Stock Market Portfolio Management: A Walk-through

Bharat V. Chawda
PhD Research Scholar,
Gujarat Technological University,
Ahmedabad, Gujarat, India
bharat.bbit@gmail.com

Dr. Jayeshkumar Madhubhai Patel
Associate Professor, MCA Programme,
Ganpat University,
Kherva, Gujarat, India
jayeshpatel_mca@yahoo.com

Abstract — Stock market portfolio management has remained successful in drawing attention of number of researchers from the fields of computer science, finance and mathematics all around the world since years. Successfully managing stock market portfolio is the prime concern for investors and fund managers in the financial markets. This paper is aimed to provide a walk-through to the stock market portfolio management. This paper deals with questions like what is stock market portfolio, how to manage it, what are the objectives behind managing it, what are the challenges in managing it. As each coin has two sides, each portfolio has two elements – risk and return. Regarding this, Markowitz's Modern Portfolio Theory, or Risk-Return Model, to manage portfolio is analyzed in detail along with its criticisms, efficient frontier, and suggested state-of-the-art enhancements in terms of various constraints and risk measures. This paper also discusses other models to manage stock market portfolio such as Capital Asset Pricing Model (CAPM) and Arbitrage Pricing Theory (APT) Model.

Keywords *– Stock market portfolio management; Efficient Frontier; Modern Portfolio Theory; Risk Measures*

*****

I. INTRODUCTION

According to theory of "Time Value of Money" [1], money must be invested to earn more money. Money lying idle loses its value gradually due to factors like inflation. A rational investment must beat the inflation to keep afloat the value of money. For investment, money is allocated among several assets. *Portfolio* refers to the allocation of wealth (or, money in hand) among several assets [2]. Assets can be anything like as gold, real estate, metals, commodities, stocks, bonds, fixed deposits. These assets can be classified either as risky assets or risk-free assets [3]. A *risky asset* is one for which return to be realized in future in uncertain. For example, stocks are risky assets. In contrast, a *risk-free* (or, risk-less) asset is one for which return to be realized in future is known with certainty today. For example, a fixed deposit in a bank is risk-free asset.

Stock market portfolio refers to the allocation of wealth among several stocks. The primary goals of stock market portfolio management (SMPM) are – maximize the return / reward / profit and minimize the risk. Stock market portfolio management consists of two major tasks: first, *Portfolio Construction*, also known as Portfolio Selection – where some particular stocks are selected to invest among a large pool of available stocks along with their proportions, and second, *Portfolio Optimization* – where portfolio is rebalanced continuously to reflect changes that occur in varying financial markets.

The remainder of this paper is organized as follows. Section II discusses scope of the research regarding stock market portfolio management, main objectives, and challenges. Section III discusses two different approaches to manage portfolio – active management and passive management. Section IV analyses elements of risk and provides risk mitigation techniques for a stock market portfolio. Section V discusses Markowitz's risk-return model along with its criticisms. Section VI extends discussion of Markowitz's model and provides enhancements in terms of constraints and risk measures. Section VII discusses other models to manage portfolio including Capital Asset Pricing Model (CAPM) and Arbitrage Pricing Theory (APT) model.

II. STOCK MARKET PORTFOLIO MANAGEMENT

Many researchers have spent hours and hours to find different ways to manage stock market portfolio. Their endeavor is justified in beginning of this section as a scope of the research. This follows discussion of the primary objectives associated with stock market portfolio. After this, challenges related with stock market portfolio management are discussed.

A. Scope of Research

Table-I depicts top 5 gainer and top 5 looser stocks for the financial year 2012-13, 2013-14 and 2014-15 for Indian Stock Market. It also shows the performance of the overall market in terms of NIFTY which is an index comprising of the well diversified 50 stocks trading on the National Stock Exchange (NSE). For this table, required data has been derived from *http://www.nseindia.com/products/content/equities/indices/historical.*

It can be seen that during all these years, some stocks have given very good performance while some have performed very poorly. There are number of factors affecting performance of stocks. It has also been observed that no any stock performs consistently good forever and no any stock performs consistently bad forever. For example, HINDALCO performed poorly during FY 2012-13 giving -28.04% returns. But it gave very good performance during FY 2013-14 giving 54.58% returns.

From this table, it can be observed that if stocks and their proportions are selected carefully, they can yield very good returns. Also performance of stocks must be tracked and whenever required portfolio should be balanced for optimization. This means management of stock market portfolio possesses high scope of the research. And, this is the reason – why number of researchers has been attracted towards management of stock market portfolio.

TABLE I. RETURNS OF TOP 5 GAINER AND TOP 5 LOOSER STOCKS ALONG WITH NIFTY DURING FY 2012-13, 2013-14, 2014-15

FY 2012-13		FY 2013-14		FY 2014-15		
Company	*Return (%)*	*Company*	*Return (%)*	*Company*	*Return (%)*	
HCLTECH	**58.60**	**HCLTECH**	**76.37**	**LUPIN**	**112.51**	**GAINERS**
ASIANPAINT	**50.68**	**TECHMAHINDRA**	**69.20**	**MARUTI**	**91.49**	
TECHMAHINDRA	**47.86**	**HINDALCO**	**54.58**	**CIPLA**	**86.75**	
SUNPHARMA	**42.82**	**MARUTI**	**53.59**	**BPCL**	**81.43**	
ITC	**36.35**	**TATAMOTORS**	**50.15**	**SUNPHARMA**	**78.97**	
NIFTY	**6.86**	**NIFTY**	**17.53**	**NIFTY**	**26.34**	**Market**
HINDALCO	**-28.04**	**TATAPOWER**	**-11.16**	**DLF**	**-9.57**	**LOSERS**
BHEL	**-32.13**	**IDFC**	**-14.09**	**RELIANCE**	**-12.37**	
TATASTEEL	**-33.68**	**JINDALSTEEL**	**-14.60**	**TATASTEEL**	**-21.03**	
JINDALSTEEL	**-35.90**	**NTPC**	**-16.45**	**CAIRN**	**-38.00**	
BPCL	**-45.04**	**DLF**	**-30.31**	**JINDALSTEEL**	**-47.02**	

B. *Objectives*

The primary objectives of the stock market portfolio management can be stated as below:

- Provided a collection of *N* stocks, select *K* stocks to be included in a portfolio and select proportions/weights of these *K* selected stocks,
- With the aim of either maximize the return for a given level of risk or minimize the risk for a given level of return, and
- Continuously assess the performance of portfolio to rebalance it whenever required.

C. *Challenges*

Stock market portfolio aims to maximize returns and minimize risks. But stock performance depends upon number of factors such as company fundamentals, future scope of business, government policies, inflation, monetary policies of Reserve Banks, international crude prices etc. So it becomes very difficult to select stocks which can give better performance consistently.

Even if better performance is ignored, to select *K* stocks out of pool of *N* available stocks, there are *N!/(K!(N-K)!)* different ways. So selecting 10 stocks out of 100 available stocks gives around 10^{13} different permutations. Complexity does not stop here. The reason is: if selection of stocks falls in a *discrete* space, selection of their weights in a portfolio falls in a *continuous* space. It can be anything from as low as 0.1 to as high as 1. This brings the problem of stock market portfolio under category of NP-Hard problems in terms of computer algorithms. With either increase in *N* or in *K*, it becomes almost impossible to evaluate each and every combination of stocks and their proportions in a reasonable time period.

Managing stock market portfolio requires thorough knowledge of the finance as well as computer science along with mathematics. This makes the problem of stock market portfolio management inter disciplinary problem.

Also, stock market portfolio management aims to achieve two objectives – maximize return and minimize risk. This makes it a multi objective combinatorial optimization problem.

Over all, managing stock market portfolio throws a very stiff challenge to researchers as well as investors and fund managers.

III. PORTFOLIO MANAGEMENT APPROACHES

Portfolio management approaches (or strategies) can be divided into three categories: A) active management, B) passive management, and C) mix strategy of active and passive management [4][5]. Active management and passive management are described in this section.

A. *Active Management*

Active management [6] believes that financial markets are not fully efficient and skillful investments can outperform the aggregate market. Active management exploits market inefficiencies by buying stocks which are undervalued or short selling stocks which are overvalued. Market timing – when to buy, when to sell – is extremely critical for the better performance of the active management. If this strategy involves frequent trading, it may generate higher transaction costs resulting in diminishing returns. The fees associated with active management will also be higher compared to the passive management.

Decisions – whether to buy or sell – are taken based on fundamental analysis and/or technical analysis [7].

Fundamental analysis searches for the real value of a share using factors such as retained earnings, earnings yield, price-earning (PE) ratio, company fundamentals, government policies etc. Contrast to this, Technical analysis [8], [9] searches for the perceptions of the real value of a share using factors such as historical price & volume data movement. For this purpose, charts of price-volume movement are used along with various technical indicators such as Moving Average (MA), Moving Average Convergence and Divergence (MACD), Relative Strength Index (RSI), On Balance Volume (OBV) etc.

This approach is also known as traditional approach. In India, most of the stock brokers follow this approach for selecting a portfolio for their clients.

B. *Passive Management*

Passive management [10] believes that financial markets are efficient and it is impossible to beat the market or timing the market consistently. Passive management strives to mimic the performance – in terms of risk and return – of a certain benchmark like a stock market index. By this, portfolio gets good diversification, extremely low management fees, low turnover and so low transaction costs. Passive management is often referred as index tracking. A passively managed fund is called an indexed exchange traded fund (ETF). Retail investors, interested in passive management, can buy one or more such ETFs.

There are two different ways to match the performance of an index: full replication and partial replication [4].

In full (or complete) replication, all the stocks comprising an index are purchased in the same proportion as in the index. By this, a perfect replica of the index is produced. But this increases the size of the portfolio, i.e. total number of stocks in a portfolio. This may also increase transaction cost comparatively. Contrast to this, in partial replication, only a subset of stocks comprising an index is purchased. Stocks can be selected such that each sector can have representation in portfolio, or, those that have the best chance of good performance. This method reduces transaction costs but introduces a tracking error – the measure of the deviation of the chosen portfolio from the index.

This approach is also known as modern approach. A model, developed by Harry Markowitz, also known as modern portfolio theory, or mean variance model, can be used for a passive management [11]. This model helps to optimize risk and return of a portfolio. This model is described in greater detail later in this paper.

IV. RISK ANALYSIS

As described earlier, stocks are risky assets to invest in. Performance of a stock depends upon a number of factors and returns that can be incurred in future by investing in stocks are uncertain. This uncertainty introduces risk for an investor. Stock price does not remain constant and varies with time. And so varies the return. This variation in returns is the base for the risk in an investment in stock market. Main factors which cause variations in stock prices are discussed in this section as elements of risk followed by risk mitigation techniques.

A. *Elements of Risk*

The elements of risk – factors causing variations in stock price – can be broadly classified into two categories, Systematic risk and Unsystematic risk, as shown in following Fig. 1.

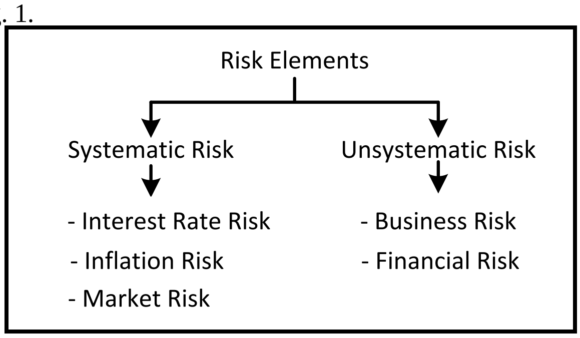

Figure 1. Risk Elements

Systematic risk [12] incorporates factors that are external to a company, macro in nature, and affects the overall market including large number of stocks simultaneously. This risk is also referred as aggregate risk or undiversified risk. It can be further subdivided into interest rate risk, inflation risk and market risk. Interest rate risk arises due to variability in the interest rates from time to time and affects the borrowing power of the company. Inflation affects the purchasing power. It erodes the realized returns. Inflation rates also vary over a time and affect the profits earned by a company. Market risk emanates from situations such as recession, natural calamity during which almost all stocks are affected negatively.

Systematic risk of some particular stock or portfolio can be known from its beta – a measure representing how volatile a stock or a portfolio is compared to the overall market. If beta is greater than one, systematic risk is high. If beta is less than one, systematic risk is low. And if beta equals to one, systematic risk is same as that of market.

Unsystematic risk [13] incorporates factors that are internal to a company, micro in nature, and affects that particular company or a sector of similar companies. This risk is also referred as specific risk, diversified risk, or residual risk. It can be further subdivided into business risk and financial risk. Business risk arises due to factors such as business cycles, technological changes etc. For example, advent of cell phones with in-built cameras has affected the industry of digital cameras very badly. Financial risk arises due to changes in the capital structure of the company. It is also referred as leveraged risk. It can be expressed in terms of debt-equity ratio. Large amount of the profit of company with high debt will be neutralized in payment of interest of debt. This will diminish the actual profit of company and so actual returns from the stock.

B. *Risk Mitigation*

Total risk associated with any stock consists of two risks: systematic risk and unsystematic risk. In other words, total risk = systematic risk + unsystematic risk.

Unsystematic risk of any individual stock can be avoided or reduced by combining more than one stocks in a portfolio. This strategy is known as risk diversification. The following figure 2 depicts how unsystematic risk and so total risk can be reduced by increasing the number of stocks in a portfolio.

In contrast to unsystematic risk, systematic risk cannot be diversified. But it can be mitigated by hedging. Simplest way to hedge is to buy derivative products such as puts. Systematic risk can also be reduced by allocating wealth among other assets along with stocks. If so, it is again diversification of different kind – in a broader context.

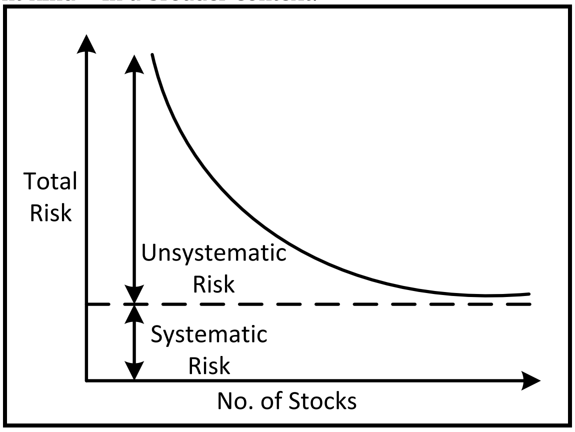

Figure 2. Risk Diversification

V. Markowitz's Model

A new era began in the field of stock market portfolio management with Harry Markowitz's 1952 revolutionary article on portfolio selection [11]. Markowitz simplified the task of portfolio management and provided a model to construct portfolio which maximizes return and minimizes risk. Markowitz, father of the modern portfolio theory, awarded with a Nobel Memorial Prize (jointly) in Economic Science in 1990.

A. Standard Model

Markowitz's standard model, also known as Mean Variance Model or Risk Return Model, is concerned with two properties of an asset: Risk and Return. The essence of this model is that, instead of risk of an individual asset, its contribution to the overall risk of a portfolio is more important. And so, Markowitz's model is also a form of diversification.

In this model, a portfolio is defined by a vector of real numbers – containing the weight corresponding to each available asset. Model then attempts to minimize the risk for a desired level of return and vice versa. *Expected return of an asset* is defined as a mean of the past returns of the asset. *Risk of an asset* is defined as a variance of the returns. *Expected return of the portfolio* is defined as a weighted sum of assets' expected return. *Risk of the portfolio* is defined as sum of the variances of the assets and covariances among the assets.

Mathematically this model can be represented as given below:

For N assets problem,

$$Minimize\ Risk, \quad Maximize\ Return \tag{1}$$

subject to

$$Return = \sum_{i=1}^{N} w_i\, r_i \tag{2}$$

$$Risk = \sum_{i=1}^{N}\sum_{j=1}^{N} \sigma_{ij}\, w_i\, w_j \tag{3}$$

$$Provided, \quad \sum_{i=1}^{N} w_i = 1, \quad and \tag{4}$$

$$0 \le w_i \le 1, \quad i = 1,2,\dots,N \tag{5}$$

where w_i is the weight assigned to asset i, r_i is the associated expected return, σ_{ij} is the covariance between asset *i* and asset *j*. Equation (4) specifies budget constraint that the sum of the asset weights should be equal to 1 (i.e. 100%). Equation (5) specifies that weights should be positive (i.e. no short selling is allowed).

B. Efficient Frontier

With high expected return comes relatively high risk and vice versa. Markowitz has established this risk-return relationship for any feasible portfolio. A *feasible portfolio* is any portfolio that can be constructed from the available set of assets. The collection of all feasible portfolios is referred as a *feasible set of portfolios*. Any feasible portfolio, now onwards simply referred as portfolio, has associated with it is some risk and return.

An *efficient portfolio* is a portfolio with highest expected return among all feasible portfolios with the same risk. It is also referred as a mean-variance efficient portfolio or non-dominated portfolio for a given level of risk. For different levels of risk, there can be different efficient portfolios. The collection of all efficient portfolios is referred as the *efficient set*. It is also referred as the *efficient frontier*, because in graphical representation all the efficient portfolios lie on the boundary of the set of feasible portfolios having maximum return for a given level of risk. An *optimal portfolio* is an efficient portfolio that suits the risk-return profile of an investor. For different investors, optimal portfolios can be different.

The following figure 3 represents feasible set of portfolios along with efficient frontier. Portfolios with blue color represent efficient (non-dominated) portfolios while remaining ones represent dominated portfolios.

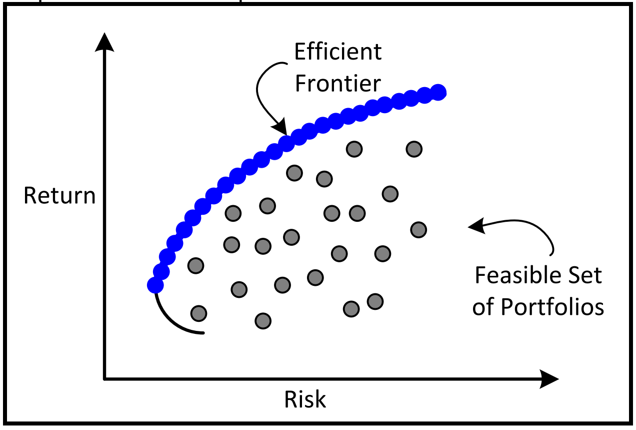

Figure 3. Efficient Frontier

C. Multi objective v/s Single objective optimization

Markowitz's standard mean variance model is a multi objective optimization model. It serves two objectives: maximizes returns as well as minimizes risks. If all possible portfolios are constructed with these two objectives keeping in mind, result will be entire efficient set of portfolios. But different investors have different risk-return profiles and an individual investor will be interested in only single optimal portfolio. So it will be costly in terms of time and effort to first construct entire efficient set and then select optimal portfolio.

To construct optimal portfolio directly, above objectives are transformed into two different single objectives as follows:

1) Maximize returns for a given level of risk, or,
2) Minimize risk for a given level of return.

First objective is applicable for a pre-selected risk level, while second objective is applicable for a pre-selected return level. Both objectives simplify the process of constructing an *optimal portfolio*.

Another way of transforming multi objective optimization into single objective is to introduce risk-return trade-off coefficient, or a risk aversion parameter, $\lambda \in [0,\ 1]$. This strategy has been used in [14]–[17]. With this new parameter, objective of the model can be expressed as:

$$Maximize \quad (1-\lambda)\sum_{i=1}^{n} w_i\, r_i - \lambda \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij}\, w_i\, w_j \tag{6}$$

If $\lambda = 0$, risk is completely avoided and return is maximized. Portfolio may end with up with single asset having maximum expected return. If $\lambda = 1$, return is completely ignored and risk is minimized. Portfolio may end up with several assets with low variances. By resolving the problem with different values of λ ranging from 0 to 1, an efficient frontier can be constructed similar to

Markowitz's standard model. An investor has to determine an optimal portfolio from this efficient frontier based on personal risk-return profile. Or, by specifying some particular value of λ based on risk-return profile of an investor, an optimal portfolio can be constructed directly.

D. Criticisms

Even though Markowitz's model has been proved as the pioneer work for the modern portfolio theory, some critical issues have been raised for directly using this standard model [18]–[21]. These criticisms are as follows:

Expected return of a stock is considered equal to mean of the past returns. But stock returns are never consistent and it is wrong to expect that if stock has performed either poor or good in past, it will continue to do so.

To calculate mean and variance as a measure of return and risk respectively for a stock, past returns are used. But length of the duration, i.e. past period, is not specified clearly. Mean and variance can be different for different past durations – such as 1 year, 6 month, 1 month etc. Also measured risk cannot be constant forever as it can change rapidly due to some catastrophic, unpredictable events.

A risk measure, variance, equally accounts for upward and downward deviation. But in reality, investors are only concerned about loses represented by downward deviation. So, upward movements are not harmful and should not be considered as a risk measure.

Model assumes that markets are efficient and all investors are rational and risk-averse which may not be true for all investors. It is also assumed that all information is available to all investors at the same time. In reality, market may contain insider trading, better informed investors, etc. Correlations between stocks are assumed to be fixed and constant forever which is also not possible for real world stocks.

Investors may not be clear with their desired returns as well as risk tolerances – two essential elements to construct portfolios with Markowitz's model.

Model does not allow short selling, i.e. selling stocks (or derivative futures) in advance – which is a necessity for hedging purposes. Also model does not put any caps on total number of stocks in a portfolio. This may result in a portfolio with large number of stocks many of them with minor proportions. Weights of stocks are represented as real values. But in reality buying or selling of fractional stocks is not possible.

Fundamental and/or technical analysis can play a crucial role in stock selection for buying/selling – which is not considered at all in this model.

Transaction costs and taxes are not considered – which may play crucial role in determining actual returns.

VI. MARKOWITZ'S MODEL – ENHANCEMENTS

Criticisms described in the previous section suggest that Markowitz's standard model cannot be applied to the real world stock market portfolio management directly. To overcome these criticisms researchers have proposed/implemented considerable enhancements to the Markowitz's standard model in terms of various constraints and different risk measures. These enhancements are discussed in this section.

A. Constraints

Markowitz's model imposes two constraints: Budget constraint and No short-selling. This model can be enhanced by including following constraints as per requirements. These enhancive constraints are summarized in [19], [22].

1) Cardinality constraint:

This constraint, introduced in [14], restricts the total number of assets to be included in a portfolio. A binary variable z_i is introduced in this constraint. If z_i = 1, asset *i* is present in portfolio. If z_i = 0, asset *i* is absent in portfolio. This constraint has two versions. The first version, referred as exact version (7), states that the total number of selected assets should be equal to *K*. The second version, referred as soft version (8), provides lower bound (K_L) and/or upper bounds (K_U) on this number.

$$\sum_{i=1}^{n} z_i = K \quad and \quad z_i = 0 \; or \; 1 \tag{7}$$

$$K_L \;\le\; \sum_{i=1}^{n} z_i \;\le\; K_U \quad and \quad z_i = 0 \; or \; 1 \tag{8}$$

This constraint facilitates portfolio management and helps to reduce its management costs.

2) Floor-ceiling constraints:

These constraints, introduced in [23], specify lower and/or upper bounds on the weight of each asset to be included in a portfolio. The floor constraint, i.e. lower bound (W_{min}), prevents very small allocations of capital to many assets in a portfolio. This helps to reduce administrative and transaction costs. The ceiling constraint, i.e. upper bound (W_{max}), prevents too large allocation of capital to single asset in a portfolio. This helps to minimize risk by sharing it among several assets and maintaining diversification. Mathematically these constraints can be represents as

$$W_{min} \;\le\; w_i \;\le\; W_{max}, \quad i = 1,2,\dots,N \tag{9}$$

3) Class/Sector weight constraint:

This constraint, adopted in [24], [25], specifies lower and/or upper bounds on the weight of class or sector of assets. For example, stocks from the Oil and Gas industry may represent one sector. This constraint is very similar to floor-ceiling constraints. The difference is, instead of restricting weights of individual assets between lower and upper bound, sum of weights of assets belonging to a same sector is restricted between some lower and upper bound. Let *M* be the set of classes or sectors $C_1, \dots, C_M$, $Wmin_m$ and $Wmax_m$ be lower and upper bounds (respectively) for class *m*, this constraint can be expressed as

$$Wmin_m \le \sum_{i \epsilon Cm} w_i \;\le\; Wmax_m, \quad m = 1, 2, \dots, M \tag{10}$$

4) Short sales constraint:

In the Markowitz's standard model, weights of assets are non-negative which means no short selling is allowed. But in real world, some markets allow investors to sell assets that are not yet owned by them in expectation of price falling. For short selling, asset weights should be allowed to have negative values. Such type of relaxation was introduced in [26]. This can be expressed as given below by replacing constraint (5).

$$w_i \,\epsilon\, R, \quad i = 1,2,\dots,N \tag{11}$$

5) Roundlot or Minimum lot constraint:

In some markets, such as Japanese one, buying and selling of assets must be done in a multiple of the minimum transaction lots. Different assets can have different minimum tradable lots. Such lots are also referred as round. In such cases, weight w_i of an asset i in a portfolio can be expressed as

$$w_i = \frac{l_i\, c_i}{\sum_{i=1}^{N} l_i\, c_i}, \quad i = 1, 2, \dots, N \tag{12}$$

where l_i is a lot size for the asset i and c_i is the buying price for asset i. This approach was first used in [27].

6) *Turnover constraint:*

This constraint controls the amount of money that can be traded – for buy as well as sell. This constraint is useful to control the transaction costs.

According to [24], if there is a change in weight w_i of asset i, the difference between current weight w_i and previous weight w_i' must be greater or equal to some threshold Δ. This constraint can be expressed as

$$|\, w_i - w_i' \,| \geq \Delta_i \quad or \quad |\, w_i - w_i' \,| = 0, \quad i = 1, 2, \dots, N \tag{13}$$

Also, the sum of the absolute change between current and previous allocation must be less than given maximum turnover ratio *TR*. This can be expressed as

$$\sum_{i=1}^{N} |\, w_i - w_i' \,| \leq TR \tag{14}$$

In the similar way, different constraints for buying and selling are proposed in [28] as given below.

$$\max(w_i - w_i', 0) \leq B_i, \quad i = 1, \dots, N \tag{15}$$

$$\max(w'_{i\,i} - w_i, 0) \leq S_i, \quad i = 1, \dots, N \tag{16}$$

where B_i and S_i represent maximum buying and selling threshold respectively.

B. Risk Measures

In Markowitz's standard mean-variance model, a variance is considered as a risk measure which equally accounts under-performance as well as over-performance. But matter of worry for investors is only under-performance. And so, variance as a risk measure remained at the center among criticisms of this model. Other risk measures, except from variance, are discussed here.

1) *Semi-Variance / Downside Risk:*

Markowitz recognized limitations of the variance as a risk measure and he himself proposed another risk measure in [18] to account only downside movements of the return distribution. This measure is known as semi-variance or down side risk. *Semi-variance* is an average of the squared deviations of returns that are less than the mean return [29]. The mathematical formula for semi-variance can be expressed as

$$Semi\text{-}variance = \frac{1}{n} \sum_{r_t < \mu}^{n} (r_t - \mu)^2 \tag{17}$$

where r_t is a return at time period t, μ is the mean return, and n is total number of returns below mean return.

2) *Value-at-Risk (VaR):*

P. Jorion defined *Value-at-Risk (VaR)* as a measure of the worst expected loss over a given time period under normal market conditions at a given level of confidence in [30]. VaR is measured in three variables: a potential loss, probability (or confidence level) of that loss, and the time horizon. As explained in [31], consider a portfolio with 1-day VaR of $1 million at the 99% confidence level. This means that there is a 1% (derived as 100-99) chance that the value of the portfolio will lose by $1 million or more during one day.

3) *Conditional Value-at-Risk (CVaR):*

Conditional Value-at-Risk (CVaR), introduced in [32] and optimized in [33], [34] can be considered to be an extension of VaR. It is also called Mean Excess Loss, Mean Shortfall or Tail VaR. Whereas VaR finds minimum level of loss to be expected, CVaR finds the expected (or average) loss, given that a loss has occurred. So, the value of CVaR will be at least as high as VaR and mostly higher. By this, it helps investors to know the extent of risk. Mathematically, CVaR is derived by taking a weighted average between the VaR and losses exceeding VaR.

The detailed comparative analysis between VaR versus CVaR is given in [35].

4) *Sharpe Ratio (SR):*

Sharpe ratio, introduced in [36] and revised in [37], helps to calculate risk-adjusted return. It is a measure of the risk premium, or excess return, per unit of total risk of a portfolio. Higher the Sharpe ratio, better will be the return with the same unit of risk. The mathematical formula for Sharpe ratio can be expressed as

$$Sharpe\ Ratio = \frac{(R_p - R_f)}{\sigma_p} \tag{18}$$

where p refers to portfolio, R_p is a mean return of the portfolio p, R_f is a return of benchmark index or risk-free return, σ_p is a standard deviation of a portfolio p. Here (R_p - R_f) represents risk premium or excess return.

5) *Mean-Absolute Deviation:*

Mean-Absolute Deviation (MAD) is the average of absolute deviations from a mean value. Let $\{x_1, x_2, \dots x_n\}$ be the data set and μ is the mean value of this data set. The mathematical expression for MAD can be expressed as

$$Mean\text{-}Absolute\ Deviation = \frac{1}{n} \sum_{i=1}^{n} |\, x_i - \mu \,| \tag{19}$$

A portfolio optimization model given in [38] uses MAD as a risk measure. In this model, risk of a portfolio is defined as

$$\frac{1}{T} \sum_{t=1}^{T} \left| \sum_{i=1}^{N} w_i\, (r_{it} - r_i) \right| \tag{20}$$

where N is a total number of assets, T is a time horizon, w_i is a weight of asset i, r_{it} is a return of asset i at time t, and r_i is a mean return of asset i. As the covariance matrix is not required to measure risk in this model, it is easier to handle compared to Markowitz's model. The main short coming of MAD is that it accounts upward and downward deviations equally as in variance.

6) *Mean Semi-Absolute Deviation(MSAD):*

Mean Semi-Absolute Deviation (MSAD) combines the concept of semi-variance with MAD to overcome the drawback of later. A portfolio selection model based on MSAD as a risk measure is given in [39]. In this model, risk of portfolio is defined as

ISSN: 2321-8169
4136 - 4143

$$\frac{1}{T}\sum_{t=1}^{T}\left|\min\left(0,\sum_{i=1}^{N}w_i\,(r_{it}-r_i)\right)\right| \tag{21}$$

where N is a total number of assets, T is a time horizon, w_i is a weight of asset i, r_{it} is a return of asset i at time t, and r_i is a mean return of asset i.

VII. Other Models

The solution for the problem of stock market portfolio management lies on the intersection of finance, mathematics and computer science, and realizing its importance, considerable amount of research within each of these three fields have been done. This section briefly summarizes some of the other models proposed by various well known researchers to manage stock market portfolio.

A. Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model, given in [40], concentrates on the risk and return of a single asset rather than entire portfolio, contrast to Markowitz's modern portfolio theory. According to this model, the expected return of an asset or a portfolio is equal to the rate of risk-free return plus a risk premium. Mathematically, this can be expressed as

$$r_a = r_f + \beta_a\,(r_m - r_f) \tag{22}$$

where r_a is expected return of an asset a, r_f is a risk-free return, r_m is expected market return, and β_a is a measure of volatility (sensitivity) of an asset a in relation to the market. The market has a β of 1.0. Individual assets' β is determined based on how volatile they are with respect to the market. If β is greater than 1 for any asset, that asset is considered to be more volatile compared to market. If β is less than 1 for any asset, that asset is considered to be less volatile compared to market. High-beta stocks may provide high returns with high risks, while low-beta stocks may provide low returns with low risks. Same is also applicable to the portfolio. The β of a portfolio p can be given as

$$\beta_p = \sum_{i=1}^{N} w_i\,\beta_i \tag{23}$$

B. Arbitrage Pricing Theory (APT) Model

In contrast to CAPM which is a single factor model (based on expected of whole market), Arbitrage Pricing Theory (APT) model, given in [41], is a multi factor model. According to this model, expected return of an asset depends upon how that asset reacts to a set of independent macro-economic factors. Four major factors identified were inflation, interest rate, industrial production and personal consumption. Mathematically, this can be expressed as

$$r_a = r_f + \sum_{i=1}^{K} \lambda_i\,\beta_{ai} \tag{24}$$

where r_a is expected return of an asset a, r_f is a risk-free return, K is the total number of macro-economic factors that are considered in the model, λ_i is average risk premium $(r_m - r_f)$ for i^{th} factor, expected market return, and β_{ai} is a measure of volatility (sensitivity) of an asset a to i^{th} factor.

VIII. Conclusion

This paper has delineated one of the most challenging and concerning problem – stock market portfolio management – for the researchers from the finance, mathematics and computer science area along with investors and fund managers. Paper began with describing basic terms such as portfolio, stock market portfolio management, risky and risk-free assets. Main activities required to manage portfolio, scope of the research behind this problem, and challenges associated with managing portfolio are described. There are mainly two approaches to manage portfolio – active management and passive management. Investor or fund manager can go with any approach based on various factors such as return targets, risk profiles, knowledge regarding financial and stock market domain, knowledge about fundamental and technical analysis, confidence level, assumptions about efficiency of markets, etc. As stocks are risky assets, investment made in stock market is risky. Risk associated with a stock can be classified as systematic risk and unsystematic risk. Systematic risk can be mitigated by hedging while unsystematic risk can be mitigated through diversification, i.e. investing in more than one stock.

Markowitz simplified the task of portfolio management and provided a mean-variance model that proved to be a pioneering work for portfolio management. This model works on a duality of features – return, measured as a mean of the past returns, and risk, measured as a variance or standard deviation of the past returns. This is a multi-objective model which maximizes returns for a given level of risk or minimizes risks for a given level of return. This model also drew serious criticisms to adopt it for real world portfolio management. To overcome this, various alternative constraints can be utilized to enhance this model such as cardinality constraint, floor-ceiling constraints, class/sector weight constraint, short sales constraint, round lot or minimum lot constraint and turnover constraint. Similarly, alternative risk measures can be used such as semi-variance or downside risk, value-at-risk, conditional value-at-risk, Sharpe ratio, mean-absolute deviation, and mean semi-absolute deviation. At end, two other well known models – capital asset pricing model and arbitrage pricing theory model – are introduced.

Even though tremendous work has been done to provide better solutions for this problem, this is ever evolving problem. There are great scopes for the future research to enhance the solution quality in terms of alternative risk and return measures, additional constraints, stock selection, their proportion selection, optimization, etc.

References

[1] "Time Value of Money (TVM) Definition," Investopedia. [Online]. Available: http://www.investopedia.com/terms/t/timevalueofmoney.asp.

[2] A. E. Yilmaz and G.-W. Weber, "Why You Should Consider Nature-Inspired Optimization Methods in Financial Mathematics," in Nonlinear and Complex Dynamics, Springer New York, 2011, pp. 241–255.

[3] F. J. Fabozzi and H. M. Markowitz, The Theory and Practice of Investment Management: Asset Allocation, Valuation, Portfolio Construction, and Strategies. John Wiley & Sons, 2011.

[4] R. Jeurissen and J. van den Berg, "Optimized index tracking using a hybrid genetic algorithm," in IEEE Congress on Evolutionary Computation, 2008. CEC 2008. (IEEE World Congress on Computational Intelligence), 2008, pp. 2327–2334.

[5] "Active vs. Passive Portfolio Styles - Series 65," Investopedia. [Online]. Available: http://www.investopedia.com/exam-guide/series-65/portfolio-management/active-passive-portfolio.asp.

[6] "Active management," Wikipedia, the free encyclopedia. 30-Dec-2014.

[7] P. Grant, "Developing risk management strategies for stock market investment portfolio management," Thesis, 2007.

[8] R. W. Colby, The Encyclopedia Of Technical Market Indicators, Second Edition, 2 edition. New York: McGraw-Hill Professional, 2004.

[9] A. Gujral, How to Make Money Trading with Charts. New Delhi, India: Vision Books, 2008.

[10] "Passive management," Wikipedia, the free encyclopedia. 08-Feb-2015.

[11] Markowitz, H. M. (1959), Portfolio Selection: Efficient Diversification. of Investments, Wiley, Yale University Press, 1970, Basil Blackwell, 1991.

[12] "Systematic Risk Definition," Investopedia. [Online]. Available: http://www.investopedia.com/terms/s/systematicrisk.asp.

[13] "Unsystematic Risk Definition," Investopedia. [Online]. Available: http://www.investopedia.com/terms/u/unsystematicrisk.asp.

[14] T.-J. Chang, N. Meade, J. E. Beasley, and Y. M. Sharaiha, "Heuristics for cardinality constrained portfolio optimisation," Comput. Oper. Res., vol. 27, no. 13, pp. 1271–1302, 2000.

[15] S. Wang and Y. Xia, "A Model for Portfolio Selection with Order of Expected Returns," in Portfolio Selection and Asset Pricing, Springer Berlin Heidelberg, 2002, pp. 23–38.

[16] D. Maringer and H. Kellerer, "Optimization of cardinality constrained portfolios with a hybrid local search algorithm," Spectr., vol. 25, no. 4, pp. 481–495, 2003.

[17] T. Cura, "Particle swarm optimization approach to portfolio optimization," Nonlinear Anal. Real World Appl., vol. 10, no. 4, pp. 2396–2406, Aug. 2009.

[18] W. R. It, "Portfolio Selection: efficient diversification of investments," 1959.

[19] A. L. J. Antonin Ponsich, "A Survey on Multiobjective Evolutionary Algorithms for the Solution of the Portfolio Optimization Problem and Other Finance and Economics Applications," Evol. Comput. IEEE Trans. On, vol. 17, no. 3, pp. 321–344, 2013.

[20] T. Fu, C. Ng, K. Wong, and F.-L. Chung, "Models for portfolio management on enhancing periodic consideration and portfolio selection," in 2011 Seventh International Conference on Natural Computation (ICNC), 2011, vol. 1, pp. 176–180.

[21] "Modern portfolio theory," Wikipedia, the free encyclopedia. 27-Apr-2015.

[22] G. Di Tollo and A. Roli, "Metaheuristics for the portfolio selection problem," Int. J. Oper. Res., vol. 5, no. 1, pp. 13–35, 2008.

[23] F. Streichert, H. Ulmer, and A. Zell, "Comparing Discrete and Continuous Genotypes on the Constrained Portfolio Selection Problem," in Genetic and Evolutionary Computation – GECCO 2004, K. Deb, Ed. Springer Berlin Heidelberg, 2004, pp. 1239–1250.

[24] T. Krink, S. Paterlini, and others, "Differential evolution for multiobjective portfolio optimization," Cent. Econ. Res. RECent, vol. 21, 2008.

[25] T. M. G. A. Vijayalakshmi Pai, "Evolutionary Optimization of Constrained -Means Clustered Assets for Diversification in Small Portfolios," Evol. Comput. IEEE Trans. On, vol. 13, no. 5, pp. 1030 – 1053, 2009.

[26] F. Black, "Capital Market Equilibrium with Restricted Borrowing," J. Bus., vol. 45, no. 3, pp. 444–55, 1972.

[27] R. Mansini and M. G. Speranza, "Heuristic algorithms for the portfolio selection problem with minimum transaction lots," Eur. J. Oper. Res., vol. 114, no. 2, pp. 219–233, 1999.

[28] M. A. Gomez, C. X. Flores, and M. A. Osorio, "Hybrid Search for Cardinality Constrained Portfolio Optimization," in Proceedings of the 8th Annual Conference on Genetic and Evolutionary Computation, New York, NY, USA, 2006, pp. 1865–1866.

[29] "Semivariance Definition," Investopedia. [Online]. Available: http://www.investopedia.com/terms/s/semivariance.asp. [Accessed: 07-May-2015].

[30] P. Jorion, Value at Risk: The New Benchmark for Managing Financial Risk, 3rd Edition, 3rd edition. New York: McGraw-Hill, 2006.

[31] N.-A. Bradshaw, C. Walshaw, C. Ierotheou, A. K. Parrott, and others, "A multi-objective evolutionary algorithm for portfolio optimisation," Proc. AISB09, 2009.

[32] P. Artzner, F. Delbaen, J.-M. Eber, and D. Heath, "Coherent Measures of Risk," Math. Finance, vol. 9, no. 3, pp. 203–228, Jul. 1999.

[33] R. T. Rockafellar and S. Uryasev, "Optimization of Conditional Value-at-Risk," J. Risk, vol. 2, pp. 21–41, 2000.

[34] R. T. Rockafellar and S. Uryasev, "Conditional value-at-risk for general loss distributions," J. Bank. Finance, pp. 1443–1471, 2002.

[35] Sergey Sarykalin, Gaia Serraino, and Stan Uryasev, "Value-at-Risk vs. Conditional Value-at-Risk in Risk Management and Optimization," in State-of-the-Art Decision-Making Tools in the Information-Intensive Age, 0 vols., INFORMS, 2008, pp. 270–294.

[36] W. F. Sharpe, "Mutual Fund Performance," J. Bus., vol. 39, no. 1, pp. 119–138, Jan. 1966.

[37] W. F. Sharpe, "The sharpe ratio," J. Portf. Manag., vol. 21, no. 1, pp. 49–58, 1994.

[38] H. Y. Hiroshi Konno, "Mean-Absolute Deviation Portfolio Optimization Model and Its Applications to Tokyo Stock Market," Manag. Sci., vol. 37, no. 5, pp. 519–531, 1991.

[39] L. Chiodi, R. Mansini, and M. G. Speranza, "Semi-Absolute Deviation Rule for Mutual Funds Portfolio Selection," Ann. Oper. Res., vol. 124, no. 1–4, pp. 245–265, Nov. 2003.

[40] W. F. Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk*," J. Finance, vol. 19, no. 3, pp. 425–442, Sep. 1964.

[41] S. A. Ross, "The arbitrage theory of capital asset pricing," J. Econ. Theory, vol. 13, no. 3, pp. 341–360, Dec. 1976.