of black points significantly decreases in the area as shown in Fig. 5.

The actual locations and the estimated locations of the users for lateration and LSL methods are illustrated in Fig. 6. It is

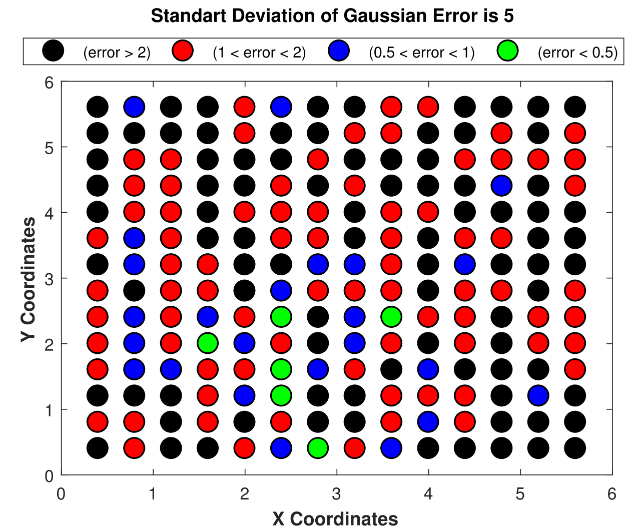

Fig. 4. Error performance of each point under high noise.

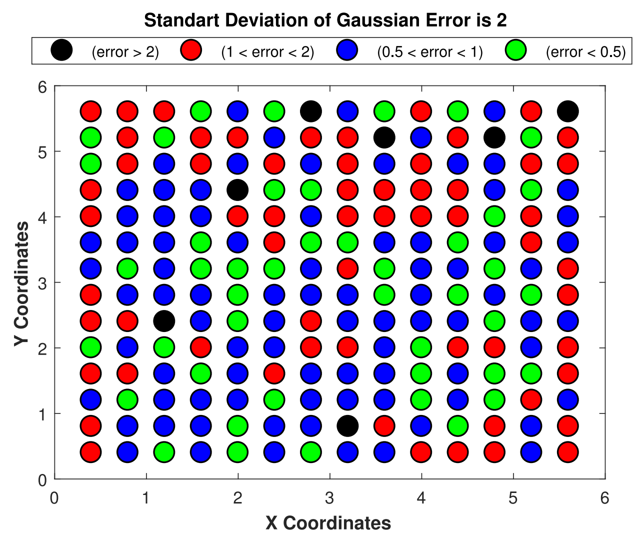

Fig. 5. Error performance of each point under low noise.

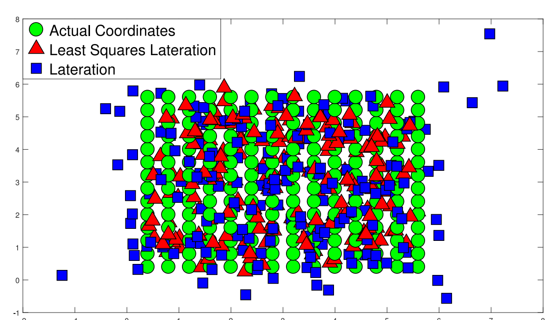

Fig. 6. Actual and estimated locations.

obtained that, LSL provides better performance than lateration for estimating the actual coordinates of the users.

B. Iteration-Based Results of Least-Squares Lateration

For least-squares estimation, the iteration-based approach is investigated in Fig. 7 to verify the analysis. For 100 independent iterations, the estimation process is performed again for

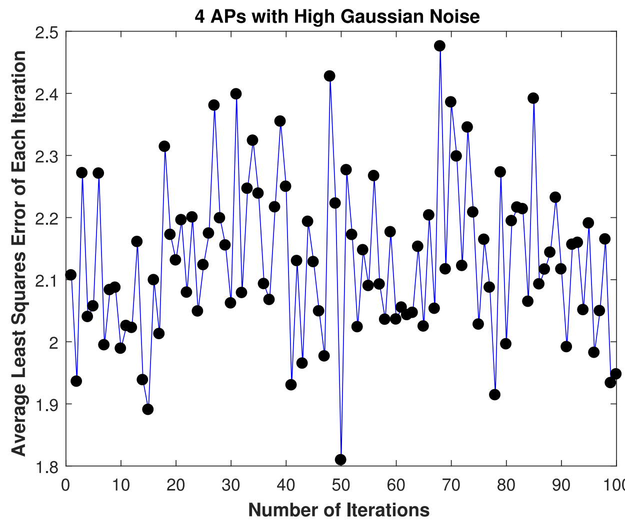

Fig. 7. Iteration-based least-square lateration error performance of four APs under high noise.

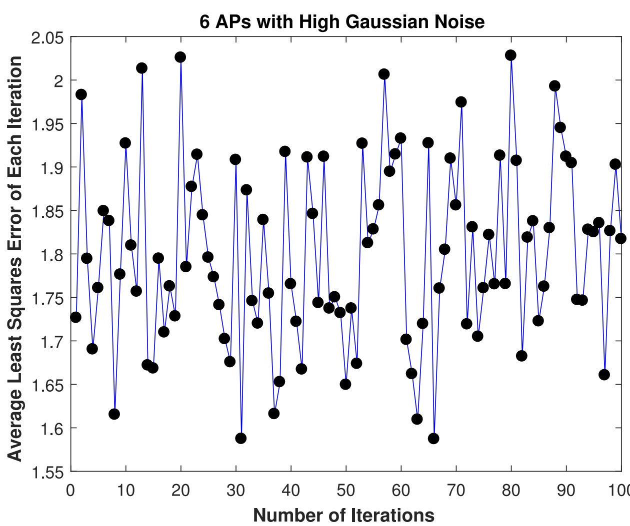

Fig. 8. Iteration-based least-square lateration error performance of six APs under high noise.

each iteration and average error of each iteration is recorded. For every iteration, the obtained average error is plotted with this figure. The average error result of 100 iterations is obtained as 2.35 m under high Gaussian noise. Note that, for PL, this average value is approximately 5 m under 100 iterations.

In Fig. 8, the results of the iteration-based LSL algorithm are shown. Differ from Fig. 4, it has six APs that are located in the measurement area. Here, it is aimed to determine the effect of number of APs on the accuracy of LSL. The average error result of all iterations is obtained as 1.99 m. Thus, when we increase the number of APs in the room, the accuracy performance of the estimation algorithm increases.

The average error value of each iteration is aimed to be shown via Fig. 9. The maximum average error is obtained as approximately 1.1 m under low noise for four APs.

To investigate and understand the effect of increase in the number of APs for LSL iterations, Fig. 10 is obtained. In Fig. 10, six APs are located in the measurement area. The

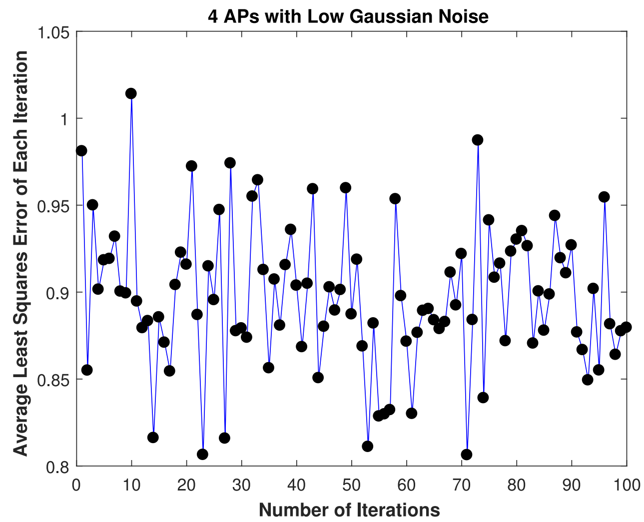

Fig. 9. Average error illustration of least squares for four APs under low noise.

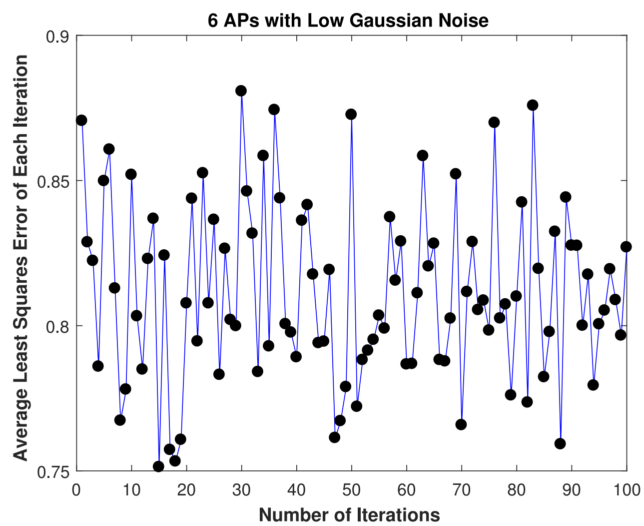

Fig. 10. Average error illustration of least squares for six APs under low noise.

maximum average estimation of this situation is approximately 0.9 m. For the same parameters, increasing the number of APs in the room provides to enhance the accuracy performance of the LSL.

C. Iteration-Based Comparisons of Least-Squares Lateration and Lateration

Fig. 11 illustrates the comparison of average error performance of each iteration of PL and LSL methods under high noise (standard deviation of 5) for four APs and 196 measurement points according to 100 iterations. LSL has significantly less average error than lateration according to this figure. It is obtained that, LSL provides significant accuracy performance for IPSs.

In Fig. 12, the comparison of the average error performance of PL and LSL based on 100 independent iterations is illustrated for four APs and 196 test points. Results demonstrate that, LSL has still significantly less average error than lateration under low noise conditions. It is obtained that LSL

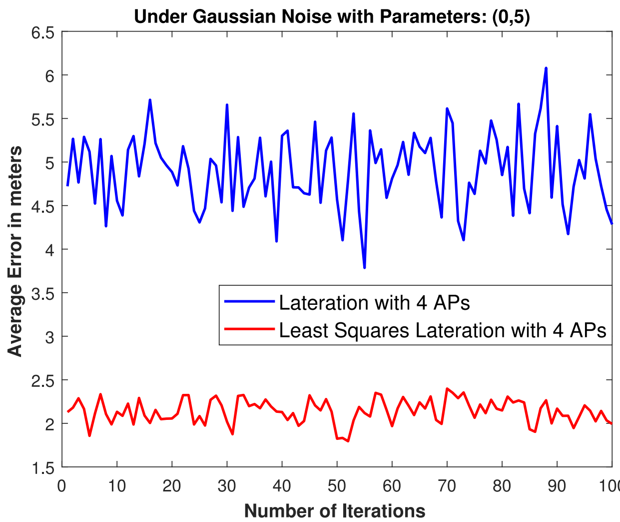

Fig. 11. Comparison of error performance of two algorithms under high noise.

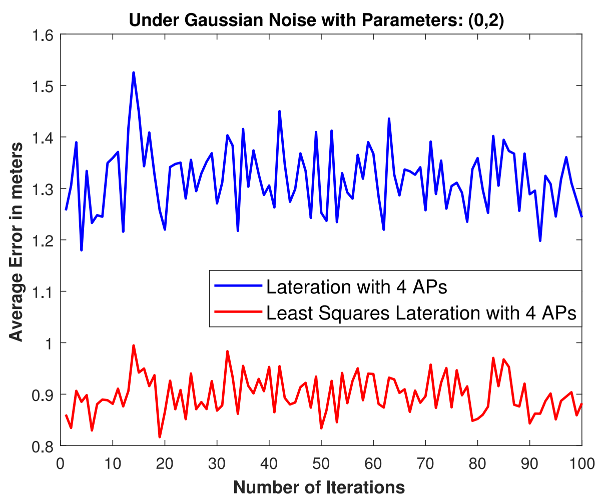

Fig. 12. Comparison of error performance of two algorithms under low noise.

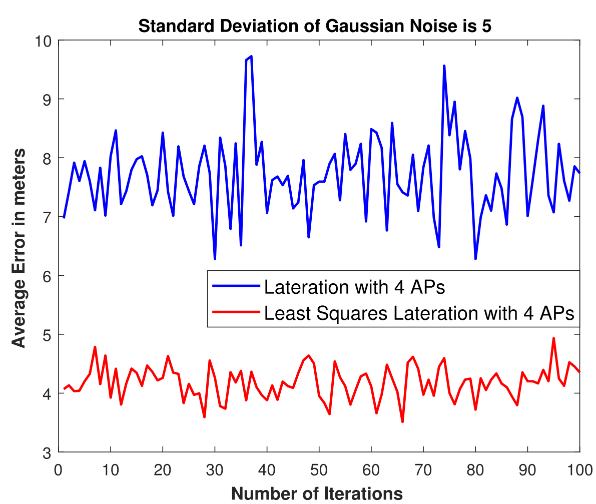

Fig. 13. Comparison of two methods for large-scale region.

provides significant accuracy performance for IPSs. Note that, the average accuracy error value of 100 iterations of LSL is approximately 0.8952 m.

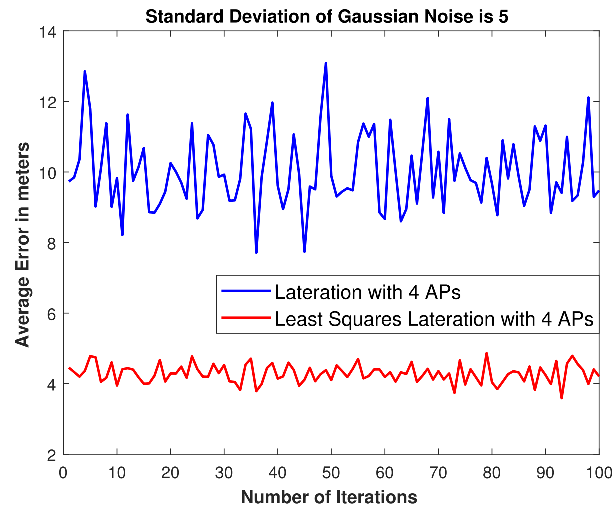

Fig. 14. Comparison of two methods for large-scale region with lower number of test points.

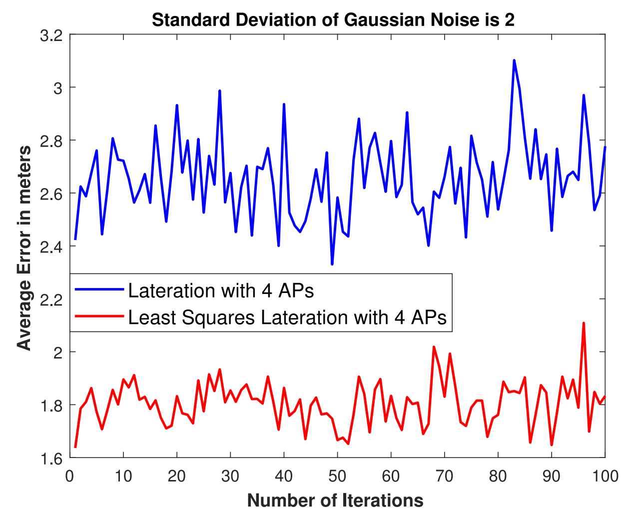

Fig. 15. Comparison of two methods for large-scale region with lower number of test points under lower Gaussian noise.

D. Iteration-Based Comparisons for Large-Scaled Regions

To determine the effect of large regions on the performance of the methods, Fig. 13 is obtained. The measurement area of this simulation is 12 m $\times$ 12 m. Here, 841 measurement points and same step-size parameter which is 0.4 m are taken into consideration. Under high Gaussian noise, for 100 independent iterations, least-squares method has 4.30-m average error, on the other hand, this value for PL is 10.24 m. When the dimensions of the field increases, LSL becomes more accurate and scalable when it is compared with lateration.

To make a fair comparison with 6 m $\times$ 6 m dimensions and obtain 196 test points under same noise parameter, the step-size parameter is determined as 0.8 m to obtain Fig. 14. According to this result, LSL provides significant accuracy performance enhancement when it is compared with lateration.

For 12 m $\times$ 12 m measurement area and same number of test points (196) of 6 m $\times$ 6 m area, under lower Gaussian noise (standart deviation of 2), from LSL for 100 iterations, 1.79-m average accuracy error is obtained as shown

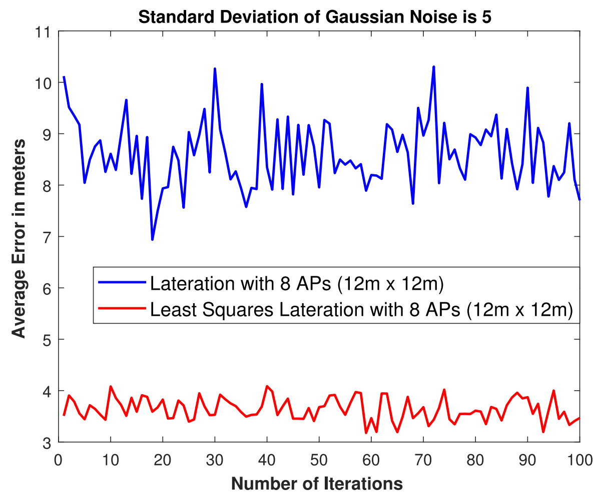

Fig. 16. Performance of two algorithms for large-scale area with redundant APs under high Gaussian noise.

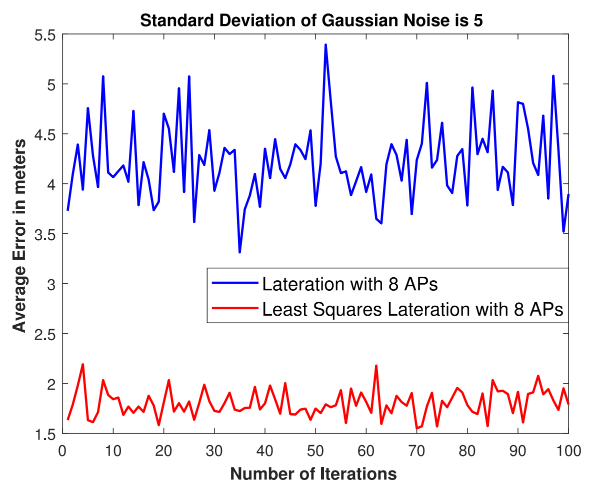

Fig. 17. Comparison of two methods for eight APs under high Gaussian noise.

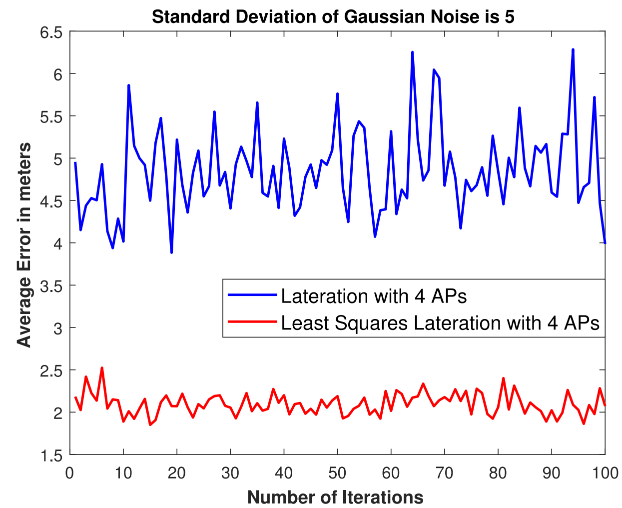

Fig. 18. Comparison of two methods for four APs under high Gaussian noise.

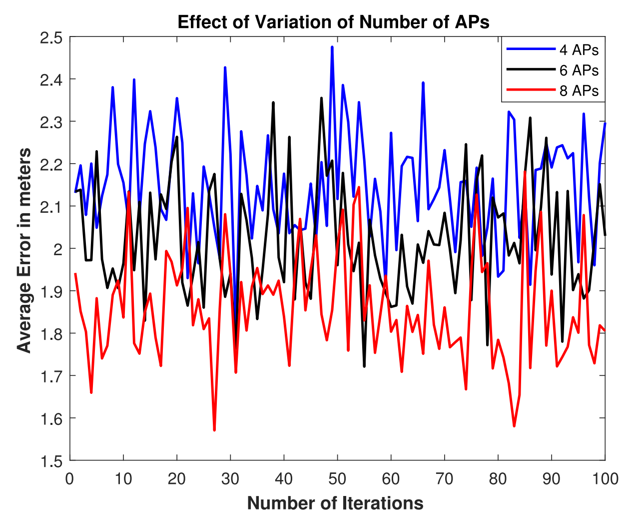

Fig. 19. Effect of the varying number of APs on the performance of accuracy.

in Fig. 15. Note that, for same parameters, average accuracy error performance of the least-squares method in 6 m $\times$ 6 m area is approximately 0.9 m. Increasing the dimensions of the field, increases the error, however, it does not highly affect the performance of LSL.

To investigate the impact of the increase in the dimensions of the field on the performance of accuracy error, Fig. 16 is obtained. Enlarging the dimensions of the measurement place and increasing the number of APs cause to enhance the error performance of the algorithms as expected. For the large-scale fields, increasing the number of APs provides to decrease the accuracy error of lateration methods.

E. Effect of Varying Number of APs

To investigate the effect of the number of APs on PL and LSL, Figs. 17 and 18 are obtained. When the number of APs is increased to 8 in the area of 6 m $\times$ 6 m with 196 test points under 100 iterations, 1.85-m average accuracy error is obtained with LSL under high Gaussian noise. It is obtained from this analysis when the number of APs is increased, the accuracy error significantly decreases.

Here, with the same parameters of Fig. 17 for four APs, the average accuracy error of iterations is obtained as 2.16 m. By increasing the number of APs, approximately 0.3-m improvement is obtained in accuracy error performance.

Fig. 19 illustrates the comparison of LSL in terms of varying number of APs. The performance of the method increases when the number of APs increases. An increase in the number of APs provides accuracy improvement, however, it causes an increase in the cost and complexity of the system.

Fig. 20 illustrates the error accuracy performance of both algorithms under varying Gaussian noise for worst case (three APs) and best case (eight APs) scenarios. Best accuracy performance is achieved with eight APs using LSL. When the noise increases, the accuracy performances of both algorithms decrease as expected. To obtain best results, it is needed to increase the number of APs in the measurement area.

The measurement area is increased to 144 m^2 in Fig. 21 to investigate the effect of enlarging the measurement area. In Fig. 20, with eight APs, the error of LSL under low noise is lower than 1 m. However, for 12 m $\times$ 12 m area, this value

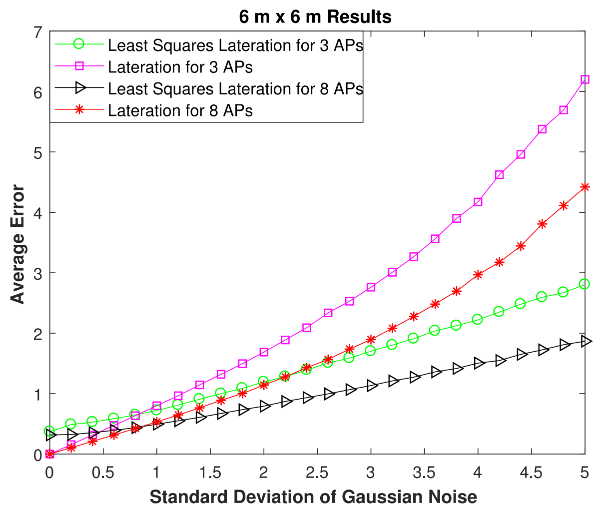

Fig. 20. Error performance comparison of PL and LSL for (three APs) and (eight APs) under varying Gaussian noise.

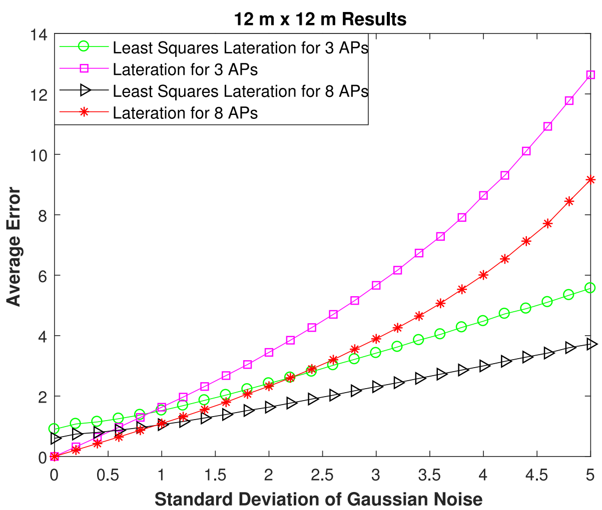

Fig. 21. Error performance comparison of PL and LSL for (three APs) and (eight APs) under varying Gaussian noise (12 m × 12 m).

becomes approximately 1.5 m. It is obtained from these figures that, when the area is enlarged, the error performance of two algorithms become worse.

If the measurement area is quadrupled according to 12 m × 12 m, Fig. 22 is obtained. The value that is considered for this case is approximately 3 m. Based on these three figures, it is obtained that, the increase in the measurement area causes the increase in the accuracy error of two methods.

Fig. 23 shows the accuracy performance of LSL and PL in terms of four APs and six APs for varying Gaussian noise in 6 m × 6 m area. With six APs, better accuracy performance is obtained in two methods.

Table III illustrates the accuracy error (in meters) performance of both approaches in terms of varying number of APs under low Gaussian noise. Increasing the number of APs significantly decreases the accuracy error. Especially, the best improvement is obtained when the number of APs is increased from three to four.

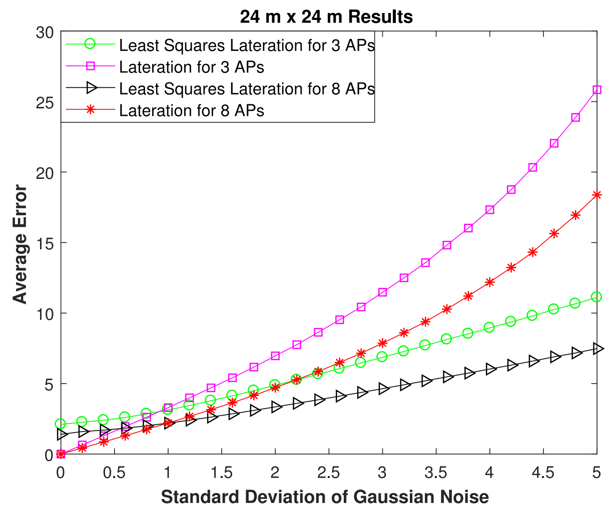

Fig. 22. Error performance comparison of PL and LSL for (three APs) and (eight APs) under varying Gaussian noise (24 m × 24 m).

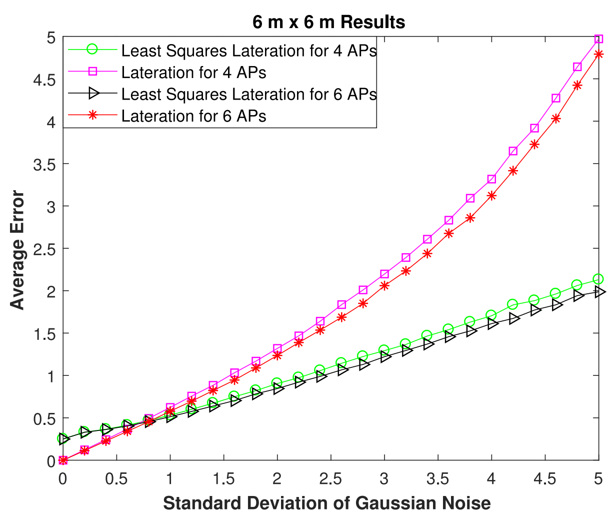

Fig. 23. Error performance comparison of PL and LSL for (four APs) and (six APs) under varying Gaussian noise.

TABLE III
EFFECT OF VARYING NUMBER OF APS ON ACCURACY WITH LOW NOISE

Number of APs	Lateration	Least Squares Lateration
3 APs	1.6858	1.1892
4 APs	1.3144	0.9019
6 APs	1.2287	0.8477
8 APs	1.1301	0.7893

Under higher Gaussian noise (standard deviation of 5), there is a significant performance improvement between lateration and LSL as shown in Table IV. Here, it is obtained that under high noise LSL provides significant error accuracy, therefore it is very suitable for environments that have high noise.

To investigate the effect of the number of grids that are used in the measurement area on the accuracy performance of LSL, Table V is obtained by conducting 100 iterations in 6 m × 6 m area. Increasing the number of test points in the measurement area causes to decrease accuracy error performance of LSL.

Finally, to confirm the effect of noise on the performance of both methods, Table VI is obtained by conducting 100 iterations. As expected, when the Gaussian noise increases, the

TABLE IV
EFFECT OF VARYING NUMBER OF APs ON ACCURACY WITH HIGH NOISE

Number of APs	Lateration	Least Squares Lateration
3 APs	6.1902	2.8100
4 APs	4.9522	2.1504
6 APs	4.7281	2.0424
8 APs	4.4178	1.8619

TABLE V
EFFECT OF VARYING NUMBER OF GRIDS ON ACCURACY WITH HIGH NOISE

Accuracy Error of LSL	Number of Test Points	Step Size
2.3518	49	0.8
2.1834	81	0.6
2.1640	196	0.4
2.1303	841	0.2
2.1273	3364	0.15

TABLE VI
EFFECT OF NOISE ON ACCURACY PERFORMANCE

Gaussian Noise	Lateration Error	LSL Error
1	0.6216	0.5357
1.5	0.9622	0.7084
2	1.3247	0.8992
3.5	2.2142	1.3079
5	4.9091	2.1467

accuracy performance of both methods decreases. However, when the noise increases, LSL provides better performance improvement when it is compared with PL.

VII. CONCLUSION

Determining the positions of the objects in indoor places has been a considerable research topic nowadays. Localization approaches can be used in numerous fields, such as supermarket applications, social media, logistics, tourism sector, traveling, and information services. On the other hand, the GPS technology designed for outdoor positioning applications, is not suitable for IPSs. Accomplishing exact position detection is a compelling problem for indoor positioning methods. In this article, the RSS-based LSL method that utilizes existing infrastructure, is proposed. By augmenting the number of APs and implementing line fitting methods to the RSS values, the triangulation method enhances the certainty of position estimation. The utilization of the existing substructure makes the designed method cheaper comparing with existing localization methods that require expensive components. The proposed least-squares triangulation approach is compared with PL in terms of accuracy error under different Gaussian noise parameters for varying number of APs and varying dimensions of the measurement area. Usage of the least-square algorithm with line fitting approaches provides significant performance improvements for all cases when it compared with PL. As future work, it is planned to use polynomial fitting algorithms to enhance the accuracy of the LSL approach.

REFERENCES

[1] H. Koyuncu and S. Yang, "A survey of indoor positioning and object locating systems," *Int. J. Comput. Sci. Netw. Security*, vol. 10, no. 5, pp. 121–128, May 2010.

[2] S. Gezici, "A survey on wireless position estimation," *Wireless Pers. Commun.*, vol. 44, no. 3, pp. 263–282, Jan. 2008.

[3] S. Saha, K. Chaudhuri, D. Sanghi, and P. Bhagwat, "Location determination of a mobile device using IEEE 802.11b access point signals," in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, Mar. 2003, pp. 1987–1992.

[4] W. Sakpere, O. Adeyeye, and N. Mlitwa, "A state-of-the-art survey of indoor positioning and navigation systems and technologies," *South African Comput. J.*, vol. 29, no. 3, pp. 145–197, Dec. 2017.

[5] A. Yassin *et al.*, "Recent advances in indoor localization: A survey on theoretical approaches and applications," *IEEE Commun. Surveys Tuts.*, vol. 19, no. 2, pp. 1327–1346, 2nd Quart., 2017.

[6] D. Zhang *et al.*, "Localization technologies for indoor human tracking," in *Proc. 5th Int. Conf. Future Inf. Technol.*, May 2010, pp. 1–6.

[7] A. Khudhair *et al.*, "Wireless indoor localization systems and techniques: Survey and comparative study," *Indonesian J. Elect. Eng. Comput. Sci.*, vol. 3, no. 2, pp. 392–412, Aug. 2016.

[8] T. Mounir *et al.*, "Positioning system for emergency situation based on RSSI measurements for WSN," in *Proc. Int. Conf. Perform. Eval. Model. Wired Wireless Netw. (PEMWN)*, Nov. 2017, pp. 1–6.

[9] Y. Lee, "Weighted-average based AOA parameter estimations for LR-UWB wireless positioning system," *IEICE Trans. Commun.*, vol. 94, no. 12, pp. 3599–3602, Dec. 2011.

[10] N. Alsindi, B. Alavi, and K. Pahlavan, "Spatial characteristics of UWB TOA based ranging in indoor multipath environments," in *Proc. 18th IEEE Int. Symp. Pers. Indoor Mobile Radio Commun.*, Sep. 2007, pp. 1–6.

[11] N. Alsindi, B. Alavi, and K. Pahlavan, "Measurement and modeling of ultrawideband TOA-based ranging in indoor multipath environments," *IEEE Trans. Veh. Technol.*, vol. 58, no. 3, pp. 1046–1058, Mar. 2009.

[12] N. Reddy and B. Sujatha, "TDOA computation using multicarrier modulation for sensor networks," *Int. J. Comput. Sci. Commun. Netw.*, vol. 1, no. 1, pp. 85–90, Sep. 2011.

[13] H. Obeidat *et al.*, "Indoor localization using received signal strength," in *Proc. 8th IEEE Design Test Symp.*, Dec. 2013, pp. 1–6.

[14] M. Youssef, A. Agrawala, and A. U. Shankar, "WLAN location determination via clustering and probability distributions," in *Proc. IEEE Int. Conf. Pervasive Comput. Commun. (PerCom)*, Mar. 2003, pp. 143–150.

[15] P. Bahl and V. N. Padmanabhan, "RADAR: An in-building RF-based user location and tracking system," in *Proc. IEEE INFOCOM*, Mar. 2000, pp. 775–784.

[16] C. Song and J. Wang, "WLAN fingerprint indoor positioning strategy based on implicit crowdsourcing and semi-supervised learning," *ISPRS Int. J. Geo Inf.*, vol. 6, no. 11, pp. 356–366, Nov. 2017.

[17] I. Bisio, F. Lavagetto, M. Marchese, and A. Sciarrone, "Smart probabilistic fingerprinting for WiFi-based indoor positioning with mobile devices," *Pervasive Mobile Comput.*, vol. 31, no. 2016, pp. 107–123, Sep. 2016.

[18] O. Baala, Y. Zheng, and A. Caminada, "The impact of AP placement in WLAN-based indoor positioning system," in *Proc. 8th Int. Conf. Netw.*, Mar. 2009, pp. 12–17.

[19] X. Hu, J. Shang, F. Gu, and Q. Han, "Improving Wi-Fi indoor positioning via AP sets similarity and semi-supervised affinity propagation clustering," *Int. J. Distrib. Sensor Netw.*, vol. 11, no. 1, pp. 1–11, Jan. 2015.

[20] W. Xue, K. Yu, X. Hua, Q. Li, W. Qiu, and B. Zhou, "APs' virtual positions-based reference point clustering and physical distance-based weighting for indoor Wi-Fi positioning," *IEEE Internet Things J.*, vol. 5, no. 4, pp. 3031–3042, Aug. 2018.

[21] X. Du, K. Yang, and D. Zhou, "MapSense: Mitigating inconsistent Wi-Fi signals using signal patterns and pathway map for indoor positioning," *IEEE Internet Things J.*, vol. 5, no. 6, pp. 4652–4662, Dec. 2018.

[22] Z. Li, X. Zhao, F. Hu, Z. Zhao, J. L. Carrera Villacrés, and T. Braun, "SoiCP: A seamless outdoor–indoor crowdsensing positioning system," *IEEE Internet Things J.*, vol. 6, no. 5, pp. 8626–8644, Oct. 2019.

[23] J. Hu, D. Liu, Z. Yan, and H. Liu, "Experimental analysis on weight K-nearest neighbor indoor fingerprint positioning," *IEEE Internet Things J.*, vol. 6, no. 1, pp. 891–897, Feb. 2019.

[24] D. Feng, C. Wang, C. He, Y. Zhuang, and X. Xia, "Kalman-Filter-based integration of IMU and UWB for high-accuracy indoor positioning and navigation," *IEEE Internet Things J.*, vol. 7, no. 4, pp. 3133–3146, Apr. 2020.

[25] H. Yang, W. Zhong, C. Chen, A. Alphones, and P. Du, "QoS-driven optimized design-based integrated visible light communication and positioning for indoor IoT networks," *IEEE Internet Things J.*, vol. 7, no. 1, pp. 269–283, Jan. 2020.

[26] Y. Zhao, W. Wong, T. Feng, and H. K. Garg, "Efficient and scalable calibration-free indoor positioning using crowdsourced data," *IEEE Internet Things J.*, vol. 7, no. 1, pp. 160–175, Jan. 2020.

[27] I. Bisio, C. Garibotto, A. Grattarola, F. Lavagetto, and A. Sciarrone, "Exploiting context-aware capabilities over the Internet of Things for industry 4.0 applications," *IEEE Netw.*, vol. 32, no. 3, pp. 101–107, May 2018.

[28] L. Chen, I. Ahriz, and D. L. Ruyet, "CSI-based probabilistic indoor position determination: An entropy solution," *IEEE Access*, vol. 7, pp. 170048–170061, 2019.

[29] Y. Chen, W. Guan, J. Li, and H. Song, "Indoor real-time 3-D visible light positioning system using fingerprinting and extreme learning machine," *IEEE Access*, vol. 8, pp. 13875–13886, 2020.

[30] S. Yang, L. Ma, S. Jia, and D. Qin, "An improved vision-based indoor positioning method," *IEEE Access*, vol. 8, pp. 26941–26949, 2020.

[31] W. C. S. S. Simões, W. A. E. Silva, M. M. D. Lucena, N. Jazdi, and V. F. D. Lucena, "A Hybrid Indoor Positioning System Using a Linear Weighted Policy Learner and Iterative PDR," *IEEE Access*, vol. 8, pp. 43630–43656, 2020.

[32] C. Cheng, T. Wang, and Y. Huang, "Indoor positioning system using artificial neural network with swarm intelligence," *IEEE Access*, vol. 8, pp. 84248–84257, 2020.

[33] S. Mazuelas *et al.*, "Robust indoor positioning provided by real-time RSSI values in unmodified WLAN networks," *IEEE J. Sel. Topics Signal Process.*, vol. 3, no. 5, pp. 821–831, Oct. 2009.

[34] Y. Gwon and R. Jain, "Error characteristics and calibration-free techniques for wireless lan-based location estimation," in *Proc. MobiWac*, Oct. 2004, pp. 2–9.

[35] M. Shchekotov, "Indoor localization methods based on Wi-Fi lateration and signal strength data collection," in *Proc. 17th Conf. Open Innov. Assoc. (FRUCT)*, Apr. 2015, pp. 186–191.

[36] O. Pathak, P. Palaskar, R. Palkar, and M. Tawar, "Wi-Fi indoor positioning system based on RSSI measurements from Wi-Fi access points—A Tri-lateration approach," *Int. J. Sci. Eng. Res.*, vol. 5, no. 4, pp. 1234–1238, Apr. 2014.

[37] M. E. Rusli, M. Ali, N. Jamil, and M. M. Din, "An improved indoor positioning algorithm based on RSSI-trilateration technique for Internet of Things (IOT)," in *Proc. Int. Conf. Comput. Commun. Eng. (ICCCE)*, Jul. 2016, pp. 72–77.

[38] L. Machado, M. Larrat, and D. Monteiro, "Performance evaluation of lateration, KNN and artificial neural networks techniques applied to real indoor localization in WSN," in *Proc. 2nd Int. Conf. Commun. Comput. Netw. Technol. (INNOV)*, Nov. 2013, pp. 1–6.

[39] F. Morgado, P. Martins, and F. Caldeira, "Beacons positioning detection, a novel approach," *Procedia Comput. Sci.*, vol. 151, no. 2019, pp. 23–30, Apr. 2019.

[40] J. Yang, Y. Li, and W. Cheng, "An improved geometric algorithm for indoor localization," *Int. J. Distrib. Sensor Netw.*, vol. 14, no. 3, pp. 1–13, Mar. 2018.

[41] A. E. M. El Ashry, and B. I. Sheta, "Wi-Fi based indoor localization using trilateration and fingerprinting methods," in *Proc. 18th Int. Conf. Aerosp. Sci. Aviation Technol.*, Oct. 2019, pp. 1–6.

[42] J. C. A. Herrera, M. Flores, and A. Ramos, "Comparative results on fingerprinting and tri-lateration positioning methods for hybrid indoor localization," in *Proc. Int. Conf. Indoor Positioning Indoor Navigation (IPIN)*, Oct. 2015, pp. 1–2.

[43] K. Huang, K. He, and X. Du, "A hybrid method to improve the BLE-based indoor positioning in a dense bluetooth environment," *Sensors*, vol. 19, no. 2, pp. 424–439, Jan. 2019.

[44] G. Retscher, "Fusion of location fingerprinting and trilateration based on the example of differential Wi-Fi positioning," *Annal. Photogrammetry Remote Sens. Spatial Inf. Sci.*, vol. IV-2, no. W4, pp. 377–384, Sep. 2017.

[45] J. Miranda *et al.*, "Path loss exponent analysis in wireless sensor networks: Experimental evaluation," in *Proc. 11th IEEE Int. Conf. Ind. Informat. (INDIN)*, Jul. 2013, pp. 1–5.

[46] Y. Hu and G. Leus, "Self-estimation of path loss exponent in wireless networks and applications," *IEEE Trans. Veh. Technol.*, vol. 64, no. 11, pp. 5091–5102, Nov. 2015.

[47] G. Wang *et al.*, "On received-signal-strength based localization with unknown transmit power and path loss exponent," *IEEE Wireless Commun. Lett.*, vol. 1, no. 5, pp. 536–539, Oct. 2012.

[48] S. Y. Seidel and T. S. Rappaport, "914MHz path loss prediction models for indoor wireless communications in multifloored buildings," *IEEE Trans. Antennas Propag.*, vol. 40, no. 2, pp. 207–217, Feb. 1992.

[49] D. Dardari, A. Conti, U. Ferner, A. Giorgetti, and M. Z. Win, "Ranging with ultrawide bandwidth signals in multipath environments," *Proc. IEEE*, vol. 97, no. 2, pp. 404–426, Feb. 2009.

[50] J. Xu, W. Liu, F. Lang, Y. Zhang, and C. Wang, "Distance measurement model based on RSSI in WSN," *Wireless Sensor Netw.*, vol. 2, no. 8, pp. 606–616, Aug. 2010.

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