9.07 Matlab Tutorial

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Aim

• Transform students into Matlab ninjas!



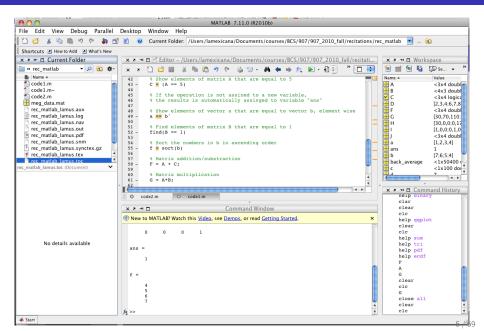
Contents

- Matrix operations: a + b, a*b, A.*B, A.^B, sort, etc
- Loops: for i=1:n, while "statement is true"
- Some useful functions: rand, randn, min, max, etc
- Function for displaying results:
 figure, plot, subplot, bar, hist, title, xlabel, etc
- Translating algorithms into Matlab code.

Getting started

- Getting Matlab: http://web.mit.edu/student-matlab/
- Matlab support: http://www.mathworks.com/support/, in search support box select "Function list for all products"
- Matlab support: Goto Help>Product Help
- In command prompt type: help <name of funct>

The Matlab interface



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• Generate the 3×4 matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$

$$A = [1 \ 2 \ 3 \ 4; 5 \ 6 \ 7 \ 8; 9 \ 10 \ 11 \ 12];$$

- ullet Accessing a portion of the matrix A
 - \bullet Take the element in the 2nd row and 3rd column of matrix A

$$d = A(2,3);$$

 Take the elements in the 1st through 2nd rows and 2nd through 4th columns

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Load data from a ".mat" file meg_data.mat

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• Multiply the matrices A and B, i.e., G = A * B

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ullet Multiply element-wise matrix G and the identity matrix I, i.e.,

$$G = \begin{bmatrix} 130 & 70 & 110 \\ 70 & 174 & 278 \\ 110 & 278 & 446 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} h_{1,1} & h_{1,2} & h_{1,3} & h_{1,4} \\ h_{2,1} & h_{2,2} & h_{2,3} & h_{2,4} \\ h_{3,1} & h_{3,2} & h_{3,3} & h_{3,4} \end{bmatrix} = \begin{bmatrix} 130 * 1 & 70 * 0 & 110 * 0 \\ 70 * 0 & 174 * 1 & 278 * 0 \\ 110 * 0 & 278 * 0 & 446 * 1 \end{bmatrix}$$

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• Find the cube of the elements of matrix A

$$J = A.^3;$$

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```
% load the meg data
load('meg_data.mat');
% Take the required values
y = back_average(1:500);
% Initialize a variable accumulate the sum
acum = 0:
for i=1:500
    % Accumulates the sum of the data
    acum = acum + v(i);
end
% Divide by number of samples
y_bar1 = acum / 500;
```

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```
% Draw 500 samples from Bernoulli distribution
p = 0.5;
b = rand(1,500) > 0.5;
```

• Computed the minimum, maximum, mean, standard deviation, and variance from simulated sample from the standard Gaussian distribution: x_i , $i=1,2,\ldots,500$, where the sample variance is $\hat{\sigma}^2=\frac{1}{500}\sum_{i=1}^{500}(x_i-\bar{x})^2$, and the sample standard deviation is $\hat{\sigma}$

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```
% The minimum
x_min = min(x);
% The maximum
x_max = max(x);
% The sample mean
x_bar = mean(x);
% The sample variance
sigma2_hat = var(x);
% The sample standard deviation
sigma_hat = sigma2_hat^(1/2);
```

Functions for displaying results

• Load and plot the MEG data

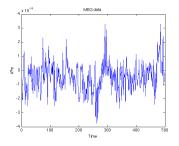
Functions for displaying results

Load and plot the MEG data

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load('meg_data.mat');
% Take the required values
y = back_average(1:500);
figure, plot(y)
title('MEG data')
xlabel('Time'), ylabel('A*m')
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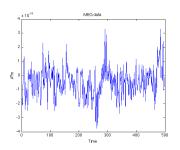
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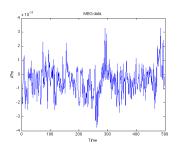
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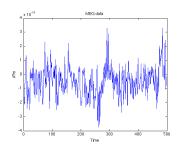


• Make a histogram of the simulated sample from the standard Gaussian distribution $x_i, i = 1, 2, \dots, 500$

```
x_i, i = 1, 2, \dots, 500
%select the number of bins
m = 25;
figure, hist(x,m)
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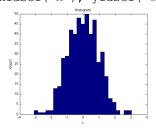
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 - With the "fair" coin we can obtain a samples from the Bernoulli distribution with parameter p=0.5. Recall that the pdf of the Bernoulli is given by:

$$f_{ber}(b) = p^b(1-p)^{(1-b)}$$
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- And from the uniform sample simulate a new sample of the exponential distribution using the Inverse Transform method!

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 - \bullet With the "fair" coin we can obtain a samples from the Bernoulli distribution with parameter p=0.5. Recall that the pdf of the Bernoulli is given by:

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- Wow! This sound impossible!
- It is possible if we could generate a sample from the uniform distribution using a coin
- And from the uniform sample simulate a new sample of the exponential distribution using the Inverse Transform method!
 - Find an algorithm
 - Write the code

$$u = \sum_{i=1}^{\infty} b_i/2^i, ext{where } b_i \in \{0,1\}$$

• To find the algorithm we should note that a number u between 0 and 1 can be represented with the binary expansion $0.b_1b_2b_3...$, i.e:

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• For example:

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- For example:
 - If u=0.75 then $b_1=1,\ b_2=1,\ b_j=0,\ j=3,4,\ldots$, since $u=1/2+1/2^2+0/2^3+\cdots$
 - If u = 0.958 then $0.b_1b_2b_3... = 0.1111010101\bar{0}$
 - If u=0.3288 then $0.b_1b_2b_3\ldots=0.01010100001010010010\bar{0}$

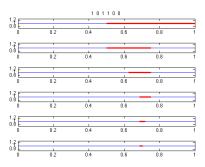
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- We will use a heuristic argument to convince our selves
- If $b_1 = 1 \to u > = 0.5$ and if $b_1 = 0 \to u < 0.5$
- More generally, with a figure we see that:



• Simulate a sample of size 1000 from the uniform distribution using samples from the Bernoulli distribution with parameter p=0.5:

$$u_j = \sum_{i=1}^{500} b_{i,j}/2^i$$
, where $j = 1, 2, \dots, 1000, \ i = 1, 2, \dots, 500$

and $b_{i,j}$ are iid Bernoulli with parameter p

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```
% Sample size
n = 1000;
% Compute the denominators expansion
m = 500;
d = 1./2.^[1:m];
% For loop to obtain the 500 uniform rvs
for i = 1:n
    % Simulate 100 Bernoulli rvs (b_{i,j})
    b = rand(1, m) > 0.5;
    % Compute the summation
    u(i) = sum(b.*d);
end
```

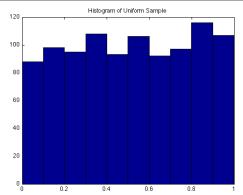
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Make a histogram of the obtained uniform sample

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figure, hist(u)
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- Recall that $f_{exp}(t) = \lambda e^{-\lambda t}$, where t > 0
- The cdf is given by:

$$F_{exp}(t) = \int_0^t \lambda e^{-\lambda \tau} d\tau$$

Make change in variable $w=-\lambda au \to d au = -dw/\lambda$

$$F_{exp}(t) = -\int_0^{-\lambda t} e^w dw = 1 - e^{-\lambda t}$$

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• Now we can obtain F_{exp}^{-1} as:

$$F_{exp}^{-1}(u) = -\frac{\log(1-u)}{\lambda}$$

• Generate a sample of size 1000 of the exponential distribution with parameter $\lambda=3$ using a sample from the uniform distribution (u_j) using the Inverse Transform method:

$$t_j = -\frac{\log(1 - u_j)}{3}$$
, where $j = 1, 2, \dots, 1000$

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Make a histogram of the sample simulated from the exponential distribution

```
figure, hist(t)
title('Histogram of Exponential Sample')
```

Did it WORK???

• Raise you hand if you think it did work

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