EE20085 Electromagnetics- CAD For Electromagnetic Devices Lab

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Author Note

This document is the required technical report for the aforementioned project above and is to be submitted for grading by Dr Manuchehr Soleimani.

Summary

This Lab report details the findings from exercises A, B, D, and E from the CAD for electromagnetic Devices document [1]. The objective of this lab was to “Learn how to use electromagnetic modelling using finite element program pdetool in MATLAB” and “learn the requirement of finite element tools in engineering applications”. Finite element method (FEM) [6], is a one of the most widely used methods for solving mathematical and engineering problems. For this lab the program used to solve using FEM was MATLAB’s finite element toolbox. The exercises were set to help the team to understand and practice FEM using real-world applicable problems. The exercises focused on getting the team acquainted with the different applications and general modelling process of draw, decompose, set boundary conditions, PDE specifications, mesh, solve and finally plot. Similarly, the exercises also produced a comparison of FEM to FDM and a final 2D plot of the electric field potential for a comb-drive electrostatic micro-actuator.

Theory and Equations

For this lab the central program used was MATLAB’s Finite Element Toolbox[2]. MATLAB’s Finite Element Toolbox is a downloadable package on MATLAB that allows the user to do finite element analysis on a given created system/model.

The magnetic field of an infinitely long straight wire can be obtained by applying Ampere's law. In order to work out the magnetic field of an infinitely long straight wire Ampere’s law[3] needed to be used. Ampere’s law is demonstrated in equation 1:

Equation 1: Where B = magnetic field strength, l= the length of the conductor, μ0 = the permeability of free space, and I = the current in the conductor.

$$\sum B_{||}\,\Delta l = \mu_0 I$$

for a circular path centered on the wire, the magnetic field is everywhere parallel to the path. The equation can be rewritten with the magnetic field strength as the subject, as seen in equation 2:

Equation 2:

$$\sum B_{||}\,\Delta l = B2\pi r$$

$$B = \frac{\mu_0 I}{2\pi r}$$

The electric field E at a point in space is the force acting on a unit positive charge placed at that point. The unit positive charge is considered a concept/ target (sometimes called test) charge. The units of E are V/m and the force acting on a charge can be calculated from the E in which it is placed using equation 3 also known as Coulomb’s Law[4]:

Equation 3: Coulobs Law.

$$\mathbf{E}_{12} = \frac{\mathbf{F}_{12}}{Q_1} = \frac{Q_2}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}_{21}$$

When wanting to calculate the potential difference at certain points across a distribution of charge the first step is to treat the volume of the charge distribution as a cube. Each face of the cube has a set of coordinates assigned to it. The following faces use D = (Dx,Dy,Dz): the left face, the bottom face and the back face. These 3 faces have the same origin. The other faces have different equations: the front face uses D = (Dx+Dx,Dy,Dz), the right face uses D = (Dx,Dy+Dy,Dz) and the top face uses D = (Dx,Dy,Dz+Dz).

Taking the faces above and applying Gauss's Law[5] gives the team a choice of Poisson's equation, when charge is present, or Laplace's equation ,when charge is not present.

Equation 4: Poisson's equation

$$\underline{\nabla}^2 V = -\frac{\rho}{\varepsilon} \quad \left(\text{in full, } \underline{\nabla}^2 V(x,y,z) = -\frac{\rho(x,y,z)}{\varepsilon}\right)$$

Equation 5: Laplace equation

$$\underline{\nabla}^2 V = 0 \quad \left(\text{in full, } \underline{\nabla}^2 V(x,y,z) = 0\right)$$

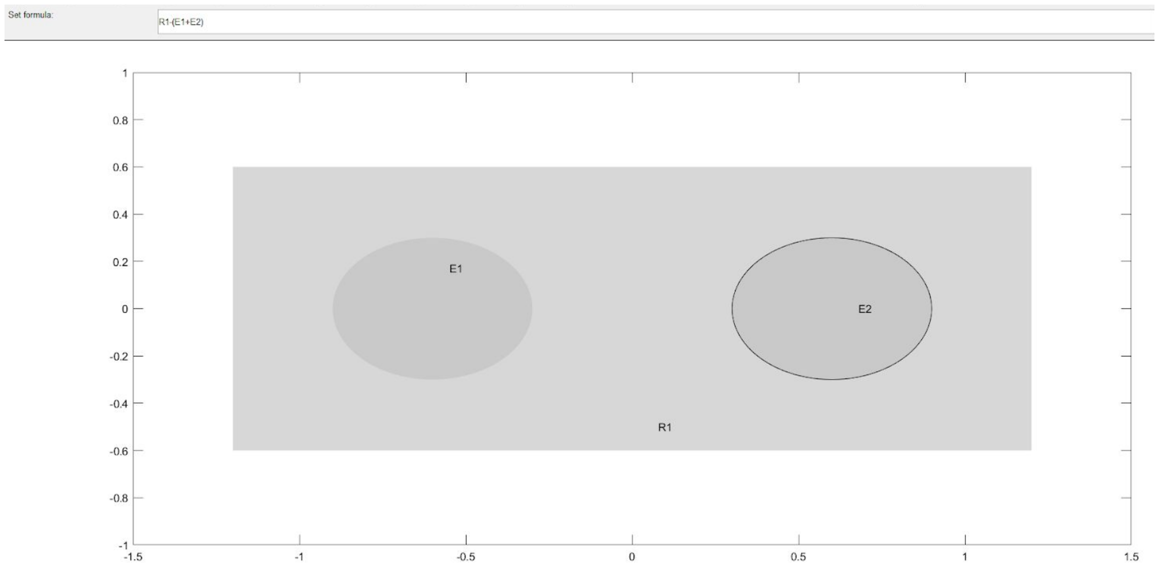
For problems that are fields within an enclosed surface. A method for solving for points within the plane section to be calculated is dividing the space into a grid. The finer the grid, the more accurate the calculation but the longer it will take to compute. From first principles it is possible to estimate V/dx and dV/dy for the grid and use these results to then estimate the next derivatives d2V/dx2 and d2V/dy2. The expressions can thus be estimated to become exactly correct as Δ→0. This iterative approach allows for precision dependent on the number of times iterated. The potential at a point can therefore be approximated as the average of the potential from 4 neighbouring points. When looking at electrical potentials within a field often time it is important to look at equipotentials. The distribution of the magnitudes of equipotentials in a field can be found and thus using the Laplace iterative method written above method it is possible to calculate the other potentials. This is often referred to as the finite difference method (FDM)[6] and is often used in magnetic and electric field calculations.

Part A:

Method:

For section A, the team were instructed to simulate two circular metallic conductors on a plane using the partial differential equation tool in Matlab. The way the team calculated this in Matlab was by drawing two circles to represent the circular conductors and a rectangle to represent the plane. The next step was to set the formula parameter to express the 2-D domain of problem; the formula required was R1-(C1+C2) giving figure 1:

Figure 1: Drawing for exercise A

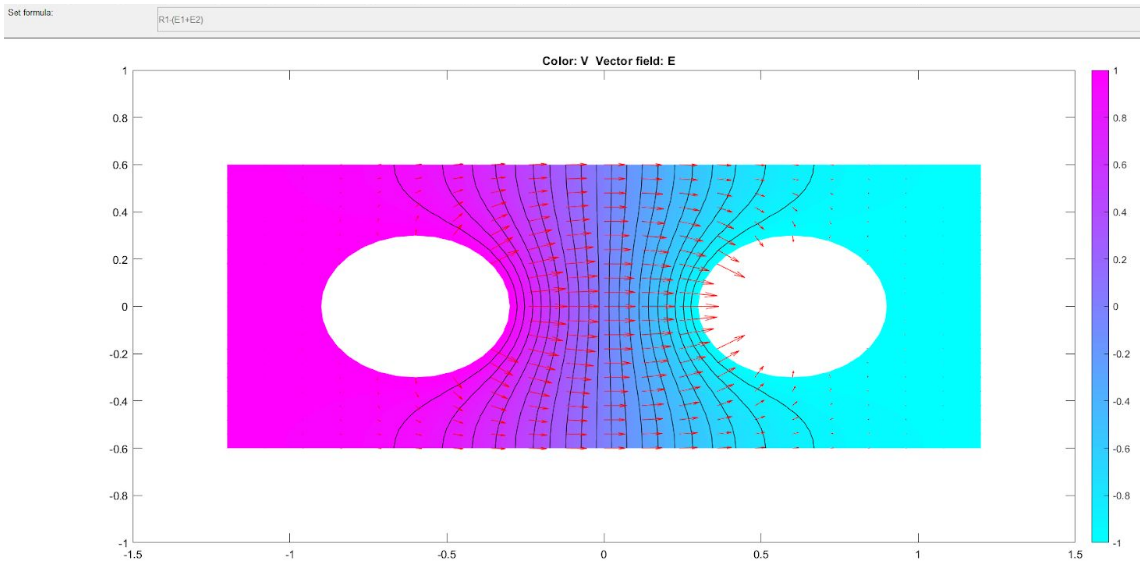


The physical model for this problem requiring FEM calculation consists of the following Laplace equation ▽ * (σ ▽ V) = 0. The boundary condition for the left circular conductor, E1, was for the electric potential, V, to be equal to 1 and the boundary condition for the right circular conductor, E2, was for the electric potential, V, to be equal to -1. As a result the team then decomposed the geometry and entered the boundary mode of the program. This is where the rectangular plane's boundary conditions for each of the four sides were set using a neumann condition. The natural Neumann boundary condition on the outer boundaries is dVd/n which is equal to 0, where V is electrical potential and n is the normal to the boundary. The Neumann condition equation is as follows: n*c*grad(u)+qu=g thus in this case both g and q were set to zero for the plane. The boundary conditions were then set for E1 and E2 which were both set using the Dirichlet condition. The Dirichlet condition equation is as follows: h*u=r. For E1 both h and r were set to 1 and for E2 h was set to 1 and r was set to -1. The final step before solving the problem was to enter PDE mode where the current source, q, was set to 0.

Results:

From the method detailed above the team used the solver tool and produced the figure 2 with the field lines and arrow produced using the plotting tool

Figure 2: 2D plot of electrical potential field for exercise A



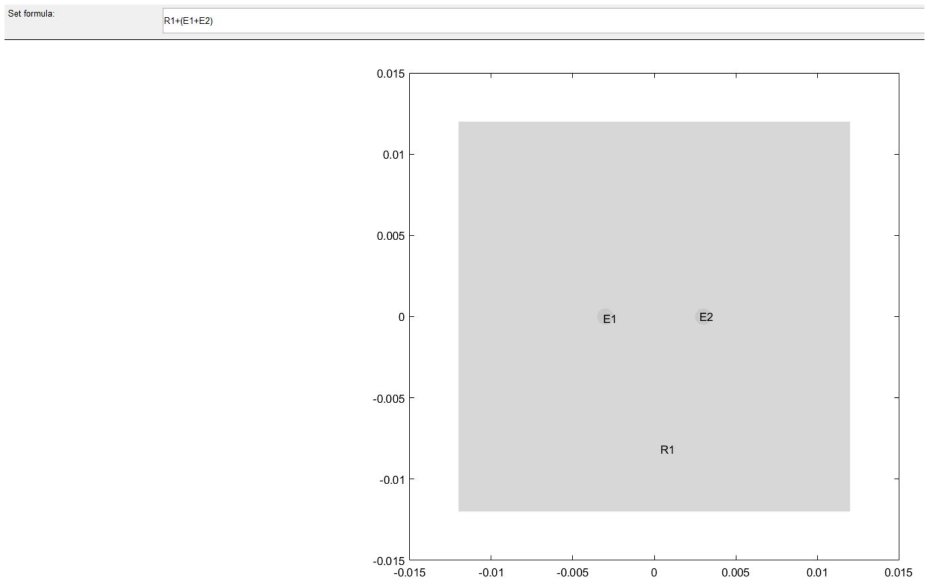
These results are accepted by the team as it matches what the theory dictates should happen in this situation. The key components of the plot that confirm the results are correct is the fact that; the plot's field lines are symmetrical, the arrows point toward the negatively charged circle (the right one), and the center is where the zero equipotential is. The plot should be symmetrical as the two conductors have the same shape and have the same electrical potential magnitude. The arrows should point towards the negatively charged circle as this follows the theory of conventional charge. Finally, the centre should be the zero equipotential point because the conductors have the same magnitude and oppositely charged electrical potentials.

Part B:

Method:

For exercise B in the lab. It was detailed to use FEM to calculate the magnetic flux density of two current carrying conductors. The objective of this exercise was to study lines of magnetic fields surrounding two long circular conductors containing go and return current. With this objective the team decided this was a magnetostatic problem and modelled it as such. For the FEM modelling the team used the team used the Finite Element Toolbox in MATLAB to constructed the figure 3 from the lab script instructions [1]

Figure 3: Drawing used for exercise A with formula of decomposition

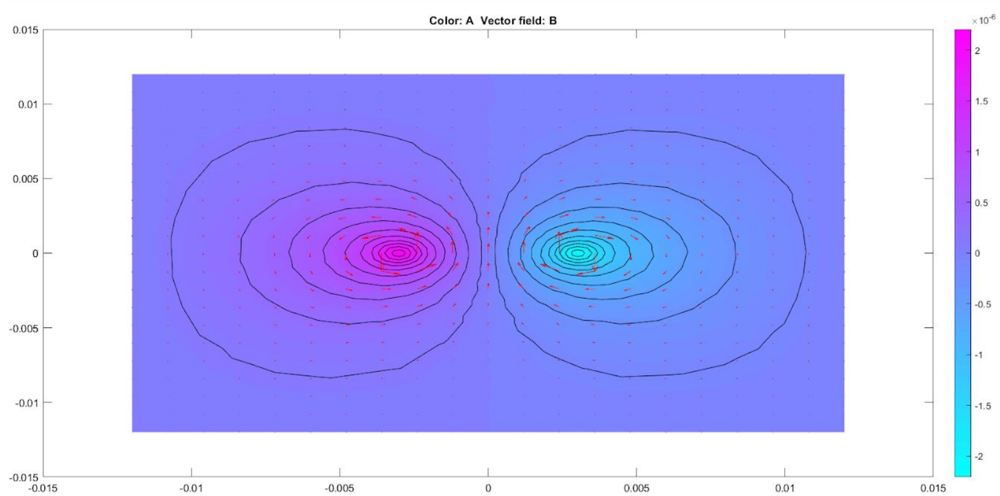


From the drawing for the system the team went to the boundary mode to set the conditions. For R1 the team set the external boundaries to the default magnetic potential zero. After this using the PDE tool the team set; R1 to μ= μ0 and J=0, E1 to μ= μ0 and J=1, and E2 to μ= μ0 and J=-1. The reason for the J values of E1 being 1 and E2 being -1 is because the current is going in opposite directions in each conductor as specified above. This difference in direction is an important difference seen in the results. The team next initialized the a triangle mesh to allow the solver to have

Results:

With the above method Used the team solver to generate the resulting figure 4

Figure 4: 2D plot of magnetic flux density of magnetic field for exercise B



The figure 4 gives the field lines required by the lab script

The next step was for the team to find the current in each conductor. This was achieved by using the equation 2 Where B is equal to magnetic flux density at the point of the left conductor (for example), I is equal to the current induced in the right conductor, r is the distance between the two conductors and μ0 is equal to the permeability of free space which is 1.25663706 × 10^-6 m kg s^-2 A^-2. By rearranging this
equation for I the current the current in each conductor can be found as shown in the figure 5 below:

Figure 5: Calculations used to find the current in each conductor

$$B = \frac{\mu_0 I}{2\pi r} \qquad I_{E1} = \frac{2\pi r B}{\mu_0}$$

$$I_{E1} = \frac{2\pi \times 0.006 \times 2.2042 \times 10^{-6}}{1.25663706 \times 10^{-6}} = 0.066126\ A$$

$$I_{E2} = \frac{2\pi \times 0.006 \times 2.2056 \times 10^{-6}}{1.25663706 \times 10^{-6}} = 0.066168\ A$$

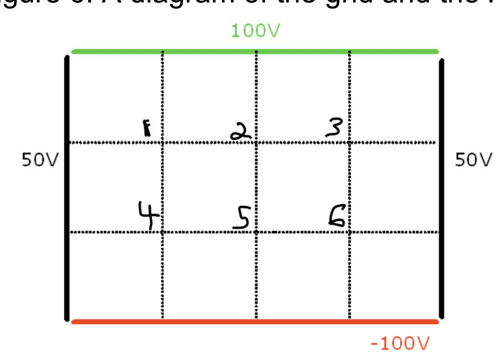
Part D:

Method:

For this exercise in the lab. It was detailed as a 16 x 12 cm rectangle of surrounding with relative dielectricity of 1. The script required the team to use both the FDM and FEM to calculate the electric potential and the electric fields of the grid points and then compare the results from each method.

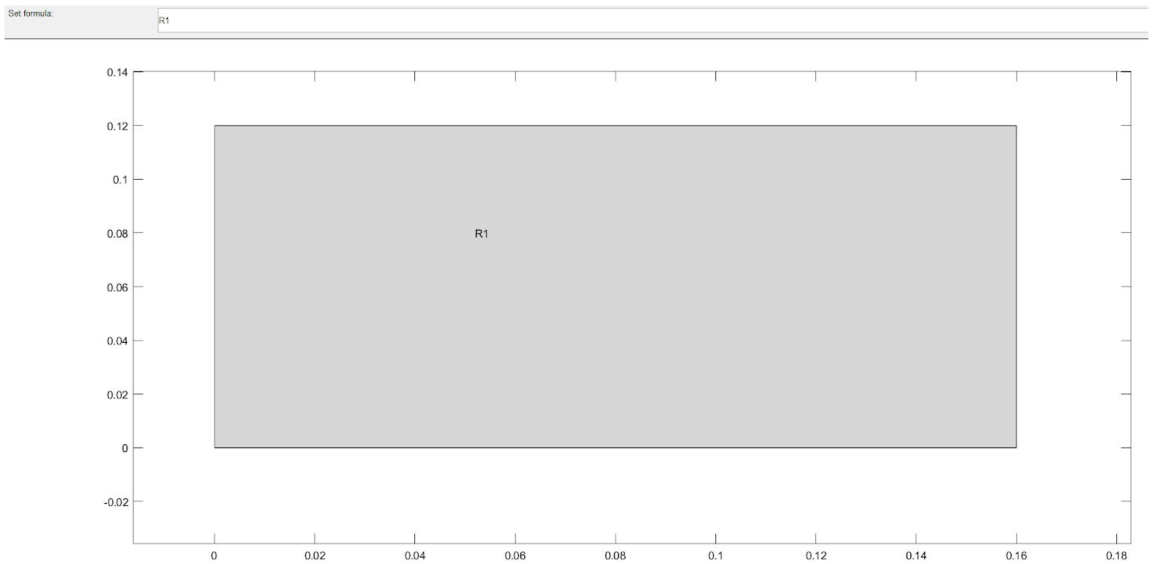
The team firstly modelled the using FEM using the finite element toolbox in MATLAB and solved the electrostatic problem. For this the 6 given in the lab script [1]

Figure 6: A diagram of the grid and the labeled coordinates



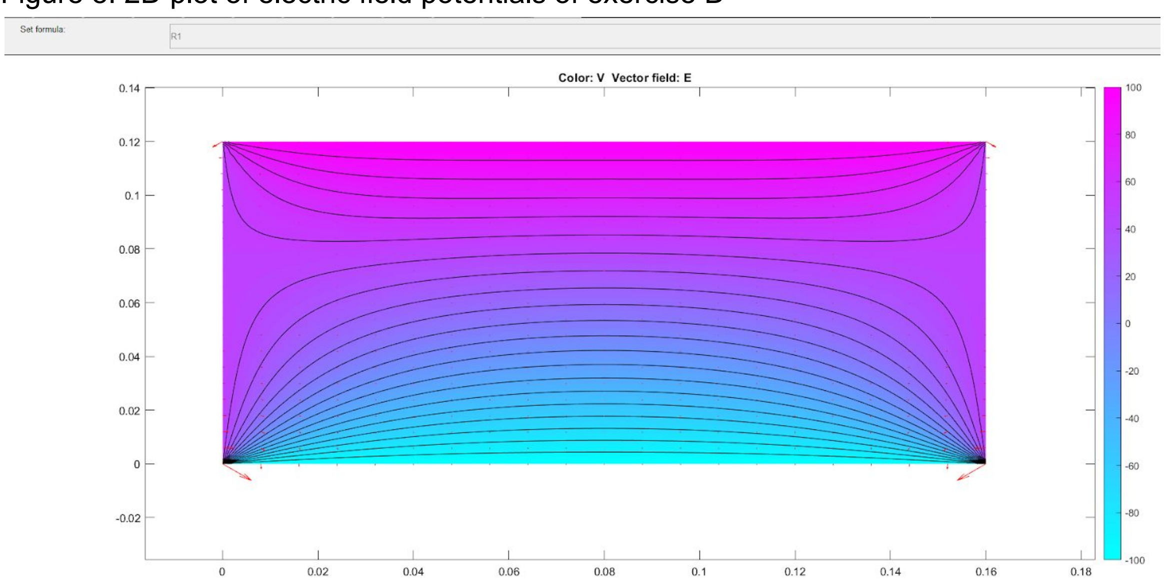
A MATLAB pdetool model was constructed giving figure 7:

Figure 7: Drawing on MATLAB for exercise D



The team used the boundary mode within the pde tool and selected each side of the rectangle and set the boundary condition to Dirchlet as these sides have electric potentials. With the boundary conditions set, the team then used the PDE tool and set the coefficient of dielectricity to 1 and the free charge value equal to zero. Before solving the team used the mesh tool to create a triangle mesh to calculate the values of the field at various nodes. This required the team to refine the mesh a few times. Next, the team noted the nodes that were closest to the coordinates on the grid. Using the solve tool the team produced the figure 8

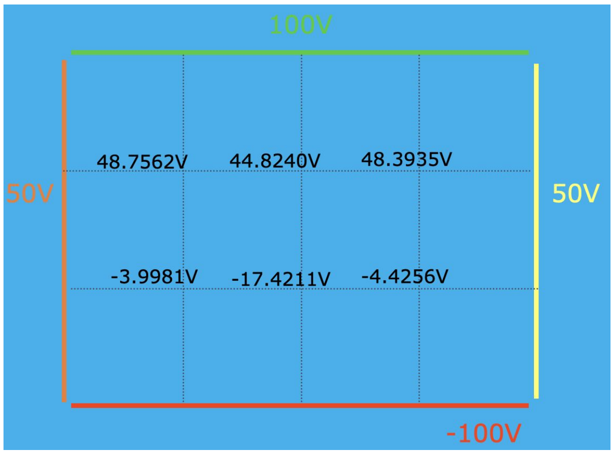
Figure 8: 2D plot of electric field potentials of exercise D



Results:

From the process above the team exported the solution giving a matrix of values of the electrical potentials at each node. Using the 6 nodes the coordinates where calculated/found to be as shown in figure 9

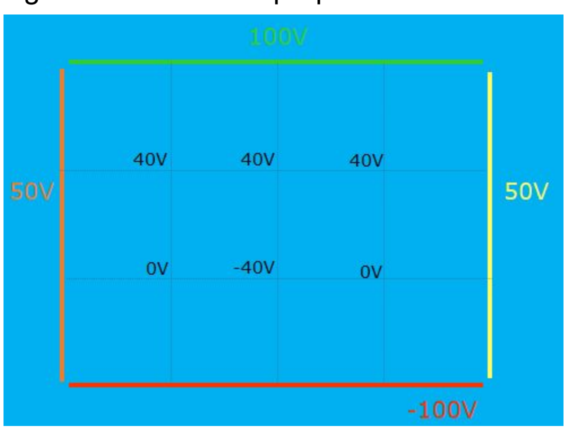
Figure 9: FEM results plotted to their specific coordinate



The Next solving method was using FDM. With this iterative process the team calculated clockwise from coordinate 1 around to coordinate 6 for each iteration, with the end goal being when all the coordinate values do not change by 1V or more.

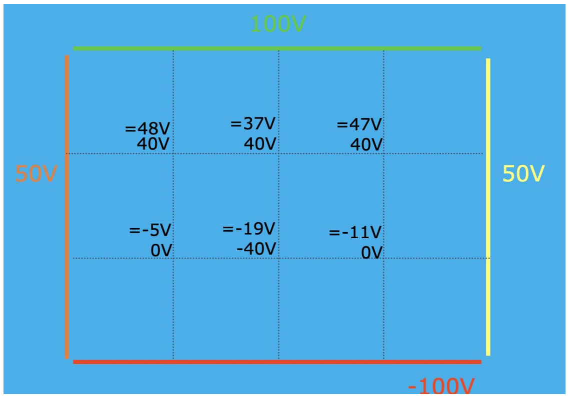
Initial Values

Figure 10: The initial proposed values for the FDM iterations



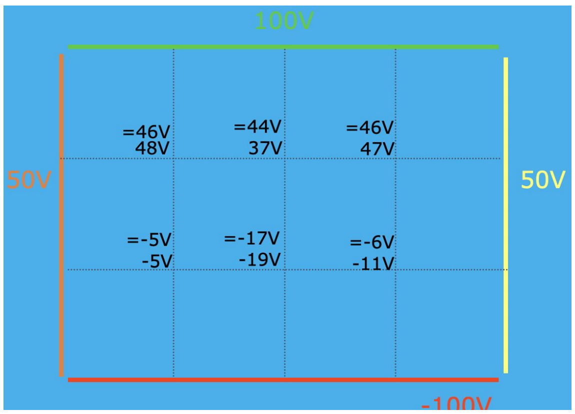
Iteration 1

Figure 11: results of iteration 1 written above previous values



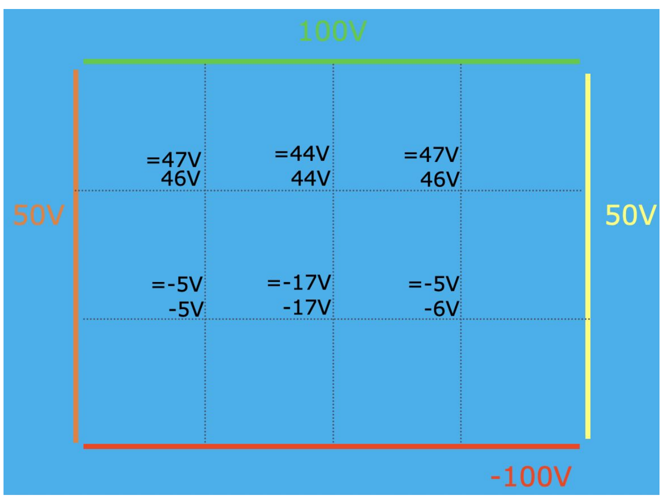
Iteration 2

Figure 12: results of iteration 2 written above previous values



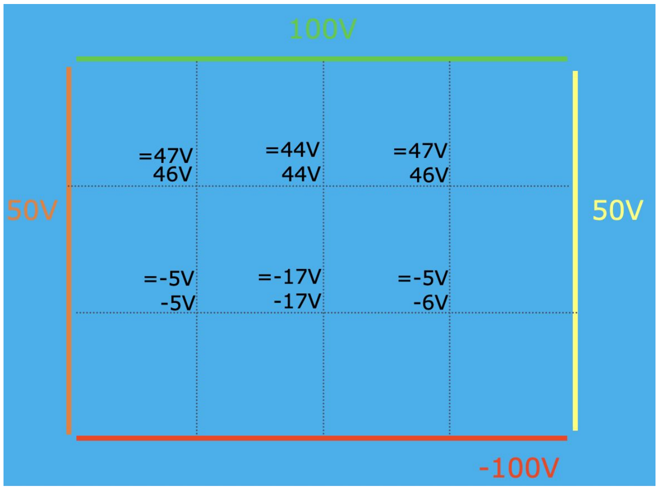
Iteration 3

Figure13: results of iteration 3 written above previous values



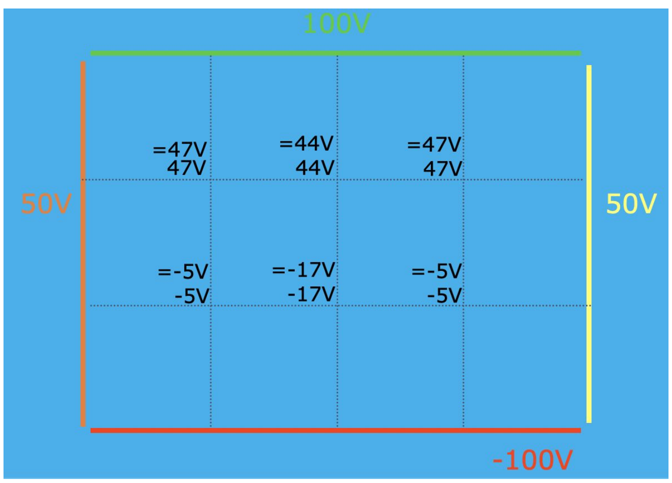
Iteration 4

Figure 14: results of iteration 4 written above previous values



Iteration 5

Figure 15: results of iteration 5 written above previous values



Both methods used in exercise D have advantages and disadvantages. For FEM this analysis is very accurate but requires a lot of calculations (normally done by a computer) and thus can require a powerful computer which may take time. Contrastingly, FDM has simpler calculations and can be easily done by hand and quickly. The factor that would determine the best method would be the degree of precision. For the calculated values from each method the FDM on average was only 8.3% off from the FEM values. Similarly, The more significant figures you use in iteration the more accurate FDM becomes.

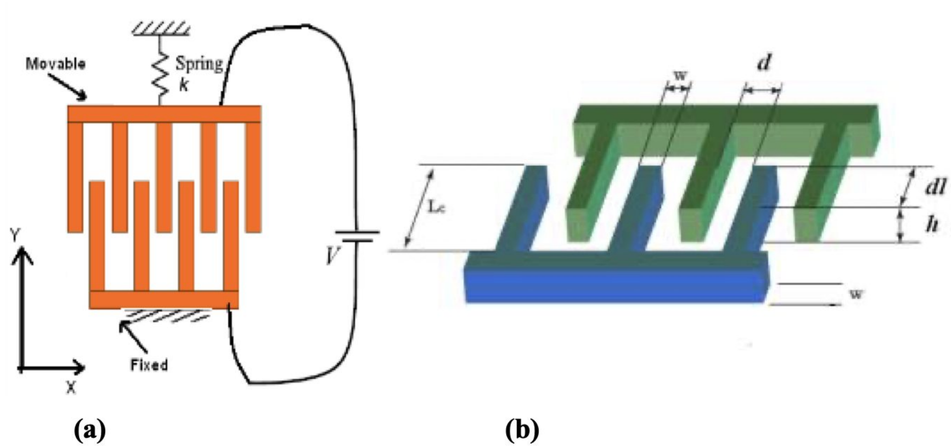
The major idea taken away from this exercise is for certain electrostatic problems there are two methods FEM and FDM. These methods are split by ease and precision. For solving electrostatic problems using these methods it is important to look at the problem's need for precision and the tools available to the team.

Part E:

Method:

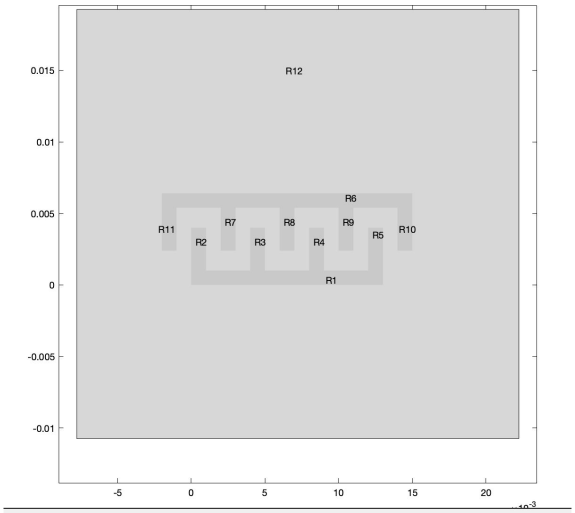
For the exercise E, the team was instructed to construct the system detailed in the figure 16:

Figure 16: (a) shows the overall design of the comb-drive electrostatic micro-actuator
(b) shows the dimensions of the combs.



The dimensions given were w=1 , d=1, dl=0.6 and Lc=3 where all are assumed to be in mm. Although the microactuator is a 3d object the team used a 2d FEM analysis. Because of this the team disregarded the thickness. The team identified the exercise as being an electrostatic problem. From the figure 16a the team assigned an electric potential of 10V to the movable comb and 0V to the fixed comb. From the dimensions above the team constructed the outline (figure 16b) with R12 being the plane on which the field to be plotted.

Figure 17: system with formula R12-(R1+R2+R3+R4+R5+R6+R7+R8+R9+R10+R11)

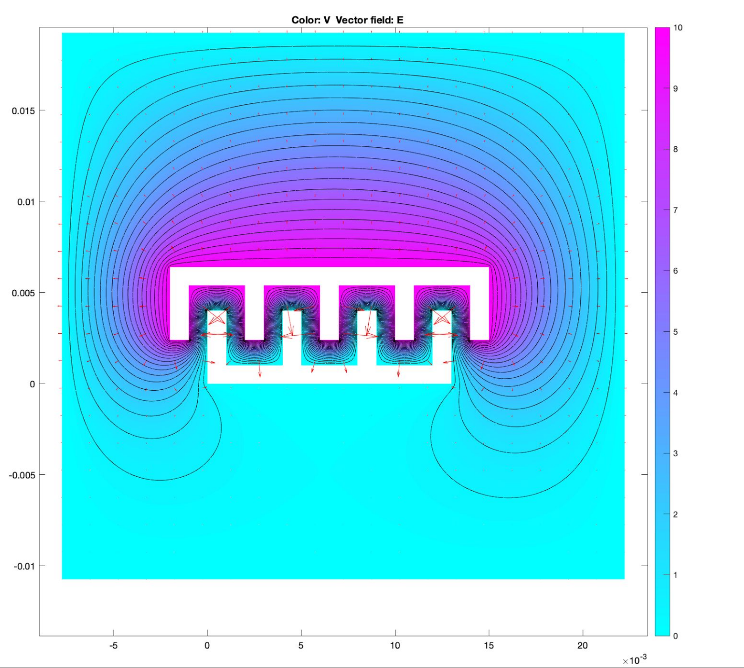


From this the system above was decomposed and assigned boundary conditions. For the moveable comb the boundaries are set to 10V using the dirichlet conditions. For the plane and the fixed comb boundaries were checked to ensure they were set to 0V with dirichlet conditions. Next using the PDE tool the team set the coefficient of dielectricity to 1 and the free charge value equal to zero. Then, the team used the mesh tool and initialized, and refined the mesh 2 times.

Results:

With the method above the team then used the solver to produce the resulting figure 18.

Figure 18: 2D plot of comb-drive electrostatic micro-actuator electric field.



The above results are what the team expected to see. These kinds of micro-actuators use the spring attached to the movable comb to resist the force the field enacts on the movable comb. The field causes a force at the movable comb in the direction of the other comb and depending on the size of this force and the knowledge of the force from the spring the distance between combs will give information about the system. This is a central reason why micro-actuators can be used as good resonators in systems like voltmeters and optical shutters.

The FEM analysis gives a good visual (figure 18) of why this setup gives a stable and reliable micro-actuator. Looking at the field arrows by each "finger" of combs The symmetry of the field and construction would resist and correct lateral displacement.

Conclusion

With the objective of this lab to "Learn how to use electromagnetic modelling using finite element program pdetool in MATLAB"[1], and "learn the requirement of finite element tools in engineering applications"[1], the team has successfully completed the lab. This was demonstrated through the completion, explanation and discussion with each of the 4 required exercises. The Team produced 2D electrical or magnetic field plots and did thorough calculations to find the values of figures 2, 4, 8 and 18. The team was further able to explore using the Finite Element Toolbox and understood how to select an application of said toolbox, as seen in exercise A and exercise B. Exercise A requires you to use the Conductive Media DC application and exercise B the magnetostatic application. Similarly, all exercises offered the team experience in going from drawing to plotting for each unique exercise. Following on from the lab, to continue through with the objective the team would also explore the AC Power Electromagnet application on the Finite Element Toolbox.

References

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