## Unit 5 Test

Name

**1.** If $\int_{4}^{-10} g(x)\,dx = -3$ and $\int_{4}^{6} g(x)\,dx = 5$, then $\int_{-10}^{6} g(x)\,dx =$

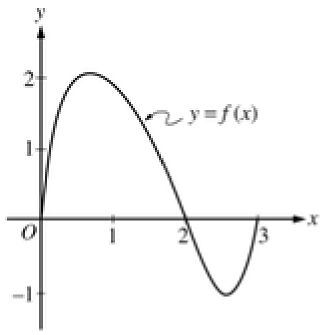
(A) $-8$

(B) $-2$

(C) $2$

(D) $8$

**2.**




The graph of a function $f$ is shown above. Which of the following expresses the relationship between $\int_{0}^{2} f(x)\,dx$, $\int_{0}^{3} f(x)\,dx$, and $\int_{2}^{3} f(x)\,dx$?

(A) $\int_{0}^{2} f(x)\,dx < \int_{0}^{3} f(x)\,dx < \int_{2}^{3} f(x)\,dx$

(B) $\int_{0}^{3} f(x)\,dx < \int_{0}^{2} f(x)\,dx < \int_{2}^{3} f(x)\,dx$

(C) $\int_{2}^{3} f(x)\,dx < \int_{0}^{2} f(x)\,dx < \int_{0}^{3} f(x)\,dx$

(D) $\int_{2}^{3} f(x)\,dx < \int_{0}^{3} f(x)\,dx < \int_{0}^{2} f(x)\,dx$

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3. $\frac{d}{dx}\int_{e}^{x^3} \ln\left(t^2+1\right)dt =$

(A) $ln(x^6+1)$

(B) $3x^2 ln\left(x^2+1\right)$

(C) $3x^2 ln\left(x^6+1\right)$

(D) $ln\left(x^6+1\right) - ln\left(e^2+1\right)$

4. $\int \frac{e^x}{1+e^x}\,dx$

(A) $\ln\left(\frac{1}{e^x}+1\right)+c$

(B) $\ln\left(1+e^x\right)+c$

(C) $x-\ln\left(1+e^x\right)+c$

(D) $e^x+x+c$

(E) $\tan^{-1}\left(e^x\right)+c$

5. $\int \frac{1}{x^2+4x+5}dx =$

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(A) $arctan\,(x+2)\ +\ C$

(B) $arcsin\,(x+2)\ +\ C$

(C) $ln\ \left|x^2+4x+5\right|\ +C$

(D) $\frac{1}{\frac{1}{3}x^3+2x^2+5x}+C$

**6.** $\int_1^2 \frac{x^2-x-5}{x+2}\,dx =$

(A) $-\frac{3}{2}+\ln\,\frac{4}{3}$

(B) $-\frac{25}{21}$

(C) $\frac{5}{2}+3\ln\,\frac{3}{4}$

(D) $\frac{23}{25}$

**7.** $\int x^2\left(x^3+5\right)^6 dx =$

(A) $\frac{1}{3}\left(x^3+5\right)^6+C$

(B) $\frac{1}{3}x^3\left(\frac{1}{4}x^4+5x\right)^6+C$

(C) $\frac{1}{7}\left(x^3+5\right)^7+C$

(D) $\frac{3}{7}x^2\left(x^3+5\right)^7+C$

(E) $\frac{1}{21}\left(x^3+5\right)^7+C$

**8.** For what value of $b$ does the integral $\int_1^b x^2\,\square x$ equal $\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{2k}{n}\right)^2\frac{2}{n}$ ?

(A) $b=2$ only

(B) $b=3$ only

(C) $b$ could be any real number.

(D) There is no such value of $b$.

**9.**

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $f(x)$ | 4 | 9 | 12 | 10 |
| $f'(x)$ | 5 | 4 | 1 | $-6$ |

Selected values of the twice-differentiable function $f$ and its derivative $f'$ are given in the table above. What is the value of $\int_0^3 f'(x)\,\square x$ ?

(A) $-11$

(B) $-1$

(C) $6$

(D) $10$

10.

| $x$ | $0$ | $a^2$ | $3a$ | $6a$ | $7a$ |
|---|---|---|---|---|---|
| $f(x)$ | $1$ | $-1$ | $-3$ | $-7$ | $-9$ |

The continuous function $f$ is decreasing for all $x$. Selected values of $f$ are given in the table above, where $a$ is a constant with $0 < a < 3$. Let $R$ be the right Riemann sum approximation for $\int_0^{7a} f(x)\,dx$ using the four subintervals indicated by the data in the table. Which of the following statements is true?

(A) $R = (a^2 - 0) \cdot 1 + (3a - a^2) \cdot (-1) + (6a - 3a) \cdot (-3) + (7a - 6a) \cdot (-7)$ and is an underestimate for $\int_0^{7a} f(x)\,dx$.

(B) $R = (a^2 - 0) \cdot 1 + (3a - a^2) \cdot (-1) + (6a - 3a) \cdot (-3) + (7a - 6a) \cdot (-7)$ and is an overestimate for $\int_0^{7a} f(x)\,dx$.

(C) $R = (a^2 - 0) \cdot (-1) + (3a - a^2) \cdot (-3) + (6a - 3a) \cdot (-7) + (7a - 6a) \cdot (-9)$ and is an underestimate for $\int_0^{7a} f(x)\,dx$.

(D) $R = (a^2 - 0) \cdot (-1) + (3a - a^2) \cdot (-3) + (6a - 3a) \cdot (-7) + (7a - 6a) \cdot (-9)$ and is an overestimate for $\int_0^{7a} f(x)\,dx$.

11. $\int \frac{x^3 + 5}{x^2} dx =$

(A) $1-\frac{10}{x^3}+C$

(B) $\frac{3x}{4}+\frac{15}{x^2}+C$

(C) $\frac{x^2}{2}-\frac{5}{x}+C$

(D) $\frac{x^2}{2}-\frac{5}{3x^3}+C$

(E) $-\frac{x^3}{4}-5+C$

**12.** $\int_2^4 \frac{dx}{5-3x} =$

(A) $-\ln 7$

(B) $-\frac{\ln 7}{3}$

(C) $\frac{\ln 7}{3}$

(D) $\ln 7$

(E) $3\ln 7$

**13.** Which of the following definite integrals has the same value as $\int_0^4 xe^{x^2}\,dx$ ?

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(A) $\frac{1}{2}\int_{0}^{4} e^{u}\,du$

(B) $\frac{1}{2}\int_{0}^{16} e^{u}\,du$

(C) $2\int_{0}^{2} e^{u}\,du$

(D) $2\int_{0}^{4} e^{u}\,du$

(E) $2\int_{0}^{16} e^{u}\,du$

**14.** If $g(x) = x^2 - 3x + 4$ and $f(x) = g'(x)$, then $\int_{1}^{3} f(x)\,dx =$

(A) $-\frac{14}{3}$

(B) -2

(C) 2

(D) 4

(E) $\frac{14}{3}$

**15.** (d) Find all $x$-values in the interval $-5 < x < 5$ at which $g$ has a critical point. Classify each critical point as the location of a local minimum, a local maximum, or neither. Justify your answers.

Please respond on separate paper, following directions from your teacher.

(c) On what open intervals, if any, is the graph of $g$ concave up? Justify your answer.

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Please respond on separate paper, following directions from your teacher.

(b) Find the instantaneous rate of change of $g$ with respect to $x$ at $x = 3$, or state that it does not exist.

Please respond on separate paper, following directions from your teacher.

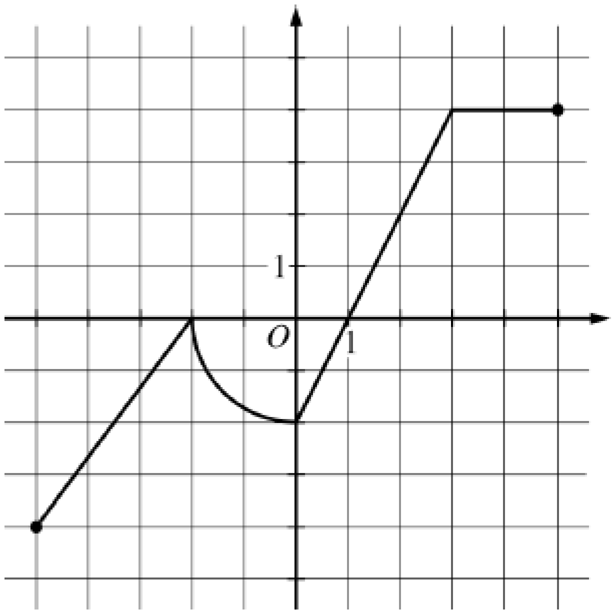
**CALCULUS BC**

**SECTION II, Part B**

**Time - 1 hour**

**Number of questions - 4**

**NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.**




Graph of $f$

3. The graph of the function $f$, consisting of three line segments and a quarter of a circle, is shown above. Let $g$ be the function defined by $g(x) = \int_1^x f(t)\,dt$.

(a) Find the average rate of change of $g$ from $x = -5$ to $x = 5$.

Please respond on separate paper, following directions from your teacher.

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**16.** (d) Find $\lim_{x\to 0} \frac{f(x)-20e^{x}}{0.5f(x)-10}$.

Please respond on separate paper, following directions from your teacher.

(c) Evaluate $\int_2^4 f''(x)\,dx$.

Please respond on separate paper, following directions from your teacher.

(b) Determine whether the actual value of $f(6)$ could be 70. Explain your reasoning.

Please respond on separate paper, following directions from your teacher.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $f'(x)$ | 4 | 3.5 | 2 | 0.8 | 1.7 | 5.8 | 7 |

4. The function $f$ satisfies $f(0)=20$. The first derivative of $f$ satisfies the inequality $0 \le f'(x) \le 7$ for all $x$ in the closed interval $[0,\ 6]$. Selected values of $f'$ are shown in the table above. The function $f$ has a continuous second derivative for all real numbers.

(a) Use a midpoint Riemann sum with three subintervals of equal length indicated by the data in the table to approximate the value of $f(6)$.

Please respond on separate paper, following directions from your teacher.

The following are related to this scenario:

A store is having a 12-hour sale. The total number of shoppers who have entered the store $t$ hours after the sale begins is modeled by the function S defined by $S(t)=0.5t^4-16t^3+144t^2$ for $0 \le t \le 12$ . At time $t=0$ , when the sale begins, there are no shoppers in the store.

**17.** At what rate are shoppers entering the store 3 hours after the start of the sale?

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Please respond on separate paper, following directions from your teacher.

**18.** Find the value of $\frac{1}{3}\int_{6}^{9} S'(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{3}\int_{6}^{9} S'(t)\,dt$ in the context of this problem.

Please respond on separate paper, following directions from your teacher.

**19.** The rate at which shoppers leave the store, measured in shoppers per hour, is modeled by the function $L$ defined by $L(t) = -80 + \frac{4400}{t^2 - 14t + 55}$ for $0 \le t \le 12$ . According to the model, how many shoppers are in the store at the end of the sale ( time $t = 12$ )? Give your answers to the nearest whole number.

Please respond on separate paper, following directions from your teacher.

**20.** Using the models, $S(t)$ and $L(t) = -80 + \frac{4400}{t^2 - 14t + 55}$, find the time $t$, $0 \le t \le 12$ , at which the number of shoppers in the store is the greatest. Justify your answer.

Please respond on separate paper, following directions from your teacher.