Mean Variance Optimization

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1 Introduction

Given a number of assets, how can we form a portfolio that gives the highest return for the least amount of risk?

Terms:

- **feasible portfolios**:
  - Any allocation that can be obtained with any asset weights. Asset weights should sum up to 1.
- **efficient portfolios** (and efficient frontier):
  - Any portfolio that comes with the least amount of risk for a given return.
- **minimum variance portfolio** (MVP):
  - The portfolio that has the smallest possible risk of all portfolios.
- **tangency portfolio**:
  - The portfolio with the greatest return-to-risk ratio.

Constraints:

- allow short-selling
  - closed form solution available
- disallow short-selling (individual weights must be between 0 and 1)
  - needs optimization (quadratic programming)

Find the efficient frontier:

- Lagrangian approach (when we allow short-selling)
- use an R package like fPortfolio, tseries, or PortfolioAnalytics
- quadratic programming in R (quadprog package)

Note: **In case that we do not restrict short-selling, there is a closed-form solution. Without short-selling quadratic programming is needed.**

2 Data

```r
library(tidyverse)
library(tidyquant)
```

```r
# data
funds <- "XLE   Energy Select Sector SPDR Fund
          XLF   Financial Select Sector SPDR Fund
          XLU   Utilities Select Sector SPDR Fund
          XLI   Industrial Select Sector SPDR Fund
          XLK   Technology Select Sector SPDR Fund
          XLV   Health Care Select Sector SPDR Fund
          XLY   Consumer Discretionary Select Sector SPDR Fund
          XLP   Consumer Staples Select Sector SPDR Fund
          XLB   Materials Select Sector SPDR Fund
          XOP   Spdr S&P Oil & Gas Exploration & Production Etf
          XHB   Spdr S&P Homebuilders Etf
          XRT   Spdr S&P Retail Etf"

funds_tbl <- tibble(funds = read_lines(funds)) %>%
  mutate(funds = str_trim(funds)) %>%
  separate(funds, into = c("ticker", "name"), sep = "\t") %>%
  mutate_all(str_trim)

prices <- tq_get(funds_tbl$ticker, from = as.Date("2010-01-01"))

monthly_rets <- prices %>%
  group_by(symbol, m = floor_date(date, "months")) %>%
  filter(date == min(date, na.rm = T)) %>%
  group_by(symbol) %>%
  mutate(date = floor_date(date, "months"),
         r = log(adjusted/lag(adjusted))) %>%
  ungroup() %>%
  select(symbol, date, r) %>%
  pivot_wider(names_from = symbol, values_from = r) %>%
  arrange(date) %>%
  filter_all(all_vars(!is.na(.)))

monthly_rets %>% saveRDS("data/monthly_rets.RDS")
```

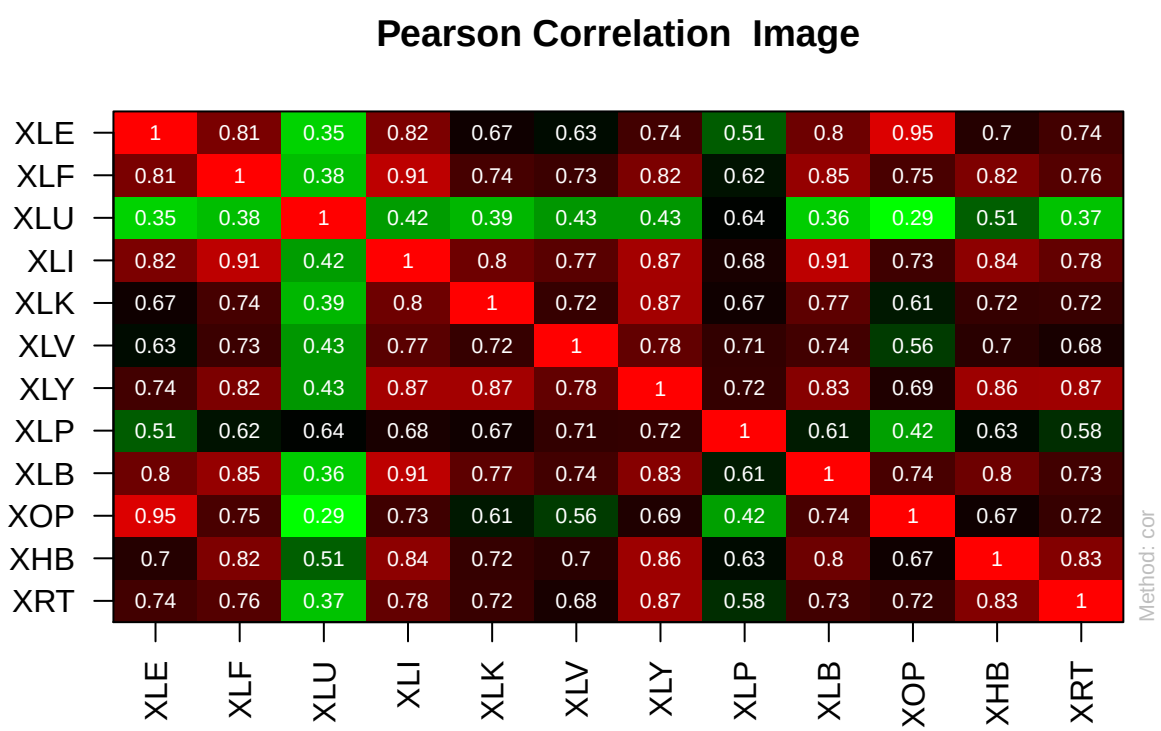
```r
# log returns
monthly_rets <- readRDS("data/monthly_rets.RDS")

library(timeSeries)

monthly_rets_ts <- column_to_rownames(monthly_rets, "date") %>%
  as.timeSeries()
```

```r
library(fPortfolio)

monthly_rets_ts %>%
  assetsCorImagePlot()
```



3 Random portfolios

```
mu_hat <- colMeans(monthly_rets_ts)*12

w <- rep(1, ncol(monthly_rets_ts))/ncol(monthly_rets_ts)

t(w) %*% mu_hat
```

```
##           [,1]
## [1,] 0.1102586
```