

$$M := \text{Matrix}([[\alpha + 2\beta \cos(\theta_-), \delta + \beta \cos(\theta_-)], [\delta + \beta \cos(\theta_-), \delta]]);$$

$$C := \text{Matrix}([[-\beta w_{-2} \sin(\theta_{-2}), -\beta \cdot (w_{-1} + w_{-2}) \sin(\theta_{-2})], [\beta w_{-1} \sin(\theta_{-2}), 0]);$$

$$B := \text{Matrix}([[b_{11}, b_{12}], [b_{21}, b_{22}]]);$$

$$\tau := \text{Matrix}([[\tau_1], [\tau_2]]);$$

$$dot\theta := Matrix([[w_1], [w_2]]);$$

$$RHS := \tau - MatrixMatrixMultiply((C + B), dot\theta);$$

$$\begin{bmatrix} \tau_1 - (-\beta_{w_2} \sin(\theta_2) + b_{11}) w_1 - (-\beta (w_1 + w_2) \sin(\theta_2) + b_{12}) w_2 \\ \tau_2 - (\beta_{w_1} \sin(\theta_2) + b_{21}) w_1 - b_{22} w_2 \end{bmatrix}$$

$$dotw := MatrixMatrixMultiply(N, RHS);$$

$$\begin{aligned} \text{dotw} := & \left[\left[\begin{aligned} & -\frac{1}{\cos(\theta_{-2})^2 \beta^2 - \alpha \delta + \delta^2} (\delta (\tau_{-1} - (-\beta w_{-2} \sin(\theta_{-2}) + b_{11}) w_{-1} - (-\beta (w_{-1} \right. \\ & \left. + w_{-2}) \sin(\theta_{-2}) + b_{12}) w_{-2})) \end{aligned} \right] \right] \end{aligned} \quad (8)$$

$$\begin{aligned}
& + \frac{(\delta + \beta \cos(\theta_{-2})) (\tau_{-2} - (\beta w_{-1} \sin(\theta_{-2}) + b_{-21}) w_{-1} - b_{-22} w_{-2})}{\cos(\theta_{-2})^2 \beta^2 - \alpha \delta + \delta^2} \Bigg], \\
& \left[\frac{1}{\cos(\theta_{-2})^2 \beta^2 - \alpha \delta + \delta^2} ((\delta + \beta \cos(\theta_{-2})) (\tau_{-1} - (-\beta w_{-2} \sin(\theta_{-2}) + b_{-11}) w_{-1} \right. \\
& - (-\beta (w_{-1} + w_{-2}) \sin(\theta_{-2}) + b_{-12}) w_{-2})) \\
& \left. - \frac{(\alpha + 2 \beta \cos(\theta_{-2})) (\tau_{-2} - (\beta w_{-1} \sin(\theta_{-2}) + b_{-21}) w_{-1} - b_{-22} w_{-2})}{\cos(\theta_{-2})^2 \beta^2 - \alpha \delta + \delta^2} \right] \Bigg]
\end{aligned}$$

$$dotw1 := dotw([1]);$$

$$dotw1 := \left[\right. \quad (9)$$

$$\begin{aligned}
& - \frac{1}{\cos(\theta_{-2})^2 \beta^2 - \alpha \delta + \delta^2} (\delta (\tau_{-1} - (-\beta w_{-2} \sin(\theta_{-2}) + b_{-11}) w_{-1} - (-\beta (w_{-1} \\
& + w_{-2}) \sin(\theta_{-2}) + b_{-12}) w_{-2})) \\
& + \frac{(\delta + \beta \cos(\theta_{-2})) (\tau_{-2} - (\beta w_{-1} \sin(\theta_{-2}) + b_{-21}) w_{-1} - b_{-22} w_{-2})}{\cos(\theta_{-2})^2 \beta^2 - \alpha \delta + \delta^2} \Bigg]
\end{aligned}$$

$$dotw2 := dotw([2]);$$

$$\begin{aligned}
dotw2 := \left[\frac{1}{\cos(\theta_{-2})^2 \beta^2 - \alpha \delta + \delta^2} ((\delta + \beta \cos(\theta_{-2})) (\tau_{-1} - (-\beta w_{-2} \sin(\theta_{-2}) \right. & (10) \\
& + b_{-11}) w_{-1} - (-\beta (w_{-1} + w_{-2}) \sin(\theta_{-2}) + b_{-12}) w_{-2})) \\
& \left. - \frac{(\alpha + 2 \beta \cos(\theta_{-2})) (\tau_{-2} - (\beta w_{-1} \sin(\theta_{-2}) + b_{-21}) w_{-1} - b_{-22} w_{-2})}{\cos(\theta_{-2})^2 \beta^2 - \alpha \delta + \delta^2} \right]
\end{aligned}$$

$$L1 := diff(dotw1, \theta_{-1});$$

$$L1 := \left[\begin{array}{c} 0 \end{array} \right] \quad (11)$$

$$L2 := diff(dotw1, \theta_{-2});$$

$$\begin{aligned}
L2 := \left[- \frac{1}{(\cos(\theta_{-2})^2 \beta^2 - \alpha \delta + \delta^2)^2} (2 \delta (\tau_{-1} - (-\beta w_{-2} \sin(\theta_{-2}) + b_{-11}) w_{-1} - (\right. & (12) \\
& - \beta (w_{-1} + w_{-2}) \sin(\theta_{-2}) + b_{-12}) w_{-2}) \cos(\theta_{-2}) \beta^2 \sin(\theta_{-2})) \\
& \left. - \frac{\delta (\beta w_{-2} \cos(\theta_{-2}) w_{-1} + \beta (w_{-1} + w_{-2}) \cos(\theta_{-2}) w_{-2})}{\cos(\theta_{-2})^2 \beta^2 - \alpha \delta + \delta^2} \right]
\end{aligned}$$

$$\begin{aligned}
& - \frac{\beta \sin(\theta_2) (\tau_2 - (\beta w_1 \sin(\theta_2) + b_{21}) w_1 - b_{22} w_2)}{\cos(\theta_2)^2 \beta^2 - \alpha \delta + \delta^2} \\
& + \frac{1}{(\cos(\theta_2)^2 \beta^2 - \alpha \delta + \delta^2)^2} (2 (\delta + \beta \cos(\theta_2)) (\tau_2 - (\beta w_1 \sin(\theta_2) \\
& + b_{21}) w_1 - b_{22} w_2) \cos(\theta_2) \beta^2 \sin(\theta_2)) \\
& - \frac{(\delta + \beta \cos(\theta_2)) \beta w_1^2 \cos(\theta_2)}{\cos(\theta_2)^2 \beta^2 - \alpha \delta + \delta^2} \Big]
\end{aligned}$$

$$L3 := \text{diff}(\text{dotw1}, w_1);$$

$$L3 := \left[- \frac{\delta (2 \beta w_2 \sin(\theta_2) - b_{11})}{\cos(\theta_2)^2 \beta^2 - \alpha \delta + \delta^2} + \frac{(\delta + \beta \cos(\theta_2)) (-2 \beta w_1 \sin(\theta_2) - b_{21})}{\cos(\theta_2)^2 \beta^2 - \alpha \delta + \delta^2} \right] \quad (13)$$

$$L4 := \text{diff}(\text{dotw1}, w_2);$$

$$L4 := \left[- \frac{\delta (\beta w_1 \sin(\theta_2) + \beta w_2 \sin(\theta_2) + \beta (w_1 + w_2) \sin(\theta_2) - b_{12})}{\cos(\theta_2)^2 \beta^2 - \alpha \delta + \delta^2} - \frac{(\delta + \beta \cos(\theta_2)) b_{22}}{\cos(\theta_2)^2 \beta^2 - \alpha \delta + \delta^2} \right] \quad (14)$$

$$M1 := \text{diff}(\text{dotw2}, \theta_1);$$

$$M1 := [0] \quad (15)$$

$$M2 := \text{diff}(\text{dotw2}, \theta_2);$$

$$\begin{aligned}
M2 := & \left[- \frac{1}{\cos(\theta_2)^2 \beta^2 - \alpha \delta + \delta^2} (\beta \sin(\theta_2) (\tau_1 - (-\beta w_2 \sin(\theta_2) + b_{11}) w_1 - (\right. \\
& - \beta (w_1 + w_2) \sin(\theta_2) + b_{12}) w_2)) + \frac{1}{(\cos(\theta_2)^2 \beta^2 - \alpha \delta + \delta^2)^2} (2 (\delta \\
& + \beta \cos(\theta_2)) (\tau_1 - (-\beta w_2 \sin(\theta_2) + b_{11}) w_1 - (-\beta (w_1 + w_2) \sin(\theta_2) \\
& + b_{12}) w_2) \cos(\theta_2) \beta^2 \sin(\theta_2)) \\
& + \frac{(\delta + \beta \cos(\theta_2)) (\beta w_2 \cos(\theta_2) w_1 + \beta (w_1 + w_2) \cos(\theta_2) w_2)}{\cos(\theta_2)^2 \beta^2 - \alpha \delta + \delta^2}
\end{aligned} \quad (16)$$

$$\begin{aligned}
& + \frac{2 \beta \sin(\theta_2) (\tau_2 - (\beta w_1 \sin(\theta_2) + b_{21}) w_1 - b_{22} w_2)}{\cos(\theta_2)^2 \beta^2 - \alpha \delta + \delta^2} \\
& - \frac{1}{(\cos(\theta_2)^2 \beta^2 - \alpha \delta + \delta^2)^2} (2 (\alpha + 2 \beta \cos(\theta_2)) (\tau_2 - (\beta w_1 \sin(\theta_2) \\
& + b_{21}) w_1 - b_{22} w_2) \cos(\theta_2) \beta^2 \sin(\theta_2)) \\
& + \frac{(\alpha + 2 \beta \cos(\theta_2)) \beta w_1^2 \cos(\theta_2)}{\cos(\theta_2)^2 \beta^2 - \alpha \delta + \delta^2} \Big]
\end{aligned}$$

$$M3 := \text{diff}(\text{dotw2}, w_1);$$

$$\begin{aligned}
M3 := & \left[\frac{(\delta + \beta \cos(\theta_2)) (2 \beta w_2 \sin(\theta_2) - b_{11})}{\cos(\theta_2)^2 \beta^2 - \alpha \delta + \delta^2} \right. \\
& \left. - \frac{(\alpha + 2 \beta \cos(\theta_2)) (-2 \beta w_1 \sin(\theta_2) - b_{21})}{\cos(\theta_2)^2 \beta^2 - \alpha \delta + \delta^2} \right]
\end{aligned} \tag{17}$$

$$M4 := \text{diff}(\text{dotw2}, w_2);$$

$$M4 := \tag{18}$$

$$\begin{aligned}
& \left[\frac{1}{\cos(\theta_2)^2 \beta^2 - \alpha \delta + \delta^2} ((\delta + \beta \cos(\theta_2)) (\beta w_1 \sin(\theta_2) + \beta w_2 \sin(\theta_2) \right. \\
& \left. + \beta (w_1 + w_2) \sin(\theta_2) - b_{12})) + \frac{(\alpha + 2 \beta \cos(\theta_2)) b_{22}}{\cos(\theta_2)^2 \beta^2 - \alpha \delta + \delta^2} \right]
\end{aligned}$$

$$\det N := \cos(\theta_2)^2 \beta^2 - \alpha \delta + \delta^2;$$

$$\det N := \cos(\theta_2)^2 \beta^2 - \alpha \delta + \delta^2 \tag{19}$$

$$ddet N := \text{diff}(\det N, \theta_2);$$

$$ddet N := -2 \cos(\theta_2) \beta^2 \sin(\theta_2) \tag{20}$$