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# Quantum State Tomography with Spiking Neural Networks

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As a part of course Fundamentals of AI/ML under supervision of Prof. Mayank Goswami

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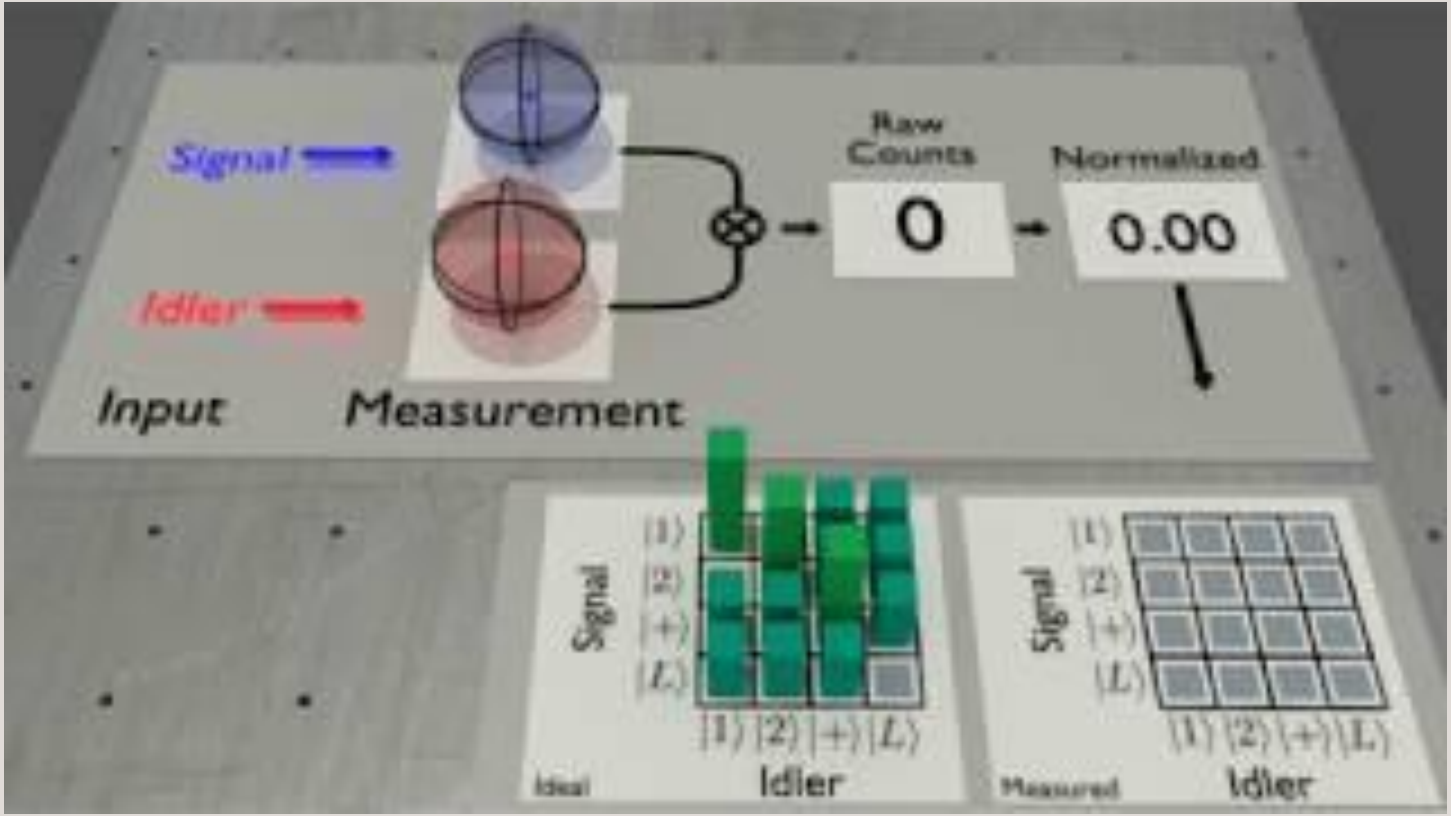
# Introduction

Quantum State Tomography (QST) reconstructs the density matrix $\rho \in \mathbb{C}^{2^N \times 2^N}$ of an $N$-qubit system from informationally complete measurements, using expectation values $\langle A \rangle = \mathrm{Tr}(\rho A)$ or outcome probabilities $P(a) = \mathrm{Tr}(\rho M_a)$ from POVMs $M_a$ with . $\sum_a M_a = I$

Neuromorphic computing implements spiking neural networks (SNNs) as mixed-signal, event-driven processors that natively generate and consume spike trains with on-chip noise and microsecond-scale sampling for probabilistic modeling and low-latency inference.

A single-chip Neural Quantum Interface (NQI) integrates spiking cores for stochastic sampling with memristor or superconducting circuit-based computation-in-memory (CiM) crossbars for dense matrix–vector multiplies, eliminating the memory wall and enabling fast, energy-efficient QST close to the quantum hardware.

# Introduction - Quantum State Tomography

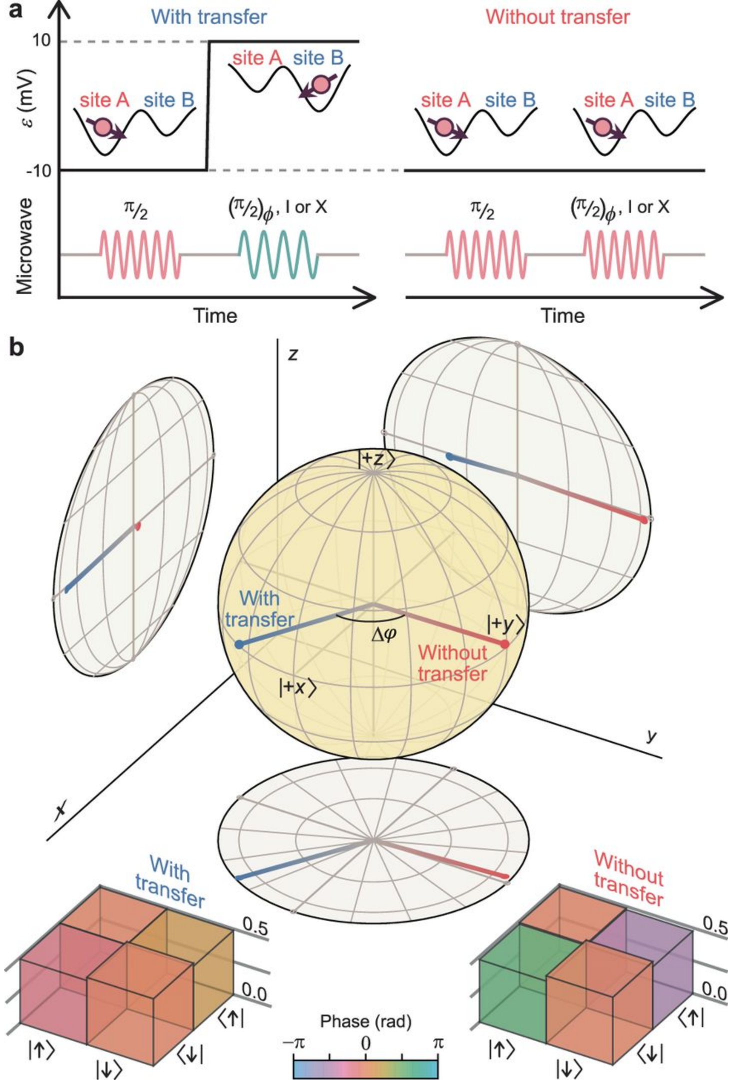


# Motivation (why?)

QST suffers from exponential scaling in Hilbert space ( $2^N$) and Pauli-basis settings ( $4^N$ ), requiring hardware-aware neural methods and in-memory acceleration to achieve practical fidelity and throughput as qubit counts grow.

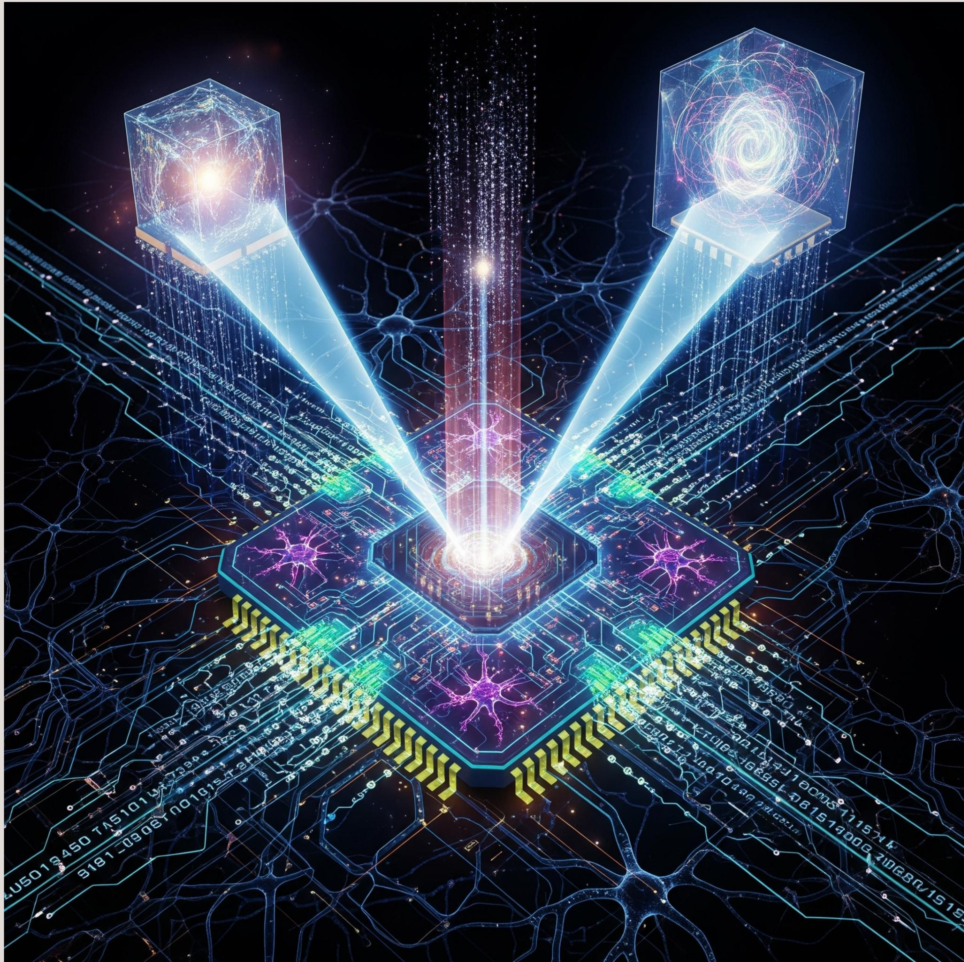
Embedded neuromorphic processors can be co-located with experiments to generate samples every few microseconds and support near-real-time diagnostics, calibration, and variance reduction for quantum devices under finite-shot noise.

Targeted applications include qubit characterization and diagnostics, quantum algorithm verification, hybrid QML feedback loops, peaked/noisy circuit readout, neural quantum interfaces for fast tomography, and quantum sensing/metrology bridging classical ML to quantum data streams.



# Applications

- The Neural Quantum Interface enables fast, low-latency quantum-classical links enabling efficient probing.
- SNNs provide ultra-fast diagnostics for Quantum Error Correction in real-time.
- Low-latency feedback from the neuromorphic chip accelerates hybrid QML algorithms by embedding and extraction of classical data into state vector.
- Quantum Sensing with fast polling rate and Optimal state reconstruction for QST is performed rapidly by the SNN.
- Event-driven SNNs offer energy-efficient edge computing near the quantum hardware.



# Methodology-Data Generation

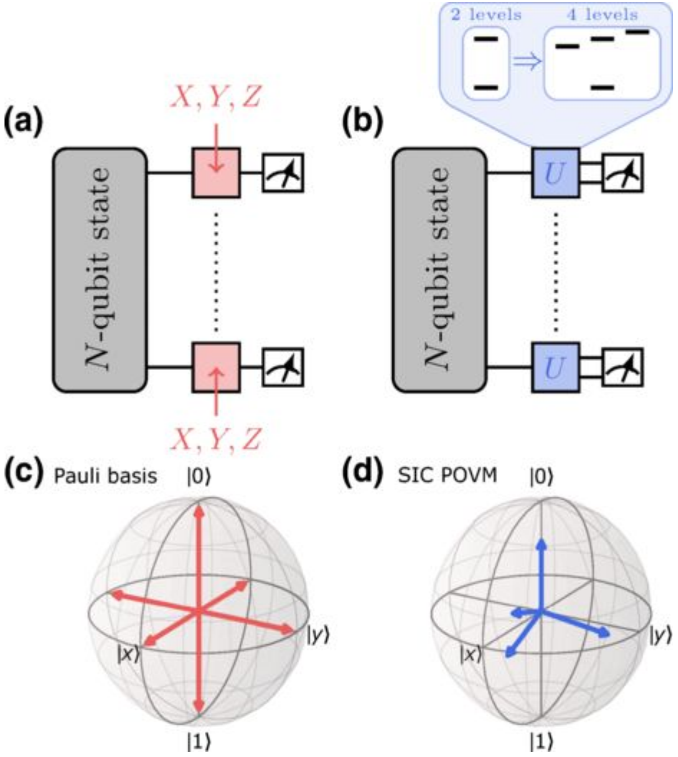
Target State Generation and Simulation

For an N-qubit system, we apply K random quantum gates selected from the set {H, RX, RY, RZ, CX, CRZ}.

The resulting final quantum state (either the state vector or density matrix) serves as our target label. This state can be:

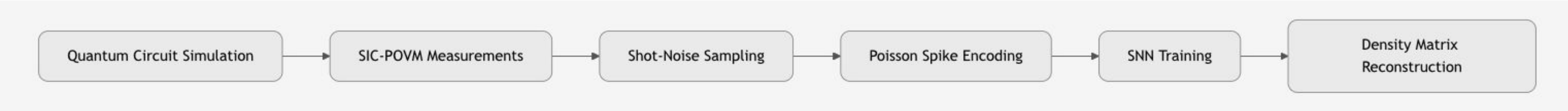
- Simulated: Extracted using a tensor network-based quantum simulator.
- Extracted: Obtained from a real quantum computer using techniques like classical shadows.

The Born rule is then applied to the final state to derive a probability distribution over all basis states. This distribution is sampled via measurements with a predefined number of shots which can be either projective measurement over pauli basis or SIC POVM for best feature information extraction.



| Feature | Pauli Projective Measurement | SIC-POVM |
| --- | --- | --- |
| **Concept** | **Projective Measurement** on orthogonal axes. | **POVM** (Symmetric Informationally Complete). |
| **Measurement Set** | **3 separate** measurement settings ( $\sigma_X, \sigma_Y, \sigma_Z$ ) needed for full QST. | **1 single setup** of $d^2$ symmetric "effects" for full QST. |
| **Implementation** | **Standard** and easy to perform on current hardware. | **Theoretically Optimal** but **Hard to Implement** physically. |
| **Goal** | Measures expectation values $\langle X\rangle, \langle Y\rangle, \langle Z\rangle.$ | Directly yields probabilities for **optimal** state reconstruction. |
| **Status** | **Practical Default.** | **Theoretical Best.** |

# Methodology-Model Pipeline



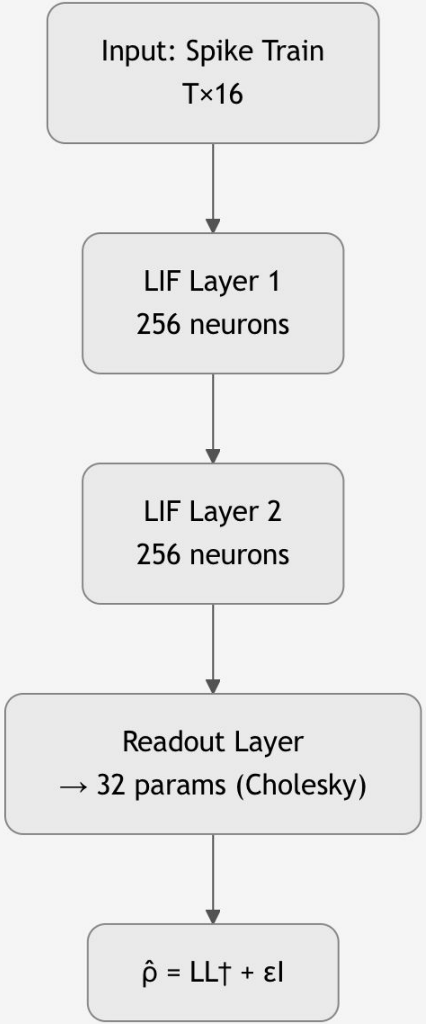

| Stage | Tool | Key Feature |
|---|---|---|
| Simulation | PennyLane | Random 2-qubit circuits |
| Measurements | SIC-POVM | 16 outcomes (4 per qubit) |
| Encoding | Poisson Spikes | T=200 timesteps |
| Model | SNN (LIF) | 256 hidden neurons |
| Output | $\hat{\rho}$ (4×4 complex) | Cholesky parameterization |

# Methodology-Model Architecture

**Neurons**: Leaky Integrate-and-Fire (LIF), β = 0.6
**Surrogate Gradient**: Fast sigmoid (slope = 70)
**Output**: 32 real params → Hermitian positive-semidefinite ρ̂
**Optimizer**: Adam (lr = 1e-3), Weight Decay = 1e-4
**Loss Function**:Physics-Aware Multi-Term Loss

$$\mathcal{L} = \underbrace{(1 - F(\rho, \hat{\rho}))}_{\text{Fidelity Loss}} + \lambda_m \underbrace{D(p \| \hat{p})}_{\text{Measurement Consistency}} + \lambda_l \underbrace{\|\hat{\rho} - \rho_{\text{lin}}\|^2}_{\text{Linear Inversion Proximity}}$$

| Term | Equation | Meaning | Weight |
|---|---|---|---|
| **Fidelity Loss** | $1 - F(\rho, \hat{\rho})$ | Maximize quantum similarity | 1.0 |
| **Meas. Consistency** | $D(p \| \hat{p})$ | Match observed probabilities | $\lambda_m = 0.5$ |
| **Linear Proximity** | $\|\hat{\rho} - \rho_{\text{lin}}\|^2$ | Stay near analytic solution | $\lambda_l = 0.05$ |



Quantum Fidelity (F) $F(\rho, \hat{\rho}) = \left(\mathrm{Tr}\left[\sqrt{\sqrt{\rho}, \hat{\rho}, \sqrt{\rho}}\right]\right)^2$

Quantum Fidelity is the gold standard metric used to measure the similarity between two quantum states, $\rho$ (true) and $\hat{\rho}$(predicted):

- Interpretation:
  - F = 1 indicates perfect reconstruction (identical states).
  - F = 0 means the states are completely orthogonal or dissimilar.
- Relevance:
  Fidelity directly quantifies how well the model learns entanglement, phase coherence, and amplitude distributions in the quantum system. It provides state-level accuracy.

Kullback–Leibler (KL) Divergence $D_{KL}(P|Q) = \sum_x P(x), \log \frac{P(x)}{Q(x)}$

KL divergence measures the difference between two probability distributions, typically comparing the true measurement probabilities (P(x)) with the model-predicted probabilities (Q(x)):

- Interpretation:
  Lower KL values signify that the predicted distribution closely matches the experimental outcomes.
- Relevance in QST:
  It captures how well the model preserves the statistical structure of the quantum measurements. It complements fidelity by comparing observable statistics rather than the complete density matrix, thus providing measurement-distribution accuracy.

Summary: Together, Fidelity and KL divergence provide a comprehensive picture of the tomography quality.

# Methodology-Other Models

## Transfomer(QSTT)

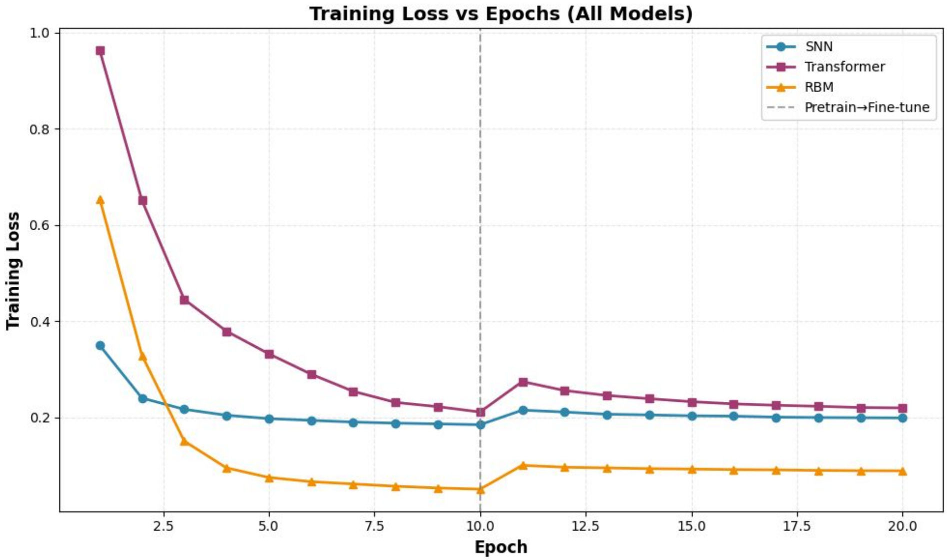
| Parameter | Value |
|---|---|
| **Input** | `[CLS] + 16 frequency tokens` |
| **Embedding** | Learned (size = `d_model=128`) |
| **Architecture** | 2-layer Transformer Encoder |
| **Attention Heads** | 4 |
| **Hidden Dim** | 128 |
| **Output Head** | `[CLS]` token → Linear(128 → 32) |
| **Optimizer** | Adam, `lr=1e-3`, `wd=1e-4` |
| **Gradient Clipping** | `clip=1.0` |

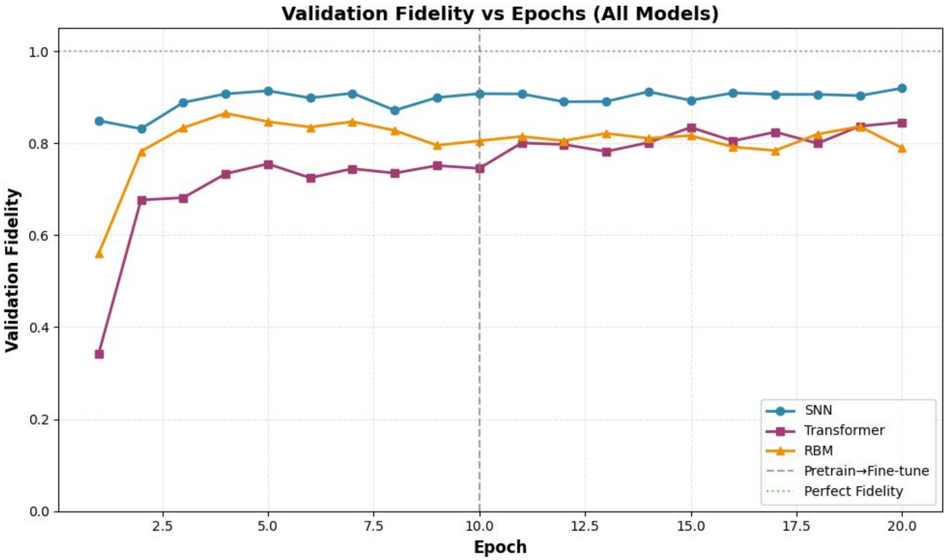
## Conditional Restricted Boltzman Machine

| Parameter | Value |
|---|---|
| **Visible Units** | 16 (one per SIC outcome) |
| **Hidden Units** | **128** |
| **Input** | soft frequencies |
| **Training** | **Contrastive Divergence (CD-1)** |
| **Learning Rate** | `1e-3` |
| **Epochs** | 20 |
| **Batch Size** | 32 |
| **Reconstruction** | Sample visible → `sic_invert_probs_to_rho()` |

# Results - Evaluation Metrics

All models exhibit highly competitive performance. The RBM is a particularly strong learner, primarily due to its advantage in probabilistic modeling.



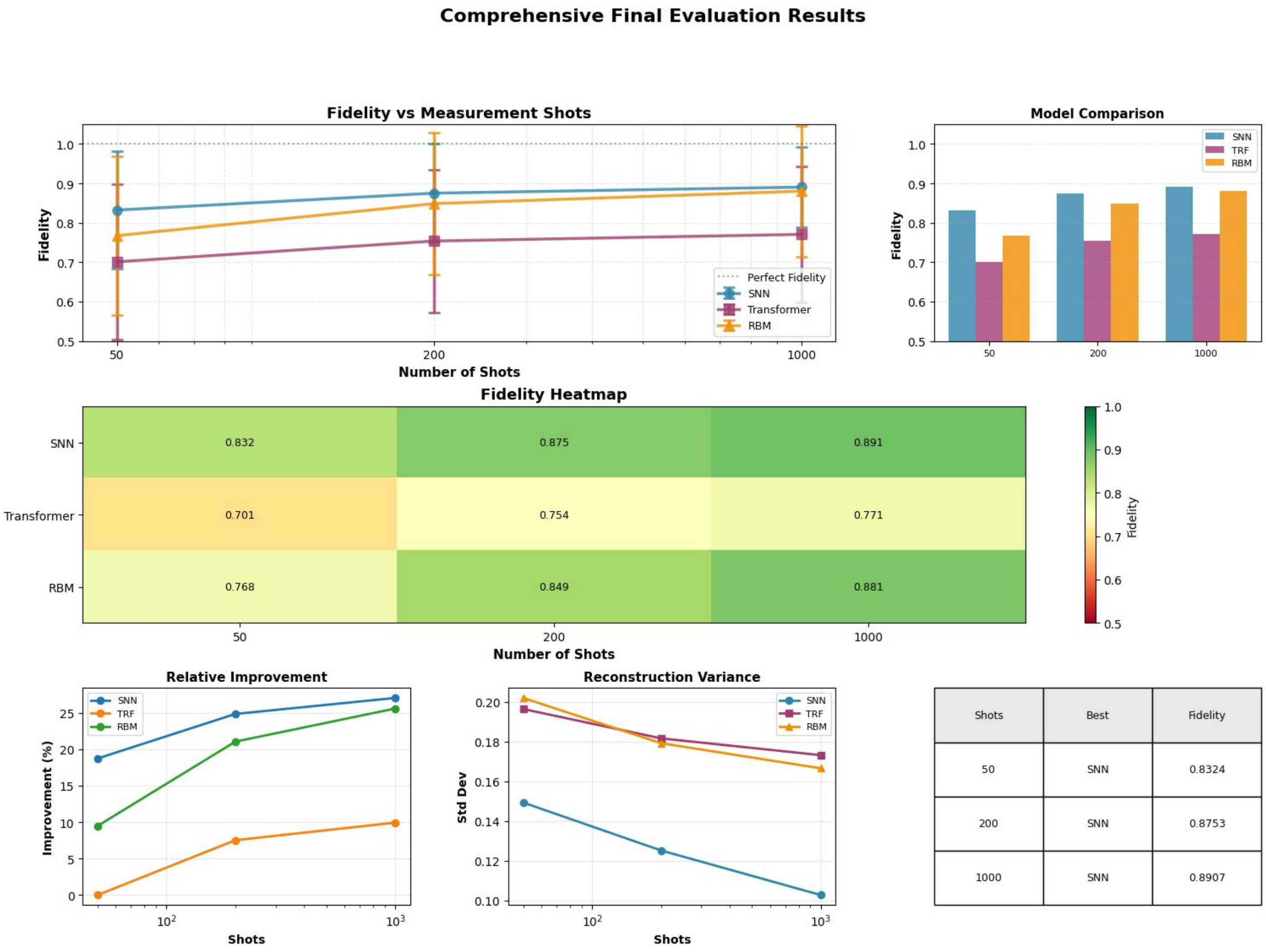



# Results

Comparison of Key Evaluation Metrics

Show that SNN is the best performing model for QST Task

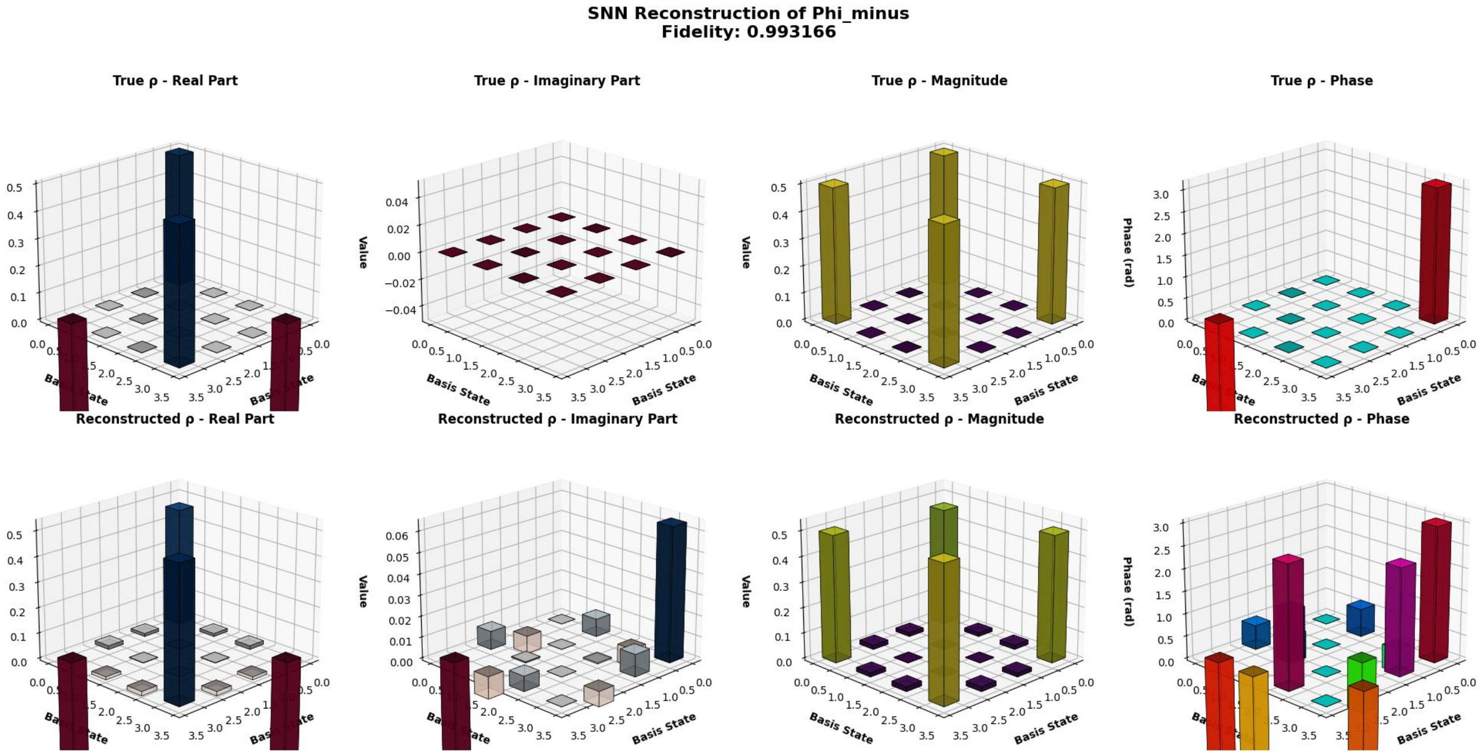
RBM model is surprisingly good given that its relatively small complexity




| Shots | Best | Fidelity |
|---|---|---|
| 50 | SNN | 0.8324 |
| 200 | SNN | 0.8753 |
| 1000 | SNN | 0.8907 |

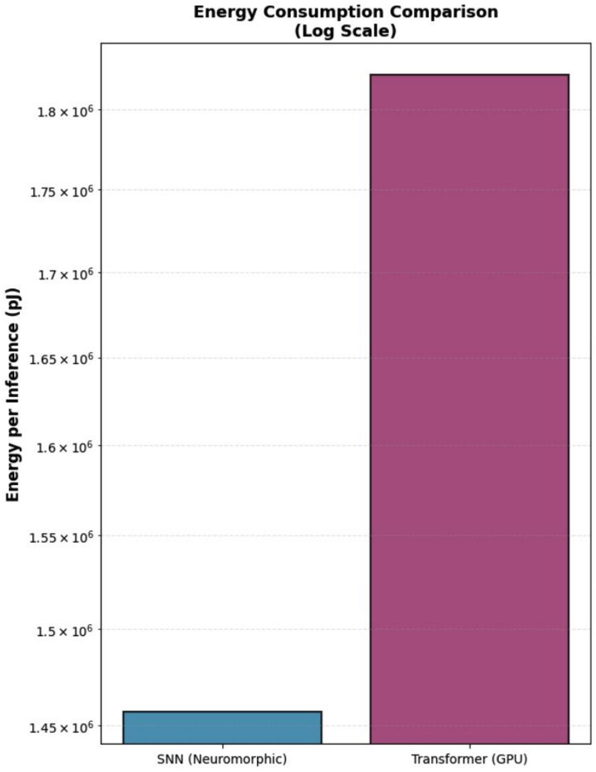
# Results - Reconstruction of Bell States

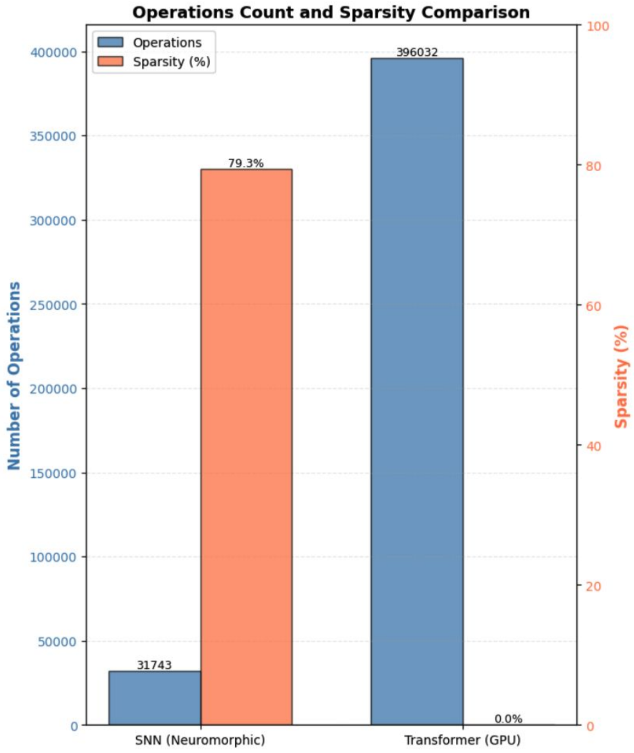
- The following graphs compare the real and reconstructed Density Matrices produced by the SNN Model.
- The model achieves very high fidelity, demonstrating its ability to learn entanglement. Note:



# Results - Computational Efficiency

The graphs illustrate that Spiking Neural Networks (SNNs) offer an energy advantage due to their sparse firing probability, as energy expenditure is avoided on non-firing neurons. In contrast, the Transformer architecture incurs a very high cost.



# Conclusion

Spiking Neural Networks (SNNs) for Quantum State Tomography (QST) deliver state-of-the-art results, offering higher fidelity, lower energy consumption, and natural alignment with quantum measurement events.

Key Advantages:

- Highest Fidelity across shot budgets.
- Energy Efficiency: 1.25× more efficient than the Transformer model.
- Robustness, especially in low-shot regimes (≤200).
- Natural Alignment: Discrete spikes map directly to quantum measurements.
- Scalability via SIC-POVM and Cholesky output methods.

| Achievement | Result |
|---|---|
| **Highest Fidelity** | **0.914** (200 shots) |
| **Energy Efficiency** | **1.46M pJ** → **1.25× better** than Transformer |
| **Sparsity** | **79.3%** → event-driven compute |
| **Robustness** | Outperforms MLP, Transformer, RBM in **low-shot regime** |
| **Physical Fit** | Spikes ↔ Measurement outcomes |

# Future Work

| # \| Direction | Code Link / Feasibility |
|---|---|
| 1 \| **Hardware Co-Design** | Deploy SNN on BrainScaleS chip with cryogenic interface |
| 2 \| **On-Chip Learning** | Real-time fine-tuning using `snntorch` + live quantum shots |
| 3 \| **Quantum Neuromorphic Accelerators** | Hybrid **Quantum-SNN GANs** for state generation |
| 4 \| **Multi-Qubit Scaling (5–10 qubits)** | Use **Tensor-Network-inspired SNNs** (e.g., MPS layers) |
| 5 \| **Quantum Error Mitigation** | SNN as **denoiser** for NISQ data (`rho_pred → rho_clean`) |
| 6 \| **Adaptive POVM Selection** | SNN learns **optimal measurement basis** online |
| 7 \| **End-to-End Spike-to-State** | Remove Poisson encoding → **direct spike input from detectors** |

# References


- Quantum State Reconstruction with Artificial and Spiking Neural Networks | Stefanie Czischek
- [2008.01039] Spiking neuromorphic chip learns entangled quantum states
- [2507.23007] Neural Network Architectures for Scalable Quantum State Tomography: Benchmarking and Memristor-Based Acceleration


**GitHub Link**

https://github.com/Heterotic-Research/Quromorphic.jl/blob/main/notebooks/QST_SNN/demo_py_visualized.ipynb

# Thank You