Template Article - Shell Model Project

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Abstract

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I. Introduction

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II. Part 1 - rename sections later, do not need the numbering of the exercises

i. Exe. 1a

Show that the unperturbed Hamiltonian $\hat{H}_0$ and $\hat{V}$ commute both with $\hat{S}_z$ and $\hat{S}^2$.

$$\hat{H}_0 = \Sigma_{p\sigma}(p-1)a^\dagger_{p\sigma}a_{p\sigma}\hat{S}_z = \frac{1}{2}\Sigma_{pq}\sigma a^\dagger_{p\sigma}a_{p\sigma}$$

first rewriting the products:

$$\begin{aligned}
\hat{S}_z\hat{H}_0 &= \frac{1}{2}\Sigma_{p\sigma}\sigma a^\dagger_{p\sigma}a_{p\sigma}\cdot\Sigma_{p\sigma}(p-1)a^\dagger_{p\sigma}a_{p\sigma} \\
&= \frac{1}{2}\Sigma_{p\sigma}\Sigma_{qb}\sigma(q-1)a^\dagger_{p\sigma}a_{p\sigma}a^\dagger_{qb}a_{qb} \\
&= \frac{1}{2}\Sigma_{p\sigma}\Sigma_{qb}\sigma(q-1)a^\dagger_{\alpha}a_{\alpha}a^\dagger_{\beta}a_{\beta} \\
\hat{H}_0\hat{S}_z &= \Sigma_{p\sigma}(p-1)a^\dagger_{p\sigma}a_{p\sigma}\cdot\frac{1}{2}\Sigma_{p\sigma}\sigma a^\dagger_{p\sigma}a_{p\sigma} \\
&= \frac{1}{2}\Sigma_{p\sigma}\Sigma_{qb}\sigma(q-1)a^\dagger_{qb}a_{qb}a^\dagger_{p\sigma}a_{p\sigma} \\
&= \frac{1}{2}\Sigma_{p\sigma}\Sigma_{qb}\sigma(q-1)a^\dagger_{\beta}a_{\beta}a^\dagger_{\alpha}a_{\alpha}
\end{aligned}$$

so that the commutation relation becomes:

$$\begin{aligned}
[\hat{S}_z,\hat{H}_0] &= \hat{S}_z\hat{H}_0 - \hat{H}_0\hat{S}_z \\
&= \frac{1}{2}\Sigma_{p\sigma}\Sigma_{qb}\sigma(q-1)a^\dagger_{\alpha}a_{\alpha}a^\dagger_{\beta}a_{\beta} - \frac{1}{2}\Sigma_{p\sigma}\Sigma_{qb}\sigma(q-1)a^\dagger_{\beta}a_{\beta}a^\dagger_{\alpha}a_{\alpha} \\
&= \frac{1}{2}\Sigma_{p\sigma}\Sigma_{qb}\sigma(q-1)\left(a^\dagger_{\alpha}a_{\alpha}a^\dagger_{\beta}a_{\beta} - a^\dagger_{\beta}a_{\beta}a^\dagger_{\alpha}a_{\alpha}\right)
\end{aligned}$$

which can only be zero if

$$a^\dagger_{\alpha}a_{\alpha}a^\dagger_{\beta}a_{\beta} - a^\dagger_{\beta}a_{\beta}a^\dagger_{\alpha}a_{\alpha} = 0$$

we can use the relations

$$\begin{aligned}
\hat{a}^\dagger_i\hat{a}^\dagger_j &= -\hat{a}^\dagger_j\hat{a}^\dagger_i \\
\hat{a}_i\hat{a}_j &= -\hat{a}_j\hat{a}_i \\
\{\hat{a}^\dagger_i,\hat{a}_j\} &= \hat{a}^\dagger_i\hat{a}_j + \hat{a}_j\hat{a}^\dagger_i = \delta_{ij} \\
\Rightarrow \hat{a}^\dagger_i\hat{a}_j &= -\hat{a}_j\hat{a}^\dagger_i, i \neq j
\end{aligned}$$

Table 1: *Example table*

Name		
First name	Last Name	Grade
John	Doe	7.5
Richard	Miles	2

to rewrite the first expression

$$
\begin{aligned}
a_\alpha^\dagger a_\alpha a_\beta^\dagger a_\beta &= a_\alpha^\dagger a_\beta^\dagger a_\alpha a_\beta \\
&= -a_\beta^\dagger a_\alpha^\dagger a_\alpha a_\beta \\
&= -a_\beta^\dagger a_\alpha^\dagger (-a_\beta a_\alpha) \\
&= a_\beta^\dagger a_\alpha^\dagger a_\beta a_\alpha \\
\Rightarrow & a_\alpha^\dagger a_\alpha a_\beta^\dagger a_\beta - a_\beta^\dagger a_\beta a_\alpha^\dagger a_\alpha = 0 \\
\Rightarrow & [\hat{S}_z, \hat{H}_0] = 0
\end{aligned}
$$

hmmm is this really a proof? indices

ii. Exe. 1b - simple model

We are only including two lowest single-particle levels and totally two particles. We want to construct the Hamiltonian matrix using second quantization and WickâĂŹs theorem for a system with no broken pairs and $S = 0$, $S_z = 0$

In this model we can only excite two particles at the same time, so we can define the ground state and the excited state which are the only two possible states:

(also include a figure with the states drawn?)

III. Methods

Text requiring further explanation[1].

IV. Results

V. Discussion

References

[Figueredo and Wolf, 2009] Figueredo, A. J. and Wolf, P. S. A. (2009). Assortative pairing and life history strategy - a cross-cultural study. *Human Nature*, 20:317–330.

[1] Example footnote