Leveraged ETF premium

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Abstract

Leveraged exchange-traded funds (*LETFs*) that allow investors to over-proportionally participate in abstract markets, such as indices, are studied in this paper. LETFs are found to suffer from a performance lag (negative alpha). The lag is categorized in three ways: fund management fees, compounding effects, and a leverage premium. The leverage premium effect intensified during the recent financial crisis (2008/09). In accordance with the approach put forward by Frazzini and Pedersen [2011], various market neutral long/short trading strategies which are sought to exploit the LETF premium are found to have delivered positive abnormal returns over the past years. While these strategies are not necessarily replicable, they teach us a lot about the relation between risk and expected return of LETFs in general.

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Contents

1 Introduction

1.1 Exchange-traded funds

An exchange-traded fund (*ETF*)[1] is a collective investment scheme that holds assets contained in the benchmark which it aims to replicate. Most ETFs are passively managed and their benchmark may be a stock or bond market index, a commodity price, or rarely a chain of returns of futures held to maturity (e.g. for ETFs tracking the (S&P 500) market volatility, an index based on the value of VIX future contracts is used as an underlying). From an investor's point of view, an ETF shares most of its characteristics with a publicly traded stock. Unlike a mutual fund, an investor can buy in at any time during market hours, using various order types, purchase them using a margin account from a retail broker, or enter into a short-selling transaction. The relatively low ETF share prices allow the straightforward participation of investors, even those with very small funds.

An ETF is created and managed by a provider (also called a sponsor). Major providers in the US are ProShares, BlackRock (iShares), Vanguard, and State Street (SPDR). An ETF has a varying number of outstanding shares. This is possible as authorized participants (such as large financial institutions) can issue and redeem shares against its asset holdings.[2] As participants are able to arbitrage out discrepancies between the market price and value of the assets contained in the ETFs, the difference to the net asset value is small at all times.

The ETF market has grown exponentially since the introduction of the first product of its kind in 1993,[3] the SPDR S&P 500 (*ticker:* `SPY`). This fund allows investors to hold a share-like security that promised to replicate the S&P 500 index at very low costs. The easy tradability, transparency, and low share price (1/10th of the S&P 500's value) were its main advantages over traditional mutual funds. The `SPY` had a market capitalization of over US$100 billion in spring 2012, and is one of the most liquid securities on the US stock market today. Other popular ETFs are the SPDR Dow Jones Industrial Average (`DIA`), the iShares MSCI Emerging Markets Index Fund (`EEM`), or the SPDR Gold Shares (`GLD`). Currently, there are roughly 1475 ETFs

[1] Occasionally the terms exchange-traded note (*ETN*), exchange-traded commodity (*ETC*), or exchange-traded product (*ETP*) are used. ETF is used as an umbrella term for all of them in the following.

[2] See `http://www.spdrs.com.sg/education/files/HowETFsareCreatedandRedeemSG-071411.pdf` for a description of the creation and redemption process of ETFs.

[3] Already in 1989, Index Participation Shares (IPS) were introduced, however, trading was stopped after a lawsuit (see Moriarty [2001]). In Canada, ETF-like products were introduced already two years earlier (see Deville [2008]).

listed in the US market[4] and the assets invested in ETFs reached US$1,186 billion recently.[5]

In 2006, a new category of ETFs emerged. So-called leveraged ETFs (abbreviated to *LETFs* from now on) allow investors to over-proportionally participate in specific markets. Thereby they magnify returns of their underlying benchmarks by amplifying factors of 2 or 3, claiming double or triple the daily performance of the benchmark. There are also negative amplifying factors of -1, -2, or -3. These are *inverse LETFs* and allow investors to have a short exposure in a market.[6] These are relatively simple products which can be bought by any retail investor like other ETF products. Notably, they do not require the use of a margin account (or to be more general, borrowing money) in order to leverage up holdings. Investors are not required to have any knowledge on derivatives, even though LETFs are normally backed by swaps and future contracts.[7] In addition, unlike when using direct leverage, an investor cannot lose more than the initial investment. If an LETF investor had wrong expectations about the market, and the underlying index were to drop (rise), the long (short) LETF investment would inevitably lose value. However, as this happens (and assuming the investor does not rebalance the LETF position), the investor's exposure is reduced, i.e. the marginal loss decreases.[8]

This apparent simplicity of LETFs comes at a price. Firstly, LETFs have quite high expense ratios, which are considerably higher than those of normal ETFs. The expense ratio is the management fee disclosed in the fund's fact sheet. It is given as a yearly percentage but is deducted from the returns in very small proportions every day, so every investor pays their share. It is below 50 basis points per year for the main equity ETFs. For example, the `SPY` has a relatively low expense ratio of just 9 basis points. On the other hand, LETFs have expense ratios of 95 basis points or more. For instance, the double S&P 500 LETF (`SSO`) charges 95 basis points, the same as its simple inverse brother (`SH`). The double VIX Short-Term LETF (`TVIX`) deducts 165 basis points yearly.[9]

Secondly, LETFs suffer from compounding effects if held over multiple days. For example, if you were to observe a two-day period where a benchmark gains 10% on the first day, and

[4] See `https://www.etfdb.com`.

[5] See `http://www.ici.org/etf_resources`.

[6] Positively leveraged ETFs are often given prefixes such as *long*, *bull*, or *ultra* by their providers. Inverse LETFs may be called *short*, *bear*, or *ultrashort*.

[7] Cheng and Madhavan [2009] explain formally how swaps allow LETF managers to achieve their return target. While bullish LETFs managers generally chose to simply leverage up their asset holdings (by being short in the money market), inverse LETFs generally have short positions in swaps and/or futures. The daily holdings of ETFs are usually revealed on the website of the responsible sponsor, e.g. on `http://www.proshares.com` for the ProShares ETF family.

[8] See `http://www.lighthouseinvestmentmanagement.com/2011/06/09/fun-with-inverse-etfs/` for some intuitive examples.

[9] Table 2 discloses the expense ratios of the funds used later in this paper.

loses 10% on the second day, a simple ETF starting at 100 would jump to 110 by the end of the first day, and then drop to 99 on the second day, in the same way as the benchmark. A double LETF under the same setup would be expected to reach a value of 120 on the first day before it falls to 96 on the second day ($100 \times (1 + 2 \times 10\%)(1 + 2 \times -10\%)$). If you are to consider the returns over those two days, you will notice that the ETF has lost 1% of its value, while the double LETF lost 4%, and not 2% as one might have expected. A triple LETF under the same scenario would have lost 9% (hitting 130 on the first day, before dropping to 91 on the second day). In addition, it is incorrect to suggest that an investor would have been better served with an inverse LETF. Under the same setup, the simple inverse, double inverse, and triple inverse funds would drop by exactly the same amounts, 1%, 4%, and 9% respectively. This effect is a consequence of compounding returns. It has mostly a negative impact on performance. Only in the case of strongly auto-correlated direction of daily returns (trends), the effect becomes positive.

Following several lawsuits in the US against LETF sponsors, who have been accused of not sufficiently disclosing compounding risks in their statements and prospectus,[10] investors are nowadays more readily warned.[11] In most LETF term sheets, a passage similar to the following one can be found.

> *Due to the compounding of daily returns, ProShares' returns over periods other than one day will likely differ in amount and possibly direction from the target return for the same period. These effects may be more pronounced in funds with larger or inverse multiples and in funds with volatile benchmarks.*[12]

To put it simply, LETFs aim to maintain their promised leverage factor constant over time. To achieve this, the LETF managers rebalance their holdings daily. An investor who wishes his exposure to remain constant too would also be required to rebalance daily. For instance, holders of a double LETF in the aforementioned example would see their LETF rise to 120 by the end of the first trading day. Subsequently, one would have to sell 20 worth of the LETF, setting back the exposure to the initial 100. On the second day, this would fall to 80 leading him to make zero profits ($-100 + 20 - 20 + 100$).

Thirdly, there are strong indications that a premium is associated with the embedded leverage

[10] See for example `http://online.wsj.com/article/SB124961014025313309.html`.

[11] For example, by the US Securities and Exchange Commission on `http://www.sec.gov/investor/pubs/leveragedetfs-alert.htm`.

[12] See `http://www.proshares.com`, where on all pages relating to LETFs this wording is stated.

property of LETFs (as well as of other financial instruments that embed leverage, e.g. options). The reason for investors to buy LETFs is to amplify their returns without directly leveraging up their holdings (e.g. using a margin account). Thus investors do not have to pay interest on money that they would need to borrow in order to leverage up. In a traditional leveraged investment, the interest rate may rise, increasing the cost of the strategy. The LETF investors do not have to fear a margin call should the securities (otherwise purchased on margin) strongly lose value. They might also have a preference for the simplicity of securities with embedded leverage over direct leverage. These effects result in what is commonly known as a premium (overvaluation) in prices.

The premium can be further divided into a tracking error of the market, i.e. the net asset value of the fund not being represented by the market valuation; and the inability of a fund manager to replicate the performance initially claimed. The former, also known as the tracking error[13] is small for most products. Additionally, the absolute deviation is mean-reverting, in other words, there is no consistent error in market valuation of the assets held by the fund in the long run (as arbitrage opportunities would arise which are quickly wiped away by those who can create and redeem ETF shares).

The inability of an ETF manager to correctly track the underlying's performance is influenced by the effectiveness of the selected replication strategy. Furthermore and lying outside the influence of the manager, the replication quality depends on the cost of leverage (interest rates), and for inverse LETFs, on the cost of borrowing shares (which consequently also increases the price of derivatives applied for the replication).[14] Mathematically, if a benchmark drops (gains) substantially on a single day (e.g. -/+33.3%), a triple LETF (triple inverse LETF) may be expected to fall by 100%. This, however, is impossible for a LETF with its share-like characteristics. A LETF therefore entails an option that guarantees limited liability. Managers of LETFs on very volatile benchmarks may hedge by holding put options.[15] In this paper, the reason for a product trading at a premium caused by any effect other than fees and compounding, is not further subclassified.

All of these three reasons together result in LETFs having more or less pronounced deviations from their targets, which may cause the products to suffer from a performance lag

[13]In investment literature, the tracking error is normally defined as the volatility of the market over- or under-valuation of the fund's net asset value (the deviation).

[14]See Genaro and Avellaneda [2012] for an empirical investigation into the influence of hard-to-borrow securities that influence the performance of LETFs. They however could not find sufficient evidence of a direct linkage.

[15]See Charupat and Miu [2011] for a further investigation of this point, as well as a further examination of the various sources of the premium.

relative to their benchmark. This is bad news for buy-and-hold LETF investors, however, it may provide opportunities for traders who are seeking to exploit the lags using specific strategies. For example, if a consistent under-performance is observed in a group of assets, then traders might consider (should they be able to do so) short-selling this group. In this paper, the under-performance of LETFs will be observed, and consequently, various strategies to exploit their deviations will be tested.

1.2 Leverage premium

Conservative investors are advised by investment consultants to put the majority of their assets in bonds and stock of well-established companies. More risk-tolerant investors may consider investing a larger proportion of their wealth into the stocks of more speculative firms. Similarly, there are mutual funds that follow more (less) aggressive strategies that are achieved by a holding a larger proportion of high-risk (low-risk) securities. This behavior contradicts the two fund separation theorem.[16] If the market were efficient, one could create any superior (in terms of efficiency) portfolio by holding the efficient market portfolio (tangency portfolio) as well as (in positive or negative amounts) the risk-free asset.

There are nonetheless some practical issues to contend with. Firstly, the CAPM framework is challenged by researchers as well as financial practitioners on the grounds that no efficient market portfolio exists. Secondly, a risk-free rate at which everyone could borrow or lend also does not exist. Additionally, the availability of the money market is non-constant over time. While in times of economic expansion, large market-participants, such as banks or investment funds, can take up liquidity easily (which may be at a high rate as a result of a central bank's smoothing policy), during heavy market trembles, the funding market can dry up completely.[17] Smaller investors can leverage up their traditional investments through a margin account. In the US, the Regulation T currently allows brokers to offer to their clients up to 50% of the securities held to be purchased on margin.[18]

The non-availability of simple leverage is paramount to the focus of this paper. Investors have found ways around these restrictions, such as trading products on the derivatives market. In the case of futures contracts, one is required to lock up a certain value per contract held on a

[16]See for example Ross [1978] for a summary of topics related to the Capital Asset Pricing Model (CAPM).

[17]See for example Cecchetti [2008] for an investigation of the reduced liquidity in December 2007 and actions undertaken by US Fed as a response.

[18]The text of the Federal Reserve Board *Regulation T* can be accessed at `http://www.federalreserve.gov/bankinforeg/reglisting.htm#T`.

margin account of the exchange. To trade an E-mini S&P 500 future (`ES`), one currently needs to deposit US$4,375 as initial margin, which currently amounts to about 6.5% of the nominal value of a long position.[19] By considering the exposure in terms the sensitivity of the instrument's value (V) relative to the sensitivity of the underlying (S) ($\frac{\partial V}{\partial S}$, known as the *delta* in options literature), this results in high leverage figures. Similar increases in leverage can be achieved using options or warrants.[20]

Furthermore, there are LETFs. These are specifically designed with embedded leverage in mind. (Leveraged) ETFs may also be preferred by investors from a tax optimization viewpoint.[21] Because the sole reason of existence for LETFs is the embedded leverage they offer, there are negligible secondary reasons why someone may purchase them. This makes them particularly interesting for in-depth analysis.

Various studies have shown that there is a premium associated with securities that embed leverage. This idea was introduced by Black [1972]. In summary, he describes a market equilibrium model under the assumption that risk-free borrowing is restricted. Under such a scenario, market participants who are willing to increase their portfolios' market exposure cannot simply leverage up their holdings (i.e. in a theoretical framework they would sell the risk-free asset and buy the efficient market portfolio). They would have to buy riskier assets with the expectation that their beta which is greater than one translates into higher returns.

As basic economic theory teaches us, a higher demand for high-beta assets inflates their prices, which, in turn, reduces their expected returns. In this case, high-beta stocks are can be said to be traded at a premium. In the CAPM framework, one can imagine the security market line being slightly concave for beta greater than one, i.e. a further increase in beta being associated with lower returns than a linear relationship between systematic market risk and excess return would imply.

Black [1972] develops a model that takes into account restricted borrowing, leading to a premium in high-risk securities. Empirically, Black et al. [1972] find that the beta describes the expected return well, although especially for some sub-periods of their data, the asset beta is not strictly proportional to the expected return. They also find, like Miller and Scholes [1972], that low-beta stocks performed better than average for a long time in the past century. In other

[19]See `http://www.cmegroup.com/clearing/margins/` for margin requirements of derivatives offered by the Chicago Merchandise Exchange (CME) Group.

[20]Frazzini and Pedersen [2011] analyze the embedded leverage premium in both LETFs and options.

[21]ETFs have a tax advantage over traditional mutual funds. See `http://etf.about.com/od/etftaxes/a/ETF_Tax_Benefit.htm` for an easy explanation or Poterba and Shoven [2002] for an academic discussion.

words, they state that the relation defined by the security market line is flatter than predicted. Later, Black [1998], using a longer sample period, empirically shows that high beta stocks offer lower returns than expected by their risk class.

The knowledge that low-beta stocks over-perform if adjusted for risk relative to high-beta stocks, creates interesting investment opportunities. Someone purchasing low-beta stocks while short-selling high-beta stocks should be able to make market risk neutral returns if holdings were adjusted such that the portfolio beta is close to zero (assuming no transaction costs). Frazzini and Pedersen [2010] first present a model in which market participants face leverage constraints. Then they back test what they call a *betting-against-beta* (*BAB*) strategy which purchases low-beta assets and short-sells high-beta assets. Their zero-beta portfolios using a wide range of international market data, and thereby deliver some significant abnormal returns. They conclude that an unconstrained arbitrageur could exploit the discrepancy.

Baker et al. [2010] provide a further possible explanation for the underperformance of high-beta stocks. Their model focuses on institutional investors that seek to maximize the Sharpe ratio of their funds, which drives up returns of low-risk assets. The average investor avoids resorting to leverage.

By the way, some large investors, for example, Warren Buffett (Berkshire Hathaway Inc.), invest primarily in low-beta stocks as can be observed in official holding reports.[22]

1.3 ETFs in academics

A large number of authors have dedicated their research to the broad area of ETFs. ETFs were approached from many different aspects, due to their large gain in popularity recently. Deville [2008] provides an excellent review on academic research in the area, highlighting the history of ETFs, trading mechanism, as well as the evaluation of their performance.

Engle and Sarkar [2006] and Ackert and Tian [2008] study the pricing of US ETFs relative to the underlying benchmark and find that they generally track their underlying index better than mutual funds.

In several papers, research has been conducted from a market point of view. For example, Svetina and Wahal [2008] examine 500 US ETFs offered at the time of their research, and analyze their use to the market and to competition within the ETF market.

[22] In the US, the Security and Exchange Commission obliges large institutional investors to disclose their holdings quarterly using Form 13F, see `http://www.sec.gov/answers/form13f.htm`. Survey results are made public online, for example at `http://whalewisdom.com`.

In addition, many authors focus on technical aspects of ETFs, i.e. tracking errors on the pricing and performance aspect of ETFs. Cheng and Madhavan [2009] describe accurately the performance issue of LETF, as well as how they are hedged by their managers. Furthermore, they find that the existence of LETFs exacerbates volatility during the final hours of a trading day (caused by the daily re-leveraging of these funds). Another remarkable study that focuses on LETFs is by Avellaneda and Zhang [2009] who develop an exact formula linking the LETF returns with the simple ETF return as a function of volatility.

More recent work on LETFs was published by Charupat and Miu [2011] who examine the performance and efficiency of these products.

As mentioned above, the most related work to this paper is by Frazzini and Pedersen [2011] who evaluate trading strategies that are short financial products that embed leverage, i.e. high-delta options and LETFs.

The remaining sections of the paper are organized as follows. In the following Section, the real market data used for analyzing the performance and modeling trading strategies is presented. In Section 3, the performance of the funds is analyzed and the causes of the premiums are discussed. In the following part, Section 4, trading strategies that seek to exploit previous findings are introduced and evaluated. In Section 5, the practical applicability of these strategies is explored, followed by a conclusion of the empirical findings and a look to future research.

2 Data

For the empirical part of this paper, three datasets are used.[23] Daily ETF (adjusted close) prices and volumes have been downloaded from `http://finance.yahoo.com`. The (leveraged) ETFs aim to replicate (a multiple) of a benchmark. For those, the total return indices prices have been obtained from Bloomberg. The expense ratios are disclosed in the fund description by the providers.

In the following, simple ETFs directly tracking a benchmark will be referred to as *1x ETFs*. Double and triple LETFs are given the prefixes *2x* and *3x*, and the inverse, double inverse, and triple inverse LETFs will be prefixed with *-1x*, *-2x*, and *-3x*. Note that a simple inverse ETF (-1x) is also a LETF with a leverage factor of negative one.

Tables 1 and 2 disclose the ticker symbols, the underlying total return index as well as

[23]For a general overview of the ETFs available on the US market, the Yahoo! Finance ETF center (`http://finance.yahoo.com/etf`), the ETF Database (`http://www.etfdb.com`), and the Bloomberg ETF Information Center (`EXTF` <`GO`>) are recommended.

the available data history, the expense ratio, the daily (US-) dollar volume, and the market capitalization. In both tables, Panel A corresponds to Set 1, Panel B to Set 2, and Panel C to Set 3. The sets are described in the following.

Set 1 consists of six major underlying indices for which (leveraged) ETFs of all six leverage categories are actively traded. The indices are: the *Dow Jones Industrial Average*, the *MSCI Emerging Markets*, the *NASDAQ-100*, the *Russell 2000*, the *S&P 500*, and the *S&P Mid Cap 400*. The five US market indices partly overlap, i.e. are strongly correlated due to commonly held assets. This, however, does not affect the analysis in this paper. These ETFs are among the most important securities on the US market today. For example, in the S&P 500 ETF products on average US$25.5 billion is traded.[24] The high market capitalization and trading volume translate into very high liquidity and interest by market participants and the financial media.

Set 2 includes a total of five benchmarks with are tracked by -2x, -1x, 1x, and 2x funds. The indices are the *Barclay's Capital U.S. 20+ years Treasury index*[25], the *S&P SmallCap 600*, as well as three *Dow Jones US* submarket indices: *Basic Materials*, *Financials*, and *Oil & Gas*. These are more niche and less important on the US market than the products of Set 1, although they still receive attention and offer decent volumes. For the more exotic benchmarks, the LETFs comprise larger parts of the capitalization in comparison to the standard ETF. For example, the -2x LETF on 20+ years Treasury (`TBT`), makes for 47% of the market capitalization of the four funds, similarely to the 2x DJ Financials ETF (`UYG`) which accumulates 52% of the assets.

Set 3 sheds light on alternative ETF categories. It is composed of the two precious metals, *gold* and *silver*, which have gained much media attention in the recent years, especially the simple gold and silver ETFs (`GLD` and `SLV`). They allow for easy direct participation in the precious metal market. Set 3 also includes the *S&P 500 VIX Short-Term Futures Expected Return index*, the *Russell 1000*, and the *NYSE Arca Gold Miners index*. Two indices from the real estate market are also considered: the *MSCI US REIT* and the *Dow Jones U.S. Real Estate*. While the 1x ETF exists for all of these benchmarks, the availability of various leverage factors is irregular. Therefore, later the strategy analysis cannot be performed simultaneously for all of

[24]The 1x `SPY` alone already makes for US$23 billion, leaving relatively little for the LETFs. There is, however, no LETF in this sample with serious lack of liquidity. The bid-ask spreads are low (The average spread of the products in set 1 is 5.5 basis point during regular market hours (Bloomberg).

[25]Price data on the Barclay's Capital indices could not be gathered, therefore, the performance analysis will not be conducted for products linked to this index.

them for obvious reasons.

For most indices, the 1x ETF is the most important product, bundling most of the volume and capitalization. However, for some of the more exotic benchmarks, the LETFs compromise larger parts of the capitalization relative to the ETF. For example, the *leveraged* products on the VIX, the DJ Financial, and also the DJ Industrial make up for the majority of the capitalization and volume. The 1x DJ Industrial (`DIA`), offers relatively small liquidity, which is due to the easy substitutability by the `SPY` which is more popular and charges a smaller expense ratio. The DJ Financial inverse LETFs were used to short bank stocks when shorting was banned by the SEC for financial stocks.[26]

Later in the analysis daily, weekly, and monthly total returns will be used.[27] Non-overlapping periods are used,[28] i.e. daily returns are between market close to the subsequent day's market close, weekly returns are Monday close to the following Monday close (or the first day the exchange is opened in a week should there be a holiday), and monthly returns are the first trading day in a month close to the following first trading day in the following month close. All of the return series used end on May 1st, 2012.

As the tables reveal, the expense ratios of LETFs are much higher then the ones of 1x ETFs. For the purpose of analysis, i.e. to determine the source of any performance lag, the expense ratios will be added back occasionally.[29] In all cases where this applies, it is highlighted specifically.

3 Fund performance

3.1 Alpha and beta

Empirically, returns of (leveraged) ETFs track their benchmark (times the leverage multiplier) more or less accurately. Minor deviations can be observed by looking at daily or longer return series. Figures 1 to 3 depict scatter plots of the daily, weekly, and monthly funds' returns versus their underlying indices. If the promised replication were perfect, the dots would lie on a straight line with slope of the inverse of their leverage factor (l^{-1}, e.g. -1/2 for the -2x LETF). As this

[26]See `http://www.sec.gov/news/press/2008/2008-211.htm` for the SEC short selling ban in 2008. Many articles promoted the use of inverse LETFs to avoid the short selling ban, see for example `http://www.thestreet.com/print/story/10440883.html`. High volumes can be observed in `SKF` and `SEF` between mid-2008 and mid-2009.

[27]Simple returns on adjusted prices which comprehend dividends are used.

[28]Shum [2011] uses overlapping multiple day periods for a similar performance analysis as done in this paper. She uses the Newey and West [1986] procedure to correct the t-statistics.

[29]As done by Frazzini and Pedersen [2011].

is not fulfilled, tracking errors must excist.

The analysis begins with an evaluation of the funds' performance. This is done by estimating the alpha and beta for each fund over the corresponding benchmark index. Different time frames are used in order to simulate corresponding holding periods, i.e. (non-overlapping) daily, weekly, and monthly periods. Market returns are looked at, so in a first stage, all deviation may those be due to compounding, management fees, or other causes (e.g. a product trading at a premium/discount) that may influence the estimated coefficients are included.

Essentially, we are interested in, i) whether or not the funds replicate well their underlyings, i.e. if the alpha of the fund over its benchmark is statistically different from zero (which would indicate an over- or underperformance); ii) whether or not the target leverage is achieved, i.e. if the beta corresponds to the promised leverage factor; and iii) if the variation in the benchmark can explain the variations in the fund well (as would be expressed by a high (adjusted) R^2).

Lu et al. [2009] and Shum [2011] perform similar analyses but use different data sets and do not examine triple (inverse) LETFs.

The regression outputs can be found in Tables 3 to 8. The analysis includes all (leveraged) ETFs for data set 1. In each table, Panel A reveals the descriptive statistics for the daily returns, including mean, minimum, maximum,[30] volatility, skewness, and kurtosis. For these summary statistics, the identical length of data history is used for all ticker symbols, i.e. starting mid-February 2010. While this allows for direct comparison of the ETFs, much of the data is neglected (especially the peak of the recent financial crisis). Panel B gives the coefficients from the regressions, and in parentheses the t-statistics are provided. The null hypothesis for the t-statistics is where alpha is zero and beta is equal to the promised leverage factor, i.e. no under- or overperformance (i), and leverage corresponding to the target multiplier (ii) are looked for. For the regressions, the complete data history available of each fund is used. The second row for each ticker includes a compounding variable (Comp.) into the regression; this will be discussed later in Section 3.2.

Dow Jones Industrial Average (Table 3)

Firstly, the six (leveraged) ETFs on the *Dow Jones Industrial Average* are examined. The summary statistics in Panel A show that the underlying index had a mean daily return of 0.07% since February 2010 while the 1x ETF returned 0.06% on average, which reveals that it suffered from a small performance lag. The -1x LETF with -0.07% seems to have replicated well the

[30] The minimum and maximum is simply the day with the lowest/highest return as daily data is looked at.

inverse of the benchmark. By looking at the minimum and maximum, it can be seen that in the extremes, on one day the index dropped by 5.5% (August 8, 2011) and on another day it spiked by 4.24% (May 5, 2010), while the 1x ETF dropped by only 5.41% and rose by 4.07% on these days. The -1x LETF changed in the same way as the 1x ETF on these days but in the opposite direction. While there is no remarkable difference in the volatility of the benchmark and the +/-1x products, the skewness is slightly more pronounced for the ETFs. It may be that on days where the benchmark return is negative, the ETF expense ratio has a stronger influence. The kurtosis is slightly higher for the -2x and -3x LETFs, which indicates that the daily returns are slightly less extreme. The other LETFs have slightly lower kurtosis than the benchmark. When considering the means, the benchmark is replicated well by the +/-2x products, with a small lag in the 2x LETF. Similarly, the daily minimum and maximum is a little less pronounced in the LETFs. As anticipated, the volatility is extended by roughly the leverage factor.[31] Similar statements can be made for the +/-3x LETFs.

The regression outputs reveal that the daily alphas are negative except for the -1x and -2x LETFs. However, they are not statistically different from zero. For the weekly regressions, all alphas of the inverse LETFs turn positive, and the one for the -1x is significantly different from zero. On the other hand, the positive LETFs have negative alphas, but only the one of the 2x LETF is significant. Except for the -1x LETF, monthly alphas are negative. For the betas, over all holding periods, many differ significantly from their targeted leverage factor. The betas are mostly lower than promised by the fund, e.g. -2.96 instead of -3, or 2.97 instead of 3 (daily).

Finally, the explained variance is relatively high with adjusted R^2 above 97%. It is higher the longer the holding period is chosen to be, which can be explained by small tracking errors being balanced out over the long term due to their mean-reverting nature.

MSCI Emerging Market index (Table 4)

For ETF products related to the *MSCI Emerging Market index*, the summary statistics give a similar picture to the previously discussed small lag in daily returns. Interestingly, the minimum and maximum daily return is larger than expected for the ETF products compared to the benchmark. The same is observable with the volatility, which is higher for the funds (1.77% in the +/-1x funds versus 1.23% in the underlying). At the same time, the skewness is reduced, meaning that there are less negative daily return outliers in the ETFs than in the index. The

[31] Lu et al. [2009] define the volatility multiplier as the ratio between the standard deviation of the fund and the standard deviation of the benchmark for various holding periods. They find a negative correlation between the holding period and the volatility multiplier.

higher kurtosis goes in line with this observation. The reason for these discrepancies may be, as Shum [2011] notes, the non-synchronicity in trading times of the ETF (in the US) and the underlying (tracking at a large proportion stocks trading in the Asian stock market).

The regression output shows that the alphas are mostly negative, however, they are not significantly different from zero. For this index, the betas are slightly above the offered leverage factors. For example, in the weekly holding periods, where the 3x and -3x LETFs have betas of 3.35 and -3.27, both are significantly different from +/-3. This is most probably also a result of the aforementioned reason of discrepancy in trading times. As a result, the explanatory value of the daily regressions is relatively small (adjusted R^2 below 50%). However, the longer holding periods result in higher R^2 values, as the exchange hour effect diminishes.

NASDAQ-100 index (Table 5)

The ETFs on the *NASDAQ-100 Index* are considered next. There is little to be said about the summary statistics, which show that the daily returns largely resemble the underlying. The kurtosis is slightly lower than the index, which indicates that the distribution of returns is more concentrated around the mean (leptokurtic). Otherwise, similar conclusions to the Dow Jones Industrial LETFs can be made.

Again, the negative alphas form the majority, although they only become significantly different from zero for weekly and monthly investment horizons. Overall, the R^2 are sufficiently high (>97%).

Russell 2000, S&P 500, S&P Mid Cap 400 indices (Table 6, 7, and 8)

The last three tables provide the summary statistics and the regression outputs for funds on the *Russell 2000*, the *S&P 500*, and lastly, the *S&P Mid Cap 400*. The outputs resemble the previous ones, which is why they are not discussed individually here. Most importantly, we again observe that the negative alphas form the majority. The alphas of the S&P 500 LETFs are a bit better (closer to zero, and more positive ones). This may be explained by the larger popularity for and liquidity of these funds. The LETFs' betas are often significantly different from their targets already for daily holding periods. The R^2 are all reasonably high, proving that their underlyings explains well the movements in the ETFs.

Conclusion

Overall, the outputs do not deliver any example of a 1x ETF with an alpha that is statistically different from zero (for any holding period up to one month). The betas differ from one for several 1x ETFs, however, not all of them differ in the same direction, i.e. some ETFs amplify while others demagnify their benchmarks. Interestingly, four of the six -1x LETFs from set 1 have positive alphas, even though only for the S&P 500 -1x LETF (`SH`) it differs significantly from zero for daily data. There is no notable difference in the output produced for the -1x LETFs from set 2 (results not reported), where positive alphas also dominate. They are not significant, but this can be partially explained due to the shorter data history for +/-2x (and +/-3x) LETFs. There, the 2x and +/-3x LETFs all have negative alphas.

The betas, on the other hand, are consistently (and mostly significantly) below the promised target beta for daily data. As holding periods lengthen, however, the betas may turn stronger positive for some LETFs, but still stay below the target on average.

Sets 2 and 3 produce similar patterns (outputs not reported) while the number of negative alphas increases further. This may be due to their lower popularity, and therefore lower efficiency in tracking the underlying. In these sets, the betas are once again significantly below the individual targets.

The mostly negative alphas indicate a performance lag which may be caused by the j) expense ratio; jj) compounding (for multiple day holding periods); or by a jjj) market premium. The latter one may be due to the inability of a manager to achieve the benchmark target, or because of an incorrect valuation of the product in the market. A mispricing in the market tends to be less of a problem, as tracking error (over the funds' asset holdings) is small for liquid ETFs.

(j) The expense ratio are known and fixed over time, as reported in Table 2. This ratio is deducted constantly in small amounts every day. Unless it is unexpectedly changed, the performance lag due to the expense ratio is a known factor. This can be remedied by simply deducting the ratio from the returns. As the expense ratio is relatively small, the alphas do not alter largely for expense ratio-adjusted returns, while betas are unaffected (output not reported).

(jj) The compounding effect is examined in the following Section 3.2. As will be shown, the compounding effect can be computed accurately, and can explain the lag in returns partially for multiple day holding periods.

(jjj) When having accounted for the two previous sources of deviations, a premium may remain. In this paper, the premium is defined as the residual of the performance difference after controlling for the expense ratio and for compounding. This cause will not be further subdivided.

3.2 Premium decomposition

When a LETF is held over more than one day without rebalancing, compounding effects arise as soon as the daily benchmark returns are non-zero. A straightforward example has already been provided in Section 1.1, where a two day period was considered. Mathematically, a two day period deviation can be stated as follows. Here, a lx LETF (amplifying the benchmark return r by the leverage factor l) deviates from the target, which is the benchmark's cumulative return over both days multiplied by l as follows.

$$[(1+lr_t)(1+lr_{t+1}))-1]-l\,[(1+r_t)(1+r_{t+1}))-1]=r_tr_{t+1}l(l-1) \tag{1}$$

For holding periods of more than two days, the computation becomes quite cumbersome, i.e. a neat expression for the deviation as in Equation 1 cannot be worked out. In the following equation, the deviation for a more general T days holding period is expressed.

$$\prod_{i=0}^{T-1}(1+lr_{t+i})-l\left(\prod_{i=0}^{T-1}(1+r_{t+i})-1\right)-1 \tag{2}$$

Thus, the question that arises is whether compounding has a positive or negative influence on the performance of the LETFs, i.e. the sign of the deviation term. This issue is addressed in Table 9 in five simple hypothetical return scenarios. A four day period is explored where the benchmark can go up or down by 10% on each day. A LETF amplifies the benchmark return (theoretically) by exactly its leverage factor, without any tracking errors or deviation from the promised leverage, even though this has been observed before. For example, the -2x LETF would rise by 20% on a day that the benchmark drops by 10%.

Firstly, the 1x ETF does not suffer from compounding as it is a one-to-one replication of its underlying. The -1x LETF, on the other hand, is exposed to the compounding effect. If the benchmark rises by 10% on each one of the four days, the -1x LETF performs as the benchmark thereby losing 10% on all of the four days. Over the whole period, performance is -34.39% ($(1-1\times 10\%)^4-1$), while the benchmark gains 46.41% ($(1+10\%)^4-1$). A holder who is unaware of the compounding in their product may be surprised not to lose the same amount, but with a reversed sign, as the benchmark. Therefore, the deviation is +12.02pp.[32] Similarly, if the benchmark drops by 10% on all four days, the 1x LETF performs relatively well with 46.41% in comparison with the benchmark that loses 34.39%. Again, the difference amounts to 12.02pp. However, in the three scenarios with mixed returns over the four days (at least one 10% or -10% return on one day), the deviation due to compounding lies in the negative.[33] 3.98pp would be lost in the case where the index goes up by 10% on two days and down by the same 10% on the other two days, i.e. the benchmark's performance is -1.99%, while the -1x LETF does not produce a positive result, but instead also loses the same 1.99%. Overall, therefore, the difference is twice the performance. When positive (negative) returns are followed by positive (negative) returns, the LETF holder benefits from the compounding. In other words, in times where the market follows a roughly constant upward or downward trend, the LETFs returns are amplified further and the compounding effect is positive. If, however, the benchmark goes up and down at random, as can be observed during most phases in the markets which evolve according to a random walk, the compounding effect has an undesired negative impact on performance.

In the scenario of +10% return on all four successive days, the 2x LETF over-performs the benchmark (times two) by 14.54pp. In a case where the benchmark drops by 10% on all days, the 2x LETF loses 9.74pp less than expected. In scenarios of mixed returns, the deviation is again negative (but for the case of three negative and one positive day, where the deviation is +1.06pp).

In case of the 3x LETF, the deviation is more amplified in both cases, but as expected, does not change its sign. In conclusion, for the positively leveraged product, the deviation is stronger in cases where the benchmark makes positive returns, as the value of the product is enlarged more strongly (value effect).

On the other hand, in the case of inverse LETFs, the deviation due to compounding becomes even more prominent. In the hypothetical scenario of the underlying returning +10% on the four sequential days, the -2x LETF deviates from its target by a sheer 33.78pp (versus 14.54pp for the 2x LETF). When the benchmark drops by 10% on each day, the -2x LETF over-performs by 38.58pp; this is again caused by the value enlargement effect. Not only are the positive deviations amplified compared to the positive LETFs, but also the negative ones are further off their targets. In cases of mixed returns, +10% on two days followed by -10% on the others, the

[32] The deviation is stated as the arithmetic difference of the two returns, hence *percentage point* (pp) is the correct unit.

[33] Note that the sequence order of positive or negative returns is irrelevant.

-2x LETF lags behind its target by 11.82pp. The -3x LETFs show the same patterns but to an even greater extent.

The compounding effect is a traceable component of the premium as defined by Equation 2. It is analyzed by being used as an additional explanatory variable in the linear regression as discussed in the previous section. For this purpose, a series of deviation values is computed. Weekly holding periods are normally composed of five trading days, while for monthly periods, the number of trading days reaches around 21. An example of the development of the deviation over time is given in Figure 4, which plots the variable as later used for the regression for the S&P 500 LETFs. It can be seen that during periods of high market volatility, especially between mid-2008 and mid-2009, the deviation caused by compounding became increasingly negative. As predicted, the deviation of the -3x LETF is the strongest. In a theoretical setup where the LETFs obey their leverage multiplier exactly (as in the hypothetical example from above), the deviations should be in line with each other. The plots seem to incorporate a lot of noise, as the lines seem unrelated to each other in several instances. This can be explained by the imperfect replication of the LETFs, especially as seen in the summary statistics in Panel A of Tables 3 to 8, where the volatility of the LETFs often differs from the one of the 1x ETF times their leverage factor.

The regression outputs including this compounding deviation variable is reported in Panel B of these tables, in the second row of each LETF for the weekly and monthly regressions. The t-statistics (for this factor) test the null-hypothesis of the coefficient being one. This would imply that the deviation is perfectly explained by the computed factor.

As is evident in the output, the estimated coefficient is close to one for all of the regressions with only a few examples of it being statistically different. Therefore, we can conclude that the explanation using the compounding effect is valuable, and leads to (even) higher adjusted R^2. The alphas in these regressions become statistically significant (mostly negatively) in many instances where they were not before. The betas change less obviously into one direction. In the majority of cases, however, they become less significantly different from their individual leverage factor. This indicates that, to some extent, compounding is responsible for the inaccurate leverage achieved. As many alphas are still negative (even if corrected for expense ratio), the conclusion can be made that there are other effects, such as a leverage premium, that drives the LETFs' performances.

The estimated betas are used to compute a fitted time series of alphas, i.e. the error term of the regressions are set to zero in order to compute a series of alphas. Figures 5 to 7 plot these alphas over time for all funds used in this paper. For the data (and estimated betas), the returns with expense ratio added back is used, and for weekly and monthly holding periods, both betas, the one for the underlying and the one for the compounding deviation, are used. The result is the performance divergence of the funds that is not caused by the expense ratio, and not by compounding. This residual is defined in this paper as the *leverage premium.* It can be observed, that the magnitude strongly enlarged by end-2008, as well as during 2011. The conclusion can be reached that the crisis further distorted the funds' performances.

The deviation from target can be exploited by adapting trading strategies as will be shown in the in the following sections. It is evident in the alphas that the LETFs lag behind the benchmarks' returns. This becomes increasingly more significant the higher the leverage factor set as a target by the fund. By short-selling these LETFs, a trader could benefit from their performance lag. On the other hand, the regressions reveal that the 1x ETFs track the indices relatively well (as indicated by alphas close to zero). Therefore, these could be used to build a long-position in order to balance out the exposure of short-positions held in LETFs on the same underlying. In the subsequent section such strategies are defined and tested.

4 Trading strategies

4.1 Definition

The trading strategies considered in this paper have some common characteristics: They all involve at least two simultaneous long/short or short/short transactions, of which at least one product traded is a LETF. The products that enter one deal must replicate the same benchmark. All trading strategies aim to keep the overall market exposure (beta) at zero. The strategies examined can be divided into two categories, plus a combination of them.[34]

The first set of strategies are based on the idea of shorting *embedded leverage.* In essence, they sell bullish LETF (which *embed* leverage), and balance out the exposure by purchasing the simple 1x ETF on the same underlying.

1 Frazzini and Pedersen [2011]-*Betting-against-beta* (BAB)

Frazzini and Pedersen [2011] describe and successfully test the strategy that longs one 1x ETF and simultaneously shorts one-half of a 2x LETF on the same underlying index, and thereby

[34] An overview of these strategies is provided in Table 10.

achieves a zero-beta exposure. To avoid compounding effects, the mentioned authors only use daily returns, and hence they would simulate daily rebalancing. In this paper, this as well as all other strategies will be run also for weekly data, simulating weekly rebalancing.

2 Frazzini and Pedersen [2011]-BAB 3x

A second strategy employed is one that changes the 2x by a 3x LETF. This time, for every one-third of a 3x LETF sold, one 1x ETF is bought in order to guarantee zero-beta.

3 Frazzini and Pedersen [2011]-BAB 2x, 3x

The third strategy is a combination of the first two. For every 1x ETF bought, one-fourth of a 2x LETF and one-sixth of a 3x LETF are sold. The contribution to the beta is, as a result, the same for both LETFs, i.e. both make up one-half. The idea behind considering more than two products parallelly is to achieve a greater diversification. While each individual fund may suffer from a random inaccuracy relative to the benchmark it aims to track, it seems that the inaccuracies are independent between funds. Chances are that volatility is reduced in comparison to the previous two strategies.

Secondly, a set of strategies that seek to exploit the performance lag in (inverse) LETFs can be employed. Essentially, one would short both an inverse and a bullish LETFs on the same benchmark. This fully balances out their individual exposure. Figure 8 charts the performance of the LETFs contained in data set 1, comparing the normal (bullish) and inverse LETFs. One observes that in most cases the average of both prices (e.g. 2x and -2x) decays over time. A trader who is short this average could be expected make positive returns. This strategy was promoted by several financial bloggers,[35], but it is yet to be scientifically tested. Technically, the strategy is expected to be more profitable the more volatile the markets are, as a trader who applies such a strategy would basically be *short volatility.*

4 Short 1x's

The most basic of these strategies makes use of the +/-1x LETFs, i.e. for every 1x ETF sold, a trader will also sell one -1x LETF. Logically, the beta of the strategy is zero.

5 Short 2x's

The +/-1x LETFs from the precedent strategy are replaced by +/-2x LETFs now.

6 Short 3x's

This strategy makes use of the +/-3x LETFs. As the triple LETFs have been proven to suffer from the largest performance lag in general, this strategy is expected to be one of the

[35] See for example `http://falkenblog.blogspot.ch/2011/10/shorting-leveraged-etf-pairs.html` or `http://matlab-trading.blogspot.ch/2011/05/problem-with-shorting-leveraged-etfs.html`.

most profitable ones.

7 Short 1x,2x's

In this approach, strategies 4 and 5 are combined, i.e. both +/-1x and +/-2x funds are sold short. This, like also the next three strategies, aim for larger diversification by using a wider range of LETFs on one benchmark.

8 Short 1x,3x's

Here, a combination of strategies 4 and 6 is used.

9 Short 2x,3x's

This strategy combines strategies 5 and 6, i.e. it does no longer make use of the simple +/-1x products.

10 Short 1x,2x,3x's

This strategy bundles all of the previous strategies into one, resulting in the largest diversification (of tracking error) possible for every underlying.

Then, there is a third set of strategies. The following two strategies combine the idea of having a long position in the 1x ETF (which is generally observed to track relatively well the benchmark) with being short various (inverse) LETFs.

11 Long 1x, short 3x's

While this strategy longs one 1x ETF, it is balanced by shorting one 3x LETF and two-third of a -3x LETF.

12 Long 1x, short 2x's

Strategy 12 is similar to strategy 11 but instead uses the +/-2x products. One is short one 2x LETF and one-half of a -2x LETF for every long position in a 1x ETF.

4.2 Risk adjustment

It is of interest if the returns generated by the strategies are abnormal. Essentially, we are interested if the returns in excess of the risk-free interest rate are constantly positive, independent of returns on the stock market, and offer a relatively low volatility.

The excess return is the total return of a fund from which the return of the one month US *Treasury* (T-) Bill (secondary market) rate is deducted. This adjustment is necessary to control the cost of the capital which is bound to any asset holding. The tied up money cannot be used for other investments. A long position would be assumed to pay net the return of the security from which the interest on the capital raised in order to make the purchase would be subtracted.

Following this procedure, a short position using the same adjustment would earn the interest rate over time. In reality, borrowing fees (required to short any share-alike security in the first place) generally outrun the accumulated interest on a short position. Therefore, using excess returns for short-transactions would distort the performance. For example, considering strategy 4 where both the +/-2x LETFs are sold short. The two positions together would return twice the T-Bill interest when excess returns were being used. In this paper, this issue has been avoided by using excess returns solely for long positions, and unadjusted returns for short positions.

The CAPM model is used to test for profitability, where a regression of (excess) returns of the strategies is performed on the excess market return. Furthermore, the two Fama-French (Fama and French [1993]) factor returns, high-minus-low (book-to-market) and small-minus-big (market capitalization), are controlled for. The regressions are carried out on a monthly basis, i.e. the (excess) return achieved by the strategies for each month as a whole is computed before the (monthly) regression is run. A similar procedure is chosen by Frazzini and Pedersen [2011].

For the purpose of evaluating the strategies, products on several benchmarks are examined simultaneously. This means that the performance of one strategy is applied to one complete data set, before the mean is taken to form an aggregated and diversified (between benchmarks) portfolio. For instance, consider strategy 2, which forms a BAB portfolio making use of the 1x ETF and the 3x LETF. The strategy as evaluated for data set 1 would be a portfolio which is formed according to this strategy on all 1x and 3x products of this set (i.e. all six underlyings). The same strategy cannot be applied for set 2 because no 3x LETFs are offered there. Contrary, set 3 contains three underlyings for which 1x as well as 3x LETFs exist. To evaluate the strategy applied to this set, a portfolio of these six funds (three underlyings) would be formed. For certain strategies, the portfolio is actually based on only on one underlying. For example, strategy 4 which shorts the +/-1x products applied to set 3 would only be able to make us of the funds on the S&P 500 VIX.

The aggregated portfolios are formed regardless of the differences in data history of the funds. For example, again by looking at strategy 2 applied to set 3. As presented in Table 1, the three 3x LETFs have different lengths of data available. The strategy can be implemented for the first time in December 2008 (when `BGU` becomes available). The funds on the Russell 1000 are the only components of the portfolio until August 2009 when the 3x LETF on the MSCI US REIT (`DRN`) starts trading. From then on, both underlyings are considered. Not before January 2011 (when `BUGT` appears) can the third benchmark be included as well. It is assumed that all funds of the theoretically formed portfolio are fully invested to equal proportions in the products belonging to each index.

For the purpose of evaluating the strategies, the part where several funds are not yet available is cut off completely. The specifically specified (in the output tables those will be highlighted with a *10*) regressions would then ignore the data where not all products of one set are available. This is for set 1 starting in March 2010, from April 2010 for set 2, and from December 2010 for set 3.

Firstly, the performance r^i (of a strategy for set i) is checked in a CAPM-style regression against the market excess return r_t^m.[36]

$$r_t^i = \alpha + \beta_m \times r_t^m + \varepsilon \tag{3}$$

Secondly, additional Fama-French factors are considered.

$$r_t^i = \alpha + \beta_m \times r_t^m + \beta_{smb} \times r_t^{smb} + \beta_{hml} \times r_t^{hml} + \varepsilon \tag{4}$$

The regression tables will also deliver the output for the regressions of return data where the expense ratio has been added back. This is to check if possible positive alpha obtained is caused by the differences in expense ratios.

4.3 Evaluation

Tables 11 to 22 present the regression outputs of all strategies as defined in Section 4.1. In each table, Panel A provides the (monthly) summary statistics including the Sharpe ratio. Panel B gives the outputs for regressions as defined in Equations 3 and 4. All stated values are in monthly terms. In the following section, the results are evaluated in detail. In Figures 10 to 33 the performances of the strategies are plotted over time, giving a graphical impression of the returns in various market conditions.

1 Frazzini and Pedersen [2011]-BAB

[36] All factors (US market excess returns, high-minus-low, and small-minus-big have been obtained from Kenneth R. French's data library, `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`. The four-week T-Bill rates have been gathered from St. Louis Fed Economic Data service, `http://research.stlouisfed.org/fred2/categories/116`.

First, the strategy originally introduced and tested by Frazzini and Pedersen [2011] is reevaluated. The summary statistics indicate that the (monthly) mean return of the strategy is positive for all data sets in case of both daily and weekly holding periods (only set 3 starting in 2010 has a negative average return). There are, however, months where the BAB portfolio return is negative, i.e. in the worst month it loses 0.29% of its value using the 1x and 2x products of set 1. The best monthly performance could be observed in October 2008, where the portfolio return of the daily rebalanced set 1 was 0.7%. In set 2 and set 3, the minimum and maximum monthly performance is slightly more extreme with -0.54% (-3.75%) and +1.32% (+5.02%) for set 2 (set 3), which is also expressed by a higher volatility. The skewness is positive trough all sets for daily rebalancing (except for set 3 starting in 2010), which indicates that there were more deviations to the positive extreme than to the negative side (right tailed). The kurtosis is quite high for some samples, which indicates that the returns are distributed rather constantly around the mean. For weekly data, the summary statistics show a weaker performance with considerably lower Sharpe ratios.

The regression results reveal that only the samples starting in 2010 of set 1 and 2 deliver significantly positive alphas (at the 5% significancy level). In general, the market beta is negative, even though this is not statistically significant except for the first regression. This means that, on average, positive returns in the portfolio formed according to the strategy goes in line with negative market returns, and vice-versa. The other Fama-French factors seem not to be able to explain any movements in the portfolios. As expected, when using the returns net of expense ratios, slightly lower alphas result.

The performance plot (Figures 10 and 11) shows how the strategy performed over time. It can be observed that between 2006 and mid-2008 one would have been losing money when applying this strategy. From then on, after a strong recovery in end of 2008, the portfolio delivered constant positive returns at a relatively low volatility.

While Frazzini and Pedersen [2011] were able to find strongly significant abnormal returns when applying this strategy on a similar selection of ETF products, the same result cannot be reached here. The reason is the difference of the use of excess returns. Until 2008, the interest rates used for the return adjustment were relatively high (see Figure 9). In this paper, it is assumed that only the long positions tie up capital in the sense that one has to pay the risk-free interest. The returns on short positions, which if calculated in the same way, would earn the interest rate over time. The returns on the short position was computed on unadjusted returns in this paper. Overall, the trader would have had to pay the rather high interest rate on the long holding until the end of 2007 without being recompensed on the short side. On the other hand, in the Frazzini and Pedersen [2011] study, the trader was to pay only half of the interest rate (because he earned half of it on the short position). The performance if calculated in this style is given by the dotted magenta line in Figures 10 to 33. Like in the mentioned paper, the alphas computed on the basis of excess returns for short *and* long positions are significantly positive (output not included).

2 Frazzini and Pedersen [2011]-BAB 3x

The same strategy is applied as before, but in this instance uses the 3x LETF instead of the 2x. This also delivers positive mean returns on average for daily and weekly rebalancing.

For the daily rebalancing periods, all alphas are positive, however, only for set 1 with the sample that starts in 2010 those are significantly positive for the CAPM as well as the three-factor model. Again, when weekly reblancing is assumed, the performances are considerably lower (as seen in the low Sharpe ratios).

Figures 12 and 13 plot the performance of the strategy over time. As the samples that include the 3x LETFs started only by end of 2008, the influence of the risk-free interest is near to irrelevant (because interests were close to zero by then). Certain products delivered exceptionally good performances, i.e. the funds on the *Russell 2000* (long `IWM`, short `TNA`), or on the *MSCI Emerging Markets* funds (long `EEM`, short `EDC`), both of which are responsible for the positive return of the equal-weighted BAB-portfolio.

3 Frazzini and Pedersen [2011]-BAB 2x, 3x

Only data set 1 offers 1x, 2x, and 3x funds on each underlying, therefore strategy 3 can solely be implemented on this set. The summary statistics show that the daily as well as weekly portfolios performed positively on average. The volatility is reduced relative to the previous two strategies, which indicates that the diversification was partially effective. The lower volatility translates into higher Sharpe ratios. The regression analysis confirms the superiority, finding significantly positive alphas for daily rebalancing periods.

Figures 14 and 15 plot the returns over time. A boost in the performance is observed in the first half of 2010 when the portfolio was composed of just funds on the *Russell 2000* (long `IWM`, short `UWM`, short `TNA`).

4 Short 1x's

Short-selling the +/-1x funds simultaneously results in positive mean returns for all data samples. The regression output indicates that the alphas of sample period starting in 2010 for set 1 and 2 delivers significantly positive CAPM alphas for daily rebalancing.

It is crucial to look at the performance plots. Those show that the until the end of 2008, the performances were bad with some of the portfolios formed according to the strategy losing 10% or more between mid-2006 and 2009. However, when excess returns are considered, the performance would be better for this period (as indicated with the dotted magenta line). I.e. the (unrealistic) interest income from the short positions would have saved the strategy from dropping into negative territory.

The regression outputs additionally reveal that the alphas are significantly lower if the returns were adjusted by the expense ratios. This leads to the conclusion that the strategy essentially cashes in the expense ratio of the funds. In reality, the cost for shorting would most probably make the strategy unprofitable (arbitrage-free argument).

The portfolio built on set 3 only includes short positions on the VIX funds `XIV` and `VIIX`, which results in a positive performance for daily rebalancing, and a strongly negative performance for weekly rebalancing. It seems that, to some extent, a mispricing of the market of these funds was responsible for this observation.[37]

5 Short 2x's

According to strategy 5, both +/-2x LETFs are sold short. The summary statistics show that the monthly mean returns for daily and weekly rebalancing is positive for all sets and sample periods. The volatility figures are comparably low for some of them, which results in relatively high Sharpe ratios.

The regression analysis gives strongly significant (CAPM and three-factor) alphas for daily holding on all sets and samples (except for the complete length of set 1 where only 10% statistical significance is reached). The other factors do not seem to have a significant influence on the stategy's returns, but still the market beta has mostly a negative sign which indicates that performance is better when the market performs worse (and vice-versa). Even when adjusted for expense ratios, the performances (under daily rebalancing) is significanlty positive.

The version of weekly rebalancing lags behind the daily one, in the sense that the alphas are lower and by the majority insignificant.

The performance plots (Figures 16 and 17) show that the performance only lifted-off by end of 2008. When excess returns were used, the performance would have been more steadily (dotted

[37] See Alexander and Korovilas [2012].

magenta line).

6 Short 3x's

Similarly to the previous two strategies, this one applies the concept of short-selling to +/-3x LETFs. Again, positive mean returns are achieved. For daily rebalancing, the lowest monthly return since 2010 was still positive with 0.01% (set 1) and 0.03% (set 3).

The regression output gives strongly significant alphas for daily rebalancing. Even when adjusted for expense ratios, the returns were mostly significantly positive. While in case of weekly rebalancing, the returns are still mostly positive, the statistical significance cannot be kept up. The reason for this is the relatively high volatility, which also results in low Sharpe ratios. The unsteadily performance can also be seen in Figure 21.

7 Short 1x,2x's

The following strategies evaluated are combinations of the previous ones that were tested. One may chose to carry out these in order to diversify over a wider range of products. The returns resulting from these strategies may be compared to the previous ones in order to assess the benefit of diversification.

The strategy that shorts +/-1x and +/-2x funds delivers returns that lie between the two previous individual strategies, while the volatility is slightly reduced. Therefore, and as expected, the Sharpe ratio is higher (but still slightly below the ones obtained by shorting +/-2x LETFs alone).

8 Short 1x,3x's

Diversifying strategy 6 (that uses only +/-3x products) by adding +/-1x funds results in higher Sharpe ratios for set 1 (the others cannot be analyzed due to lack of data). For daily rebalancing, all alphas are significantly positive. Figure 24 shows that the performance is a lot more constant over time than the strategy using uniquely the +/-3x LETF.

9 Short 2x,3x's

The combination of +/-2x and +/-3x products leads also to slightly higher Sharpe ratios than the two strategies involving the specific LETFs separately. For daily rebalancing, all alphas are significantly positive.

The summary statistics show that the average monthly return for the complete sample period was 1.29%. This adds up to 16.63% per year, which is a remarkable return, especially when considering that in the worst month (August 2009), the loss was only 0.11%.

10 Short 1x,2x,3x's

The last combining strategy involves all funds of all six leverage categories. Due to lack of data, the strategy can only be back tested for the data set 1. There, the strategy delivers strongly positive alphas when rebalanced daily. The Sharpe ratios are also considerably higher than when using the funds with each leverage factor independently.

The monthly returns when rebalanced daily do not become negative for the analyzed sample. This is a further proof that diversification was beneficial. The good performance can also be observed in Figure 28.

11 Long 1x, short 3x's

The idea of strategy 11 is to combine the good performance received from short-selling the LETFs with the benefit of being long the unleveraged ETF on the same underlying. However, compared to strategy 6 that shorts +/-3x funds, the performance in terms of Sharpe ratios cannot be improved. While the alphas for daily rebalancing are still significantly positive, no improvement to strategy 6 can be achieved.

12 Long 1x, short 2x's

This last strategy seeks to improve strategy 5 by including the 1x ETF. Again, the performance cannot be improved for any of the historical data samples.

5 Discussion

5.1 Practical applicability

Some of the evaluated trading strategies have been found to have delivered constantly positive returns for the data looked at. Some of them even show astonishing performances. Furthermore, during the substantial drop in the world's financial markets in 2008, many of the strategies are found to have performed very strongly, as can be seen in the performance plots. The question that arises is, firstly, if the strategies would have been applicable under real world conditions, and secondly and even more importantly, if the strategies would work out of sample. If both of these questions could be affirmed, this would imply that a trader can make profits from applying the strategies henceforth.

The problem is that various cost components have been neglected in the analysis, which in reality are paramount for the successful implementation of any trading strategy. Firstly, straightforward transaction commissions were disregarded. This is the fee that a broker charges to cover the exchange fee as well as the effort of placing the order in the first place. This can reach significant amounts if a strategy requires frequent rebalancing. For the strategies that require daily rebalancing to ensure profitability, transaction costs may be a major component that can influence performance. Transaction costs are often assumed to be a function of the transaction volume.

In principle, the strategies described in the previous sections are passive strategies. The only adjustment of the holdings is essentially to balance out the exposure of the portfolio. For example, when the benchmark gains 10% of its value on one day, in the original BAB-strategy (1), the 2x LETF should gain 20% (and the 1x gains 10%). According to this strategy, a market participant sells 2x LETFs, which amount to half the value of what the participant bought in 1x ETFs. For instance, the participant starts with 100 (US$) invested in the 1x and -50 in the 2x fund. After one day of the underlying having returned 10%, the value in the 1x ETF reaches 110, while the 2x LETF goes down to -60. As can be seen, the exposure has been completely balanced, i.e. the 10 lost on the 2x LETF is balanced out by the 10 gained on the 1x ETF. In order to stay balanced during the following day as well, and to have the same amount of capital bound in the strategy, a market participant needs to sell 10 worth of 1x ETF and buy 10 worth of 2x LETF, which together reaches a sum of 20 (US$ trading volume).

Studies often state that they fully neglect transaction costs, while others[38] generally assume transaction costs between 5 and 30 basis point of the transaction (dollar) volume. The transaction volume of 20 is 13.33% of the invested capital, however, this was for the rare case of a 10% change on one day. The S&P 500 was found to have a daily volatility of 1.25% in the sample used in this paper. Therefore, on an average day, the transaction volume of the strategy would be 2.5 (when 150 was invested as before), amounting to 1.66% of the invested capital. Subtracting the assumed transaction costs from this figure results in a very small residual amount, even when annualized. Of course, as a consequence, for more volatile underlyings, or during volatile periods, the transaction cost caused by any strategy increases. Strategies that involve more products (especially with higher leverage) will simultaneously also face increased transaction costs. Logically, when the rebalancing period is chosen to be less frequent than daily, the transaction expenses shrink.

Secondly, security borrowing fees are an issue. When a market participant short-sells a stock (or an ETF which is traded just like a stock on the same markets), that trader is required to

[38] See for example Goldstein et al. [2009].

locate the security first.[39] In practice, anyone engaging in a short-selling transaction is asked to find a counterparty that holds the stock and who will agree to lend it out. Only once such a lending counterparty has been found, the borrower can sell the security on the market, and is required to purchase it back later in order for it to be handed back to the lender. For the time the security is borrowed, the lender demands a fee called the borrowing fee.[40] The borrowing fees rise when interest rates rise. However, they may also change as a result of other factors. There may also be a shortage of certain assets available for borrowing. During the financial crisis in 2008, the security borrowing market partially froze.[41]

In practice, a broker is generally commissioned to locate the securities a trader wishes to sell short. Brokers may have a pool of stock held by other customers, who have agreed that their stock can be lent out for a small fee. There is little transparency in the securities lending market, and there is no unique price for each transaction because of their two-sided nature (deal between the borrower and lender, or broker). Data for borrowing fees cannot be retrieved easily over financial information services.[42,43]

Thirdly, several characteristics of the data used in this paper require discussion. Market close prices are used to compute the returns, which is only a rough approximation of the actual realizable return. No trader can be sure to trade at the market close price, but trades during market hours with prices that may be near the market close price. In addition, a trader who wishes his order to be executed immediately sells/buys at the best offered/demanded price (bid/ask). As a result, further costs relating to every transaction will be incurred. The bid-ask spreads are relatively low for the products considered in this paper. During an average trading day, they range between 0.7 basis points (for the most liquid securities, e.g. `SPY` or `DIA`) to 40 basis point (for less liquid ones, e.g. `SSA` or `SDD`).[44] 7.9 basis points is the average spread during market hours for all 77 products examined in this paper.

Lastly, in this paper, the cost of capital was assumed to be the one-month US T-Bill (sec-

[39] The US Security Exchange Commission introduced Regulation SHO in 2005 which (among other restrictions) forbids naked short-selling. I.e. it requires a short-seller to locate a sufficient amount of the security before executing a transaction. Key points about Regulation SHO can be found on `http://www.sec.gov/spotlight/keyregshoissues.htm`.

[40] The lender also demands that dividends are repaid, however, this does not need to be investigated further when analysing total return data which accounts for such events by definition.

[41] Apparently, during the financial crisis in 2009, the 1x S&P 500 ETF became hard-to-borrow, see `http://zerohedge.blogspot.ch/2009/04/spy-have-become-hard-to-borrow.html`.

[42] Historical data is very expensive. A provider of such data is Data Explorers, `http://www.dataexplorers.com`.

[43] Interactive Brokers Group, Inc. is a retail broker who allows its customers to execute short-selling transactions. A snap-shot of borrowing fees in Spring 2012 shows that major securities, including many 1x ETFs such as the `SPY` or `EEM` cost 0.25% in (yearly) fees to short. For more exotic funds, such the -2x Silver `ZSL` or -2x DJ Financials `SKF`, the (retail) short-seller is charged 5.8%, or 6.2% respectively.

[44] The bid-ask spread is calculated as $(\text{ask} - \text{bid})/\text{bid}$.

ondary market) rate.[45] However, the returns were adjusted for the cost of capital only for long positions. On the other hand, all returns from short holdings were unadjusted because the interest would have otherwise been accumulated on the open short positions. This would lead to strong and unrealistic biases especially in the strategies that are fully hedged by opening multiple short positions.

Would the findings be comparable out of sample? There is no guarantee that any strategy designed using past data and back tested for the identical sample period would perform in the same way out of sample. There are, however, strong indications that the premium in LETFs is caused by the embedded leverage component, for which investors are ready to pay a surplus. In addition, there are strong indications that unconstrained investors may be able to exploit this premium. Yet it cannot be ruled out that this is a result of data-snooping, i.e. that the unprofitable strategies are omitted.[46,47]

Does the sobering conclusion that the strategies are probably inapplicable in real-life conditions render the work of this paper irrelevant? This is not the case because the trading strategy back testing, as well as the performance evaluation, reveal a lot about the relation between risk and expected return of products that embed leverage. The high profitability of certain trading strategies leads to strong evidence that embedded leverage is priced in the funds, which essentially drives them up in value.

5.2 Concluding remarks

In this paper, the performance characteristics of LETFs have been examined. LETFs are a relatively new class of alternative ETFs. The first of their kind only appeared in mid-2006, and thus, previous studies were forced to use shorter samples of empirical data. This may have resulted in reduced significance of findings in general. While the theoretical background of LETFs' performance had been described before (e.g. Lu et al. [2009]), empirical work has been limited to 1x and 2x LETFs (Frazzini and Pedersen [2011]), or additionally to inverse LETFs (Shum [2011]). Triple (inverse) LETFs have not been investigated before.

In this paper, the theoretical background on (leveraged) ETFs provided by other researchers, and the observation that high-beta assets underperform if adjusted for risk (Frazzini and Ped-

[45] Other authors use the three-month T-Bill rate, however, there is litte difference between them for the sample period used in this paper.

[46] For a discussion on and possible solutions to data-spoofing/snooping/mining issues, see Lo and MacKinlay [1990].

[47] For example, the strategies that used monthly rebalancing periods delivered thoroughly unsatisfying results, therefore they were omitted in this paper on purpose.

ersen [2010]), was combined with empirical data, i.e. LETFs. For the practical part, three data sets were examined, which cover a wide range of US listed LETFs on stock market indices, as well as on alternatives (i.e. commodities, real estate, and volatility). The total number of funds that have been included is 77, which is a considerably greater amount than what was used by previous studies. Thereby it allows to partially control for sample specific results. The recent financial crisis and heavy drop in markets occurring in the middle of the data sample, allowed for a closer study of the behavior of LETFs in volatile phases.

In a first step, the performance of the products was analyzed. Negative alphas were found for most products, even when adjusted for expense ratio and compounding. This indicates that LETFs (especially those that offer high leverage factors) lag behind their benchmarks when adjusted for their leverage multiplier. A not further specified residual premium (that other authors explain to be caused by the embedded leverage component in the products) is found to drive up the prices, which in turn results in a lower expected returns. The negative alphas were more extreme during periods of increased volatility in the market. Additionally, during these periods, the deviation of the alphas from their means (volatility) increased.

In the same step, betas are found to follow only inaccurately the leverage promised by the funds. For many funds, the betas are (partly significantly) below the claimed leverage even for daily returns. For example, the S&P 500 1x ETF (`SPY`) has a beta of 0.98, the 2x (`SSO`) has one of 1.91, and the 3x (`UPRO`) one of 2.94 (instead of 1, 2, and 3). Nevertheless, the funds are found to replicate their underlyings well, which means that while there are some performance issues, little to no deviation between the underlyings' changes and the funds' changes are observed (i.e. the volatility of the underlyings comprehensively explains the volatility of the funds). This is indicated by a large adjusted coefficient of determination.

Longer holding periods result in larger performance lags and deviation from the promised leverage. Lu et al. [2009], whose own performance analysis results are highly similar to the ones reported in this paper, conclude that a long-term investor can safely assume that LETFs return the desired multiple of the benchmark for holding periods of up to one month. Contrary to the total return indices assumed as the benchmark in this paper, the authors use the 1x ETF as the benchmark. Therefore, they only assess the LETFs' performance in relation to the 1x ETF, thereby neglecting the performance lag already comprised in the 1x ETF (as found in the performance analysis of the 1x ETFs vs. the indices).

In a second step, specifically designed trading strategies are applied to exploit the discovered performance issue. Various trading strategies are proposed, which in essence cover two areas; firstly, strategies that hold long positions in the simple ETF and short positions in various bullish LETFs, and secondly, strategies that hold short positions in both normal and inverse LETFs simultaneously. The common goal of applying these strategies is to generate profit while reducing the market risk exposure to a minimum. Essentially, the positions are adjusted such that the portfolio beta remains at zero. Additionally, some strategies that are evaluated short-sell 2x and/or 3x LETFs and along with that purchases the 1x ETFs on the same underlying. Similar to Frazzini and Pedersen [2011], these strategies are found to deliver positive returns for all data sets if rebalanced daily. Then, strategies that make use of inverse LETFs are explored. Again, positive performances are found. Interestingly, while daily rebalancings leads to partially strong performances, weekly (and longer) holding periods lower the profitability.

The performance plots show that the returns of the strategies changed in 2009, which was at the peak of the financial crisis. Sudden amplitudes in the LETFs' alphas led to the exceptional performances of some strategies. By 2009, a regime shift seems to have taken place. Various strategies have shown better performance in the most recent three years than previously. This is partially caused by the strongly lowered interest rates as a result of the quantitative easing policy by the US Fed to fight the global recession. Following the framework of excess returns used in this paper, entering a long position is nearly free of (capital) cost.

The embedded leverage component is a further explanation for the abnormal returns achieved by the strategies recently. Even though interest rates have been at record lows for the past couple of years, investors may face constraints which limit the use of direct leverage. Being unable to leverage up stock holdings directly, investors may turn toward LETFs. While there are strong indications that embedded leverage plays a role, this proposition cannot be tested.

While this paper was able to use a slightly longer sample period than previous authors (see above), future studies will benefit from even longer ones. The development of the ETF market is anything but over. While LETFs have received a lot of public attention over the past six years, and many new LETFs have been issued, partly on highly exotic niche markets, it seems that recently market participants have become better informed and concerned about the drawbacks of such products. The compounding effects in LETFs have been discussed in mainstream financial press.[48] On the other hand, many investors are unaware of other causes of lags observed in LETFs' returns, i.e. the leverage premium, on which no popular article can be found. It is

[48]See for example `http://online.wsj.com/article/SB10001424052702304543904577394261225920548.html`.

interesting to see whether the popularity of these products will remain high in the coming years.

A further interesting and worthy area of research would be the contribution of the LETFs performance to the overall market dynamics. For example, an analysis of those strategies that involve parallel holdings in LETFs as well as in options written on them (or on the underlying) would allow a researcher to explore and understand their dynamics in a broader context. Like any other stock, there are options traded on ETFs (and on LETFs).

While the strategies looked at may not be applicable in reality, as discussed in the previously, they teach us a lot about the relation between expected returns and the risks of assets with an embedded leverage component.

Due to compounding, investors are advised against the use of LETFs to amplify their performances over periods longer than a few days. Furthermore, the high expense ratios of LETFs decrease their attractiveness. Lastly, there are strong indications that a *leverage premium* inflates the prices of LETFs, which further reduces the performance of LETFs.

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B Tables and figures

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Table 1: **Data Sets (1/2).**

This table provides the ticker symbols of the funds used in each of the three data sets. In italics the corresponding first month of complete data is indicated. The ticker below the index specification is the one used on Bloomberg for the total return index. * Daily dollar volume (DDV) is the average daily US$ volume of all the ETFs on one underlying together scaled by 10^8. For example, in the S&P 500 products on average US$25.5 billion is traded over the exchange per day.

Panel A: Set 1

Leverage	-3	-2	-1	1	2	3	DDV*
Dow Jones Industrial	SDOW	DXD	DOG	DIA	DDM	UDOW	17.20
DJITR	*Mar10*	*Aug06*	*Jul06*	*Jul06*	*Jul06*	*Mar10*	
MSCI Emerging Markets	EDZ	EEV	EUM	EEM	EET	EDC	24.45
NDUEEGF	*Jan09*	*Nov07*	*Nov07*	*Jul06*	*Jul09*	*Jan09*	
NASDAQ-100	SQQQ	QID	PSQ	QQQ	QLD	TQQQ	62.14
XNDX	*Mar10*	*Aug06*	*Jul06*	*Jul96*	*Jul96*	*Mar10*	
Russell 2000	TZA	TWM	RWM	IWM	UWM	TNA	60.89
RU20INTR	*Dec08*	*Feb07*	*Feb07*	*Jul06*	*Feb07*	*Dec08*	
S&P 500	SPXU	SDS	SH	SPY	SSO	UPRO	255.51
SPXT	*Jul09*	*Aug06*	*Jul06*	*Jul06*	*Jul06*	*Jul09*	
S&P Mid Cap 400	SMDD	MZZ	MYY	MDY	MVV	UMDD	6.54
SPTRMDCP	*Mar10*	*Aug06*	*Jul06*	*Jul06*	*Jul06*	*Mar10*	

Panel B: Set 2

Leverage	-3	-2	-1	1	2	3	DDV*
Barclays U.S. 20+yr T		TBT	TBF	TLT	UBT		8.32
(not available)		*Jun08*	*Sep09*	*Feb07*	*Feb10*		
Dow Jones Basic Mat.		SMN	SBM	IYM	UYM		1.50
DJUSBMT		*Feb07*	*Apr10*	*Feb07*	*May07*		
Dow Jones Financials		SKF	SEF	IYF	UYG		14.13
DJUSFNT		*Feb07*	*Aug08*	*Feb07*	*Feb07*		
Dow Jones Oil/Gas		DUG	DDG	IYE	DIG		11.60
DJUSENT		*Feb07*	*Aug08*	*Feb07*	*Feb07*		
S&P SmallCap 600		SDD	SBB	IJR	SAA		4.08
SPTRSMCP		*Feb07*	*Feb07*	*Feb07*	*Feb07*		

Panel C: Set 3

Leverage	-3	-2	-1	1	2	3	DDV*
Silver		ZSL		SLV	AGQ		5.96
SLVRLN		*Dec08*		*Jul06*	*Dec08*		
Gold		GLL		GLD	UGL		14.94
GOLDLNPM		*Dec08*		*JUl06*	*Dec08*		
S&P 500 VIX st Futures			XIV	VIIX	TVIX		1.69
SPVXSP			*Dec10*	*Dec10*	*Dec10*		
MSCI US REIT	DRV			VNQ		DRN	1.55
RMZ	*Aug09*			*Jul06*		*Aug09*	
Dow Jones Real Estate		SRS		IYR	URE		8.91
DJUSRET		*Feb07*		*Jul06*	*Feb07*		
Russell 1000	BGZ			IWB		BGU	5.15
RU10INTR	*Dec08*			*Jul06*		*Dec08*	
NYSE Arca Gold Miners	DUST			GDX		NUGT	3.81
GDM	*Jan11*			*Jul06*		*Jan11*	

Table 2: **Data Sets (2/2).**

This table indicates the ticker symbols and the corresponding yearly expense ratios (in %) in italics. * Mktcap is the sum of assets under management (in million US$) as of May 1st, 2012, in all of the ETFs for one underlying. Generally, the 1x ETF accounts for the largest proportion (usually above 85%).

Panel A: Set 1

Leverage	-3	-2	-1	1	2	3	Mktcap*
Dow Jones Industrial	SDOW	DXD	DOG	DIA	DDM	UDOW	53.44
DJITR	*0.95*	*0.95*	*0.95*	*0.16*	*0.95*	*0.95*	
MSCI Emerging Markets	EDZ	EEV	EUM	EEM	EET	EDC	39.36
NDUEEGF	*0.95*	*0.95*	*0.95*	*0.67*	*0.95*	*0.95*	
NASDAQ-100	SQQQ	QID	PSQ	QQQ	QLD	TQQQ	34.71
XNDX	*0.95*	*0.95*	*0.95*	*0.2*	*0.95*	*0.95*	
Russell 2000	TZA	TWM	RWM	IWM	UWM	TNA	17.87
RU20INTR	*0.95*	*0.95*	*0.95*	*0.28*	*0.95*	*0.95*	
S&P 500	SPXU	SDS	SH	SPY	SSO	UPRO	109.94
SPXT	*0.95*	*0.95*	*0.95*	*0.09*	*0.95*	*0.95*	
S&P Mid Cap 400	SMDD	MZZ	MYY	MDY	MVV	UMDD	9.95
SPTRMDCP	*0.95*	*0.95*	*0.95*	*0.25*	*0.95*	*0.95*	

Panel B: Set 2

Leverage	-3	-2	-1	1	2	3	Mktcap*
Barclays U.S. 20+yr T		TBT	TBF	TLT	UBT		7.43
(not available)		*0.95*	*0.95*	*0.15*	*0.95*		
Dow Jones Basic Mat.		SMN	SBM	IYM	UYM		0.86
DJUSBMT		*0.95*	*0.95*	*0.48*	*0.95*		
Dow Jones Financials		SKF	SEF	IYF	UYG		1.74
DJUSFNT		*0.95*	*0.95*	*0.48*	*0.95*		
Dow Jones Oil/Gas		DUG	DDG	IYE	DIG		1.29
DJUSENT		*0.95*	*0.95*	*0.48*	*0.95*		
S&P SmallCap 600		SDD	SBB	IJR	SAA		7.75
SPTRSMCP		*0.95*	*0.95*	*0.2*	*0.95*		

Panel C: Set 3

Leverage	-3	-2	-1	1	2	3	Mktcap*
Silver		ZSL		SLV	AGQ		10.45
SLVRLN		*0.95*		*0.5*	*0.95*		
Gold		GLL		GLD	UGL		68.53
GOLDLNPM		*0.95*		*0.4*	*0.95*		
S&P 500 VIX ST Futures			XIV	VIIX	TVIX		0.89
SPVXSP			*1.35*	*0.89*	*1.65*		
MSCI US REIT	DRV			VNQ		DRN	12.61
RMZ	*0.95*			*0.12*		*0.95*	
Dow Jones Real Estate		SRS		IYR	URE		4.57
DJUSRET		*0.95*		*0.48*	*0.95*		
Russell 1000	BGZ			IWB		BGU	7.34
RU10INTR	*0.95*			*0.15*		*0.95*	
NYSE Arca Gold Miners	DUST			GDX		NUGT	8.43
GDM	*0.95*			*0.53*		*0.95*	

Table 3: **Performance (1/6): Dow Jones Industrial Average (DJITR).**
In Panel A, the summary statistics are presented (daily data since February 2010). In Panel B, the regression output is provided, firstly, for the (leveraged) ETF return on the benchmark index, and secondly, including the compounding factor (Comp., explained in Section 3.2). The data history may differ between products (see Table 1). 5% statistical significance is indicated in bold.

Panel A

		Mean	Min	Max	Std Dev	Skewness	Kurtosis
Index	DJITR	0.0007	-0.0550	0.0424	0.0112	-0.3654	6.1205
-3	SDOW	-0.0020	-0.1229	0.1633	0.0333	0.3840	6.1625
-2	DXD	-0.0014	-0.0818	0.1082	0.0223	0.3876	6.1236
-1	DOG	-0.0007	-0.0407	0.0542	0.0112	0.3882	6.0218
1	DIA	0.0006	-0.0541	0.0407	0.0112	-0.3849	6.1147
2	DDM	0.0012	-0.1075	0.0807	0.0223	-0.3909	6.0796
3	UDOW	0.0019	-0.1612	0.1223	0.0334	-0.3859	6.0231

Panel B

		Daily			Weekly				Monthly			
Lvg.	Ticker	Alpha	Beta	Adj.Rsq	Alpha	Beta	Comp.	Adj.Rsq	Alpha	Beta	Comp.	Adj.Rsq
-3	SDOW	-0.0001	**-2.9618**	0.9976	0.0000	-2.9997		0.9932	-0.0070	**-2.7829**		0.9806
		(-1.26)	(6.2)		(.01)	(.01)			(-1.91)	(2.78)		
					-0.0003	**-2.9828**	1.0222	0.9997	**-0.0010**	-2.9891	0.9736	0.9998
					(-2.17)	(3.69)	(1.13)		(-2.62)	(1.28)	(-1.38)	
-2	DXD	0.0001	**-1.9189**	0.9838	0.0002	**-1.9510**		0.9857	-0.0024	**-1.8255**		0.9652
		(1.02)	(12.57)		(.51)	(3.63)			(-1.01)	(4.15)		
					0.0006	**-1.9805**	0.9717	0.9953	**0.0025**	-1.9755	0.9400	0.9966
					(2.83)	(2.5)	(-.73)		(3.27)	(1.7)	(-1.59)	
-1	DOG	0.0001	**-0.9662**	0.9790	**0.0003**	**-0.9851**		0.9922	0.0004	**-0.9544**		0.9836
		(1.76)	(9.14)		(2.07)	(2.97)			(.53)	(3.07)		
					0.0004	-0.9936	0.9156	0.9957	**0.0019**	-0.9981	0.9727	0.9950
					(3.95)	(1.71)	(-1.46)		(4.1)	(.21)	(-.35)	
1	DIA	0.0000	1.0013	0.9743	-0.0001	**1.0120**		0.9934	-0.0002	**0.9915**		0.9995
		(-.32)	(.31)		(-.38)	(2.53)			(-1.15)	(-3.24)		
2	DDM	-0.0002	**1.8968**	0.9736	**-0.0008**	1.9901		0.9911	**-0.0038**	2.0063		0.9973
		(-1.33)	(-12.68)		(-2.38)	(-.91)			(-5.3)	(.5)		
					-0.0006	**1.9788**	**1.5684**	0.9933	**-0.0027**	1.9848	1.1161	0.9990
					(-2.07)	(-2.25)	(3.66)		(-6.18)	(-1.95)	(1.17)	
3	UDOW	0.0000	**2.9691**	0.9969	-0.0001	**2.9679**		0.9975	-0.0037	3.0639		0.9954
		(-.28)	(-4.38)		(-.42)	(-2.3)			(-1.9)	(1.54)		
					-0.0003	**2.9813**	1.0798	0.9993	**-0.0015**	2.9955	**1.0687**	0.9999
					(-1.59)	(-2.62)	(1.33)		(-5.56)	(-.78)	(2.36)	

Table 4: **Performance (2/6): MSCI Emerging Markets (NDUEEGF).**

In Panel A, the summary statistics are presented (daily data since February 2010). In Panel B, the regression output is provided, firstly, for the (leveraged) ETF return on the benchmark index, and secondly, including the compounding factor (Comp., explained in Section 3.2). The data history may differ between products (see Table 1). 5% statistical significance is indicated in bold.

Panel A

		Mean	Min	Max	Std Dev	Skewness	Kurtosis
Index	ND...	0.0004	-0.0631	0.0493	0.0123	-0.4126	5.2098
-3	EDZ	-0.0014	-0.2167	0.2433	0.0522	0.2001	5.6055
-2	EEV	-0.0009	-0.1459	0.1662	0.0351	0.2005	5.7073
-1	EUM	-0.0005	-0.0723	0.0826	0.0177	0.1820	5.5843
1	EEM	0.0004	-0.0835	0.0718	0.0177	-0.2132	5.6306
2	EET	0.0007	-0.1591	0.1382	0.0344	-0.1926	5.7171
3	EDC	0.0009	-0.2444	0.1967	0.0512	-0.2388	5.5849

Panel B

		Daily			Weekly				Monthly			
Lvg.	Ticker	Alpha	Beta	Adj.Rsq	Alpha	Beta	Comp.	Adj.Rsq	Alpha	Beta	Comp.	Adj.Rsq
-3	EDZ	-0.0002	-3.0015	0.4945	0.0014	**-3.2697**		0.8630	-0.0001	-2.8640		0.9335
		(-.17)	(-.01)		(.36)	(-2.72)			(-.01)	(1.11)		
					-0.0018	**-3.3162**	1.1589	0.8743	-0.0101	-3.2149	1.0588	0.9640
					(-.49)	(-3.3)	(.55)		(-1.27)	(-1.98)	(.32)	
-2	EEV	-0.0007	-1.9864	0.4097	-0.0035	-2.0906		0.7723	**-0.0285**	**-1.4997**		0.6800
		(-.55)	(.19)		(-.98)	(-1.22)			(-2.17)	(3.52)		
					-0.0062	-2.0938	1.1824	0.7990	**-0.0358**	-1.8108	1.8577	0.7312
					(-1.84)	(-1.34)	(.87)		(-2.92)	(1.17)	(1.51)	
-1	EUM	-0.0003	-0.9846	0.3793	-0.0012	-1.0533		0.8030	-0.0085	-0.9080		0.8703
		(-.42)	(.41)		(-.76)	(-1.56)			(-1.9)	(1.89)		
					-0.0023	-1.0575	1.3603	0.8187	**-0.0101**	-0.9504	0.9114	0.8728
					(-1.45)	(-1.75)	(1.21)		(-2.21)	(.88)	(-.14)	
1	EEM	0.0001	0.9621	0.3691	0.0000	**1.0702**		0.8454	-0.0008	**1.0904**		0.9292
		(.27)	(-1.15)		(-.03)	(2.67)			(-.25)	(2.5)		
2	EET	0.0000	1.9119	0.4779	-0.0011	2.1324		0.8668	-0.0060	**2.2023**		0.9717
		(.01)	(-1.19)		(-.5)	(1.94)			(-1.34)	(3.09)		
					-0.0011	2.1323	-0.0216	0.8659	-0.0076	**2.1909**	1.1800	0.9727
					(-.48)	(1.93)	(-.8)		(-1.67)	(2.95)	(.23)	
3	EDC	-0.0002	2.9895	0.5007	-0.0028	**3.3771**		0.8772	-0.0158	**3.4176**		0.9697
		(-.13)	(-.1)		(-.77)	(3.92)			(-1.9)	(4.32)		
					-0.0040	**3.3517**	0.9042	0.8782	**-0.0203**	**3.2955**	0.7067	0.9722
					(-1.09)	(3.61)	(-.16)		(-2.45)	(2.69)	(-.86)	

Table 5: **Performance (3/6): NASDAQ-100 (XNDX).**

In Panel A, the summary statistics are presented (daily data since February 2010). In Panel B, the regression output is provided, firstly, for the (leveraged) ETF return on the benchmark index, and secondly, including the compounding factor (Comp., explained in Section 3.2). The data history may differ between products (see Table 1). 5% statistical significance is indicated in bold.

Panel A

		Mean	Min	Max	Std Dev	Skewness	Kurtosis
Index	XNDX	0.0009	-0.0610	0.0505	0.0131	-0.2579	5.5147
-3	SQQQ	-0.0027	-0.1453	0.1810	0.0383	0.2593	5.4305
-2	QID	-0.0018	-0.0995	0.1202	0.0257	0.2545	5.4629
-1	PSQ	-0.0009	-0.0502	0.0583	0.0129	0.2473	5.4066
1	QQQ	0.0009	-0.0601	0.0520	0.0130	-0.2517	5.5136
2	QLD	0.0017	-0.1171	0.0998	0.0257	-0.2529	5.4279
3	TQQQ	0.0026	-0.1757	0.1492	0.0382	-0.2656	5.3846

Panel B

		Daily			Weekly				Monthly			
Lvg.	Ticker	Alpha	Beta	Adj.Rsq	Alpha	Beta	Comp.	Adj.Rsq	Alpha	Beta	Comp.	Adj.Rsq
-3	SQQQ	-0.0002	**-2.9240**	0.9924	0.0005	-3.0014		0.9899	**-0.0132**	**-2.6207**		0.9791
		(-1.11)	(7.03)		(.53)	(-.05)			(-3.17)	(4.96)		
					-0.0004	-2.9746	0.9661	0.9978	**-0.0012**	-2.9980	0.9869	0.9998
					(-1.07)	(1.92)	(-.71)		(-2.9)	(.21)	(-.73)	
-2	QID	0.0000	**-1.8930**	0.9807	0.0002	**-1.9529**		0.9855	**-0.0043**	**-1.7953**		0.9820
		(.22)	(15.38)		(.47)	(3.45)			(-2.17)	(6.94)		
					0.0005	**-1.9754**	0.9911	0.9959	0.0017	**-1.9668**	**0.8948**	0.9969
					(2.06)	(3.38)	(-.25)		(1.96)	(2.14)	(-2.12)	
-1	PSQ	0.0001	**-0.9484**	0.9670	0.0003	-0.9888		0.9894	0.0001	**-0.9553**		0.9869
		(.85)	(11.29)		(1.56)	(1.9)			(.12)	(3.37)		
					0.0004	-0.9960	1.0367	0.9941	**0.0022**	-1.0161	1.1725	0.9949
					(2.66)	(.9)	(.55)		(3.77)	(-1.59)	(1.54)	
1	QQQ	0.0000	**0.9557**	0.9850	0.0000	**0.9873**		0.9949	-0.0001	**0.9936**		0.9996
		(-.24)	(-14.39)		(-.1)	(-3.13)			(-.61)	(-2.67)		
2	QLD	-0.0002	**1.8891**	0.9782	**-0.0007**	1.9908		0.9950	**-0.0035**	2.0082		0.9974
		(-1.26)	(-15.07)		(-2.3)	(-1.13)			(-4.22)	(.67)		
					-0.0006	1.9855	0.8839	0.9958	**-0.0026**	1.9859	1.0179	0.9982
					(-2.3)	(-1.94)	(-.97)		(-3.62)	(-1.29)	(.1)	
3	TQQQ	0.0000	**2.9187**	0.9947	0.0002	**2.9668**		0.9969	**-0.0054**	**3.1406**		0.9959
		(-.23)	(-9.02)		(.39)	(-2.14)			(-2.44)	(3.48)		
					-0.0003	**2.9840**	1.0685	0.9992	**-0.0011**	2.9896	1.0649	0.9999
					(-1.32)	(-2.08)	(1.2)		(-2.34)	(-1.08)	(1.55)	

Table 6: **Performance (4/6): Russell 2000 (RU20INTR).**

In Panel A, the summary statistics are presented (daily data since February 2010). In Panel B, the regression output is provided, firstly, for the (leveraged) ETF return on the benchmark index, and secondly, including the compounding factor (Comp., explained in Section 3.2). The data history may differ between products (see Table 1). 5% statistical significance is indicated in bold.

Panel A

		Mean	Min	Max	Std Dev	Skewness	Kurtosis
Index	RU...	0.0007	-0.0890	0.0695	0.0178	-0.1483	5.4555
-3	TZA	-0.0025	-0.2030	0.2602	0.0518	0.1597	5.5672
-2	TWM	-0.0016	-0.1381	0.1745	0.0347	0.1716	5.6296
-1	RWM	-0.0008	-0.0696	0.0850	0.0175	0.1332	5.5345
1	IWM	0.0007	-0.0870	0.0667	0.0173	-0.1629	5.5083
2	UWM	0.0014	-0.1719	0.1328	0.0345	-0.1607	5.5269
3	TNA	0.0021	-0.2600	0.1993	0.0516	-0.1620	5.5709

Panel B

		Daily			Weekly				Monthly			
Lvg.	Ticker	Alpha	Beta	Adj.Rsq	Alpha	Beta	Comp.	Adj.Rsq	Alpha	Beta	Comp.	Adj.Rsq
-3	TZA	**-0.0004**	**-2.8825**	0.9928	-0.0013	**-2.9177**		0.9766	**-0.0142**	**-2.7307**		0.9668
		(-2.56)	(14.06)		(-.87)	(2.43)			(-2.05)	(3.37)		
					-0.0013	**-2.9623**	**1.0817**	0.9985	**-0.0061**	**-2.9261**	**0.9444**	0.9994
					(-3.33)	(4.41)	(3.87)		(-6.47)	(6.4)	(-2.71)	
-2	TWM	0.0000	**-1.9563**	0.9842	-0.0001	-1.9727		0.9774	**-0.0078**	**-1.8427**		0.9798
		(-.05)	(6.41)		(-.08)	(1.51)			(-2.81)	(4.68)		
					0.0001	-1.9923	1.0361	0.9973	0.0000	**-1.9760**	0.9523	0.9981
					(.47)	(1.21)	(1.55)		(-.04)	(2.05)	(-1.21)	
-1	RWM	0.0000	**-0.9788**	0.9890	0.0000	**-0.9802**		0.9903	**-0.0019**	**-0.9611**		0.9913
		(.12)	(7.48)		(.04)	(3.38)			(-2.04)	(3.39)		
					0.0001	**-0.9861**	0.9481	0.9977	0.0003	-0.9916	0.9436	0.9976
					(.53)	(4.84)	(-1.62)		(.64)	(1.31)	(-.76)	
1	IWM	0.0000	**0.9672**	0.9806	0.0000	**0.9858**		0.9960	-0.0001	**0.9852**		0.9992
		(-.39)	(-9.25)		(-.13)	(-3.97)			(-.22)	(-4.54)		
2	UWM	-0.0001	**1.9528**	0.9840	-0.0005	1.9860		0.9953	**-0.0035**	**1.9703**		0.9972
		(-.69)	(-6.9)		(-1.37)	(-1.69)			(-3.23)	(-2.26)		
					-0.0005	**1.9810**	0.8438	0.9966	**-0.0019**	**1.9810**	1.0567	0.9987
					(-1.49)	(-2.7)	(-1.9)		(-2.43)	(-2.11)	(.46)	
3	TNA	0.0000	**2.8471**	0.9908	-0.0002	**2.9333**		0.9961	-0.0037	**2.8828**		0.9906
		(-.18)	(-16.41)		(-.27)	(-4.84)			(-.96)	(-2.64)		
					-0.0001	**2.9250**	**0.7041**	0.9982	-0.0007	**2.9234**	1.0333	0.9973
					(-.34)	(-8.02)	(-6.09)		(-.31)	(-3.15)	(.32)	

Table 7: **Performance (5/6): S&P 500 (SPXT).**
In Panel A, the summary statistics are presented (daily data since February 2010). In Panel B, the regression output is provided, firstly, for the (leveraged) ETF return on the benchmark index, and secondly, including the compounding factor (Comp., explained in Section 3.2). The data history may differ between products (see Table 1). 5% statistical significance is indicated in bold.

Panel A

		Mean	Min	Max	Std Dev	Skewness	Kurtosis
Index	SPXT	0.0006	-0.0665	0.0474	0.0125	-0.3649	6.3721
-3	SPXU	-0.0020	-0.1412	0.1929	0.0369	0.3705	6.2976
-2	SDS	-0.0013	-0.0948	0.1290	0.0248	0.3499	6.3002
-1	SH	-0.0007	-0.0480	0.0648	0.0124	0.3536	6.3143
1	SPY	0.0006	-0.0651	0.0465	0.0124	-0.3837	6.2812
2	SSO	0.0012	-0.1286	0.0934	0.0247	-0.3750	6.3113
3	UPRO	0.0018	-0.1975	0.1391	0.0369	-0.3917	6.4076

Panel B

		Daily			Weekly				Monthly			
Lvg.	Ticker	Alpha	Beta	Adj.Rsq	Alpha	Beta	Comp.	Adj.Rsq	Alpha	Beta	Comp.	Adj.Rsq
-3	SPXU	-0.0001	**-2.9429**	0.9917	0.0003	-2.9937		0.9882	**-0.0084**	**-2.7594**		0.9768
		(-.8)	(5.69)		(.4)	(.23)			(-2.27)	(3.25)		
					-0.0002	-2.9784	0.9446	0.9954	-0.0005	-3.0030	**1.0301**	0.9999
					(-.44)	(1.28)	(-.89)		(-1.68)	(-.44)	(2.07)	
-2	SDS	0.0001	**-1.9224**	0.9855	0.0003	**-1.9609**		0.9842	-0.0031	**-1.7979**		0.9736
		(1.2)	(12.71)		(.64)	(2.73)			(-1.38)	(5.63)		
					0.0007	**-1.9839**	0.9595	0.9961	**0.0029**	-1.9797	0.9348	0.9969
					(3.2)	(2.24)	(-1.28)		(3.58)	(1.38)	(-1.57)	
-1	SH	**0.0001**	**-0.9790**	0.9843	**0.0004**	-0.9966		0.9912	0.0004	**-0.9515**		0.9880
		(2.15)	(6.51)		(2.08)	(.64)			(.45)	(3.84)		
					0.0005	-1.0038	0.9856	0.9964	**0.0021**	-1.0015	0.9387	0.9953
					(4.51)	(-1.1)	(-.31)		(4.07)	(-.17)	(-.68)	
1	SPY	0.0000	**0.9838**	0.9774	0.0000	1.0030		0.9945	-0.0001	**0.9922**		0.9995
		(-.27)	(-4.16)		(-.13)	(.7)			(-.44)	(-3.03)		
2	SSO	-0.0002	**1.9050**	0.9826	**-0.0007**	2.0020		0.9923	**-0.0038**	1.9935		0.9978
		(-1.59)	(-14.38)		(-2.21)	(.2)			(-5.5)	(-.58)		
					-0.0006	1.9943	**1.3471**	0.9944	**-0.0026**	**1.9739**	1.0416	0.9988
					(-2.)	(-.65)	(2.78)		(-4.99)	(-3.02)	(.31)	
3	UPRO	0.0000	**2.9413**	0.9922	-0.0001	2.9698		0.9948	**-0.0041**	3.0596		0.9959
		(-.39)	(-6.03)		(-.11)	(-1.7)			(-2.36)	(1.74)		
					-0.0003	2.9808	1.0582	0.9970	**-0.0014**	2.9922	1.0180	0.9999
					(-.9)	(-1.41)	(.57)		(-4.77)	(-1.3)	(.6)	

Table 8: **Performance (6/6): S&P Mid Cap 400 (SPTRMDCP).**
In Panel A, the summary statistics are presented (daily data since February 2010). In Panel B, the regression output is provided, firstly, for the (leveraged) ETF return on the benchmark index, and secondly, including the compounding factor (Comp., explained in Section 3.2). The data history may differ between products (see Table 1). 5% statistical significance is indicated in bold.

Panel A

		Mean	Min	Max	Std Dev	Skewness	Kurtosis
Index	SP...	0.0008	-0.0824	0.0639	0.0152	-0.3062	6.2652
-3	SMDD	-0.0024	-0.1935	0.2475	0.0454	0.2927	6.3123
-2	MZZ	-0.0016	-0.1288	0.1635	0.0301	0.2999	6.3308
-1	MYY	-0.0008	-0.0634	0.0816	0.0151	0.3224	6.2353
1	MDY	0.0007	-0.0807	0.0625	0.0151	-0.3107	6.2534
2	MVV	0.0015	-0.1635	0.1288	0.0301	-0.3056	6.3208
3	UMDD	0.0023	-0.2483	0.1889	0.0452	-0.3096	6.2459

Panel B

		Daily			Weekly				Monthly			
Lvg.	Ticker	Alpha	Beta	Adj.Rsq	Alpha	Beta	Comp.	Adj.Rsq	Alpha	Beta	Comp.	Adj.Rsq
-3	SMDD	-0.0002	-2.9889	0.9949	0.0018	-3.0795		0.9806	-0.0108	**-2.7232**		0.9647
		(-1.19)	(1.22)		(1.17)	(-1.96)			(-1.6)	(2.66)		
					-0.0003	-2.9989	1.0224	0.9992	**-0.0011**	-2.9966	0.9737	0.9998
					(-.97)	(.14)	(1.15)		(-2.2)	(.39)	(-1.8)	
-2	MZZ	0.0001	**-1.9720**	0.9882	0.0007	-1.9791		0.9806	-0.0036	**-1.8032**		0.9834
		(1.17)	(4.98)		(1.06)	(1.3)			(-1.74)	(6.94)		
					0.0007	-1.9976	0.9923	0.9976	**0.0025**	-1.9797	1.0008	0.9978
					(3.12)	(.44)	(-.36)		(3.03)	(1.53)	(.02)	
-1	MYY	0.0000	**-0.9662**	0.9411	0.0000	-0.9912		0.9516	-0.0010	-1.0103		0.9703
		(-.1)	(5.36)		(-.05)	(.69)			(-.65)	(-.48)		
					0.0000	-0.9968	0.9108	0.9574	-0.0006	-1.0211	**0.2390**	0.9701
					(-.06)	(.26)	(-.64)		(-.38)	(-.82)	(-2.37)	
1	MDY	0.0000	1.0069	0.9704	-0.0001	**1.0130**		0.9953	-0.0004	**0.9890**		0.9991
		(-.43)	(1.51)		(-.62)	(3.27)			(-1.41)	(-3.17)		
2	MVV	-0.0002	**1.9778**	0.9857	**-0.0006**	2.0149		0.9960	**-0.0035**	**1.9789**		0.9981
		(-1.53)	(-3.58)		(-2.06)	(2.01)			(-4.6)	(-2.03)		
					-0.0006	2.0097	**0.8223**	0.9971	**-0.0030**	**1.9735**	0.7358	0.9986
					(-2.5)	(1.53)	(-2.31)		(-4.47)	(-2.95)	(-1.83)	
3	UMDD	0.0000	**2.9769**	0.9964	0.0006	**2.9505**		0.9961	-0.0039	3.0491		0.9938
		(-.05)	(-3.04)		(.88)	(-2.88)			(-1.26)	(1.02)		
					-0.0003	**2.9844**	0.9491	0.9997	**-0.0015**	2.9901	**1.0513**	0.9999
					(-1.86)	(-3.15)	(-1.95)		(-4.45)	(-1.91)	(2.29)	

Table 9: **Hypothetical performance of LETFs and deviation due to compounding.** This table provides the hypothetical performance of a benchmark and LETFs tracking its performance multiplied by their target leverage on a daily basis. Four examples of benchmark returns are assumed; +10% on every one of the four days, +10% on three days and -10% on the fourth day, +10% on two days followed by -10% on the remaining two, +10% only on the first day, and the benchmark losing -10% every day. The deviation due to compounding (dev., in *percentage points* (pp)) is the LETF's four-day performance minus the benchmark's four-day performance multiplied by the LETF's leverage factor.

Benchmark											
r_t	r_{t+1}	r_{t+2}	r_{t+3}	$r_{t,t+4}$							
10%	10%	10%	10%	46.41%							
10%	10%	10%	-10%	19.79%							
10%	10%	-10%	-10%	-1.99%							
10%	-10%	-10%	-10%	-19.81%							
-10%	-10%	-10%	-10%	-34.39%							
1x ETF						**-1x LETF**					
r_t	r_{t+1}	r_{t+2}	r_{t+3}	$r_{t,t+4}$	dev.	r_t	r_{t+1}	r_{t+2}	r_{t+3}	$r_{t,t+4}$	dev.
10%	10%	10%	10%	46.41%	0.00pp	-10%	-10%	-10%	-10%	-34.39%	12.02pp
10%	10%	10%	-10%	19.79%	0.00pp	-10%	-10%	-10%	10%	-19.81%	-0.02pp
10%	10%	-10%	-10%	-1.99%	0.00pp	-10%	-10%	10%	10%	-1.99%	-3.98pp
10%	-10%	-10%	-10%	-19.81%	0.00pp	-10%	10%	10%	10%	19.79%	-0.02pp
-10%	-10%	-10%	-10%	-34.39%	0.00pp	10%	10%	10%	10%	46.41%	12.02pp
2x LETF						**-2x LETF**					
r_t	r_{t+1}	r_{t+2}	r_{t+3}	$r_{t,t+4}$	dev.	r_t	r_{t+1}	r_{t+2}	r_{t+3}	$r_{t,t+4}$	dev.
20%	20%	20%	20%	107.36%	14.54pp	-20%	-20%	-20%	-20%	-59.04%	33.78pp
20%	20%	20%	-20%	38.24%	-1.34pp	-20%	-20%	-20%	20%	-38.56%	1.02pp
20%	20%	-20%	-20%	-7.84%	-3.86pp	-20%	-20%	20%	20%	-7.84%	-11.8pp
20%	-20%	-20%	-20%	-38.56%	1.06pp	-20%	20%	20%	20%	38.24%	-1.38pp
-20%	-20%	-20%	-20%	-59.04%	9.74pp	20%	20%	20%	20%	107.36%	38.58pp
3x LETF						**-3x LETF**					
r_t	r_{t+1}	r_{t+2}	r_{t+3}	$r_{t,t+4}$	dev.	r_t	r_{t+1}	r_{t+2}	r_{t+3}	$r_{t,t+4}$	dev.
30%	30%	30%	30%	185.61%	46.38pp	-30%	-30%	-30%	-30%	-75.99%	63.24pp
30%	30%	30%	-30%	53.79%	-5.58pp	-30%	-30%	-30%	30%	-55.41%	3.96pp
30%	30%	-30%	-30%	-17.19%	-11.2pp	-30%	-30%	30%	30%	-17.19%	-23.2pp
30%	-30%	-30%	-30%	-55.41%	4.02pp	-30%	30%	30%	30%	53.79%	-5.64pp
-30%	-30%	-30%	-30%	-75.99%	27.18pp	30%	30%	30%	30%	185.61%	82.44pp

Table 10: **Strategies.**

This table indicate the position held in the ETF/LETFs of each leverage category according to the strategies. For example strategy 2, named *F/P11-BAB 3x*, requires to hold a long position of one 1x ETF for each third of a 3x LETF sold on the same underlying.

	Leverage	-3	-2	-1	1	2	3
1	F/P11-BAB				1	- 1/2	
2	F/P11-BAB 3x				1		- 1/3
3	F/P11-BAB 2x,3x				1	- 1/4	- 1/6
4	Short 1x's			-1	-1		
5	Short 2x's		-1			-1	
6	Short 3x's	-1					-1
7	Short 1x,2x's		-1	-1	-1	-1	
8	Short 1x,3x's	-1		-1	-1		-1
9	Short 2x,3x's	-1	-1			-1	-1
10	Short 1x,2x,3x's	-1	-1	-1	-1	-1	-1
11	Long 1x, short 3x's	- 2/3			1		-1
12	Long 1x, short 2x's		- 1/2		1	-1	

Table 11: **1 Frazzini and Pedersen [2011]-BAB.**
Panel A shows the descriptive statistics of the betting-against-beta (BAB) strategy as defined by Frazzini and Pedersen [2011], applied to 1x and 2x funds. The mean is the simple excess return. †SR is the Sharpe ratio. Panel B gives the strategies' alphas, firstly as for a regression on market excess returns (capm), and secondly accounting for the three Fama-French factors (3fac). All betas are from the latter model, first on the market excess return (mkt), and for the small-minus-big (smb) and high-minus-low (hml) factors. The last two columns indicated with an asterisk * use returns with the funds' expense ratio added back. All values are in monthly terms.

Panel A

	Mean	Min	Max	Std Dev	Skewness	Kurtosis	SR†
daily							
Set 1	0.0002	-0.0029	0.0070	0.0017	1.0191	6.4199	0.1349
Set 1, 10	0.0005	-0.0004	0.0015	0.0006	0.2689	1.8849	0.8937
Set 2	0.0006	-0.0054	0.0132	0.0026	1.6569	10.2886	0.2257
Set 2, 10	0.0004	-0.0010	0.0021	0.0006	0.3919	4.8413	0.6620
Set 3	0.0012	-0.0375	0.0502	0.0097	0.9577	15.4101	0.1289
Set 3, 10	0.0015	-0.0138	0.0115	0.0063	-1.0185	3.7844	0.2344
weekly							
Set 1	0.0002	-0.0083	0.0145	0.0025	2.1245	16.9597	0.0763
Set 1, 10	0.0003	-0.0083	0.0030	0.0020	-3.1008	14.9267	0.1290
Set 2	0.0003	-0.0137	0.0145	0.0041	-0.1218	6.9763	0.0793
Set 2, 10	0.0000	-0.0114	0.0048	0.0027	-2.8304	13.5516	-0.0057
Set 3	0.0011	-0.0340	0.0310	0.0101	-0.6615	6.3548	0.1110
Set 3, 10	-0.0018	-0.0292	0.0125	0.0108	-1.1669	4.0268	-0.1693

Panel B

	$\text{Alpha}_{\text{capm}}$	$\text{Alpha}_{\text{3fac}}$	Beta_{mkt}	Beta_{smb}	Beta_{hml}	$\text{Alpha*}_{\text{capm}}$	$\text{Alpha*}_{\text{3fac}}$
daily							
Set 1	0.0003	0.0002	**-0.0109**	0.0176	-0.0079	0.0001	0.0000
	(1.36)	(1.12)	(-2.64)	(1.91)	(-1.03)	(.32)	(.08)
Set 1, 10	**0.0005**	**0.0004**	0.0021	-0.0009	**-0.0156**	**0.0003**	0.0002
	(4.36)	(3.38)	(.76)	(-.16)	(-2.51)	(3.01)	(2.03)
Set 2	0.0006	0.0006	-0.0127	0.0075	0.0038	0.0005	0.0005
	(1.92)	(1.85)	(-1.86)	(.47)	(.29)	(1.69)	(1.63)
Set 2, 10	**0.0004**	**0.0005**	-0.0048	0.0034	0.0066	**0.0003**	**0.0004**
	(3.56)	(3.38)	(-1.35)	(.42)	(.82)	(2.78)	(2.75)
Set 3	0.0012	0.0012	-0.0238	0.0963	0.0371	0.0012	0.0012
	(1.01)	(.98)	(-.94)	(1.64)	(.76)	(1.01)	(.98)
Set 3, 10	0.0017	0.0021	-0.0552	0.0645	0.0429	0.0017	0.0021
	(1.06)	(1.06)	(-.88)	(.44)	(.28)	(1.06)	(1.06)
weekly							
Set 1	0.0002	0.0002	-0.0104	0.0154	-0.0074	0.0000	0.0000
	(.76)	(.61)	(-1.57)	(1.05)	(-.6)	(.09)	(-.05)
Set 1, 10	0.0002	0.0003	-0.0075	0.0347	0.0031	0.0001	0.0001
	(.57)	(.6)	(-.71)	(1.55)	(.14)	(.17)	(.24)
Set 2	0.0003	0.0004	-0.0044	0.0023	0.0084	0.0003	0.0003
	(.64)	(.67)	(-.4)	(.09)	(.4)	(.5)	(.53)
Set 2, 10	0.0000	0.0001	-0.0113	0.0493	0.0035	-0.0001	0.0000
	(-.07)	(.13)	(-.69)	(1.31)	(.09)	(-.24)	(-.01)
Set 3	0.0012	0.0009	-0.0178	0.0450	-0.0335	0.0012	0.0009
	(.91)	(.72)	(-.66)	(.72)	(-.65)	(.91)	(.72)
Set 3, 10	-0.0024	-0.0014	0.0016	0.2604	0.0447	-0.0024	-0.0014
	(-.93)	(-.45)	(.02)	(1.13)	(.19)	(-.92)	(-.45)

Table 12: **2 Frazzini and Pedersen [2011]-BAB 3x.**
Panel A shows the descriptive statistics of a betting-against-beta (BAB) strategy similar to the one defined by Frazzini and Pedersen [2011], applied to 1x and 3x funds. The mean is the simple excess return. †SR is the Sharpe ratio. Panel B gives the strategies' alphas, firstly as for a regression on market excess returns (capm), and secondly accounting for the three Fama-French factors (3fac). All betas are from the latter model, first on the market excess return (mkt), and for the small-minus-big (smb) and high-minus-low (hml) factors. The last two columns indicated with an asterisk * use returns with the funds' expense ratio added back. All values are in monthly terms.

Panel A

	Mean	Min	Max	Std Dev	Skewness	Kurtosis	SR†
daily							
Set 1	0.0009	-0.0093	0.0143	0.0035	1.5420	10.3765	0.2491
Set 1, 10	0.0007	-0.0005	0.0027	0.0009	0.9088	3.0061	0.8064
Set 3	0.0005	-0.0334	0.0122	0.0072	-2.6192	13.2947	0.0719
Set 3, 10	0.0023	-0.0117	0.0122	0.0066	-0.4241	2.7137	0.3530
weekly							
Set 1	0.0002	-0.0181	0.0258	0.0056	1.3496	13.8409	0.0436
Set 1, 10	0.0001	-0.0181	0.0057	0.0042	-3.2194	15.4532	0.0318
Set 3	0.0000	-0.0264	0.0173	0.0073	-0.7606	6.4492	-0.0020
Set 3, 10	0.0014	-0.0132	0.0173	0.0085	0.2741	2.6023	0.1712

Panel B

	$\text{Alpha}_{\text{capm}}$	$\text{Alpha}_{\text{3fac}}$	Beta_{mkt}	Beta_{smb}	Beta_{hml}	$\text{Alpha*}_{\text{capm}}$	$\text{Alpha*}_{\text{3fac}}$
daily							
Set 1	0.0006	0.0005	0.0232	-0.0008	-0.0160	0.0006	0.0005
	(1.11)	(.82)	(1.66)	(-.03)	(-.73)	(1.11)	(.82)
Set 1, 10	**0.0007**	**0.0006**	0.0003	0.0080	-0.0149	**0.0006**	**0.0005**
	(3.9)	(3.09)	(.07)	(.83)	(-1.5)	(3.72)	(2.91)
Set 3	0.0005	0.0003	-0.0211	**0.1064**	-0.0001	0.0004	0.0002
	(.4)	(.25)	(-.74)	(2.09)	(-.)	(.31)	(.16)
Set 3, 10	0.0020	0.0029	0.0580	-0.0162	0.1535	0.0019	0.0029
	(1.24)	(1.5)	(.97)	(-.12)	(1.06)	(1.21)	(1.48)
weekly							
Set 1	0.0000	-0.0001	0.0039	0.0513	0.0042	0.0000	-0.0001
	(-.01)	(-.06)	(.17)	(1.27)	(.12)	(-.02)	(-.07)
Set 1, 10	0.0001	0.0001	-0.0143	0.0576	-0.0070	0.0001	0.0000
	(.14)	(.09)	(-.63)	(1.2)	(-.14)	(.1)	(.05)
Set 3	-0.0002	-0.0003	-0.0098	0.0902	-0.0046	-0.0003	-0.0004
	(-.13)	(-.28)	(-.34)	(1.73)	(-.1)	(-.22)	(-.36)
Set 3, 10	0.0009	0.0028	0.0675	0.0080	0.2797	0.0009	0.0028
	(.47)	(1.19)	(.94)	(.05)	(1.6)	(.45)	(1.17)

Table 13: **3 Frazzini and Pedersen [2011]-BAB 2x, 3x.**
Panel A shows the descriptive statistics of a betting-against-beta (BAB) strategy similar to the one defined by Frazzini and Pedersen [2011], applied to 1x, 2x and 3x funds. The mean is the simple excess return. †SR is the Sharpe ratio. Panel B gives the strategies' alphas, firstly as for a regression on market excess returns (capm), and secondly accounting for the three Fama-French factors (3fac). All betas are from the latter model, first on the market excess return (mkt), and for the small-minus-big (smb) and high-minus-low (hml) factors. The last two columns indicated with an asterisk * use returns with the funds' expense ratio added back. All values are in monthly terms.

Panel A

	Mean	Min	Max	Std Dev	Skewness	Kurtosis	SR
daily							
Set 1	0.0010	-0.0018	0.0125	0.0023	3.3925	17.5133	0.4473
Set 1, 10	0.0006	-0.0003	0.0021	0.0007	0.8268	2.6899	0.8935
weekly							
Set 1	0.0007	-0.0132	0.0157	0.0038	0.5103	11.3955	0.1838
Set 1, 10	0.0002	-0.0132	0.0044	0.0031	-3.2440	15.5964	0.0634

Panel B

	$Alpha_{capm}$	$Alpha_{3fac}$	$Beta_{mkt}$	$Beta_{smb}$	$Beta_{hml}$	$Alpha^*_{capm}$	$Alpha^*_{3fac}$
daily							
Set 1	**0.0010**	0.0007	0.0080	0.0177	-0.0275	**0.0009**	0.0006
	(2.65)	(1.94)	(.92)	(1.13)	(-2.01)	(2.42)	(1.72)
Set 1, 10	**0.0006**	**0.0005**	0.0012	0.0035	-0.0153	**0.0005**	**0.0004**
	(4.34)	(3.4)	(.35)	(.48)	(-2.01)	(3.65)	(2.74)
weekly							
Set 1	0.0005	0.0004	0.0066	0.0398	-0.0069	0.0004	0.0003
	(.82)	(.59)	(.44)	(1.49)	(-.3)	(.68)	(.46)
Set 1, 10	0.0002	0.0002	-0.0109	0.0461	-0.0020	0.0001	0.0001
	(.28)	(.25)	(-.66)	(1.31)	(-.05)	(.12)	(.11)

Table 14: **4 Short 1x's.**

Panel A shows the descriptive statistics of a shorting strategy, using +/-1x funds. The mean is the simple excess return. †SR is the Sharpe ratio. Panel B gives the strategies' alphas, firstly as for a regression on market excess returns (capm), and secondly accounting for the three Fama-French factors (3fac). All betas are from the latter model, first on the market excess return (mkt), and for the small-minus-big (smb) and high-minus-low (hml) factors. The last two columns indicated with an asterisk * use returns with the funds' expense ratio added back. All values are in monthly terms.

Panel A

	Mean	Min	Max	Std Dev	Skewness	Kurtosis	SR
daily							
Set 1	-0.0011	-0.0090	0.0275	0.0052	1.8746	13.7428	-0.2014
Set 1, 10	0.0011	0.0001	0.0023	0.0006	0.0400	2.3059	1.8386
Set 2	0.0012	-0.0180	0.1272	0.0170	6.4771	49.1812	0.0713
Set 2, 10	0.0011	-0.0027	0.0065	0.0024	0.6151	2.8556	0.4554
Set 3	0.0086	-0.0453	0.0487	0.0197	-0.5011	5.3397	0.4365
Set 3, 10	0.0085	-0.0453	0.0487	0.0204	-0.4747	5.0207	0.4189
weekly							
Set 1	-0.0005	-0.0212	0.0311	0.0075	1.1738	8.1347	-0.0727
Set 1, 10	0.0007	-0.0212	0.0059	0.0048	-3.7530	18.1915	0.1371
Set 2	0.0026	-0.0283	0.1236	0.0188	4.6284	29.2323	0.1408
Set 2, 10	0.0005	-0.0283	0.0080	0.0068	-3.1643	14.3922	0.0738
Set 3	-0.0032	-0.1519	0.0836	0.0532	-0.9838	4.8933	-0.0596
Set 3, 10	-0.0001	-0.1519	0.0836	0.0534	-1.1660	5.3775	-0.0020

Panel B

	Alpha_{capm}	Alpha_{3fac}	Beta_{mkt}	Beta_{smb}	Beta_{hml}	Alpha*_{capm}	Alpha*_{3fac}
daily							
Set 1	-0.0011	-0.0012	0.0030	**0.0653**	-0.0132	**-0.0021**	**-0.0022**
	(-1.77)	(-1.99)	(.22)	(2.21)	(-.53)	(-3.37)	(-3.6)
Set 1, 10	**0.0011**	**0.0012**	0.0003	0.0043	0.0075	0.0001	0.0002
	(9.)	(8.62)	(.08)	(.63)	(1.04)	(1.03)	(1.48)
Set 2	0.0011	0.0004	0.0054	**0.2186**	-0.0829	0.0001	-0.0007
	(.54)	(.17)	(.12)	(2.17)	(-.98)	(.05)	(-.32)
Set 2, 10	**0.0011**	0.0012	-0.0017	-0.0032	0.0170	0.0000	0.0002
	(2.21)	(2.05)	(-.11)	(-.1)	(.51)	(.06)	(.32)
Set 3	0.0092	0.0066	-0.0856	0.0499	-0.5140	0.0075	0.0048
	(1.88)	(1.09)	(-.5)	(.13)	(-1.16)	(1.53)	(.81)
Set 3, 10	0.0091	0.0058	-0.1367	0.1932	-0.5880	0.0073	0.0041
	(1.74)	(.93)	(-.71)	(.43)	(-1.27)	(1.41)	(.66)
weekly							
Set 1	-0.0004	-0.0006	**-0.0419**	**0.0978**	-0.0102	-0.0014	-0.0016
	(-.51)	(-.69)	(-2.23)	(2.35)	(-.29)	(-1.64)	(-1.84)
Set 1, 10	0.0006	0.0006	-0.0149	0.0651	-0.0096	-0.0004	-0.0004
	(.64)	(.54)	(-.57)	(1.17)	(-.17)	(-.36)	(-.36)
Set 2	0.0028	0.0022	-0.0801	0.2057	-0.0448	0.0017	0.0011
	(1.19)	(.92)	(-1.65)	(1.83)	(-.48)	(.74)	(.48)
Set 2, 10	0.0005	0.0012	-0.0400	0.1247	0.0556	-0.0006	0.0002
	(.35)	(.74)	(-.98)	(1.33)	(.6)	(-.4)	(.13)
Set 3	-0.0056	0.0057	-0.3381	**2.0347**	0.4451	-0.0074	0.0040
	(-.43)	(.4)	(-.83)	(2.2)	(.42)	(-.57)	(.28)
Set 3, 10	-0.0024	0.0060	-0.3137	1.9663	0.4804	-0.0042	0.0044
	(-.18)	(.4)	(-.68)	(1.8)	(.43)	(-.31)	(.29)

Table 15: **5 Short 2x's.**

Panel A shows the descriptive statistics of a shorting strategy, using +/-2x funds. The mean is the simple excess return. †SR is the Sharpe ratio. Panel B gives the strategies' alphas, firstly as for a regression on market excess returns (capm), and secondly accounting for the three Fama-French factors (3fac). All betas are from the latter model, first on the market excess return (mkt), and for the small-minus-big (smb) and high-minus-low (hml) factors. The last two columns indicated with an asterisk * use returns with the funds' expense ratio added back. All values are in monthly terms.

Panel A

	Mean	Min	Max	Std Dev	Skewness	Kurtosis	SR
daily							
Set 1	0.0011	-0.0117	0.0168	0.0056	0.0123	3.4268	0.1954
Set 1, 10	0.0029	-0.0001	0.0055	0.0012	-0.1881	3.3353	2.4219
Set 2	0.0040	-0.0122	0.0450	0.0097	1.8448	8.4686	0.4134
Set 2, 10	0.0032	0.0002	0.0059	0.0013	0.1976	3.1235	2.5162
Set 3	0.0135	-0.0297	0.1751	0.0266	3.9170	23.3709	0.5070
Set 3, 10	0.0168	-0.0004	0.0702	0.0209	1.7159	4.7377	0.8040
weekly							
Set 1	0.0015	-0.0833	0.0707	0.0174	-0.3026	13.3577	0.0862
Set 1, 10	0.0010	-0.0833	0.0221	0.0186	-3.6705	17.7271	0.0559
Set 2	0.0047	-0.1043	0.1389	0.0304	1.6765	13.1784	0.1531
Set 2, 10	0.0000	-0.1043	0.0254	0.0232	-3.7437	17.7138	0.0006
Set 3	0.0183	-0.0862	0.3720	0.0578	3.8234	23.9583	0.3164
Set 3, 10	0.0163	-0.0394	0.0843	0.0309	1.0078	4.1077	0.5272

Panel B

	$\text{Alpha}_{\text{capm}}$	$\text{Alpha}_{\text{3fac}}$	Beta_{mkt}	Beta_{smb}	Beta_{hml}	$\text{Alpha*}_{\text{capm}}$	$\text{Alpha*}_{\text{3fac}}$
daily							
Set 1	0.0011	0.0010	-0.0126	0.0440	-0.0083	-0.0004	-0.0006
	(1.66)	(1.46)	(-.86)	(1.3)	(-.29)	(-.66)	(-.8)
Set 1, 10	**0.0030**	**0.0029**	0.0008	-0.0041	-0.0091	**0.0014**	**0.0014**
	(11.95)	(10.4)	(.11)	(-.28)	(-.62)	(6.51)	(5.57)
Set 2	**0.0041**	**0.0041**	-0.0595	0.0617	0.0331	**0.0025**	**0.0025**
	(3.46)	(3.39)	(-2.4)	(1.07)	(.69)	(2.15)	(2.11)
Set 2, 10	**0.0033**	**0.0032**	0.0012	-0.0173	0.0006	**0.0017**	**0.0017**
	(12.36)	(10.18)	(.16)	(-.96)	(.03)	(7.32)	(6.11)
Set 3	**0.0135**	**0.0134**	-0.0760	0.2233	0.0661	**0.0119**	**0.0118**
	(4.01)	(3.9)	(-1.08)	(1.38)	(.49)	(3.55)	(3.45)
Set 3, 10	**0.0178**	**0.0141**	-0.1623	0.0643	-0.5901	**0.0163**	0.0127
	(3.46)	(2.29)	(-.86)	(.14)	(-1.28)	(3.19)	(2.07)
weekly							
Set 1	0.0018	0.0015	**-0.0989**	0.1605	0.0005	0.0002	-0.0001
	(.86)	(.71)	(-2.23)	(1.57)	(.01)	(.1)	(-.04)
Set 1, 10	0.0009	0.0009	-0.0630	0.2645	-0.0132	-0.0006	-0.0006
	(.25)	(.22)	(-.62)	(1.23)	(-.06)	(-.16)	(-.14)
Set 2	0.0050	0.0048	-0.1428	0.0936	0.0223	0.0034	0.0033
	(1.33)	(1.25)	(-1.8)	(.51)	(.15)	(.91)	(.85)
Set 2, 10	-0.0002	0.0007	-0.0852	0.3932	0.0188	-0.0018	-0.0008
	(-.05)	(.13)	(-.61)	(1.22)	(.06)	(-.36)	(-.13)
Set 3	**0.0184**	**0.0185**	0.0054	-0.1715	-0.0507	**0.0168**	**0.0169**
	(2.5)	(2.44)	(.03)	(-.48)	(-.17)	(2.29)	(2.23)
Set 3, 10	0.0139	0.0112	0.1740	0.5998	-0.6641	0.0124	0.0098
	(2.04)	(1.42)	(.71)	(1.05)	(-1.13)	(1.83)	(1.25)

Table 16: **6 Short 3x's.**

Panel A shows the descriptive statistics of a shorting strategy, using +/-3x funds. The mean is the simple excess return. †SR is the Sharpe ratio. Panel B gives the strategies' alphas, firstly as for a regression on market excess returns (capm), and secondly accounting for the three Fama-French factors (3fac). All betas are from the latter model, first on the market excess return (mkt), and for the small-minus-big (smb) and high-minus-low (hml) factors. The last two columns indicated with an asterisk * use returns with the funds' expense ratio added back. All values are in monthly terms.

Panel A

	Mean	Min	Max	Std Dev	Skewness	Kurtosis	SR
daily							
Set 1	0.0077	-0.0103	0.0558	0.0117	3.0535	12.8258	0.6578
Set 1, 10	0.0051	0.0001	0.0149	0.0031	1.2390	4.9781	1.6457
Set 3	0.0060	-0.0932	0.0544	0.0182	-3.4046	23.3120	0.3305
Set 3, 10	0.0085	0.0003	0.0544	0.0126	3.2364	12.4171	0.6715
weekly							
Set 1	0.0022	-0.1859	0.1306	0.0397	-1.7318	15.4677	0.0557
Set 1, 10	0.0006	-0.1859	0.0487	0.0417	-3.5018	16.8884	0.0143
Set 3	0.0024	-0.1688	0.0784	0.0382	-1.9012	11.1472	0.0618
Set 3, 10	0.0006	-0.1688	0.0572	0.0478	-2.9123	11.3407	0.0124

Panel B

	$\text{Alpha}_{\text{capm}}$	$\text{Alpha}_{\text{3fac}}$	Beta_{mkt}	Beta_{smb}	Beta_{hml}	$\text{Alpha*}_{\text{capm}}$	$\text{Alpha*}_{\text{3fac}}$
daily							
Set 1	**0.0070**	**0.0066**	0.0619	0.0075	-0.0478	**0.0054**	**0.0050**
	(3.77)	(3.31)	(1.32)	(.09)	(-.65)	(2.95)	(2.55)
Set 1, 10	**0.0052**	**0.0050**	-0.0106	0.0118	-0.0230	**0.0037**	**0.0035**
	(8.31)	(7.17)	(-.62)	(.33)	(-.61)	(6.12)	(5.22)
Set 3	**0.0066**	**0.0066**	-0.1148	0.2468	0.0493	0.0051	0.0051
	(2.26)	(2.2)	(-1.61)	(1.92)	(.44)	(1.74)	(1.69)
Set 3, 10	**0.0085**	**0.0115**	0.0056	-0.0561	0.4817	**0.0070**	0.0100
	(2.57)	(3.03)	(.05)	(-.2)	(1.71)	(2.13)	(2.68)
weekly							
Set 1	0.0009	0.0001	0.0166	0.3481	-0.0267	-0.0007	-0.0014
	(.14)	(.02)	(.1)	(1.21)	(-.11)	(-.1)	(-.21)
Set 1, 10	0.0006	0.0002	-0.1544	0.5751	-0.0720	-0.0010	-0.0013
	(.07)	(.02)	(-.68)	(1.2)	(-.14)	(-.11)	(-.14)
Set 3	0.0016	0.0006	-0.0304	0.4143	-0.0358	0.0000	-0.0009
	(.25)	(.09)	(-.2)	(1.5)	(-.15)	(.)	(-.14)
Set 3, 10	-0.0016	0.0093	-0.0319	1.0299	1.2498	-0.0031	0.0079
	(-.14)	(.67)	(-.07)	(1.03)	(1.21)	(-.26)	(.57)

Table 17: **7 Short 1x,2x's.**
Panel A shows the descriptive statistics of a shorting strategy, using +/-1x and +/-2x funds. The mean is the simple excess return. †SR is the Sharpe ratio. Panel B gives the strategies' alphas, firstly as for a regression on market excess returns (capm), and secondly accounting for the three Fama-French factors (3fac). All betas are from the latter model, first on the market excess return (mkt), and for the small-minus-big (smb) and high-minus-low (hml) factors. The last two columns indicated with an asterisk * use returns with the funds' expense ratio added back. All values are in monthly terms.

Panel A

	Mean	Min	Max	Std Dev	Skewness	Kurtosis	SR
daily							
Set 1	0.0001	-0.0206	0.0487	0.0107	0.9930	7.6009	0.0130
Set 1, 10	0.0041	0.0000	0.0073	0.0015	-0.2506	3.8364	2.7140
Set 2	0.0044	-0.0381	0.1476	0.0231	3.7176	24.7789	0.1921
Set 2, 10	0.0043	-0.0004	0.0124	0.0031	0.6473	3.0532	1.4034
weekly							
Set 1	0.0007	-0.1038	0.0928	0.0236	0.1892	11.3824	0.0308
Set 1, 10	0.0017	-0.1038	0.0276	0.0233	-3.6820	17.7977	0.0741
Set 2	0.0087	-0.1308	0.2387	0.0479	2.4027	13.3388	0.1827
Set 2, 10	0.0006	-0.1308	0.0273	0.0292	-3.8025	17.9725	0.0201

Panel B

	$Alpha_{capm}$	$Alpha_{3fac}$	$Beta_{mkt}$	$Beta_{smb}$	$Beta_{hml}$	$Alpha^*_{capm}$	$Alpha^*_{3fac}$
daily							
Set 1	0.0001	-0.0002	-0.0035	0.0996	-0.0216	-0.0024	**-0.0027**
	(.08)	(-.12)	(-.13)	(1.55)	(-.4)	(-1.89)	(-2.06)
Set 1, 10	**0.0041**	**0.0041**	0.0011	0.0002	-0.0016	**0.0015**	**0.0016**
	(13.29)	(11.68)	(.13)	(.01)	(-.09)	(6.04)	(5.35)
Set 2	0.0044	0.0036	-0.0344	0.2637	-0.0681	0.0018	0.0010
	(1.52)	(1.23)	(-.57)	(1.89)	(-.58)	(.63)	(.34)
Set 2, 10	**0.0044**	**0.0045**	-0.0004	-0.0205	0.0176	**0.0018**	**0.0019**
	(6.84)	(5.77)	(-.02)	(-.47)	(.41)	(2.84)	(2.55)
weekly							
Set 1	0.0010	0.0005	**-0.1289**	0.2656	-0.0144	-0.0015	-0.0020
	(.37)	(.18)	(-2.15)	(1.93)	(-.12)	(-.54)	(-.72)
Set 1, 10	0.0016	0.0015	-0.0787	0.3283	-0.0227	-0.0009	-0.0009
	(.34)	(.29)	(-.62)	(1.22)	(-.08)	(-.19)	(-.18)
Set 2	0.0093	0.0090	**-0.2689**	0.2483	0.0671	0.0066	0.0064
	(1.58)	(1.49)	(-2.18)	(.87)	(.28)	(1.14)	(1.06)
Set 2, 10	0.0004	0.0020	-0.1269	0.5136	0.0735	-0.0022	-0.0005
	(.06)	(.28)	(-.72)	(1.27)	(.18)	(-.37)	(-.06)

Table 18: **8 Short 1x,3x's.**

Panel A shows the descriptive statistics of a shorting strategy, using +/-1x and +/-3x funds. The mean is the simple excess return. †SR is the Sharpe ratio. Panel B gives the strategies' alphas, firstly as for a regression on market excess returns (capm), and secondly accounting for the three Fama-French factors (3fac). All betas are from the latter model, first on the market excess return (mkt), and for the small-minus-big (smb) and high-minus-low (hml) factors. The last two columns indicated with an asterisk * use returns with the funds' expense ratio added back. All values are in monthly terms.

Panel A

	Mean	Min	Max	Std Dev	Skewness	Kurtosis	SR
daily							
Set 1	0.0092	-0.0078	0.0587	0.0122	3.1133	13.0009	0.7513
Set 1, 10	0.0062	0.0002	0.0164	0.0034	0.9323	4.3982	1.8399
weekly							
Set 1	0.0032	-0.2054	0.1381	0.0433	-1.9448	15.9347	0.0741
Set 1, 10	0.0013	-0.2054	0.0546	0.0461	-3.5117	16.9419	0.0285

Panel B

	$\text{Alpha}_{\text{capm}}$	$\text{Alpha}_{\text{3fac}}$	Beta_{mkt}	Beta_{smb}	Beta_{hml}	$\text{Alpha*}_{\text{capm}}$	$\text{Alpha*}_{\text{3fac}}$
daily							
Set 1	**0.0084**	**0.0080**	0.0646	0.0187	-0.0459	**0.0058**	**0.0054**
	(4.34)	(3.86)	(1.32)	(.21)	(-.6)	(3.03)	(2.64)
Set 1, 10	**0.0063**	**0.0062**	-0.0103	0.0161	-0.0154	**0.0038**	**0.0037**
	(9.17)	(8.)	(-.55)	(.4)	(-.37)	(5.89)	(5.1)
weekly							
Set 1	0.0018	0.0008	0.0232	0.3767	-0.0452	-0.0008	-0.0018
	(.26)	(.11)	(.13)	(1.2)	(-.17)	(-.11)	(-.24)
Set 1, 10	0.0013	0.0009	-0.1712	0.6372	-0.0815	-0.0012	-0.0016
	(.14)	(.08)	(-.68)	(1.2)	(-.15)	(-.13)	(-.16)

Table 19: **9 Short 2x,3x's.**

Panel A shows the descriptive statistics of a shorting strategy, using +/-2x and +/-3x funds. The mean is the simple excess return. †SR is the Sharpe ratio. Panel B gives the strategies' alphas, firstly as for a regression on market excess returns (capm), and secondly accounting for the three Fama-French factors (3fac). All betas are from the latter model, first on the market excess return (mkt), and for the small-minus-big (smb) and high-minus-low (hml) factors. The last two columns indicated with an asterisk * use returns with the funds' expense ratio added back. All values are in monthly terms.

Panel A

	Mean	Min	Max	Std Dev	Skewness	Kurtosis	SR
daily							
Set 1	0.0129	-0.0011	0.0790	0.0150	2.8176	11.4731	0.8575
Set 1, 10	0.0081	-0.0001	0.0196	0.0040	0.9247	4.2237	2.0032
weekly							
Set 1	0.0095	-0.2629	0.1804	0.0589	-1.6231	13.6566	0.1619
Set 1, 10	0.0019	-0.2629	0.0704	0.0592	-3.4946	16.8379	0.0313

Panel B

	$Alpha_{capm}$	$Alpha_{3fac}$	$Beta_{mkt}$	$Beta_{smb}$	$Beta_{hml}$	$Alpha^*_{capm}$	$Alpha^*_{3fac}$
daily							
Set 1	**0.0122**	**0.0115**	0.0521	0.0833	-0.0668	**0.0090**	**0.0084**
	(5.06)	(4.49)	(.86)	(.77)	(-.71)	(3.8)	(3.31)
Set 1, 10	**0.0082**	**0.0079**	-0.0098	0.0078	-0.0321	**0.0051**	**0.0049**
	(10.04)	(8.69)	(-.44)	(.17)	(-.66)	(6.75)	(5.77)
weekly							
Set 1	0.0068	0.0052	0.1071	0.4815	-0.1002	0.0037	0.0021
	(.72)	(.52)	(.45)	(1.13)	(-.27)	(.39)	(.21)
Set 1, 10	0.0018	0.0014	-0.2246	0.8287	-0.0842	-0.0013	-0.0016
	(.15)	(.11)	(-.7)	(1.21)	(-.12)	(-.1)	(-.12)

Table 20: **10 Short 1x,2x,3x's.**

Panel A shows the descriptive statistics of a shorting strategy, using +/-1x, +/-2x and +/-3x funds. The mean is the simple excess return. †SR is the Sharpe ratio. Panel B gives the strategies' alphas, firstly as for a regression on market excess returns (capm), and secondly accounting for the three Fama-French factors (3fac). All betas are from the latter model, first on the market excess return (mkt), and for the small-minus-big (smb) and high-minus-low (hml) factors. The last two columns indicated with an asterisk * use returns with the funds' expense ratio added back. All values are in monthly terms.

Panel A

	Mean	Min	Max	Std Dev	Skewness	Kurtosis	SR
daily							
Set 1	0.0144	0.0000	0.0818	0.0160	2.8160	11.0398	0.8995
Set 1, 10	0.0092	0.0000	0.0212	0.0043	0.7422	4.0265	2.1392
weekly							
Set 1	0.0109	-0.2817	0.1871	0.0627	-1.7429	13.8756	0.1741
Set 1, 10	0.0026	-0.2817	0.0760	0.0636	-3.4957	16.8462	0.0409

Panel B

	$\text{Alpha}_{\text{capm}}$	$\text{Alpha}_{\text{3fac}}$	Beta_{mkt}	Beta_{smb}	Beta_{hml}	$\text{Alpha*}_{\text{capm}}$	$\text{Alpha*}_{\text{3fac}}$
daily							
Set 1	**0.0134**	**0.0128**	0.0650	0.0802	-0.0595	**0.0093**	**0.0087**
	(5.29)	(4.74)	(1.01)	(.7)	(-.59)	(3.71)	(3.25)
Set 1, 10	**0.0093**	**0.0091**	-0.0095	0.0121	-0.0245	**0.0052**	**0.0051**
	(10.64)	(9.25)	(-.4)	(.24)	(-.47)	(6.58)	(5.67)
weekly							
Set 1	0.0080	0.0062	0.1206	0.5004	-0.1294	0.0039	0.0021
	(.8)	(.58)	(.48)	(1.11)	(-.33)	(.39)	(.19)
Set 1, 10	0.0026	0.0021	-0.2423	0.8896	-0.0936	-0.0015	-0.0018
	(.2)	(.15)	(-.7)	(1.21)	(-.12)	(-.12)	(-.13)

Table 21: **11 Long 1x, short 3x's.**

Panel A shows the descriptive statistics of a long/short strategy, using 1x and +/-3x funds. The mean is the simple excess return. †SR is the Sharpe ratio. Panel B gives the strategies' alphas, firstly as for a regression on market excess returns (capm), and secondly accounting for the three Fama-French factors (3fac). All betas are from the latter model, first on the market excess return (mkt), and for the small-minus-big (smb) and high-minus-low (hml) factors. The last two columns indicated with an asterisk * use returns with the funds' expense ratio added back. All values are in monthly terms.

Panel A

	Mean	Min	Max	Std Dev	Skewness	Kurtosis	SR
daily							
Set 1	0.0060	-0.0163	0.0500	0.0111	2.7692	12.2731	0.5380
Set 1, 10	0.0041	0.0002	0.0126	0.0028	1.2638	4.5002	1.4625
Set 3	0.0045	-0.0956	0.0447	0.0183	-3.6959	22.9824	0.2474
Set 3, 10	0.0080	-0.0065	0.0447	0.0122	1.6147	6.1574	0.6526
weekly							
Set 1	0.0018	-0.1429	0.1143	0.0320	-1.1755	14.4477	0.0550
Set 1, 10	0.0005	-0.1429	0.0376	0.0321	-3.4949	16.8409	0.0155
Set 3	0.0015	-0.1269	0.0592	0.0304	-1.6412	9.3341	0.0505
Set 3, 10	0.0018	-0.1269	0.0557	0.0372	-2.6276	10.5346	0.0476

Panel B

	$\text{Alpha}_{\text{capm}}$	$\text{Alpha}_{\text{3fac}}$	Beta_{mkt}	Beta_{smb}	Beta_{hml}	$\text{Alpha*}_{\text{capm}}$	$\text{Alpha*}_{\text{3fac}}$
daily							
Set 1	**0.0053**	**0.0049**	0.0645	0.0045	-0.0475	**0.0042**	**0.0038**
	(2.99)	(2.58)	(1.45)	(.06)	(-.68)	(2.41)	(2.04)
Set 1, 10	**0.0041**	**0.0039**	-0.0067	0.0158	-0.0302	**0.0031**	**0.0029**
	(7.29)	(6.23)	(-.44)	(.49)	(-.9)	(5.59)	(4.71)
Set 3	0.0049	0.0047	-0.0975	**0.2711**	0.0327	0.0038	0.0036
	(1.65)	(1.56)	(-1.36)	(2.11)	(.29)	(1.27)	(1.19)
Set 3, 10	**0.0076**	**0.0106**	0.0622	-0.0537	0.4737	0.0066	**0.0095**
	(2.43)	(2.97)	(.57)	(-.21)	(1.79)	(2.11)	(2.71)
weekly							
Set 1	0.0006	0.0000	0.0158	0.2863	-0.0142	-0.0004	-0.0010
	(.12)	(.)	(.12)	(1.24)	(-.07)	(-.08)	(-.19)
Set 1, 10	0.0004	0.0002	-0.1163	0.4426	-0.0550	-0.0006	-0.0009
	(.07)	(.02)	(-.67)	(1.2)	(-.14)	(-.09)	(-.12)
Set 3	0.0009	0.0000	-0.0295	0.3692	-0.0292	-0.0003	-0.0011
	(.17)	(.)	(-.24)	(1.69)	(-.15)	(-.06)	(-.22)
Set 3, 10	-0.0002	0.0089	0.0469	0.7005	1.1202	-0.0013	0.0080
	(-.03)	(.86)	(.15)	(.93)	(1.44)	(-.14)	(.77)

Table 22: **12 Long 1x, short 2x's.**
Panel A shows the descriptive statistics of a long/short strategy, using 1x and +/-2x funds. The mean is the simple excess return. †SR is the Sharpe ratio. Panel B gives the strategies' alphas, firstly as for a regression on market excess returns (capm), and secondly accounting for the three Fama-French factors (3fac). All betas are from the latter model, first on the market excess return (mkt), and for the small-minus-big (smb) and high-minus-low (hml) factors. The last two columns indicated with an asterisk * use returns with the funds' expense ratio added back. All values are in monthly terms.

Panel A

	Mean	Min	Max	Std Dev	Skewness	Kurtosis	SR
daily							
Set 1	0.0008	-0.0083	0.0139	0.0042	0.1541	3.8146	0.1919
Set 1, 10	0.0020	0.0000	0.0042	0.0011	0.2013	2.4225	1.8665
Set 2	0.0026	-0.0107	0.0352	0.0070	1.9071	10.1199	0.3687
Set 2, 10	0.0020	0.0002	0.0049	0.0010	0.8378	3.9667	1.9875
Set 3	0.0081	-0.0629	0.1362	0.0221	2.7695	20.5673	0.3641
Set 3, 10	0.0108	0.0002	0.0444	0.0131	1.7321	4.7828	0.8246
weekly							
Set 1	0.0010	-0.0502	0.0401	0.0110	-0.0496	11.8943	0.0887
Set 1, 10	0.0008	-0.0502	0.0141	0.0113	-3.6476	17.5826	0.0680
Set 2	0.0027	-0.0640	0.0861	0.0190	1.4401	11.8567	0.1422
Set 2, 10	0.0000	-0.0640	0.0175	0.0144	-3.6413	17.2192	-0.0017
Set 3	0.0116	-0.0694	0.2154	0.0337	3.5188	22.9063	0.3428
Set 3, 10	0.0101	-0.0239	0.0506	0.0185	0.9006	3.9228	0.5448

Panel B

	$\text{Alpha}_{\text{capm}}$	$\text{Alpha}_{\text{3fac}}$	Beta_{mkt}	Beta_{smb}	Beta_{hml}	$\text{Alpha*}_{\text{capm}}$	$\text{Alpha*}_{\text{3fac}}$
daily							
Set 1	0.0008	0.0007	-0.0172	0.0376	-0.0105	-0.0001	-0.0002
	(1.71)	(1.47)	(-1.59)	(1.51)	(-.51)	(-.26)	(-.47)
Set 1, 10	**0.0020**	**0.0018**	0.0025	-0.0030	-0.0202	**0.0011**	**0.0009**
	(9.17)	(7.93)	(.45)	(-.25)	(-1.63)	(5.21)	(4.23)
Set 2	**0.0027**	**0.0027**	**-0.0421**	0.0383	0.0205	**0.0018**	**0.0018**
	(3.11)	(3.03)	(-2.34)	(.92)	(.59)	(2.11)	(2.06)
Set 2, 10	**0.0021**	**0.0021**	-0.0042	-0.0052	0.0069	**0.0012**	**0.0013**
	(10.1)	(8.51)	(-.68)	(-.38)	(.5)	(6.24)	(5.43)
Set 3	**0.0081**	**0.0080**	-0.0555	0.2054	0.0723	**0.0073**	**0.0072**
	(2.87)	(2.8)	(-.96)	(1.53)	(.64)	(2.59)	(2.53)
Set 3, 10	**0.0115**	**0.0093**	-0.1102	0.0438	-0.3580	**0.0107**	**0.0086**
	(3.6)	(2.42)	(-.93)	(.16)	(-1.25)	(3.37)	(2.23)
weekly							
Set 1	0.0011	0.0009	**-0.0601**	0.0948	-0.0072	0.0002	0.0000
	(.88)	(.72)	(-2.13)	(1.46)	(-.13)	(.13)	(-.03)
Set 1, 10	0.0007	0.0007	-0.0387	0.1674	-0.0035	-0.0002	-0.0002
	(.3)	(.28)	(-.63)	(1.29)	(-.03)	(-.11)	(-.08)
Set 2	0.0029	0.0028	-0.0781	0.0500	0.0199	0.0020	0.0020
	(1.22)	(1.16)	(-1.57)	(.43)	(.21)	(.85)	(.81)
Set 2, 10	-0.0002	0.0004	-0.0536	0.2471	0.0132	-0.0010	-0.0004
	(-.06)	(.12)	(-.62)	(1.24)	(.07)	(-.35)	(-.11)
Set 3	**0.0117**	**0.0115**	-0.0164	-0.0442	-0.0598	**0.0109**	**0.0107**
	(2.73)	(2.6)	(-.18)	(-.21)	(-.34)	(2.54)	(2.42)
Set 3, 10	0.0086	0.0070	0.1074	0.3794	-0.3993	0.0078	0.0063
	(2.15)	(1.53)	(.76)	(1.14)	(-1.17)	(1.97)	(1.38)

Figure 1: **Daily (leveraged) ETF returns vs. benchmarks.**
These graphs plot the daily return of the benchmark (labeled with *Bloomberg tickers*) on the vertical axis against the returns of the (leveraged) ETFs on the horizontal axis. The dots that form a positive slope are long (leveraged) ETFs while the ones that form a negative slope are inverse LETFs.

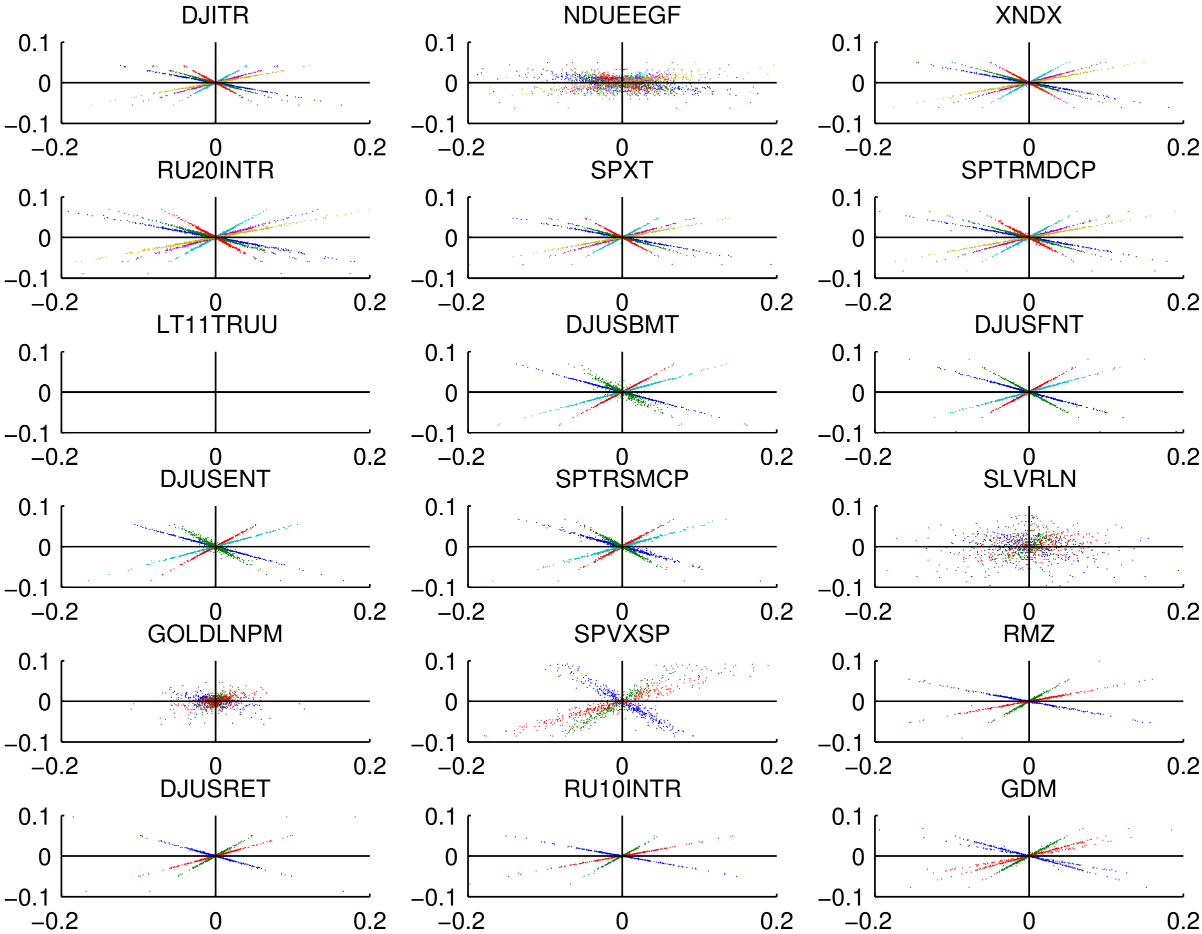


Figure 2: **Weekly (leveraged) ETF returns vs. benchmarks.**

These graphs plot the weekly return of the benchmark (labeled with *Bloomberg tickers*) on the vertical axis against the returns of the (leveraged) ETFs on the horizontal axis. The dots that form a positive slope are long (leveraged) ETFs while the ones that form a negative slope are inverse LETFs.

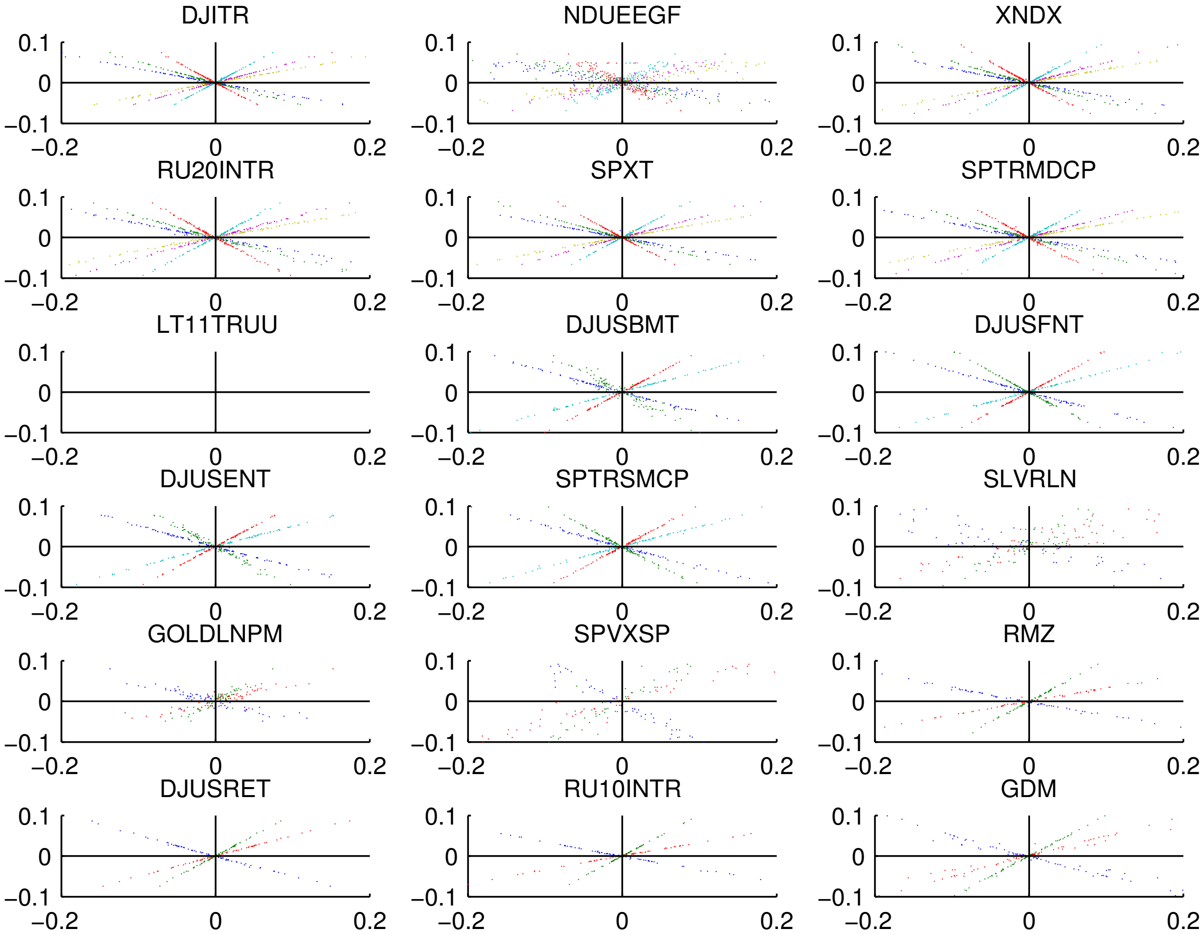


Figure 3: **Monthly (leveraged) ETF returns vs. benchmarks.**
These graphs plot the monthly return of the benchmark (labeled with *Bloomberg tickers*) on the vertical axis against the returns of the (leveraged) ETFs on the horizontal axis. The dots that form a positive slope are long (leveraged) ETFs while the ones that form a negative slope are inverse LETFs.

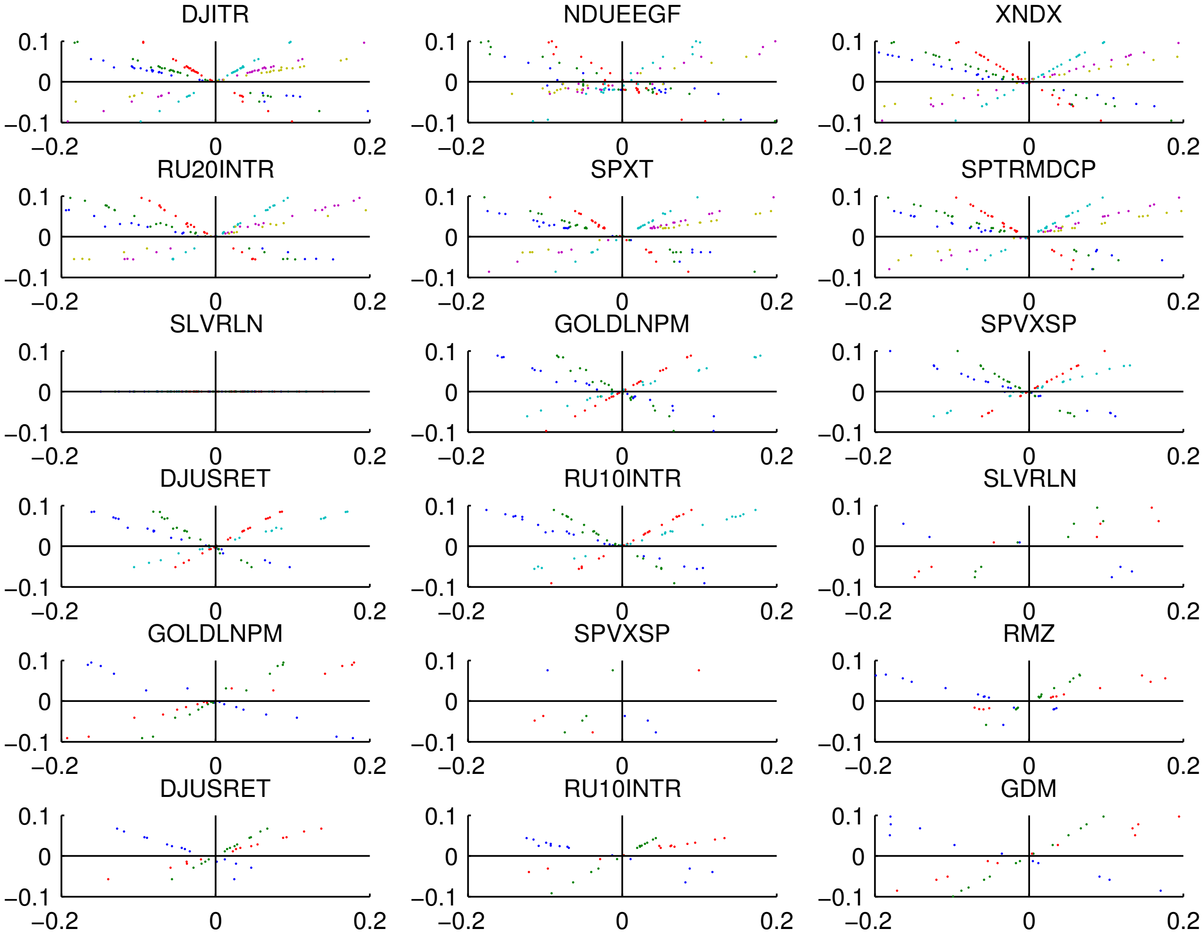
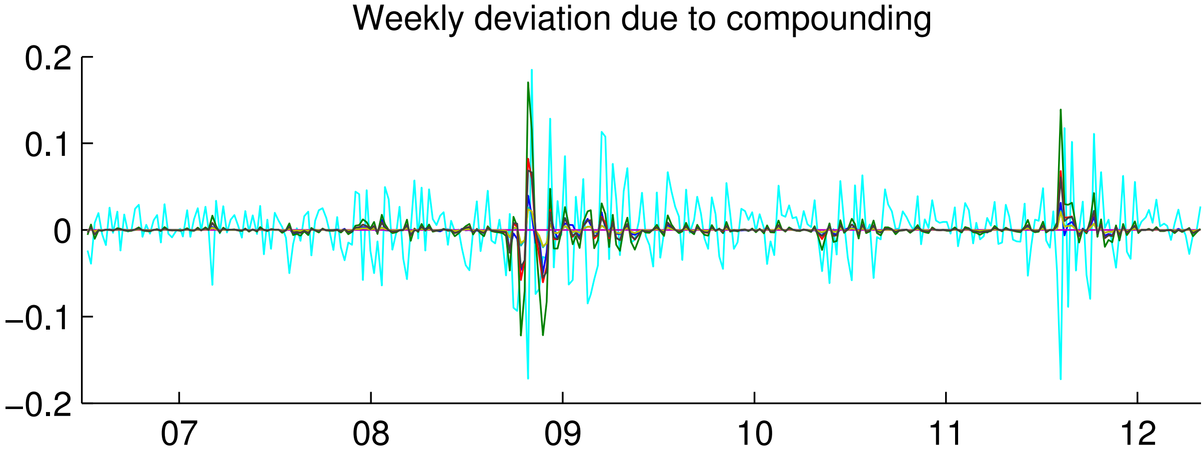


Figure 4: **Compounding deviation over time (S&P 500 LETFs).**
The top figure plots the deviation of the weekly return. The bottom figure uses monthly returns. In both of them, the return of the S&P 500 is depicted in cyan for comparison.



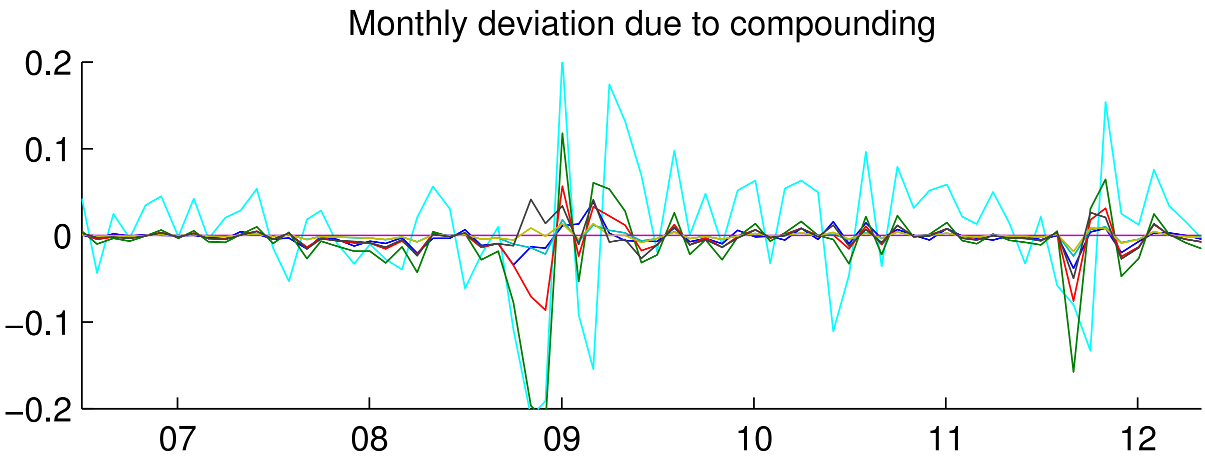


Figure 5: **Alpha over time (daily calculation).**
These graphs plot the fitted time series of alphas (daily). This alpha is the premium over the benchmark not caused by the expense ratio or compounding. All funds, as defined in Table 1 for each *Bloomberg ticker* is plotted.

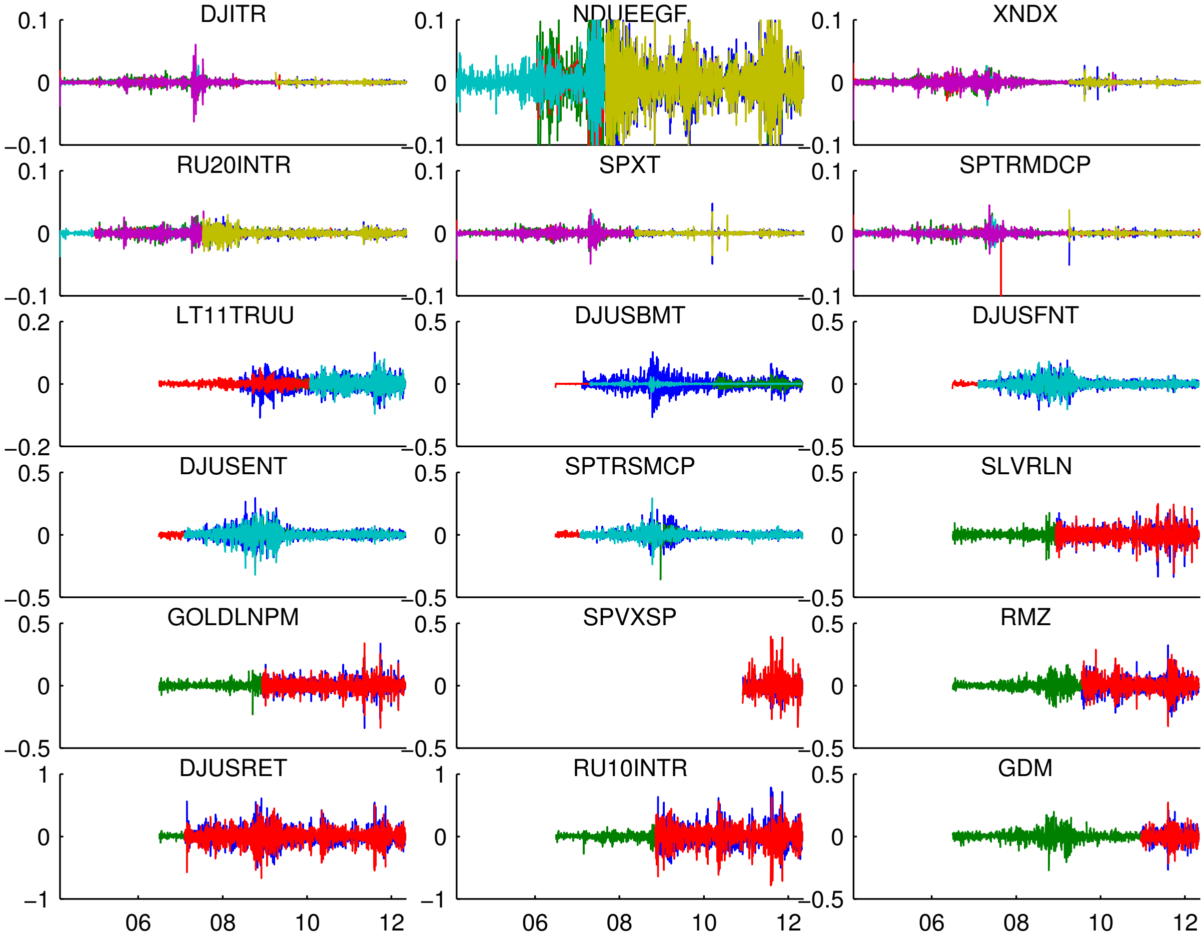


Figure 6: **Alpha over time (weekly calculation).**

These graphs plot the fitted time series of alphas weekly. This alpha is the premium over the benchmark not caused by the expense ratio or compounding. All funds, as defined in Table 1 for each *Bloomberg ticker* is plotted.

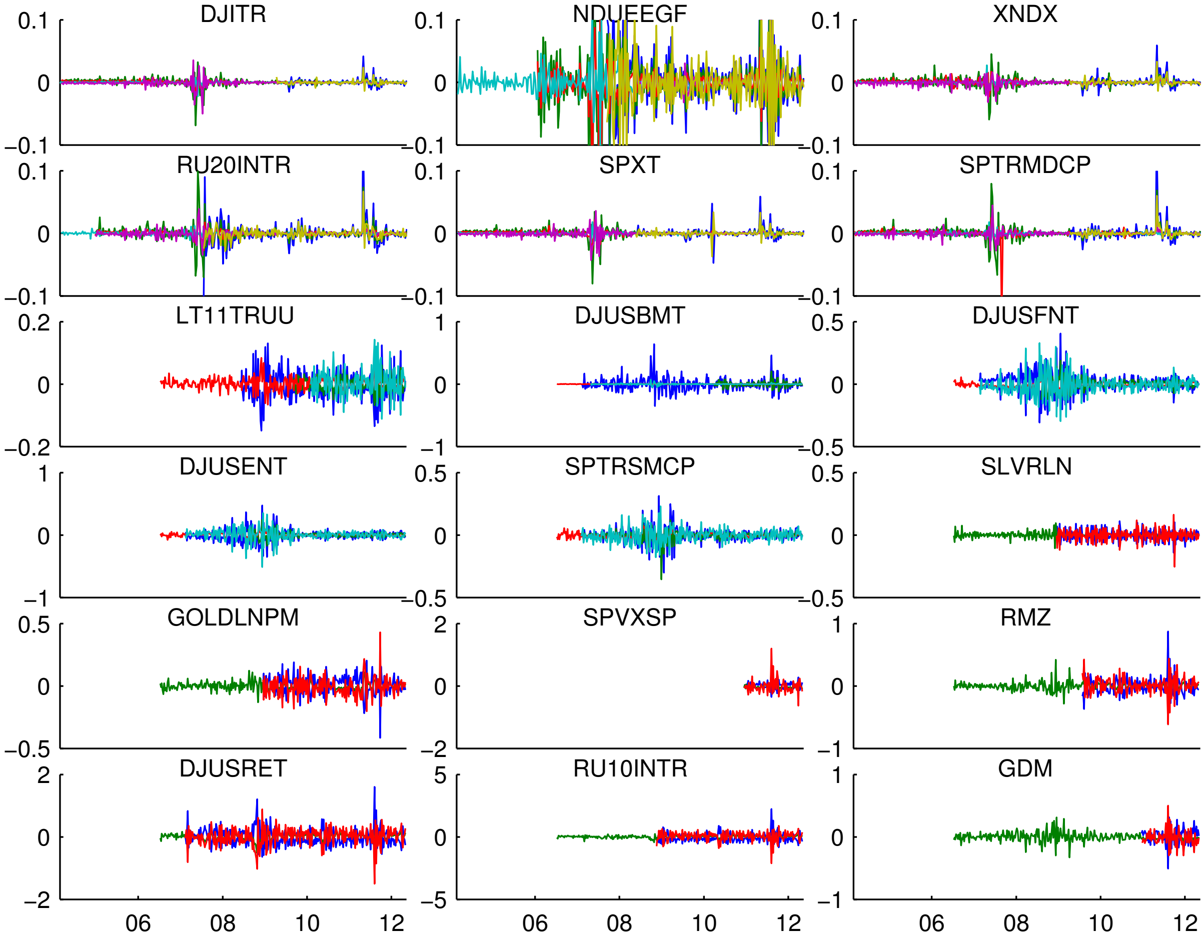


Figure 7: **Alpha over time (monthly calculation).**
These graphs plot the fitted time series of alphas (monthly). This alpha is the premium over the benchmark not caused by the expense ratio or compounding. All funds, as defined in Table 1 for each *Bloomberg ticker* is plotted.

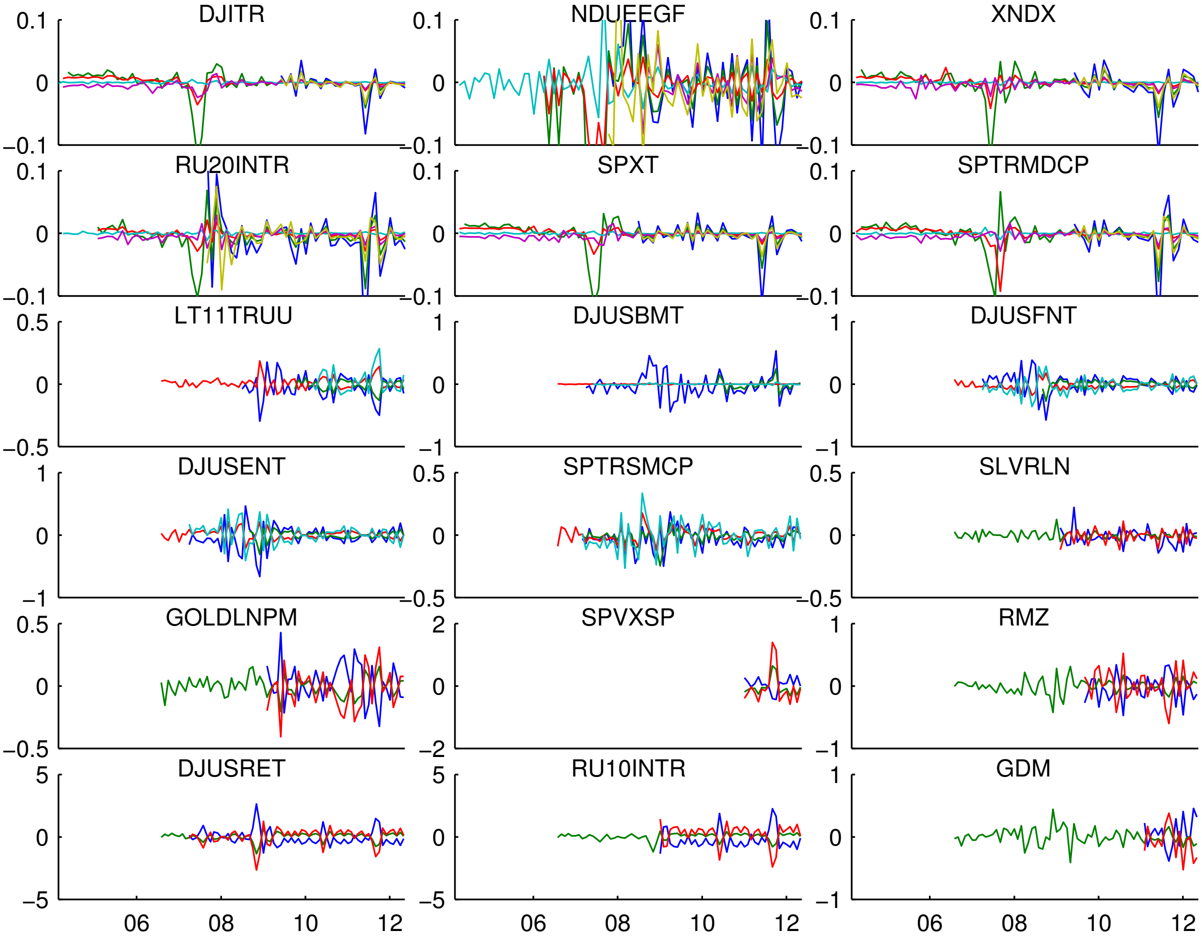


Figure 8: **Prices over time.**

These graphs plot the price performance of LETFs (out of set 1) over time. The vertical axis is in percent. For example, the lower left graph plots the performances of the products on the S&P 500 Index (*Bloomberg ticker:* `SPXT`). There, in red, the development of the +/-2x LETFs is displayed. While they mirror each other, together they seem to decrease over time.

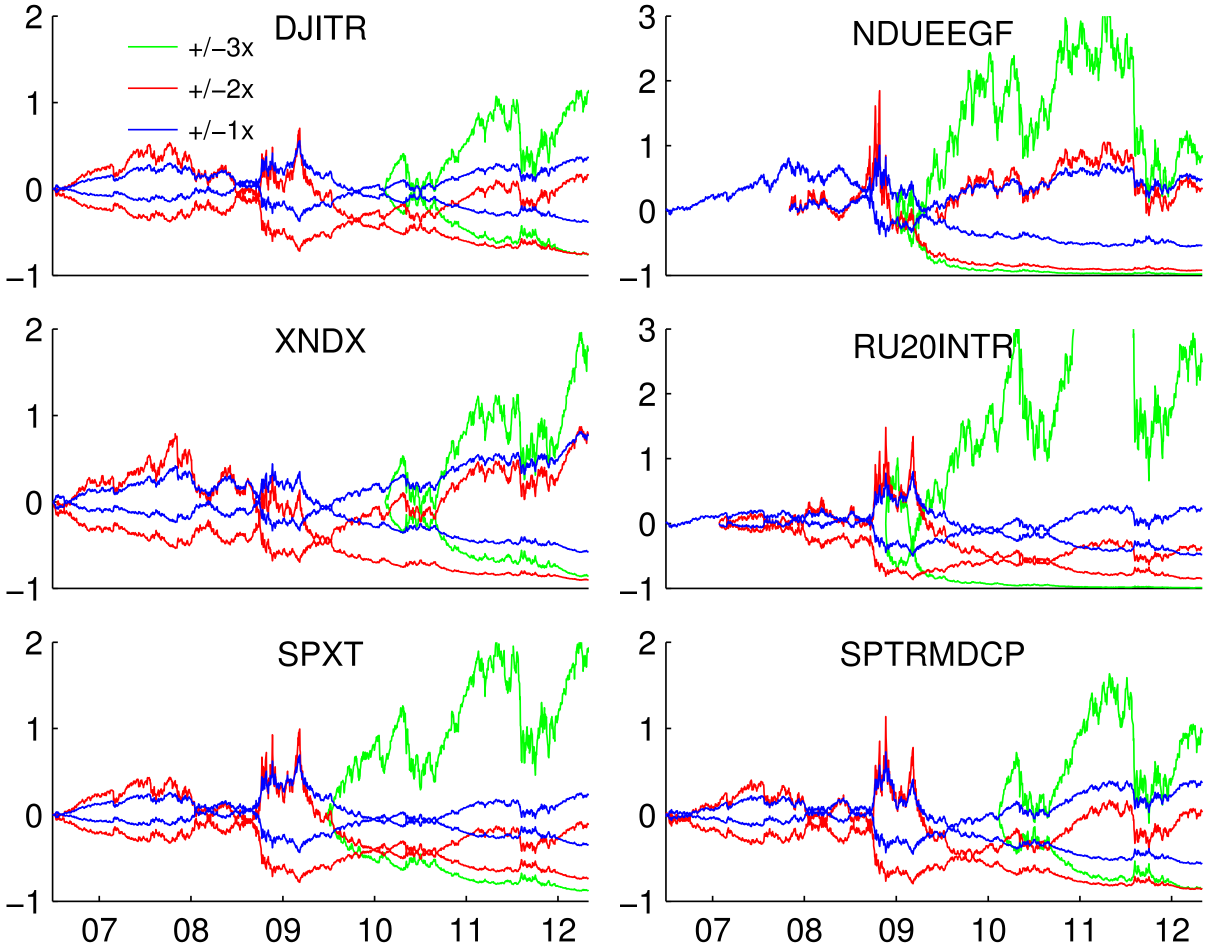


Figure 9: **Four-week US Treasury Bill rate, secondary-market, daily.**
This plot gives the risk-free interest over the sample period of the data used in sets 1 to 3. Excess returns are computed over this rate.

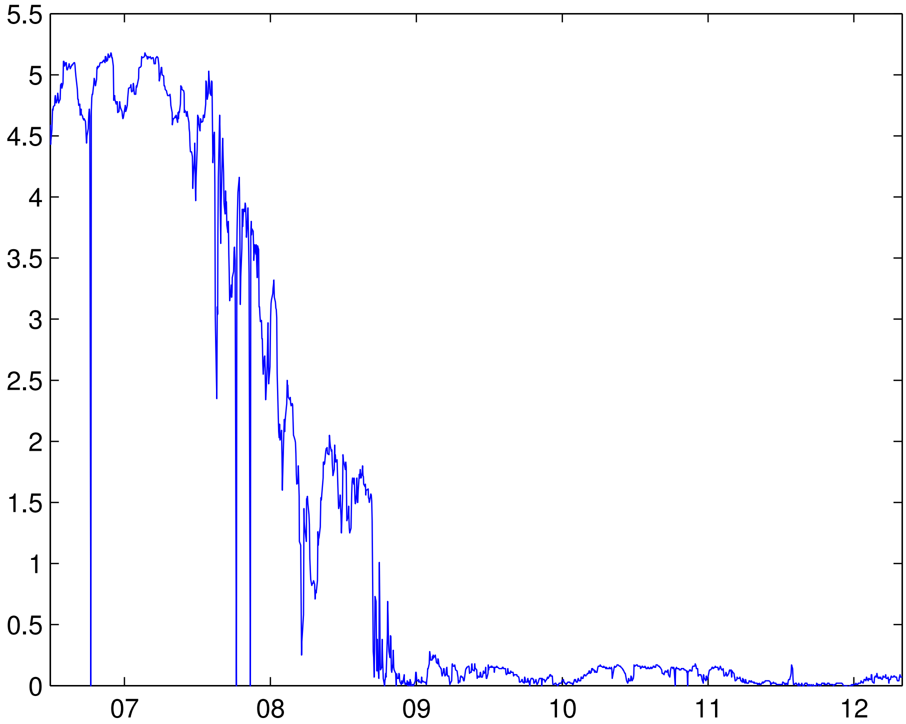


Figure 10: **Strategy 1: P/F11-BAB, daily.**
These graphs plot the performance (vertical axis in %) of the original BAB trading strategy that shorts one-half of a 2x LETF for every long position in a 1x ETF on the same underlying, rebalanced daily. The colored lines are labeled with the exchange tickers. The bold magenta line is the performance of the equal-weighted portfolio of all available products used according to the strategy. The dotted magenta line is performance of the equal-weighted portfolio that used the excess returns for both long and short holdings.

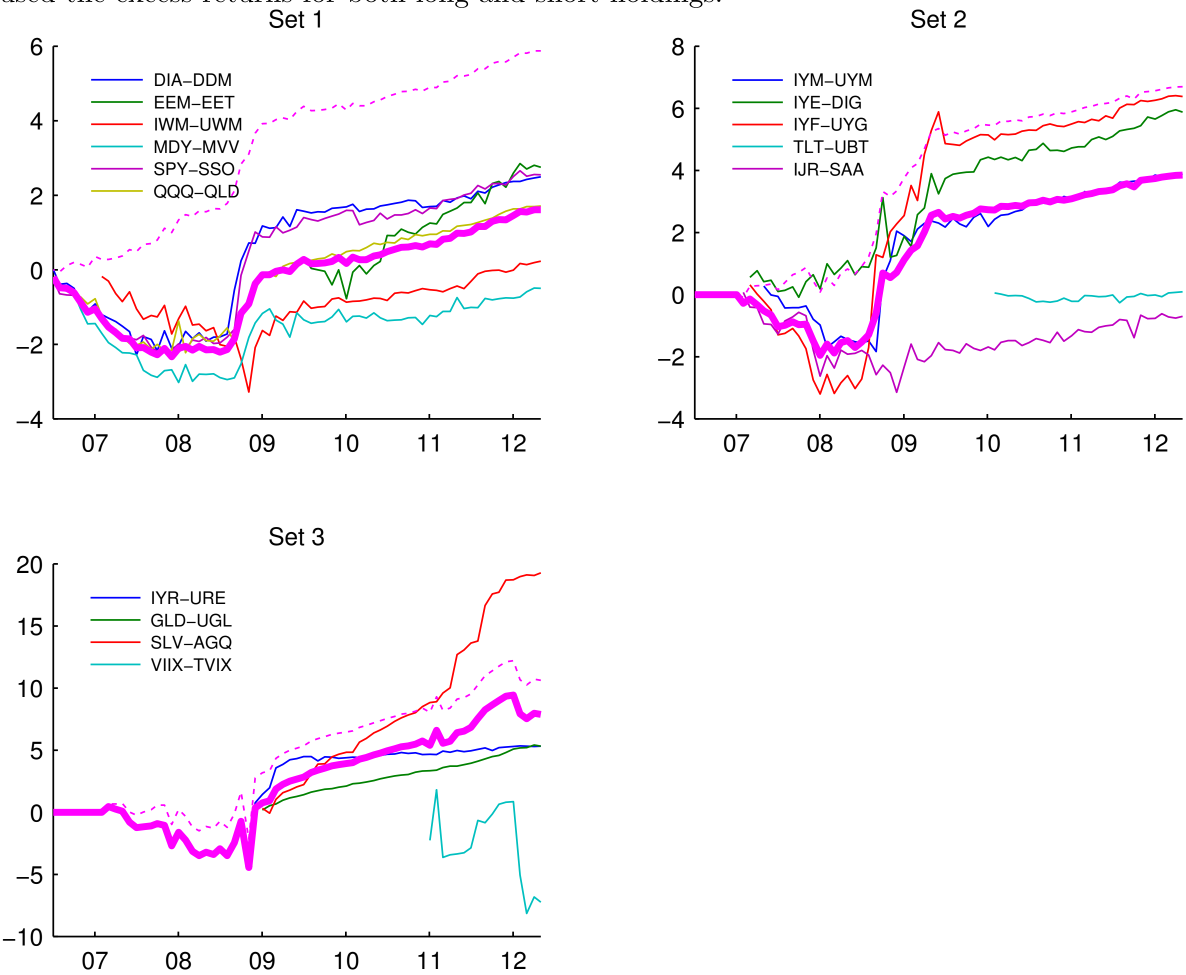


Figure 11: **Strategy 1: P/F11-BAB, weekly.**

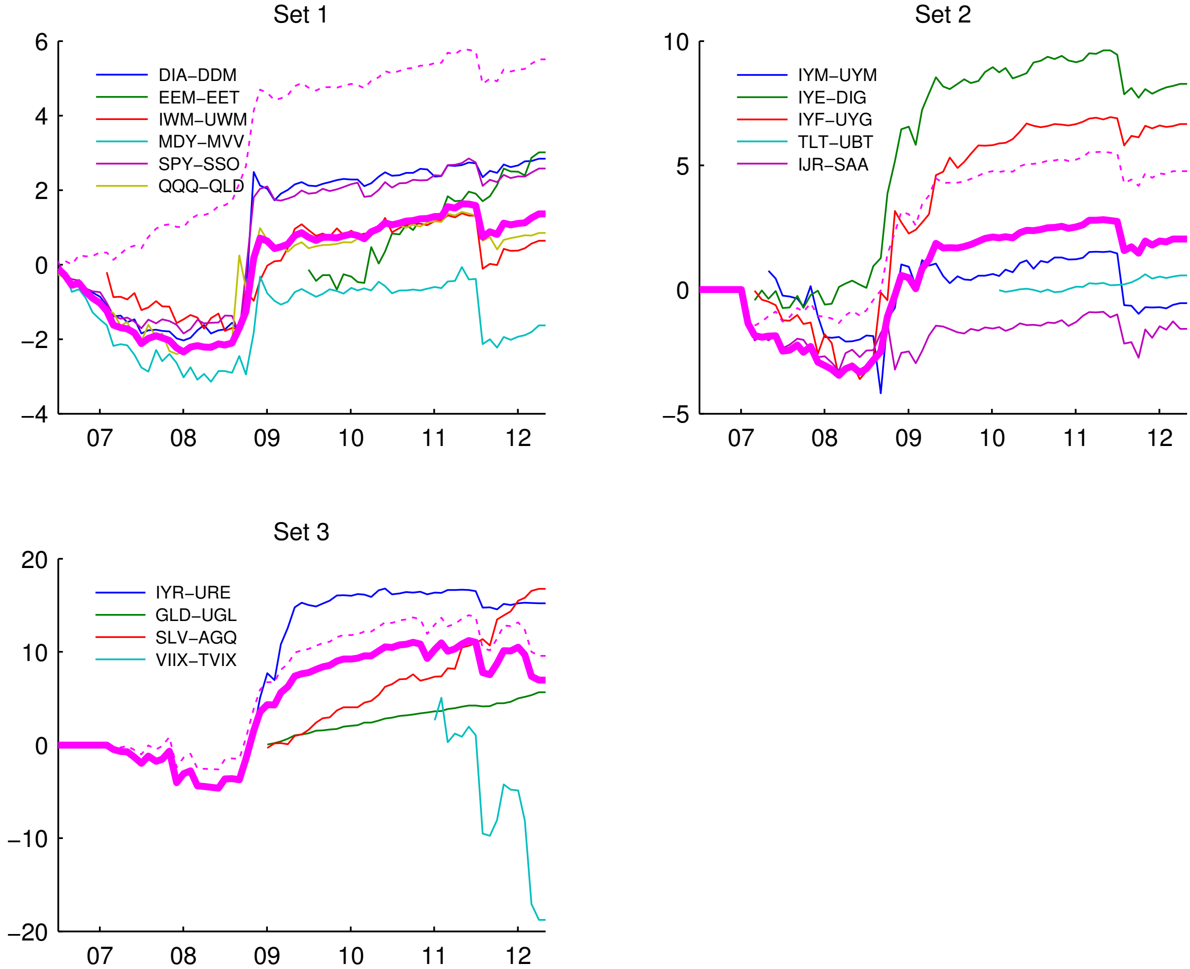


Figure 12: **Strategy 2: P/F11-BAB 3x, daily.**

These graphs plot the performance (vertical axis in %) of the BAB trading strategy that shorts one-third of a 3x LETF for every long position in a 1x ETF on the same underlying, rebalanced daily. The colored lines are labeled with the exchange tickers. The bold magenta line is the performance of the equal-weighted portfolio of all available products used according to the strategy. The dotted magenta line is performance of the equal-weighted portfolio that used the excess returns for both long and short holdings.

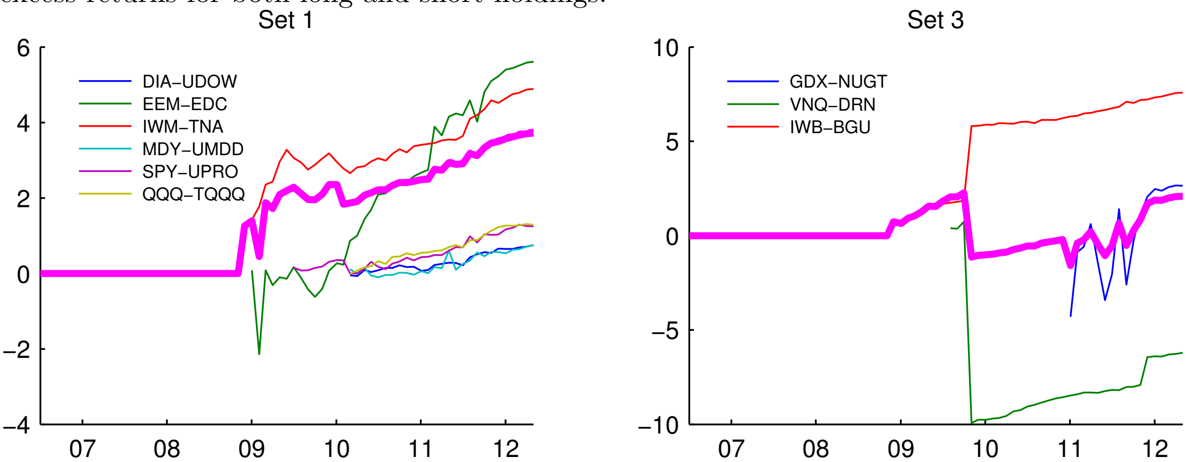


Figure 13: **Strategy 2: P/F11-BAB 3x, weekly.**

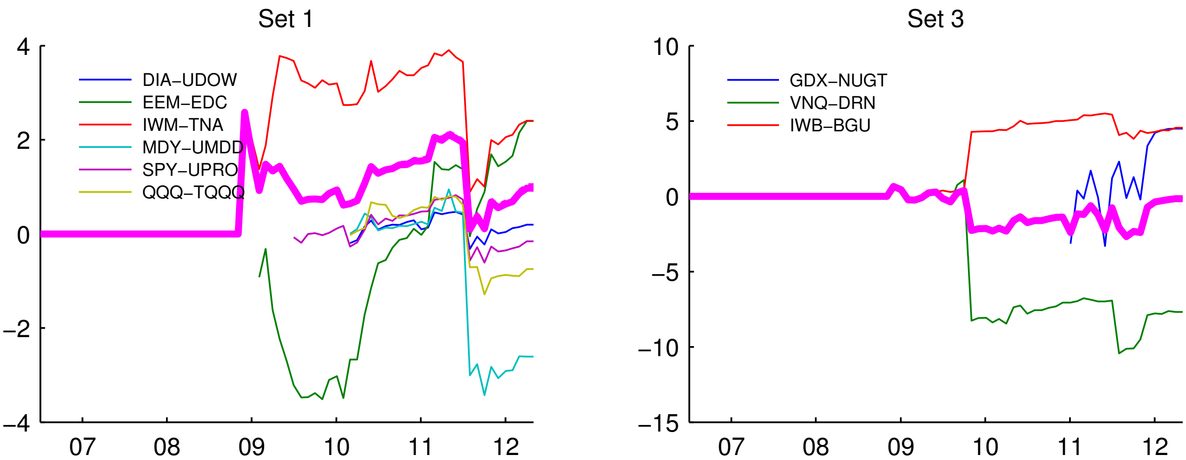


Figure 14: **Strategy 3: P/F11-BAB 2x, 3x, daily.**

These graphs plot the performance (vertical axis in %) of the BAB trading strategy that shorts one-fourth of a 2x and one-sixth of a 3x LETF for every long position in a 1x ETF on the same underlying, rebalanced daily. The colored lines are labeled with the exchange tickers. The bold magenta line is the performance of the equal-weighted portfolio of all available products used according to the strategy. The dotted magenta line is performance of the equal-weighted portfolio that used the excess returns for both long and short holdings.

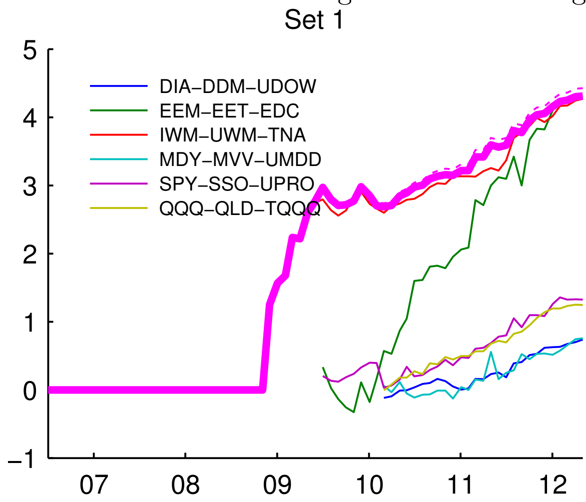


Figure 15: **Strategy 3: P/F11-BAB 2x, 3x, weekly.**

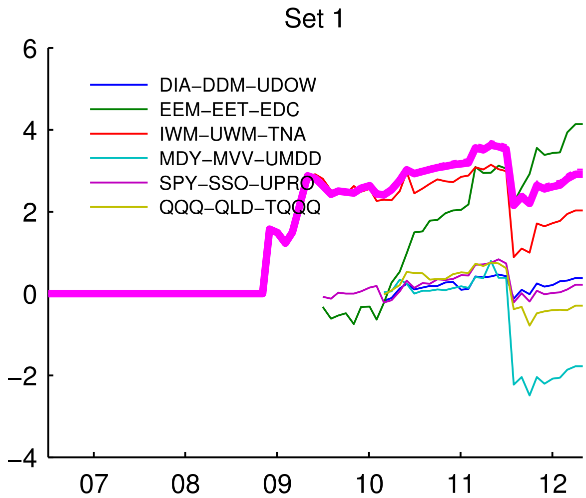


Figure 16: **Strategy 4: short 1x, daily.**
These graphs plot the performance (vertical axis in %) of the trading strategy that shorts both 1x and -1x funds on the same underlying, rebalanced daily. The colored lines are labeled with the exchange tickers. The bold magenta line is the performance of the equal-weighted portfolio of all available products used according to the strategy. The dotted magenta line is performance of the equal-weighted portfolio that used the excess returns for both long and short holdings.

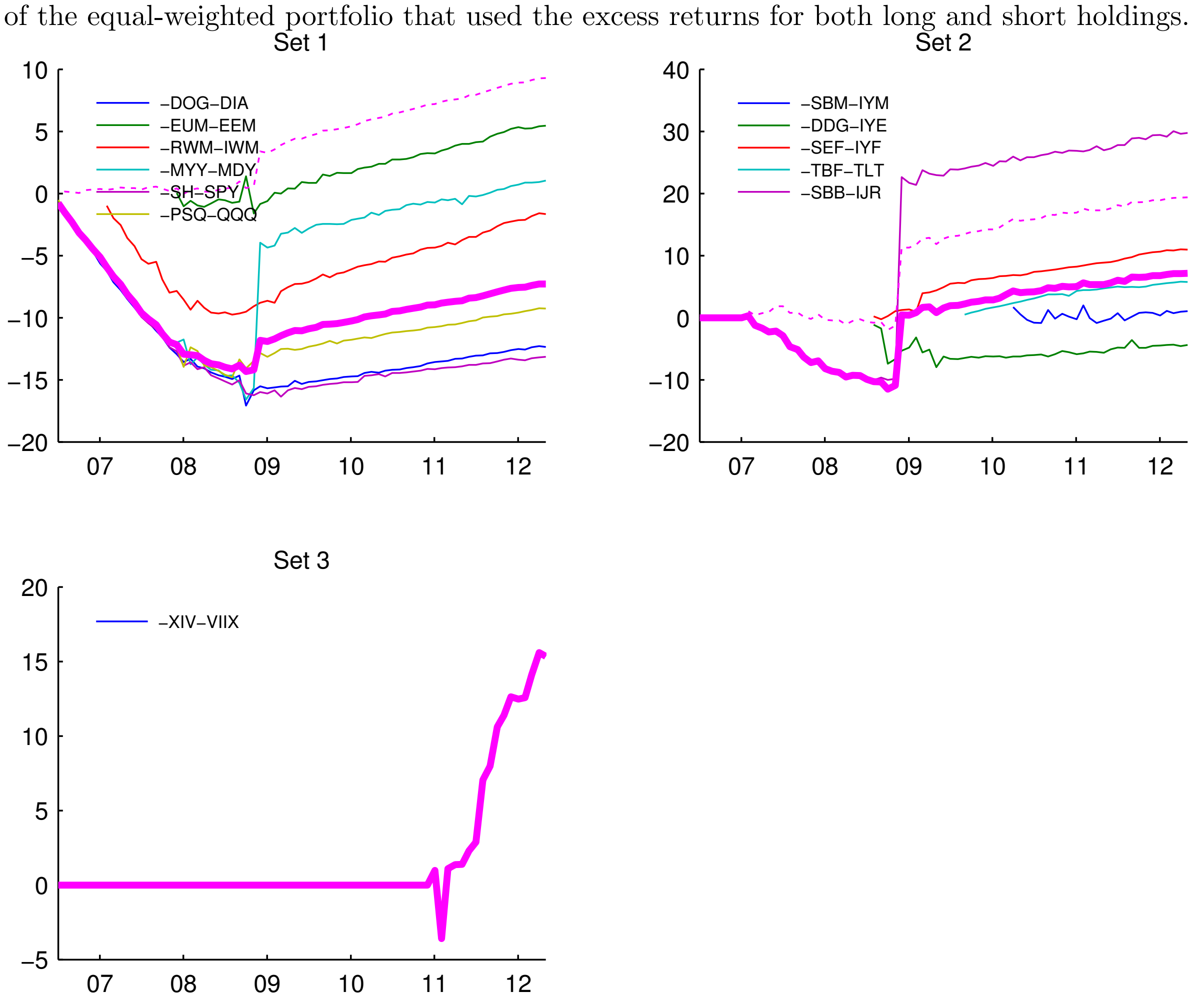


Figure 17: **Strategy 4: short 1x, weekly.**

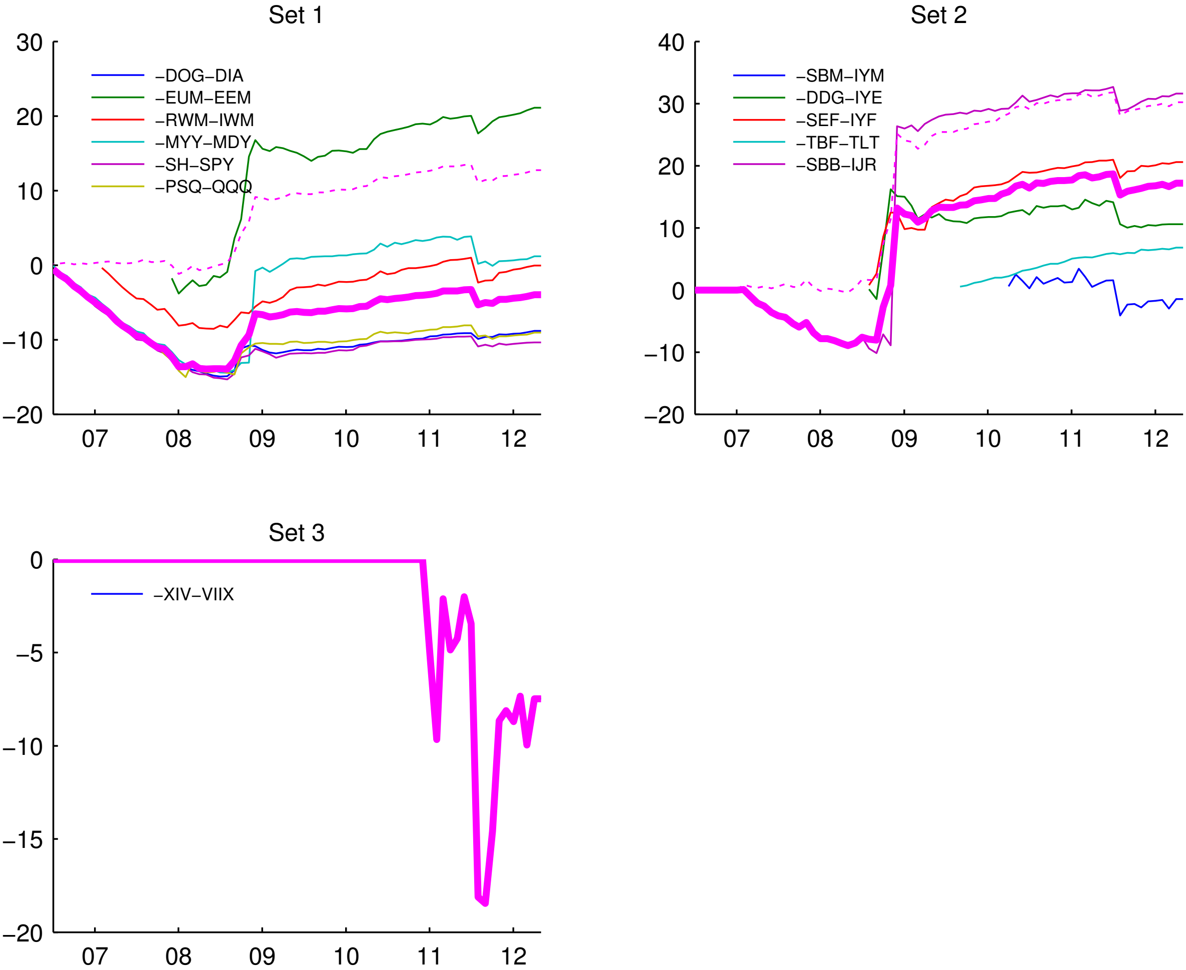


Figure 18: **Strategy 5: short 2x, daily.**
These graphs plot the performance (vertical axis in %) of the trading strategy that shorts both 2x and -2x funds on the same underlying, rebalanced daily. The colored lines are labeled with the exchange tickers. The bold magenta line is the performance of the equal-weighted portfolio of all available products used according to the strategy. The dotted magenta line is performance of the equal-weighted portfolio that used the excess returns for both long and short holdings.

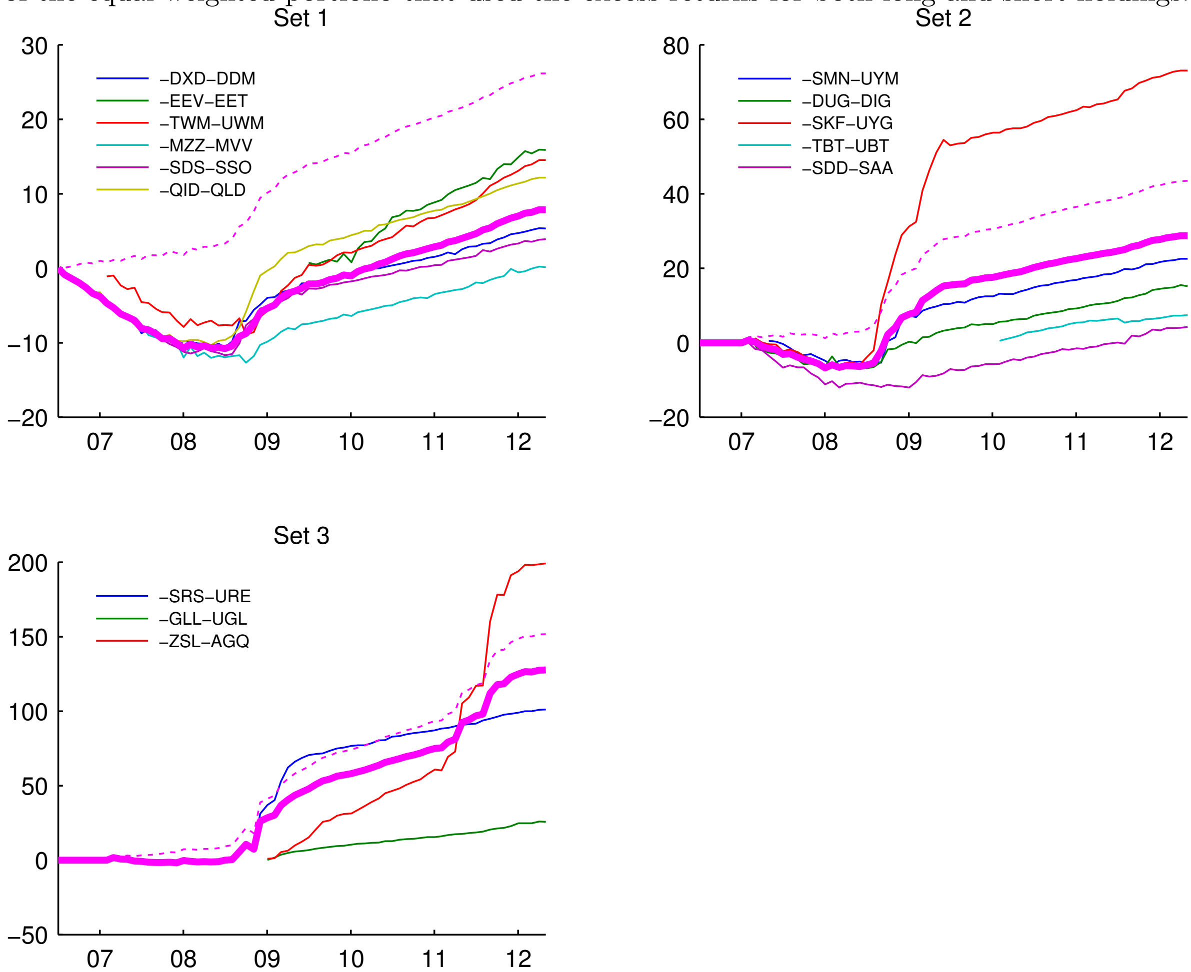


Figure 19: **Strategy 5: short 2x, weekly.**

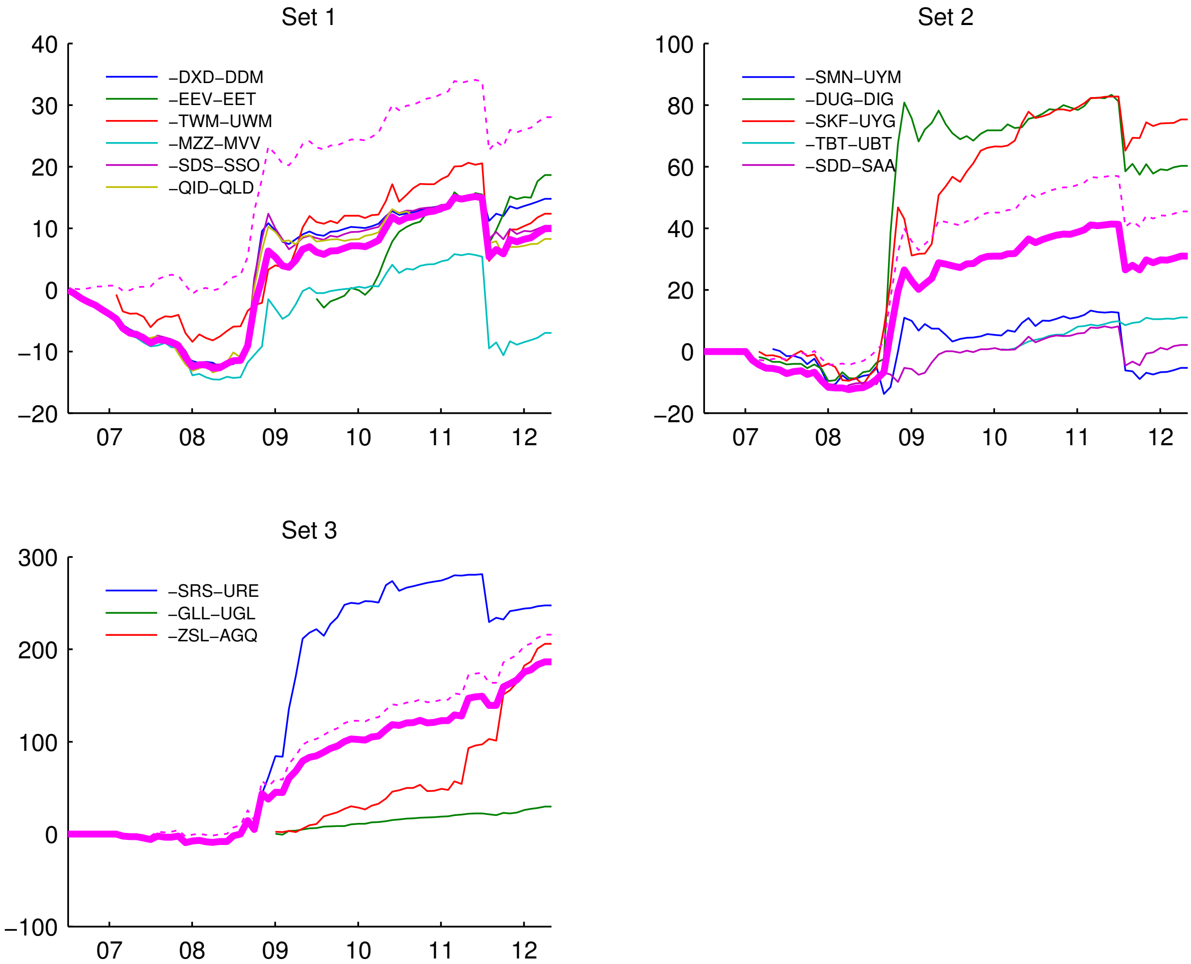


Figure 20: **Strategy 6: short 3x, daily.**
These graphs plot the performance (vertical axis in %) of the trading strategy that shorts both 3x and -3x funds on the same underlying, rebalanced daily. The colored lines are labeled with the exchange tickers. The bold magenta line is the performance of the equal-weighted portfolio of all available products used according to the strategy. The dotted magenta line is performance of the equal-weighted portfolio that used the excess returns for both long and short holdings.

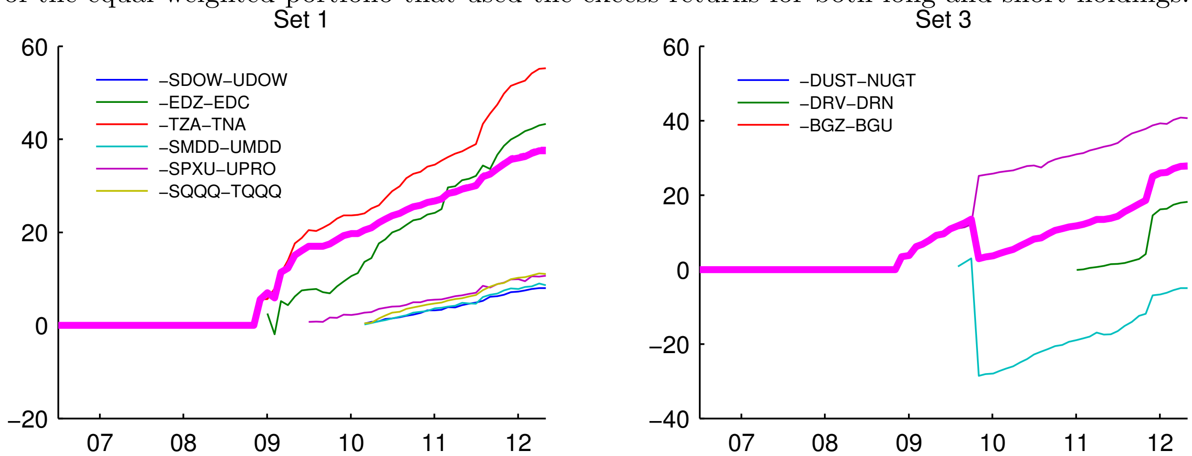


Figure 21: **Strategy 6: short 3x, weekly.**

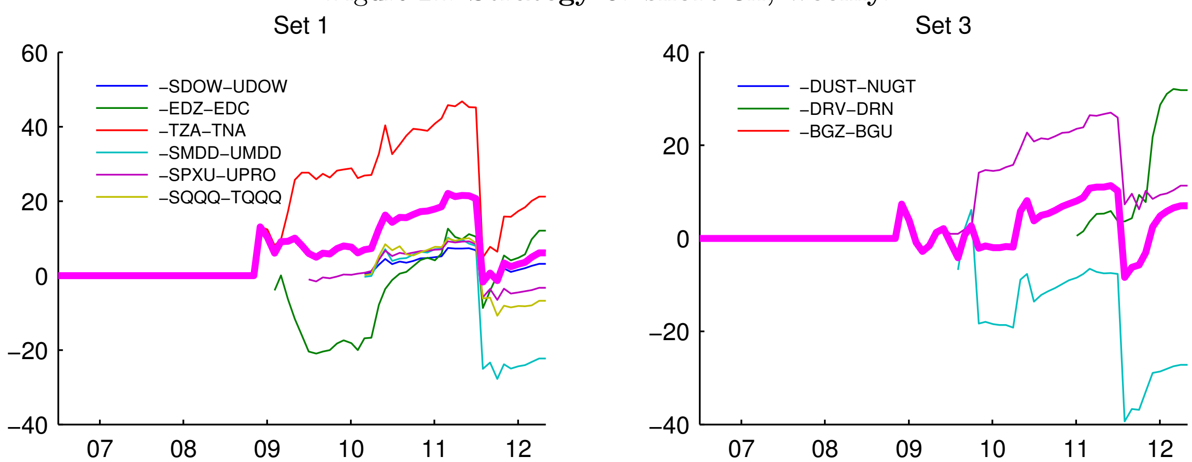


Figure 22: **Strategy 7: short 1x, 2x, daily.**

These graphs plot the performance (vertical axis in %) of the trading strategy that shorts both +/-1x and +/-2x funds on the same underlying, rebalanced daily. The colored lines are labeled with the exchange tickers. The bold magenta line is the performance of the equal-weighted portfolio of all available products used according to the strategy. The dotted magenta line is performance of the equal-weighted portfolio that used the excess returns for both long and short holdings.

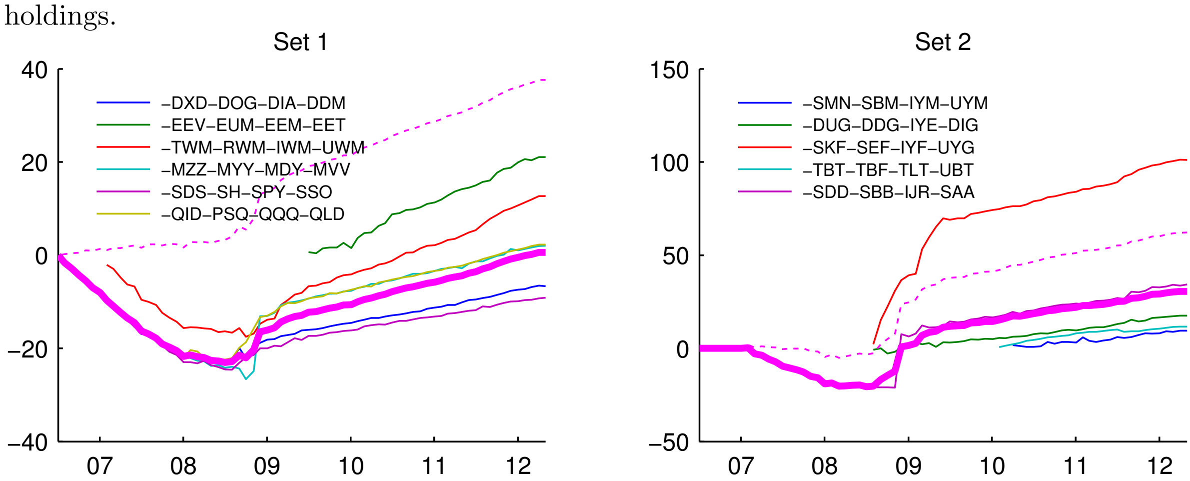


Figure 23: **Strategy 7: short 1x, 2x, weekly.**

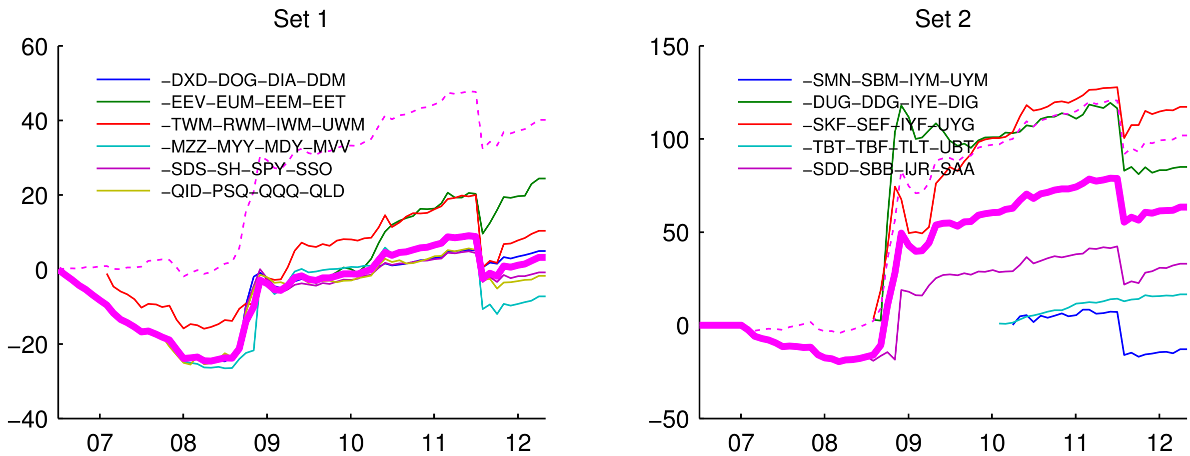


Figure 24: **Strategy 8: short 1x, 3x, daily.**
These graphs plot the performance (vertical axis in %) of the trading strategy that shorts both +/-1x and +/-3x funds on the same underlying, rebalanced daily. The colored lines are labeled with the exchange tickers. The bold magenta line is the performance of the equal-weighted portfolio of all available products used according to the strategy. The dotted magenta line is performance of the equal-weighted portfolio that used the excess returns for both long and short holdings.

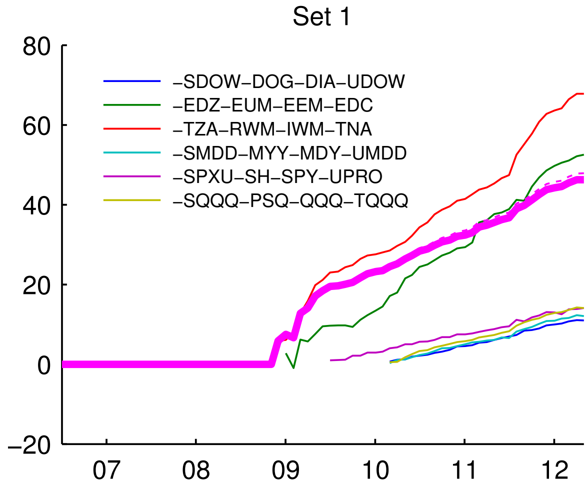


Figure 25: **Strategy 8: short 1x, 3x, weekly.**

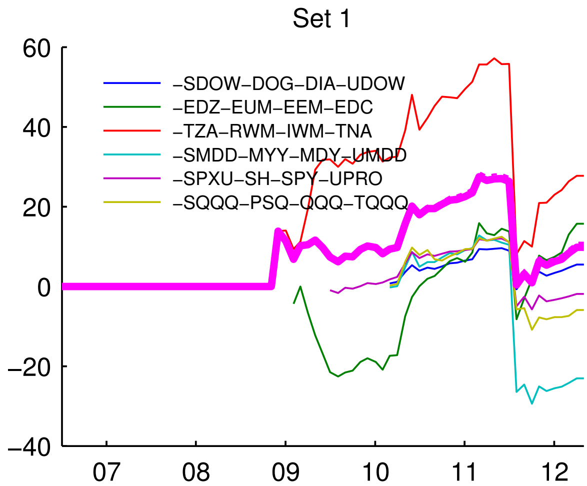


Figure 26: **Strategy 9: short 2x, 3x, daily.**
These graphs plot the performance (vertical axis in %) of the trading strategy that shorts both +/-2x and +/-3x funds on the same underlying, rebalanced daily. The colored lines are labeled with the exchange tickers. The bold magenta line is the performance of the equal-weighted portfolio of all available products used according to the strategy. The dotted magenta line is performance of the equal-weighted portfolio that used the excess returns for both long and short holdings.

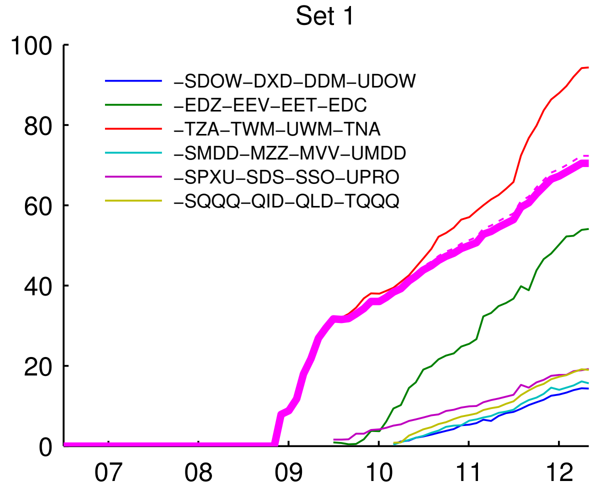


Figure 27: **Strategy 9: short 2x, 3x, weekly.**

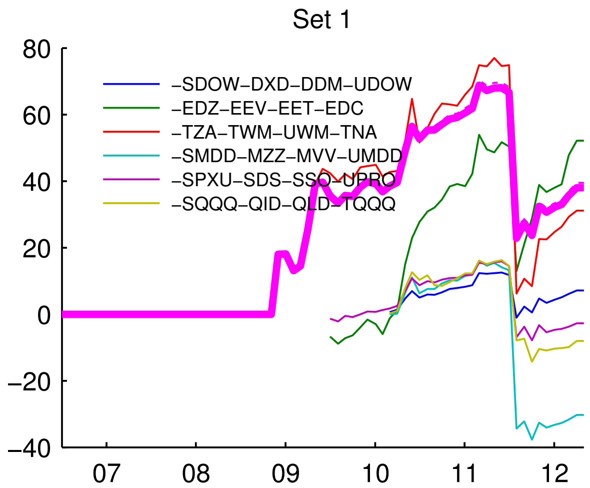


Figure 28: **Strategy 10: short 1x, 2x, 3x, daily.**
These graphs plot the performance (vertical axis in %) of the trading strategy that shorts both +/-2x and +/-3x funds on the same underlying, rebalanced daily. The colored lines are labeled with the exchange tickers. The bold magenta line is the performance of the equal-weighted portfolio of all available products used according to the strategy. The dotted magenta line is performance of the equal-weighted portfolio that used the excess returns for both long and short holdings.

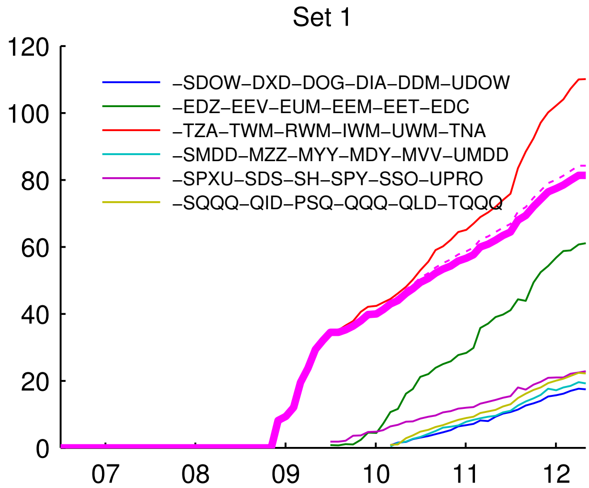


Figure 29: **Strategy 10: short 1x, 2x, 3x, weekly.**
These graphs plot the performance (vertical axis in %) of the trading strategy that shorts both +/-2x and +/-3x funds on the same underlying, rebalanced weekly. The colored lines are labeled with the exchange tickers. The bold magenta line is the performance of the equal-weighted portfolio of all available products used according to the strategy. The dotted magenta line is performance of the equal-weighted portfolio that used the excess returns for both long and short holdings.

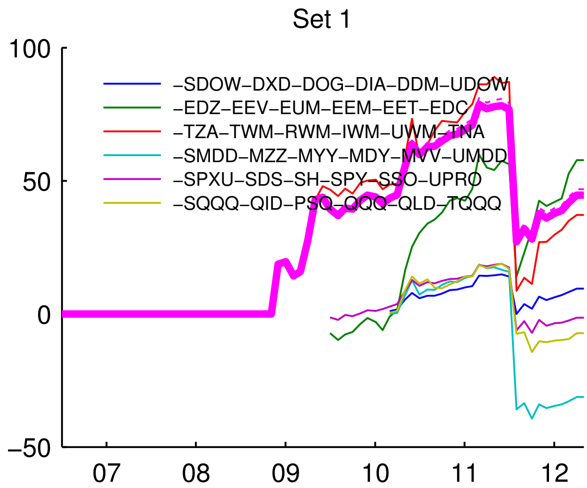


Figure 30: **Strategy 11: short 3x, long 1x, daily.**
These graphs plot the performance (vertical axis in %) of the trading strategy that shorts the +/-3x LETFs balanced out by holding a long position on the 1x ETF on the same underlying, rebalanced daily. The colored lines are labeled with the exchange tickers. The bold magenta line is the performance of the equal-weighted portfolio of all available products used according to the strategy. The dotted magenta line is performance of the equal-weighted portfolio that used the excess returns for both long and short holdings.

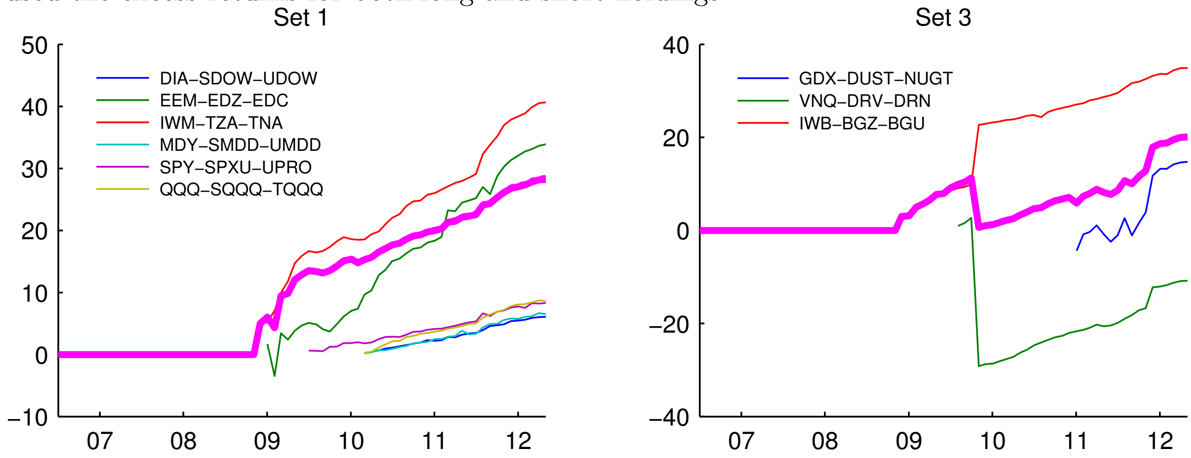


Figure 31: **Strategy 11: short 3x, long 1x, weekly.**

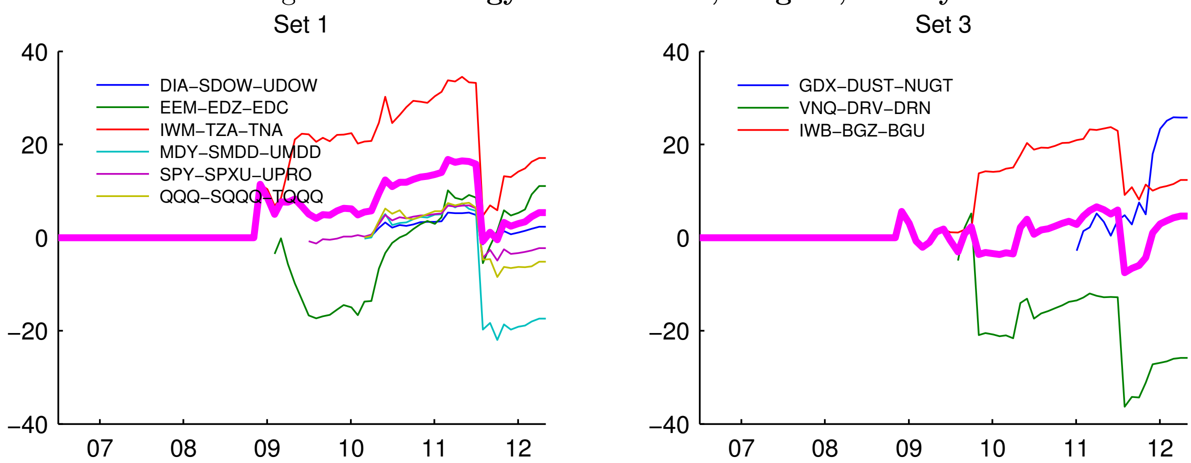


Figure 32: **Strategy 8: short 2x, long 1x, daily.**
These graphs plot the performance (vertical axis in %) of the trading strategy that shorts the +/-2x LETFs balanced out by holding a long position on the 1x ETF on the same underlying, rebalanced daily. The colored lines are labeled with the exchange tickers. The bold magenta line is the performance of the equal-weighted portfolio of all available products used according to the strategy. The dotted magenta line is performance of the equal-weighted portfolio that used the excess returns for both long and short holdings.

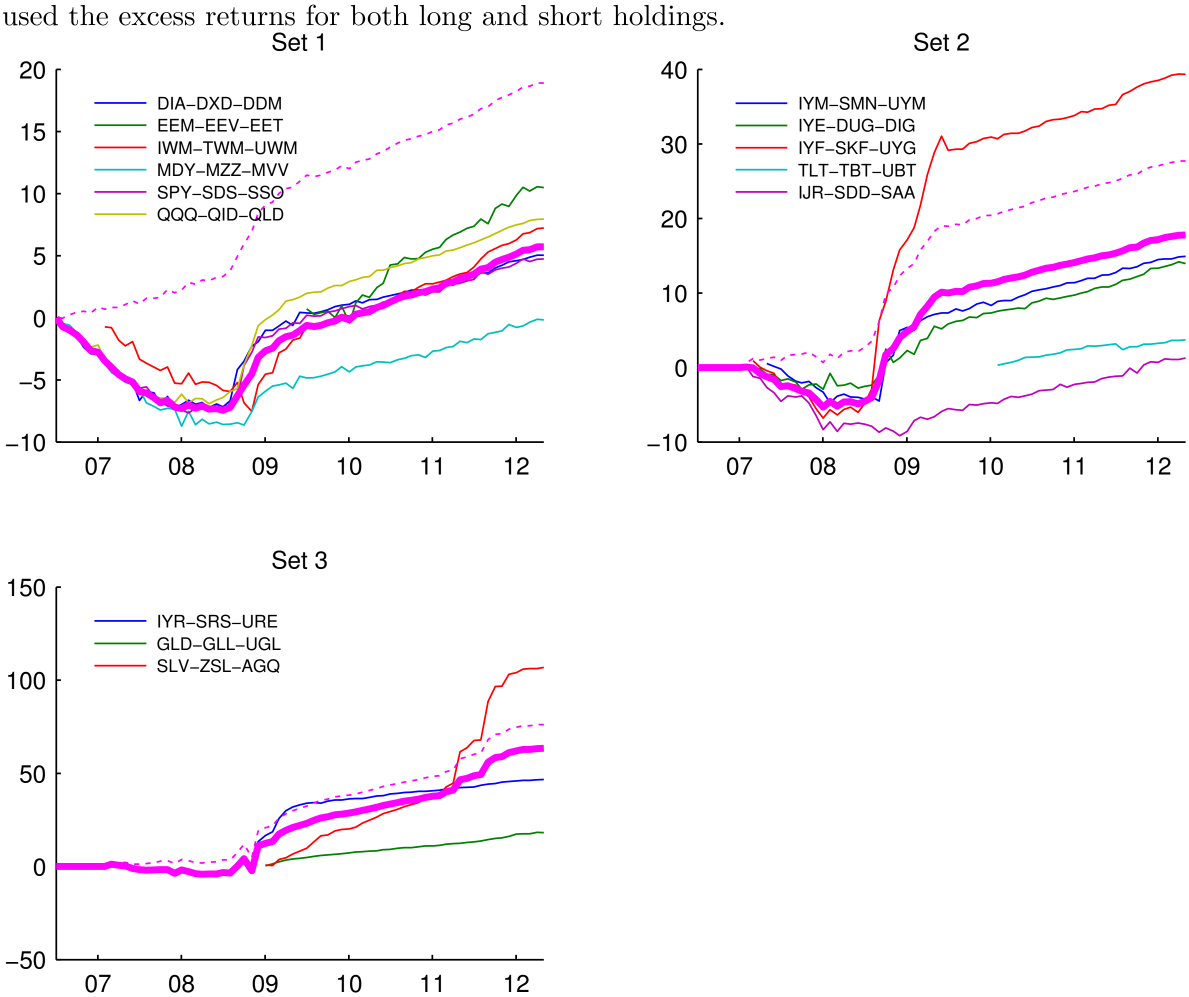


Figure 33: **Strategy 8: short 2x, long 1x, weekly.**