Reference Selection for Hybrid TOA/RSS Linear Least Squares Localization

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***Abstract*—Linear least squares (LLS) estimation is a sub-optimum but low-complexity localization method based on measurements of location-related parameters. It has been proved that selection of the reference anchor influences the LLS localization accuracy. In addition, hybridization of different types of measurements can fix the deficiencies of one type of measurements. In this paper, we proposed a new reference selection criterion for the hybrid TOA/RSS LLS localization technique (called *H-LLS-RS*), which considers both measured ranges and the information about their coarse variances. Moreover, we consider a general scenario that variances of range measurements are different, and derive a weighted LLS (WLLS) estimator for hybrid TOA/RSS localization according to the information about the accurate ranging variances and the correlations among the observations. Simulation results show that if the RSS-based ranging variances are considerably larger than the TOA-based ranging variances, the *H-LLS-RS* localization technique yields better accuracy than the conventional LLS localization techniques. Furthermore, reference selection has no effect on the accuracy of WLLS localization technique.**

Keywords*—hybrid localization, received-signal-strength (RSS), reference selection, time-of-arrival (TOA), ultra-wideband (UWB), weighted linear least squares (WLLS).

I. INTRODUCTION

Real-time and high-accuracy position information is essential to a variety of wireless applications [1]. In two-step localization method [2], the first step is to estimate location-related parameters, such as range and angle information, and the second step is to estimate location of the agent based on these location-related parameters using geometric and statistical techniques. Specifically, range information is usually adopted in the first step of localization and it can be measured from time-of-arrival (TOA) or received-signal-strength (RSS) estimates [3]. For the second step of range-based localization, it is well known that the maximum likelihood (ML) estimator can be adopted if the variances of range measurements are available and it is an optimum estimator since it can asymptotically achieve the Cramer-Rao lower bound (CRLB) for high signal-to-noise ratios (SNRs) and/or large signal bandwidths [3]. If the information about the measured range variances is unavailable, a nonlinear least squares (NLS) estimator can be obtained by assuming the variances are identical and using uniform weighting. Solving the NLS problem requires an explicit minimization of a nonlinear cost function, which cannot in general be solved analytically. Therefore, numerical search methods, such as the Newton-Raphson, the Gauss-Newton, or the steepest descent techniques, are usually employed to approximate the NLS estimate. Such techniques may be computationally intensive and they typically require good initialization to avoid erroneously converging to the local minima of the cost function [1, Ch. 3].

In order to obtain a closed form solution and avoid explicit minimization of the NLS cost function, the nonlinear expressions of observations can be linearized using the least squares (LS) calibration method [4]. An alternative way for linear least squares (LLS) solution based on subtraction of the reference measured range is initially proposed in [5] and its reference selection issue is discussed in [6], where the shortest measured range among all the range measurements is selected as the reference. Moreover, a maximum likelihood estimator is proposed to further improve the LLS localization accuracy and it assumes all the measurement variances are the same [6]. In addition, the LLS localization performance for both line-of-sight (LOS) and non-line-of-sight (NLOS) cases is studied in [6], [7]. Although LLS estimation is a sub-optimum localization technique [8], it usually has a reasonable localization accuracy and lower implementation complexity compared with iterative techniques (e.g., ML and NLS approaches), which is very important for self-localization of nodes in internet of things networks and wireless sensor networks due to their constraints on power consumption, signal processing capability, cost and so on. Moreover, LLS localization technique can be used to provide an initial location estimate for high-accuracy localization techniques [9], such as the ML approach, which may considerably decrease their computational complexity and prevent large localization error. Therefore, LLS localization technique carries importance from multiple perspectives.

Hybrid localization techniques [10], [11], such as the hybrid TOA/RSS technique, are proposed to enhance localization accuracy and fix the deficiencies of one type of measurements, such as the lack of measurements to uniquely localize the agent. Although the variances of the measurements for closer ranges between the agent and the anchor is likely smaller than that for non-hybrid localization techniques, it is possible that the variance of the shortest range measurement based on one type ranging technique is considerably larger than some longer range measurements based on another type ranging technique, e.g., RSS-based ranging commonly has larger variance than TOA-based ranging [3]. Therefore, the variances of range measurements should be included as a second criterion in selecting the reference for hybrid LLS localization techniques when their coarse information is available.

In this paper, we proposed a new reference selection criterion for the hybrid TOA/RSS LLS localization technique (called

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H-LLS-RS), which considers both measured ranges and the information about their coarse variances. Moreover, we consider a general scenario that the variances of range measurements are different, and derive a weighted LLS (WLLS) estimator for hybrid TOA/RSS localization according to the accurate ranging variances information and the correlations among the observations. Simulation results show that if the RSS-based ranging variances are considerably larger than the TOA-based ranging variances, the *H-LLS-RS* localization technique yields better accuracy than the traditional LLS localization techniques. Furthermore, reference selection has no effect on the accuracy of WLLS localization technique.

The rest of the paper is organized as follows. Section II introduces the system model. Section III briefly reviews the prior art reference selection techniques and proposes a new reference selection criterion. The WLLS location estimator is given in Section IV. Section V provides simulation results and discussions. A conclusion in Section VI wrap up this report.

II. System Model

We assume that the agent is connected to different anchors, which are able to measure the range between the agent and anchors via two types of parameters, i.e., TOA and RSS. Let N be the total number of all anchors in the hybrid localization system. Without loss of generality, we assume that the agent can measure TOA-based ranges from anchors with indexes $i \in \{1, 2, \ldots, S\}$ and measure RSS-based ranges from anchors with indexes $i \in \{S+1, S+2, \ldots, N\}$.

In the first step of two-step localization method, the range measurement between the agent and the ith $(i = 1, 2, \ldots, N)$ anchor is denoted as $\hat{d}_i$. Let $\mathbf{p} = [x\ y]^T$ be the unknown two-dimensional (2-D) position of the agent, which is to be estimated, and let $\mathbf{p}_i = [x_i\ y_i]^T$ be the known 2-D coordinate of the ith anchor. The error-free range between the agent and the ith anchor is given by

$$\begin{aligned} d_i &= \|\mathbf{p} - \mathbf{p}_i\| \\ &= \sqrt{(x - x_i)^2 + (y - y_i)^2}. \end{aligned}$$

The range measurement is modeled as

$$\hat{d}_i = d_i + n_i \tag{1}$$

where n_i is the ranging error in $\hat{d}_i$, which results from TOA or RSS estimation disturbance. It is assumed that $\{n_i\}$ are zero-mean independent Gaussian processes with variances $\{\sigma_i^2\}$.

Obtaining all the range estimates in (1) leads to the following inconsistent equations

$$(x - x_i)^2 + (y - x_i)^2 = \hat{d}_i^2,\ i = 1, 2, \ldots, N. \tag{2}$$

For TOA-based ranging, we can adopt two-way TOA ranging protocol [3] and the range can be calculated as

$$\hat{d} = c \cdot \frac{\hat{\tau}_{\text{RTT}} - \hat{\tau}_{\text{TAT}}}{2}$$

where c is the propagation speed of ranging signals, $\hat{\tau}_{\text{RTT}}$ is measured round-trip-time at the ranging start node, and $\hat{\tau}_{\text{TAT}}$ is measured turn-around-time at the ranging respond node. Ideally, the CRLB of mean square ranging error from the ith $(i \in \{1, 2, \ldots, S\})$ anchor in additive white Gaussian noise (AWGN) channel is [2]

$$\sigma^2_{i,\text{CRLB,TOA}} = \frac{c^2}{8\pi^2 \text{SNR}_i \beta^2}$$

where $\text{SNR}_i = \mathcal{E}_i / \mathcal{N}_0$ is SNR from the ith anchor with $\mathcal{E}_i$ denoting the received signal energy and $\mathcal{N}_0$ denoting the one-side power spectral density of AWGN, and β is the effective signal bandwidth defined by

$$\beta = \left(\frac{\int_{-\infty}^{+\infty} f^2 |S(f)|^2 df}{\int_{-\infty}^{+\infty} |S(f)|^2 df} \right)^{1/2}$$

with $S(f)$ denoting the Fourier transform of the ranging signal. When adopting ultra-wideband (UWB) signals for ranging, high-accuracy range measurements can be achieved even in harsh multipath environments, such as in buildings, in urban canyons, under tree canopies, and in caves [12].

For RSS-based ranging, a common model used to calculate range from path loss is given by

$$\bar{P}_r(d) = P_0 - 10\gamma \log_{10}(d/d_0) + S$$

where $\bar{P}_r(d)$ is the average received power in decibels at a distance d, P_0 is the received power in decibels at a short reference distance d_0, γ is the path-loss exponent that typically assumes values between 2 and 6, and S represents the large-scale fading variations (i.e., shadowing) in decibels, which is commonly modeled as a Gaussian random variable with zero mean and standard deviation σ_{sh}. The CRLB of mean square ranging error from the ith $(i \in \{S+1, S+2, \ldots, N\})$ anchor is [2]

$$\sigma^2_{i,\text{CRLB,RSS}} = \left(\frac{(\ln 10)\sigma_{\text{sh}} d_i}{10\gamma} \right)^2.$$

Although the relation between average received power and distance seems simple, it is quite challenging to obtain the exact relation between them in a practical wireless environment due to complicated propagation mechanisms such as reflection, scattering, and diffraction, which can cause significant fluctuations in RSS even over short distances and/or small time intervals. Therefore, RSS-based ranging commonly has low accuracy, which is also true for UWB systems. For example, in a LOS residential environment, modeled according to the IEEE 802.15.4a UWB channel model [13], with $\gamma = 1.79$ and $\sigma_{\text{sh}} = 2.22$ dB, the lower bound $\sigma_{i,\text{CRLB,RSS}}$ is about 2.86 m at $d_i = 10$ m.

In the second step of two-step localization method, we adopt the LLS localization technique based on subtraction of the reference measured range [5], [6] to convert the non-linear range measurements in (2) into linear models in $\mathbf{p}$ and give a close form location estimate $\hat{\mathbf{p}}$.

We choose the rth anchor as the reference anchor for example without loss of generality. By subtracting the non-linear expression of the rth anchor in (2) from the rest of the expressions and substituting (1) into (2), we have

$$\begin{aligned} &2(x_i - x_r)x + 2(y_i - y_r)y \\ &= \left(d_r^2 - d_i^2 - k_r + k_i\right) + \left(2d_r n_r - 2d_i n_i + n_r^2 - n_i^2\right) \end{aligned} \tag{3}$$

where we define $k_i \triangleq x_i^2 + y_i^2,\ i = 1, 2, \ldots, N$.

Rewrite (3) in matrix form, we have

$$\mathbf{A}\mathbf{p} = \mathbf{b} \tag{4}$$

where $\mathbf{b} = \mathbf{b}_c + \mathbf{b}_n$, $\mathbf{b}_c$ represents the constant components, and $\mathbf{b}_n$ represents the noisy components. Expressions of all parts in (4) are shown at the bottom of the page.

Given the linear model in (4), the LLS estimator to obtain the agent location is given by [14]

$$\hat{\mathbf{p}}_{\text{LLS}} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b}. \tag{5}$$

III. Reference Selection for LLS Localization

A. Prior Art Reference Selection Techniques

The simplest way for reference selection is random selection (call it *LLS-1*) [5], which arbitrarily selects one anchor as the reference to realize LLS localization. The second LLS approach (call it *LLS-2*) obtains $N\times(N-1)/2$ linear equations by selecting each anchor as the reference for the other $N-1$ measurements [7]. In the third LLS technique (call it *LLS-3*), the average of all measurements is obtained as the reference range, thus resulting in N linear equations [8]. The fourth LLS technique (call it *LLS-RS*) selects the reference anchor with the shortest measured range among all the range measurements [6], and the index of the reference anchor is given by

$$r = \arg\min_i\{\hat{d}_i\},\ i = 1, 2, \ldots, N. \tag{6}$$

B. Reference Selection for Hybrid TOA/RSS LLS Localization

Observing the noisy terms in $\mathbf{b}_n$, we can see that all the rows of the vector $\mathbf{b}_n$ depend not only on the error-free range between the agent and the reference anchor, but also on the ranging error in the range measurement between them. Since $n_i \sim \mathcal{N}(0, \sigma_i^2)\ \forall i$, it is the variance of the reference range measurement, i.e., σ_r^2, actually influences the $\mathbf{b}_n$. If the reference anchor is away from the agent position, all the elements of vector $\mathbf{b}$ will be more noisy, thus degrading the localization accuracy. Similarly, the larger variance of the reference range measurement also causes vector $\mathbf{b}$ be more noisy and degrades the localization accuracy. Hence, the reference selection may considerably affect the performance of the LLS location estimator.

For non-hybrid LLS localization with one type ranging technique, such as TOA-based ranging or RSS-based ranging, it is likely that the variances of the measurements for closer ranges between the agent and the anchor are smaller, which further motivates the adoption of (6) for reference selection. However, this reference selection criterion may be not suitable for hybrid LLS localization techniques, such as the hybrid TOA/RSS localization technique, since it is possible that a RSS-based range measurement is the shortest among all the range measurements but has considerably larger variance than some TOA-based rang measurements, which may lead to that the RSS-based anchor is a sub-optimal choice. Therefore, for the hybrid TOA/RSS localization technique, it is better to consider both range measurements and their variances when designing the reference selection criteria. If coarse information of ranging variances is available, e.g., the RSS-based ranging variances are considerably larger than the TOA-based ranging variances, a simple method to select the reference anchor for improved location accuracy is to choose the TOA-based anchor with the shortest measured range. The index of the reference anchor that has the shortest measured range among all the TOA-based range measurements is given by

$$r = \arg\min_i\{\hat{d}_i\},\ i \in \mathcal{C}_{\text{TOA}}$$

where $\mathcal{C}_{\text{TOA}}$ denotes the index set for all the TOA-based anchors. We refer to the resulting estimator as the hybrid LLS with reference selection (*H-LLS-RS*).

IV. Weighted LLS Localization

Since the rows of the vector $\mathbf{b}_n$ become correlated during the linearization process, this correlation is utilized in [6] assuming all the ranging variances are the same to further improve the localization accuracy. We consider a more general scenario that the variances of range measurements are different, then the WLLS location estimate can be written as [14]

$$\hat{\mathbf{p}}_{\text{WLLS}} = (\mathbf{A}^T\mathbf{C}^{-1}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{C}^{-1}\mathbf{b},$$

where $\mathbf{C} = \text{Cov}(\mathbf{b}_n)$ is the covariance matrix of vector $\mathbf{b}_n$, and its elements can be derived as (see Appendix A)

$$\begin{aligned}[\mathbf{C}]_{ij} =& 4d_r^2\sigma_r^2 + 3\sigma_r^4 - \sigma_r^2(\sigma_i^2 + \sigma_j^2) + \sigma_i^2\sigma_j^2 \\ &+ I(i,j)(4d_i^2\sigma_i^2 + 2\sigma_i^4)\end{aligned}$$

with $i, j = 1, 2, \ldots, N$, $i \neq r$, $j \neq r$, and $I(i,j)$ is an indicator function which is 1 for $i = j$, and is 0 otherwise. As a special case, if all the variances are the same, i.e., $\sigma_i^2 = \sigma^2\ \forall i$, the elements of the covariance matrix can be simplified as

$$[\mathbf{C}]_{ij} = 4d_r^2\sigma^2 + 2\sigma^4 + I(i,j)(4d_i^2\sigma^2 + 2\sigma^4),$$

which coincides with the result in [6]. Note that since the error-free ranges $\{d_i\}$ are not available in practice, the noisy measurements $\{\hat{d}_i\}$ are used to evaluate the covariance matrix. According to the reference selection methods discussed above, the WLLS localization technique can be classified as *WLLS-1*, *WLLS-RS*, and *H-WLLS-RS* techniques.

$$\mathbf{A} = 2\begin{bmatrix} x_1 - x_r & y_1 - y_r \\ \vdots & \vdots \\ x_{r-1} - x_r & y_{r-1} - y_r \\ x_{r+1} - x_r & y_{r+1} - y_r \\ \vdots & \vdots \\ x_N - x_r & y_N - y_r \end{bmatrix},\ \mathbf{b}_c = \begin{bmatrix} d_r^2 - d_1^2 - k_r + k_1 \\ \vdots \\ d_r^2 - d_{r-1}^2 - k_r + k_{r-1} \\ d_r^2 - d_{r+1}^2 - k_r + k_{r+1} \\ \vdots \\ d_r^2 - d_N^2 - k_r + k_N \end{bmatrix},\ \mathbf{b}_n = \begin{bmatrix} 2d_rn_r - 2d_1n_1 + n_r^2 - n_1^2 \\ \vdots \\ 2d_rn_r - 2d_{r-1}n_{r-1} + n_r^2 - n_{r-1}^2 \\ 2d_rn_r - 2d_{r+1}n_{r+1} + n_r^2 - n_{r+1}^2 \\ \vdots \\ 2d_rn_r - 2d_Nn_N + n_r^2 - n_N^2 \end{bmatrix}.$$

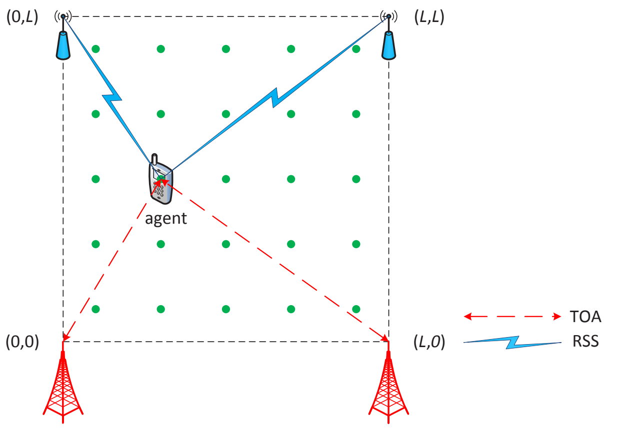

Fig. 1. Hybrid TOA/RSS LLS localization network.

V. Numerical Simulation Results

The simulation scenario is depicted in Fig. 1, where a hybrid network with $N = 4$ anchors is used to localize one agent. The area is a square of $L \times L$ m^2 and L is fixed to 10 m. There are two sets of anchors (set I and set II) in Fig. 1. Set I contains $S = 2$ TOA-based ranging anchors located at $\mathbf{p}_1 = (0, 0)$ and $\mathbf{p}_2 = (L, 0)$ respectively. Set II contains $N - S = 2$ RSS-based ranging anchors located at $\mathbf{p}_3 = (L, L)$ and $\mathbf{p}_4 = (0, L)$ respectively. The agent location $\mathbf{p}$ is changed with 2 meter intervals within $[1,\ 9]$ m both in x and y directions, yielding a 5×5 grid of possible agent locations. The mean square position error (MSPE) of different techniques are simulated at each location on the grid, and then average over all the agent locations on the grid. For *LLS-1*, the anchor-1 is selected as the reference anchor.

For simplicity, we assume that the agent and anchors have the same configuration and let TOA-based ranging error variance $\sigma^2_{\mathrm{TOA},i}$ be reversely proportional to SNR_i, which is

$$\sigma^2_{\mathrm{TOA},i} = \frac{1}{\mathrm{SNR}_i} = \frac{1}{\mathrm{SNR}_0}\left(\frac{d_i}{d_0}\right)^{\gamma},\ i \in \{1, 2, \ldots, S\}$$

where SNR_0 is the SNR at the reference distance d_0, and γ is the path-loss exponent. Similarly, we simply assume RSS-based ranging error variance $\sigma^2_{\mathrm{RSS},i}$ as

$$\sigma^2_{\mathrm{RSS},i} = \frac{\eta^2}{\mathrm{SNR}_i} = \frac{\eta^2}{\mathrm{SNR}_0}\left(\frac{d_i}{d_0}\right)^{\gamma},\ i \in \{S+1, S+2, \ldots, N\}$$

where η ($\eta \geq 1$) controls the relation between the two ranging error variances since TOA-based ranging usually has higher accuracy than RSS-based ranging, especially when adopting UWB signals. In the following simulations, we assume $d_0 = 1$, $\mathrm{SNR}_0 \in [20 : 30]$ dB with increasing step $\Delta = 2$ dB, and $\gamma = 2$ unless otherwise specified. For each simulation setting, 10^3 simulations are run to get the average performance.

If the RSS-based ranging variances are the same as the TOA-based ranging variances, i.e., $\eta = 1$, simulation results are depicted in Fig. 2. The *LLS-1* technique performs the worst among all the techniques and the CRLB provides the lower bound for them. The *LLS-2* and *LLS-3* techniques perform

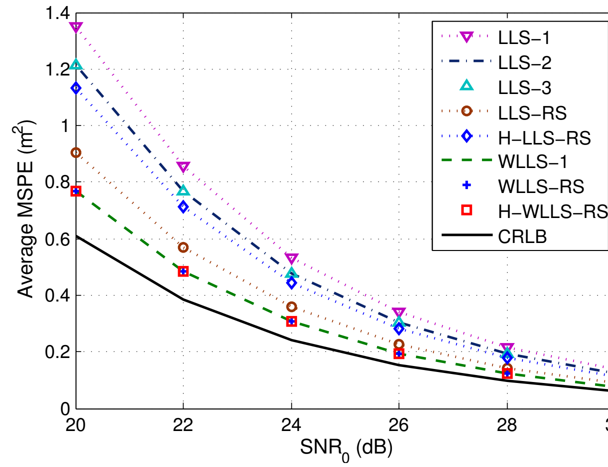

Fig. 2. Comparison of different LLS localization techniques ($\eta = 1$).

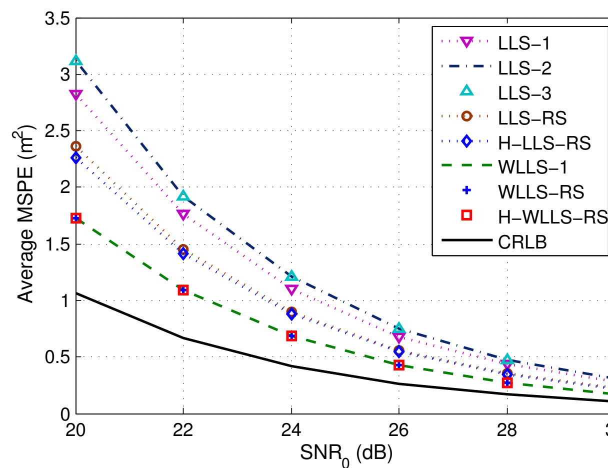

Fig. 3. Comparison of different LLS localization techniques ($\eta = 2$).

slightly better than the *LLS-1* technique and their performances are identical, since their reference selection methods utilize all anchors. The *H-LLS-RS* technique performs slightly better than *LLS-2* and *LLS-3* techniques since it considers the value of TOA-based range measurements. The *LLS-RS* technique beats all the other LLS techniques, since the *H-LLS-RS* technique never selects a RSS-based anchor as the reference even if its range measurement is the shortest. Moreover, the accuracy of the WLLS technique is not affected by the reference selection and it outperforms all the LLS techniques, since it weights the effect of all the measurements on the whole localization accuracy according to their reliability (i.e., ranging variances). As SNR_0 increases, the differences between all the techniques become smaller and the performance of the WLLS technique asymptotically approaches the CRLB.

If the RSS-based ranging variances are slightly larger than the TOA-based ranging variances, e.g., $\eta = 2$, simulation results are depicted in Fig. 3. Different from Fig. 2, the *LLS-1* technique outperforms both *LLS-2* and *LLS-3* techniques. The reason is that the *LLS-1* technique selects a TOA-based anchor with small ranging variance as the reference, while *LLS-2* and *LLS-3* techniques incorporating RSS-based anchors with larger ranging variances with TOA-based anchors as the reference,

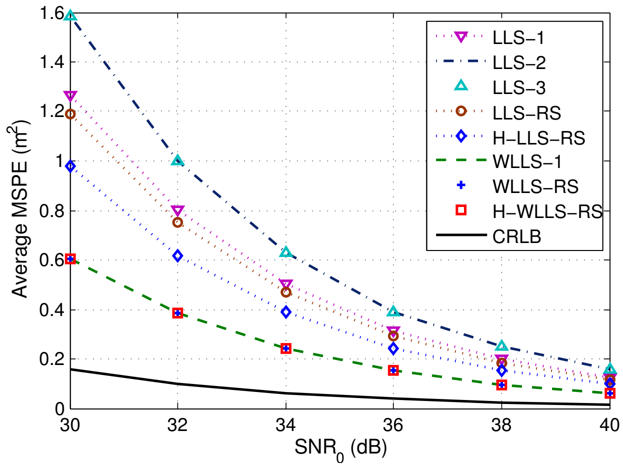

Fig. 4. Comparison of different LLS localization techniques ($\eta = 5$).

thus degrading their performance. Moreover, *LLS-RS* and *H-LLS-RS* techniques have nearly identical localization accuracy, since the effect of larger RSS-based ranging variances plays an almost identical role as that of range measurements on the final localization performance.

If the RSS-based ranging variances are considerably larger than the TOA-based ranging variances, e.g., $\eta = 5$, simulation results are depicted in Fig. 4. Since the variances of RSS-based ranging become larger, the SNR_0 range is changed to $[30 : 40]$ dB to get reasonable localization results. The localization performances of both *LLS-1* and *LLS-RS* techniques are nearly identical and the *H-LLS-RS* technique beats all the other LLS techniques, since the effect of larger RSS-based ranging variances plays a more important role than that of range measurements on the final localization performance.

VI. Conclusions

If coarse information of the ranging variances is available and the RSS-based ranging variances are considerably larger than the TOA-based ranging variances, the *H-LLS-RS* localization technique should be adopted to enhance the localization accuracy. If accurate information of the ranging variances is available, the WLLS location estimator should be utilized to further improve the localization performance. Although only LOS scenarios are considered in the paper, the idea could be extended to NLOS scenarios.

Appendix

A. Derivation of the Covariance Matrices

The elements of the covariance matrix of a vector $\mathbf{b}_n$ are calculated as

$$\begin{aligned}[\mathbf{C}]_{ij} =& \mathbb{E}\left\{([\mathbf{b}_n]_i - \mathbb{E}\{[\mathbf{b}_n]_i\})([\mathbf{b}_n]_j - \mathbb{E}\{[\mathbf{b}_n]_j\})\right\} \\ =& \mathbb{E}\{(2d_r n_r - 2d_i n_i + n_r^2 - n_i^2) \\ & \times (2d_r n_r - 2d_j n_j + n_r^2 - n_j^2)\},\end{aligned}$$

with $i, j = 1, 2, \ldots, N,\ i \neq r,\ j \neq r$, and $\mathbb{E}\{\cdot\}$ denotes the expectation operation. Since $n_i \sim \mathcal{N}\left(0, \sigma_i^2\right)\ \forall i$, we have

$$\mathbb{E}\{n_i^p\} = \begin{cases} 0, & \text{if } p \text{ is odd}, \\ \sigma_i^p (p-1)!!, & \text{if } p \text{ is even}, \end{cases}$$

where p is a positive integer and $(p-1)!!$ denotes the double factorial that is the product of all odd numbers from 1 to $p-1$; and

$$\mathbb{E}\{n_i^m n_j^n\} = \mathbb{E}\{n_i^m\}\mathbb{E}\{n_j^n\},\ i \neq j$$

where m and n are positive integers.

Therefore, the elements of the covariance matrix are

$$\begin{aligned}[\mathbf{C}]_{ij} =& 4d_r^2\sigma_r^2 + 3\sigma_r^4 - \sigma_r^2(\sigma_i^2 + \sigma_j^2) + \sigma_i^2\sigma_j^2 \\ & + I(i,j)(4d_i^2\sigma_i^2 + 2\sigma_i^4)\end{aligned}$$

where $I(i, j)$ is an indicator function which is 1 for $i = j$, and is 0 otherwise.

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References

[1] D. Dardari, M. Luise, and E. Falletti, *Satellite and Terrestrial Radio Positioning Techniques: A Signal Processing Perspective*. Oxford: Elsevier, 2012.

[2] S. Gezici and H. V. Poor, "Position estimation via ultra-wide-band signals," *Proc. IEEE*, vol. 97, no. 2, pp. 386–403, Feb. 2009.

[3] D. Dardari, A. Conti, U. Ferner, A. Giorgetti, and M. Z. Win, "Ranging with ultrawide bandwidth signals in multipath environments," *Proc. IEEE*, vol. 97, no. 2, pp. 404–426, Feb. 2009.

[4] J. C. Chen, R. E. Hudson, and K. Yao, "Maximum-likelihood source localization and unknown sensor location estimation for wideband signals in the near field," *IEEE Trans. Signal Process.*, vol. 50, no. 8, pp. 1843–1854, Aug. 2002.

[5] J. J. Caffery, "A new approach to the geometry of TOA location," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, vol. 4, Boston, Sep. 2000, pp. 1943–1949.

[6] I. Guvenc, S. Gezici, F. Watanabe, and H. Inamura, "Enhancements to linear least squares localization through reference selection and ML estimation," in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, Las Vegas, Mar. 2008, pp. 284–289.

[7] S. Venkatesh and R. M. Buehrer, "A linear programming approach to NLOS error mitigation in sensor networks," in *Proc. IEEE Int. Symp. Info. Process. Sensor Netw. (IPSN)*, Nashville, TN, Apr. 2006, pp. 301–308.

[8] Z. Li, W. Trappe, Y. Zhang, and B. Nath, "Robust statistical methods for securing wireless localization in sensor networks," in *Proc. IEEE Int. Symp. Info. Process. Sensor Netw. (IPSN)*, Los Angeles, CA, Apr. 2005, pp. 91–98.

[9] Y. T. Chan, H. Y. C. Hang, and P. C. Ching, "Exact and approximate maximum likelihood localization algorithms," *IEEE Trans. Veh. Technol.*, vol. 55, no. 1, pp. 10–16, Jan. 2006.

[10] A. Catovic and Z. Sahinoglu, "The Cramer-Rao bounds for hybrid TOA/RSS and TDOA/RSS location estimation schemes," *IEEE Commun. Lett.*, vol. 8, no. 10, pp. 626–628, Oct. 2004.

[11] M. Laaraiedh, S. Avrillon, and B. Uguen, "Cramer-Rao lower bounds for nonbybrid and bybrid localisation techniques in wireless networks," *Trans. Emerging Telecommun. Technol.*, vol. 23, no. 3, pp. 268–280, Apr. 2012.

[12] Y. Shen and M. Z. Win, "Fundamental limits of wideband localization-Part I: A general framework," *IEEE Trans. Info. Theory*, vol. 56, no. 10, pp. 4956–4980, Oct. 2010.

[13] A. F. Molisch *et al.*, "A comprehensive standardized model for ultrawideband propagation channels," *IEEE Trans. Antennas Propag.*, vol. 54, no. 11, pp. 3151–3166, Nov. 2006.

[14] S. M. Kay, *Fundamental of Statistical Signal Processing: Estimation Theory*. Upper Saddle River, NJ: Prentice Hall, Inc., 1993.