STATISTICS FOR NEUROSCIENCE RESEARCH

9.073/HST 460

Simple Regression

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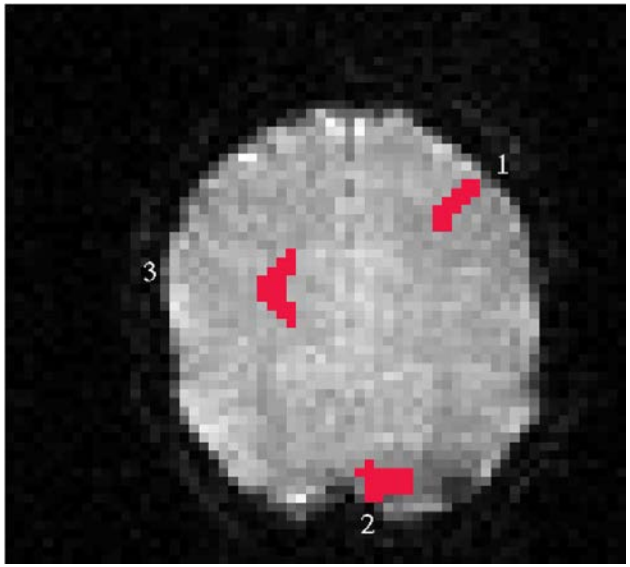
Stimulus-Response Experiments

The activity of a neural system or a component of neural systems is recorded in response to an input stimulus, usually under the experimenter's control, that is believed to be specific for that system or some subset of its components. The objective of these investigations is usually to characterize the strength and time course of the stimulus response relation. Many neuroscience experiments fall into this category.

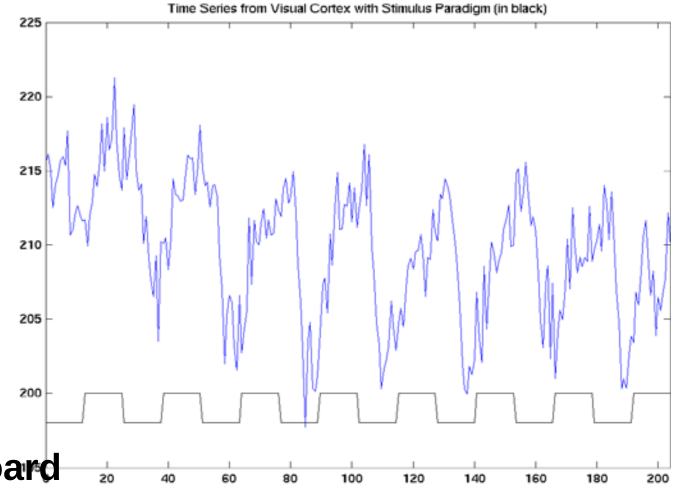
1. **Sensory-Motor (fMRI Block design)**
2. **Neurophysiology**
3. **Behavioral Learning**
4. **Pharmacology**
5. **Cognitive/Psychophysics**

fMRI Sensory-Motor Experiment



Purdon et al. Neuroimage (2001)

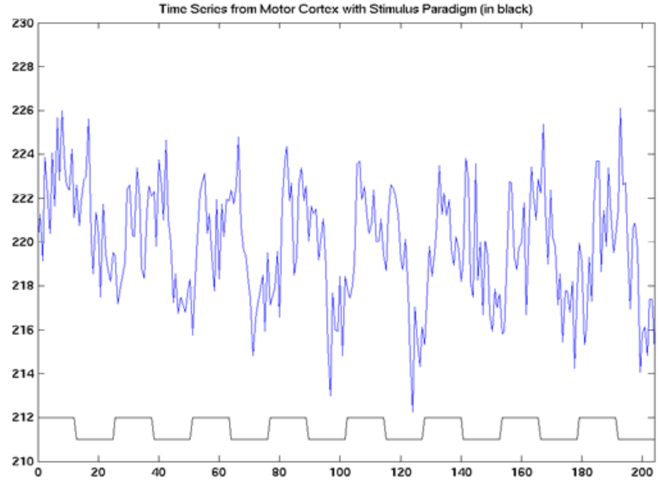
Response 1: fMRI activation in visual cortex



Stimulus 1:

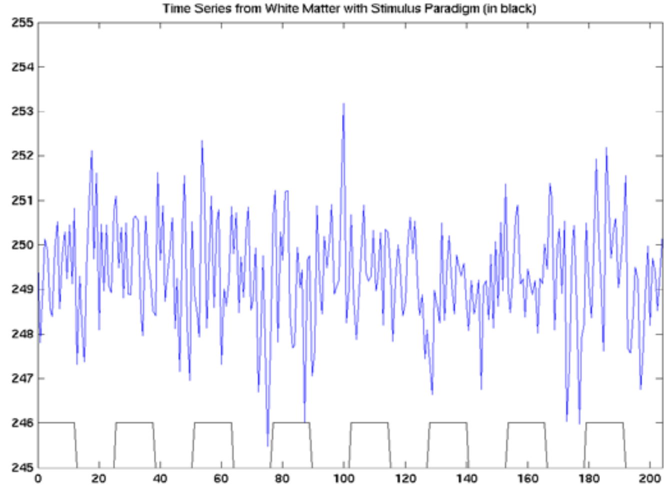
Flickering Checkerboard

Response 2: fMRI activation in motor cortex



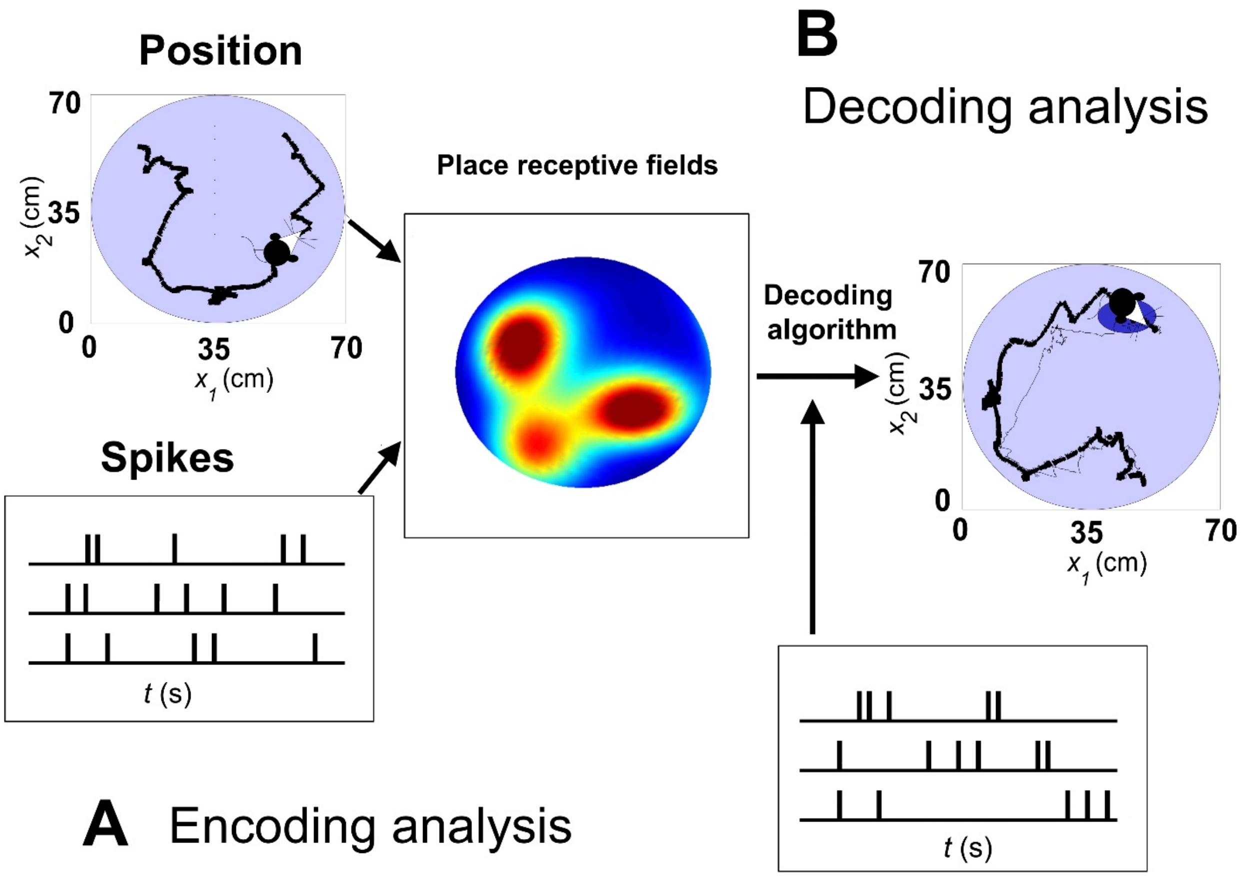
Stimulus 2: Finger Tapping

Response 3: fMRI activation in white matter track



Stimulus-Response Experiment: Rat Hippocampal Experiment

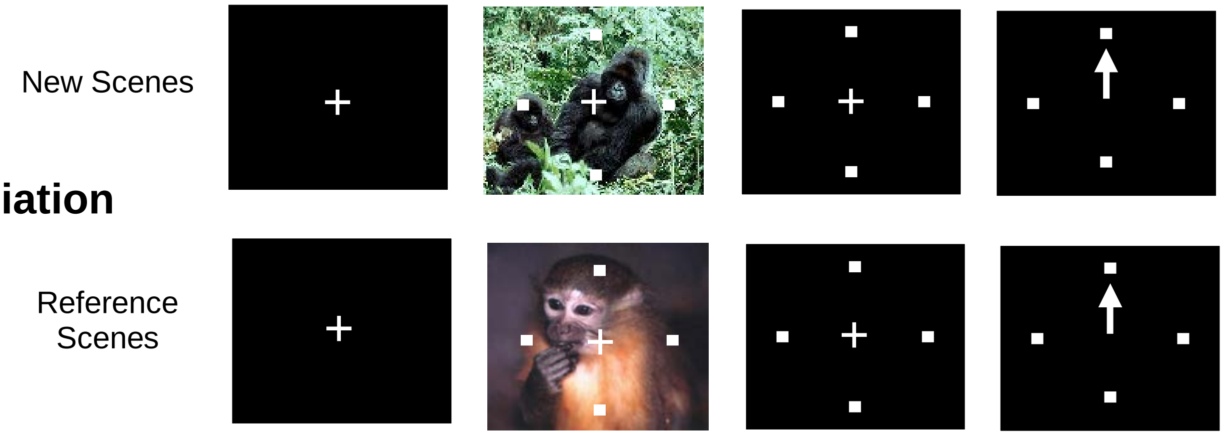
Stimulus: Position **Response: Ensemble Neural Spiking Activity**



Brown Kass, Mitra, Nature Neurosci. (2004)

Stimulus-Response Behavioral Learning Experiment

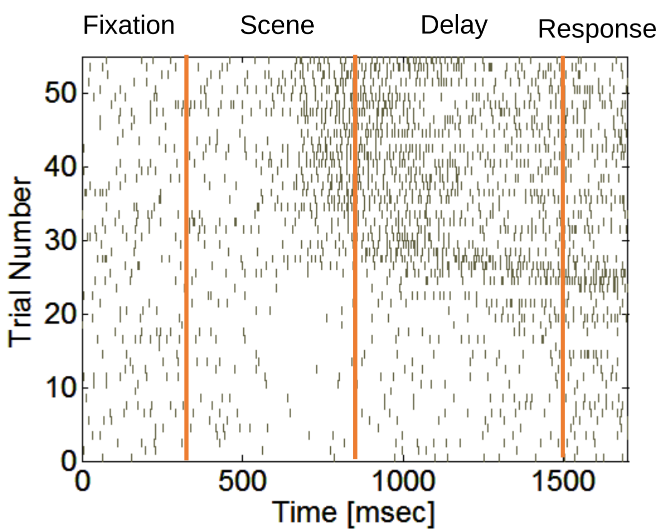
Stimulus: Location-Scene Association Task



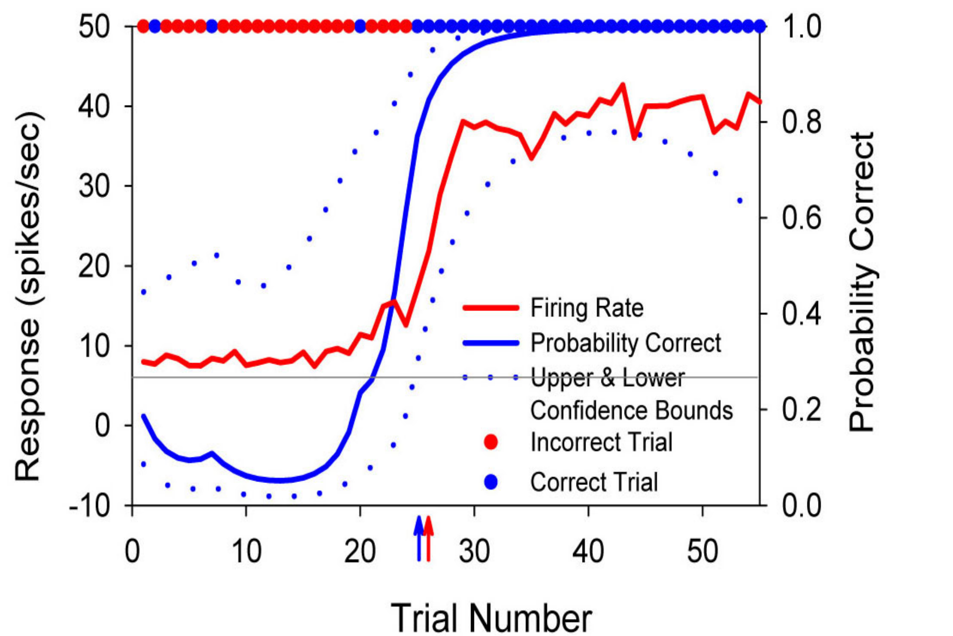
Fixation/ Baseline (300 ms) → Scene/Target Presentation (500 ms) → Delay (700 ms) → Eye Movement Response

Wirth et al., Science (2003)

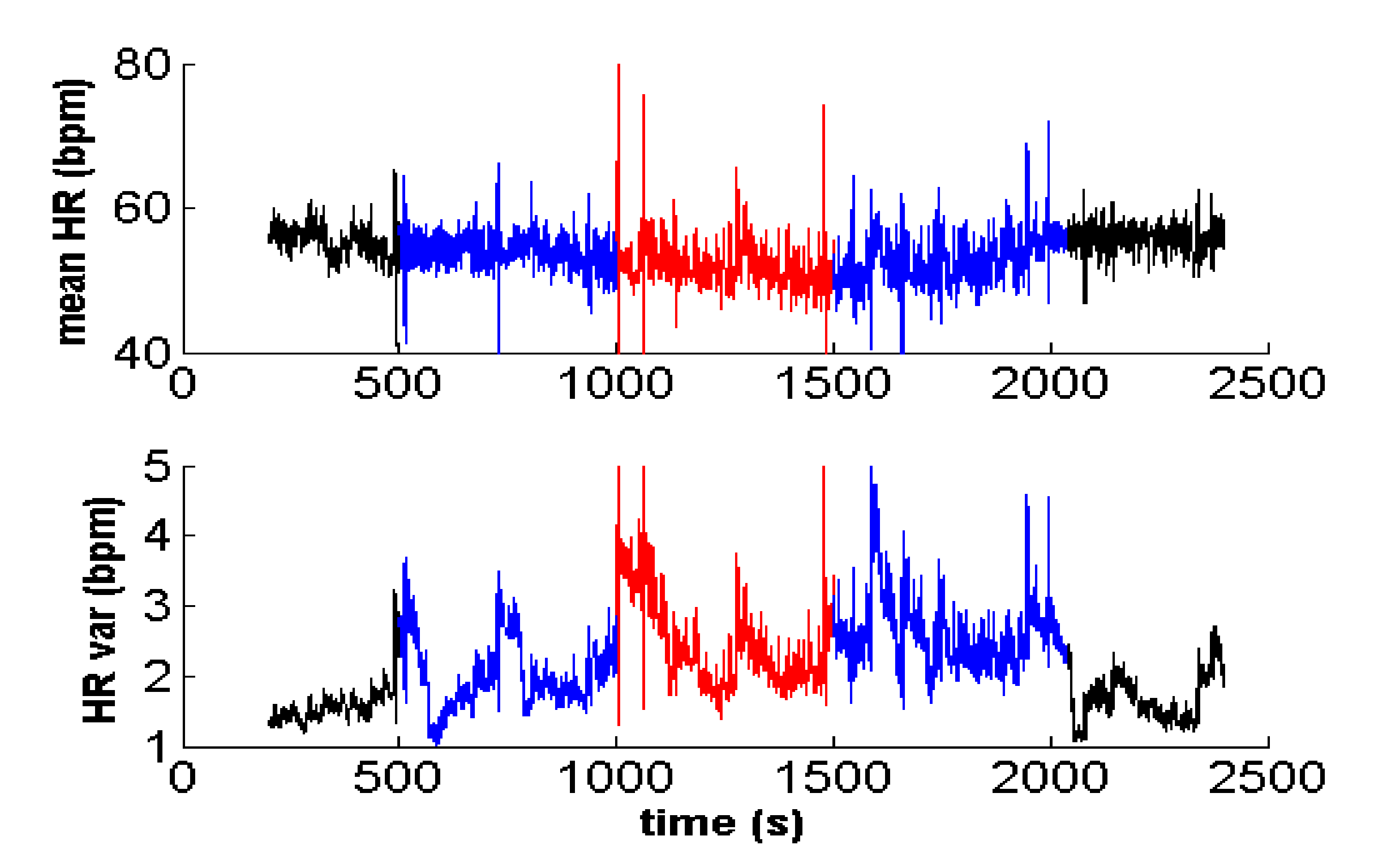
Response 1: Hippocampal Neural Activity



Response 2: Correct/Incorrect Response



Cognitive-Stimulus Response Experiment: Mediation
Stimulus: Mediation, Response: HR and HR Variability
(Courtesy of Sara Lazar, MGH)



	Baseline	Early	Middle	Late	Numbers
Mean HR	55	53	52	52	55
HRV	1.58	2.01	2.42	2.58	1.69

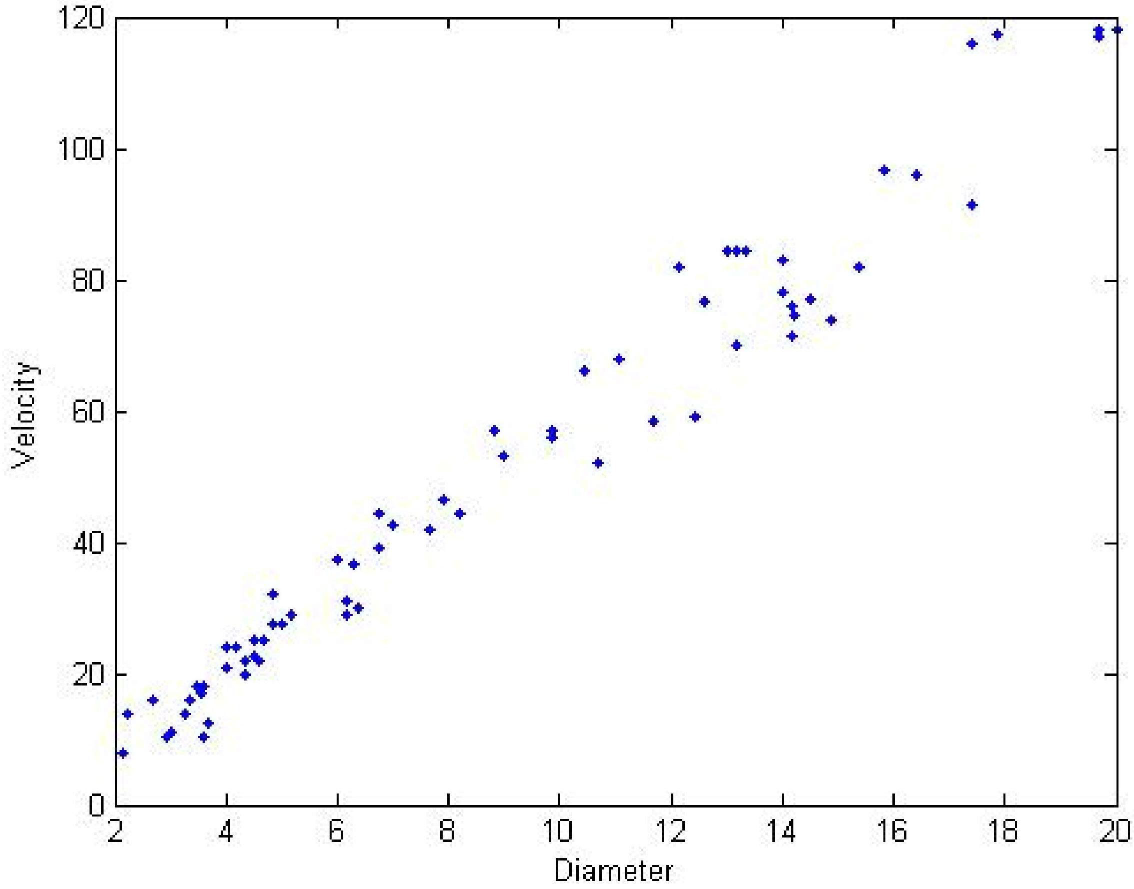
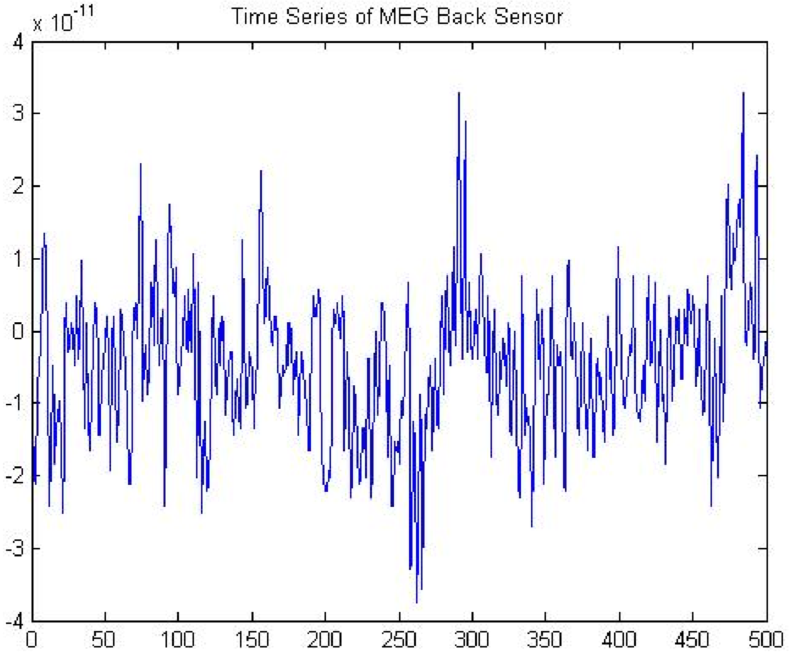
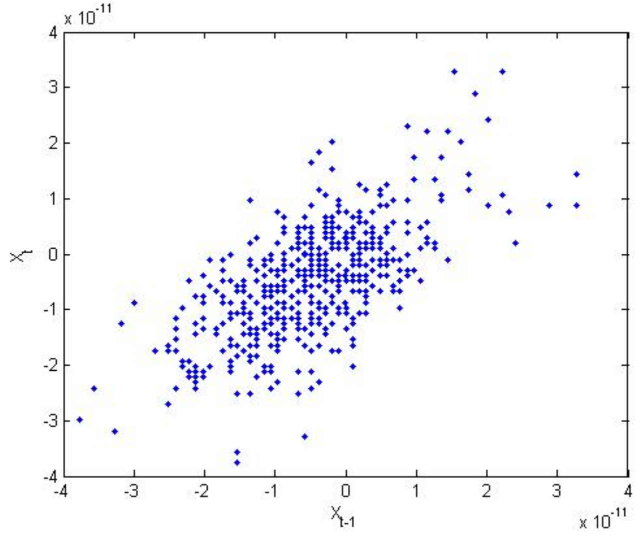


Figure 14.1. Relation between Conduction Velocity and Axon Diameter. Replotted from Hursh (1939).





Figures 14.2 and 14.3. Time-Series Plot of first 500 observations of the MEG sensor background noise measurements. Plot of x_t vs x_t-1.

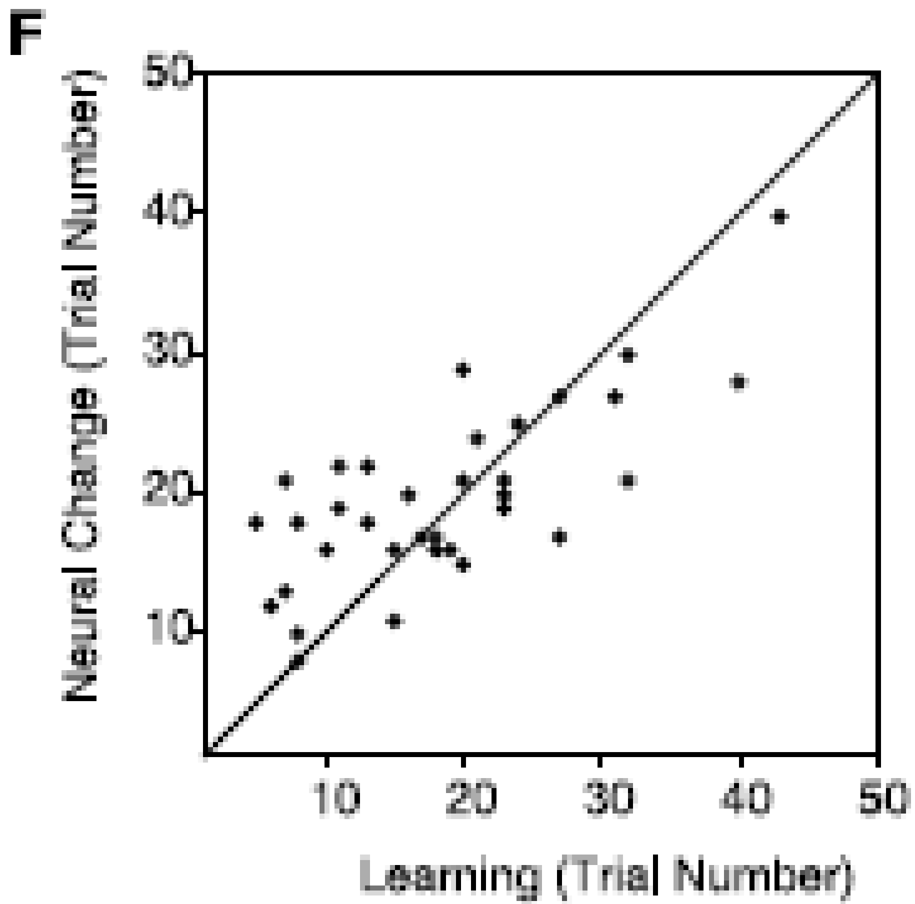
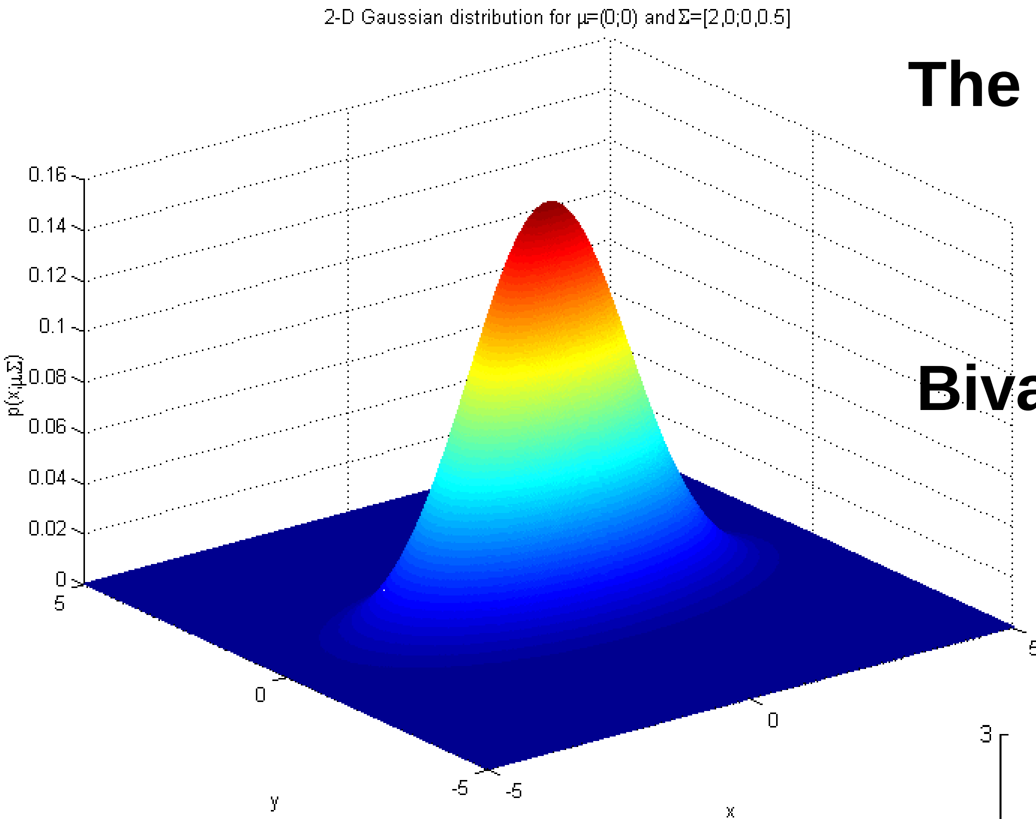


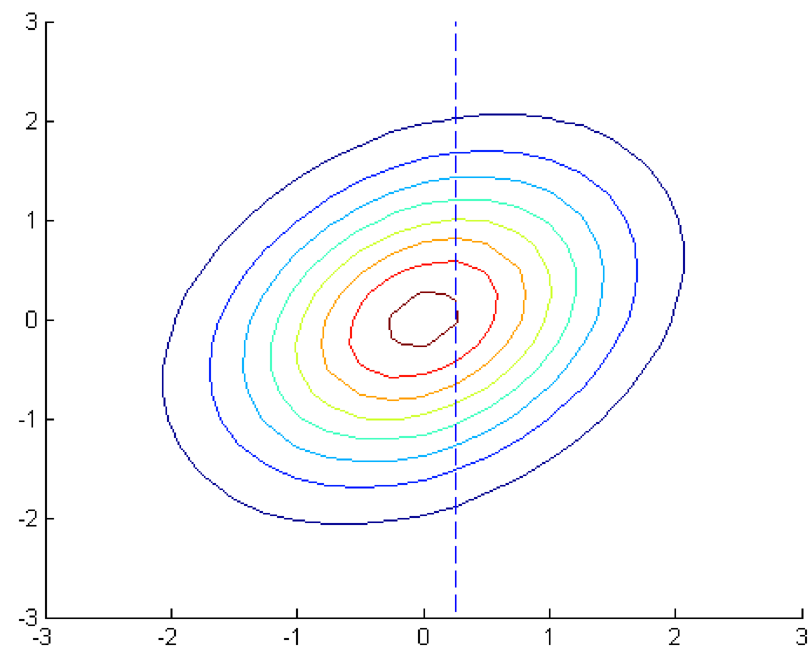
Figure 14.4. Plot of Change in Neural Activity during Learning Experiments versus Learning Trial (Wirth et al. 2003).

The Meaning of Regression



Bivariate Gaussian Density

Cross-Sections are Ellipses.



Bivariate Gaussian (Joint) Density of (X, Y)

$$f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y(1-\rho^2)^{\frac{1}{2}}} \times$$

$$\exp\{-\frac{1}{2(1-\rho^2)}[\frac{(x-\mu_x)^2}{\sigma_y^2} + \frac{(y-\mu_y)^2}{\sigma_y^2} - \frac{2\rho(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma y}]\}$$

Conditional Density of Y Given X

$$\Pr(B \mid A) = \frac{\Pr(A \cap B)}{\Pr(A)}$$

$$f(y \mid x) = \frac{f(x,y)}{f(y)}$$

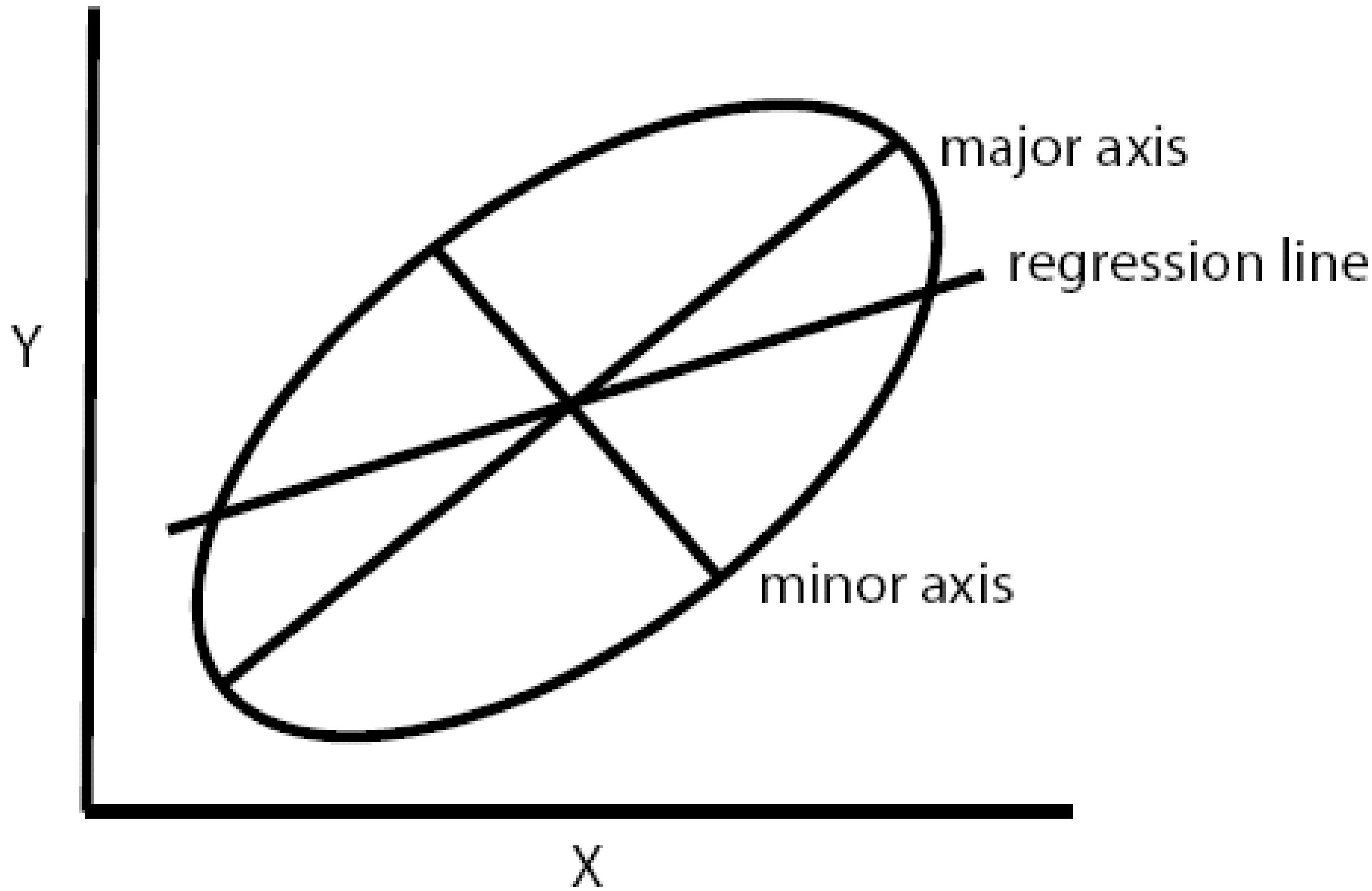
Conditional Expectation: Theoretical Regression Line

$$E(Y \mid X = x) = \mu_y + \rho\frac{\sigma_y}{\sigma_x}(x-\mu_x)$$

Conditional Variance

$$Var(Y \mid X = x) = \sigma_y - \rho\sigma_{xy}\sigma_y$$

The Geometry of the Regression Line



The Meaning of Regression

Conditional Expectation: Theoretical Regression Line

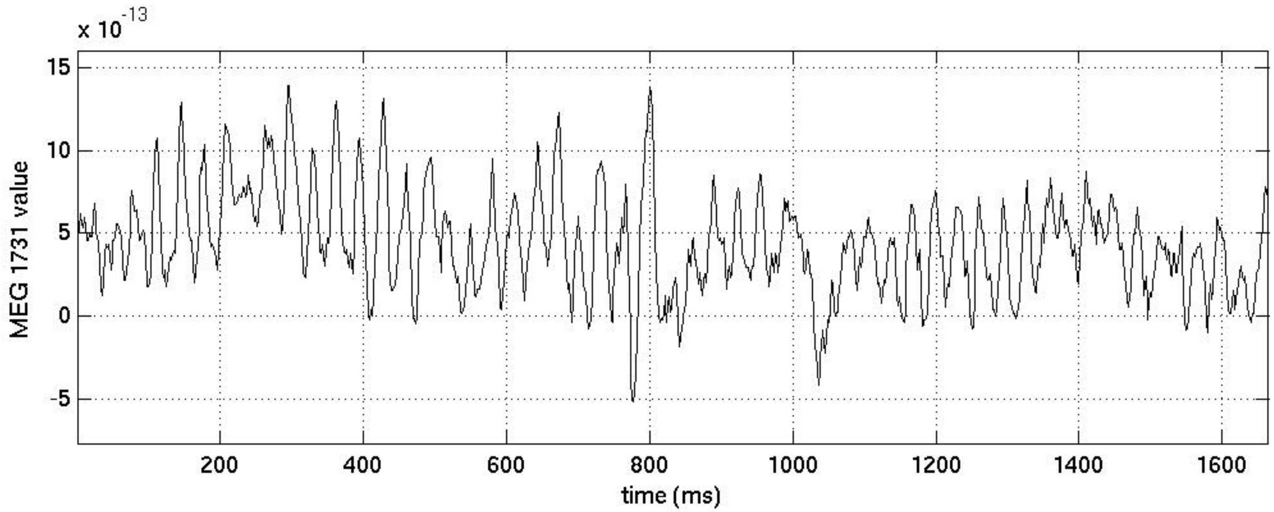
$$E(Y \mid X = x) = \mu_y + \rho \frac{\sigma_y}{\sigma_x}(x - \mu_x)$$

The Empirical Regression Line Variance

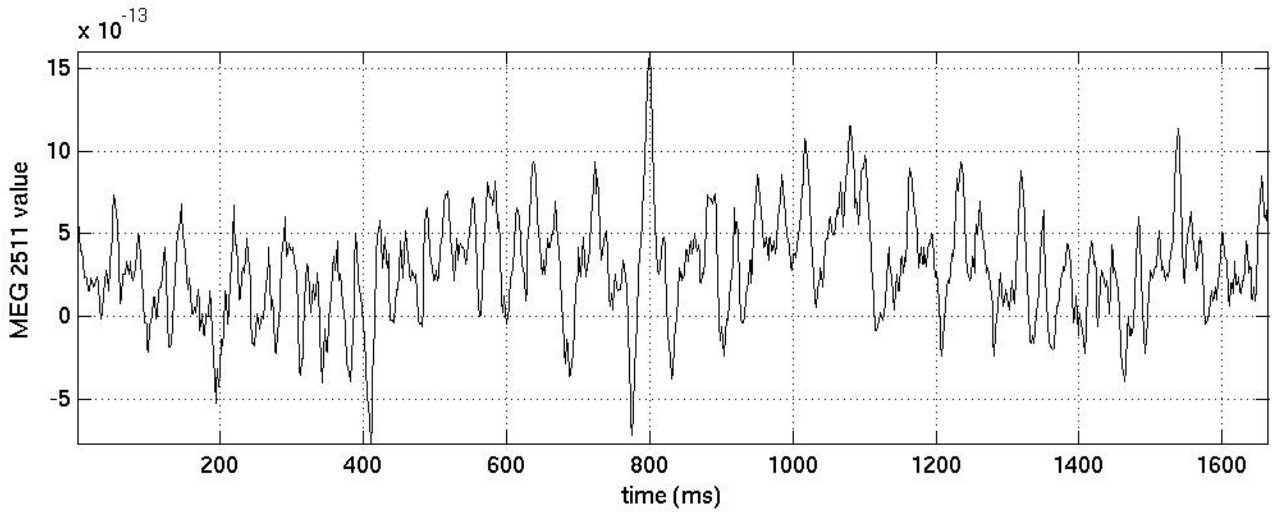
$$\begin{aligned}
\hat{y} &= \overline{y} + \hat{\rho} \frac{\hat{\sigma}_y}{\hat{\sigma}_x}(x - \overline{x}) \\
&= \overline{y} + \hat{\beta}(x - \overline{x}) \\
&= \overline{y} - \hat{\beta}\overline{x} + \hat{\beta}x \\
&= \hat{\alpha} + \hat{\beta}x
\end{aligned}$$

Front (Y) and Back (X) MEG Sensor Background Noise Recordings

Y

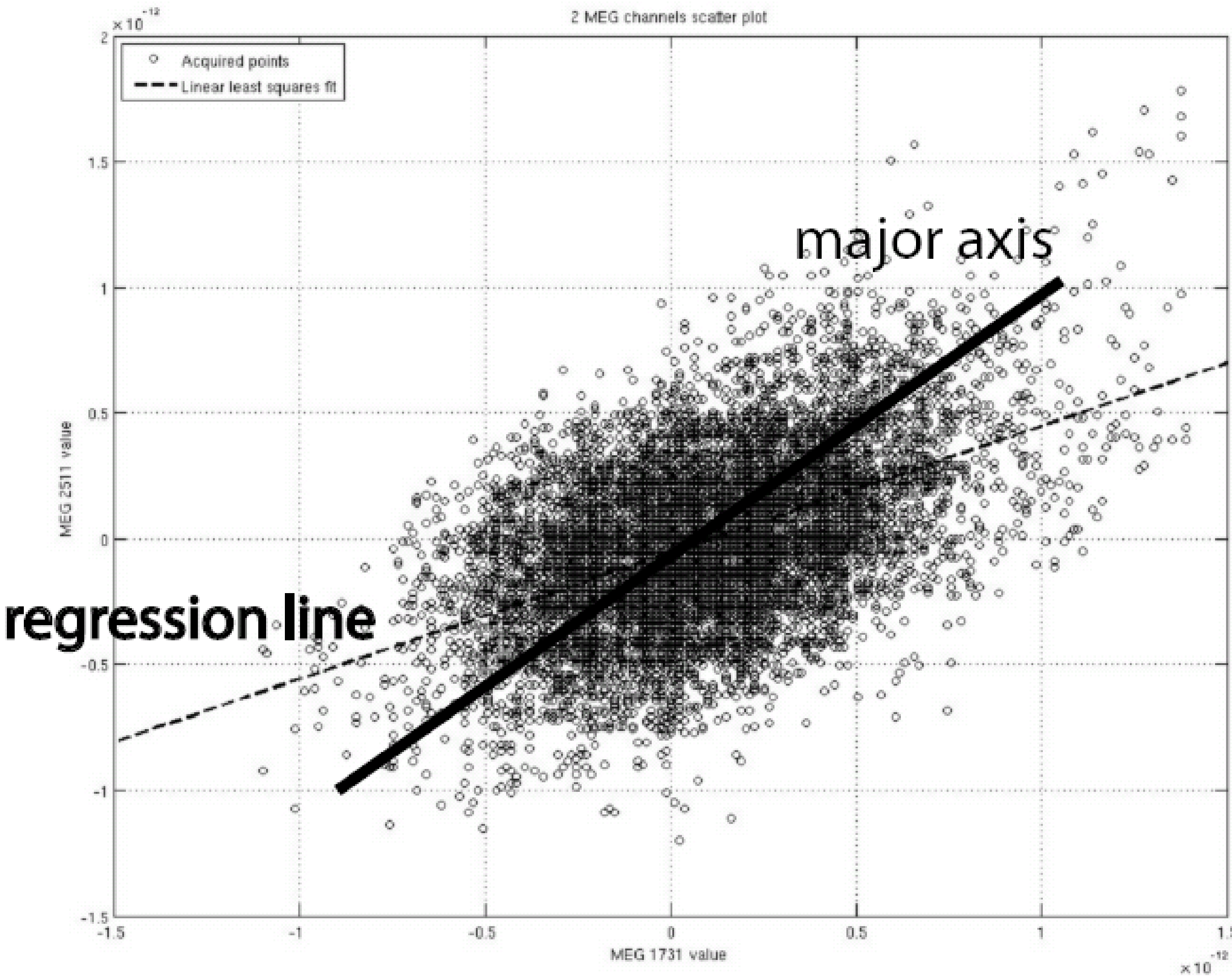


X



Time (ms)

The Meaning of Regression

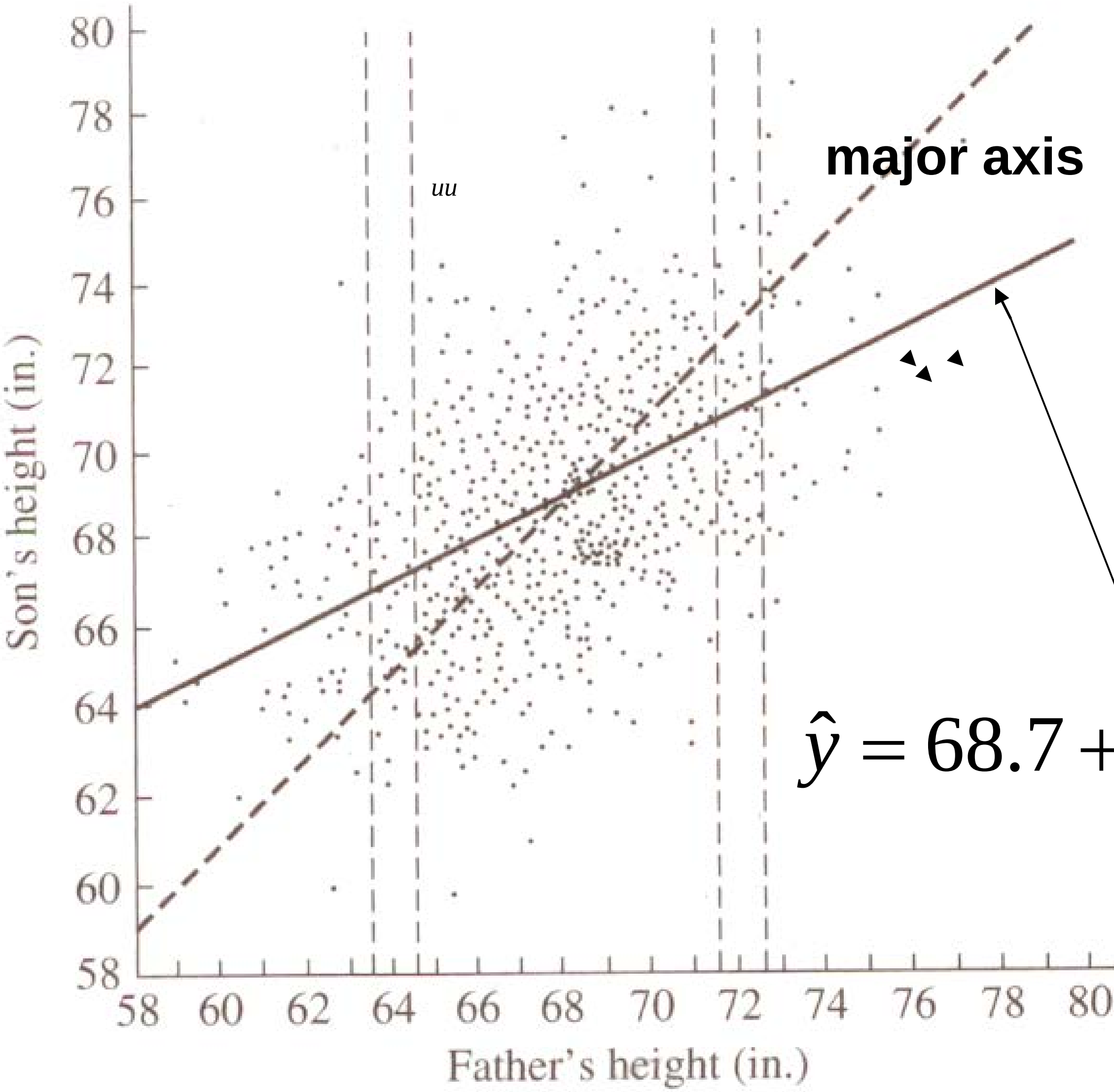


The Meaning of Regression

The geneticist Sir Francis Galton (1822-1911) observed that the sons of fathers who were taller than average tended to be shorter than average and that the sons of fathers who were shorter than average tended to be taller than average. He termed the phenomenon "regression towards mediocrity". More recently it has been termed "regression to the mean".

This relation is given exactly by the regression line.

Heights of 1078 Pairs of Fathers and Sons



$$\overline{x} = 67.7$$

$$\hat{\sigma}_x = 2.74$$

$$\overline{y} = 68.7$$

$$\hat{\sigma}_y = 2.81$$

$$\hat{\rho} = 0.5$$

$$\hat{y} = 68.7 + 0.5 \times \frac{2.81}{2.74}(x - 67.7)$$

$$x = 72 \Rightarrow \hat{y} = 70.9$$

$$x = 64 \Rightarrow \hat{y} = 66.8$$

Rice (2007)

Summary

Simple regression is our most basic technique for relating one variable to another.

It is useful to be able to think of simple regression in terms of likelihood, method-of-moments and least squares analyses.

The geometry of the Pythagorean relations carries over to the case of multiple regression.

The concepts discussed here form the basis for our formulations of more complex models.