Radial RSD "toy-survey" calculation

After fixing typo, we have

$$\hat{J} = \int_V \frac{d^3x}{V} \left\{ (\hat{h}\cdot\hat{d})^2 + \mathcal{L}_2(\hat{h}\cdot\hat{d}) \left[-\frac{\hat{x}^2+1}{4} + \frac{(\hat{x}^2-1)^2}{16\hat{x}} \ln\left(\frac{\hat{x}+1}{\hat{x}-1}\right)^2 \right] \right\}$$

$$= (\hat{h}\cdot\hat{d})^2 + \mathcal{L}_2(\hat{h}\cdot\hat{d}) \int_V \frac{d^3x}{V} \left[-\frac{\hat{x}^2+1}{4} + \frac{(\hat{x}^2-1)^2}{16\hat{x}} \ln\left(\frac{\hat{x}+1}{\hat{x}-1}\right)^2 \right] \qquad \hat{x} \equiv \frac{x}{d}, \quad \hat{R} \equiv R/d$$

$$= (\hat{h}\cdot\hat{d})^2 + \mathcal{L}_2(\hat{h}\cdot\hat{d}) \left[-\frac{1}{4} - \frac{1}{4}\frac{3}{5}\hat{R}^2 + \frac{1}{16}\int_V \frac{d^3x}{V} \frac{(\hat{x}^2-1)^2}{\hat{x}} \ln\left(\frac{\hat{x}+1}{\hat{x}-1}\right)^2 \right]$$

$$\int_V \frac{d^3x}{V} = \frac{3}{\hat{R}^3} \int_0^{\hat{R}} \hat{x}^2\, d\hat{x}$$

$$\Rightarrow \frac{1}{16}\int_V \frac{d^3x}{V} \frac{(\hat{x}^2-1)^2}{\hat{x}} \ln\left(\frac{\hat{x}+1}{\hat{x}-1}\right)^2 = \frac{3}{16}\int_0^{\hat{R}} \frac{\hat{x}\, d\hat{x}}{\hat{R}^3} (\hat{x}^2-1)^2 \ln\left(\frac{\hat{x}+1}{\hat{x}-1}\right)^2$$

Now:

$$\int_0^{\hat{R}} \frac{\hat{x}\, d\hat{x}}{\hat{R}^3} (\hat{x}^2-1)^2 \ln\left(\frac{\hat{x}+1}{\hat{x}-1}\right)^2 = 2\frac{(15-10\hat{R}^2+3\hat{R}^4)}{45\hat{R}^2} + \frac{(\hat{R}^2-1)^3}{6\hat{R}^3} \ln\left(\frac{\hat{R}+1}{\hat{R}-1}\right)^2$$

Then:

$$\hat{J} = (\hat{h}\cdot\hat{d})^2 + \mathcal{L}_2(\hat{h}\cdot\hat{d}) \left[-\frac{1}{4} - \frac{3}{20}\hat{R}^2 + \frac{(15-10\hat{R}^2+3\hat{R}^4)}{120\hat{R}^2} + \frac{(\hat{R}^2-1)^3}{32\hat{R}^3} \ln\left(\frac{\hat{R}+1}{\hat{R}-1}\right)^2 \right]$$

$$\Rightarrow \boxed{\hat{J} = (\hat{h}\cdot\hat{d})^2 + \mathcal{L}_2(\hat{h}\cdot\hat{d}) \left[\frac{1}{8\hat{R}^2} - \frac{1}{3} - \frac{1}{8}\hat{R}^2 + \frac{(\hat{R}^2-1)^3}{32\hat{R}^3} \ln\left(\frac{\hat{R}+1}{\hat{R}-1}\right)^2 \right]}$$

expanding in $\hat{R} = \frac{R}{d} \ll 1$ we get

$$[\ \ ] \simeq -\frac{2}{5}\left(\frac{R}{d}\right)^2 + \frac{2}{35}\left(\frac{R}{d}\right)^4 + \frac{2}{315}\left(\frac{R}{d}\right)^6$$

(same as in old notes!)

The power then looks like, in diagonal

$$P_s = P\left[1 + 2f\hat{J} + f^2\hat{J}^2\right] + \sum_{q\neq k} \cdots$$

where $\hat{J} \simeq \mu^2 - \frac{2}{5}\mathcal{L}_2(\mu)\hat{R}^2$

$$\Rightarrow P_s = P_s^{\text{kaiser}} + P\left[-\frac{4}{5}\hat{R}^2 f\mathcal{L}_2(\mu) - \frac{4}{5}f^2\hat{R}^2\mu^2\mathcal{L}_2(\mu)\right] + \cdots$$

$$\Rightarrow \Delta P_s = P_s - P_s^{\text{kaiser}} \simeq -\frac{4}{5}P\,\mathcal{L}_2(\mu)\hat{R}^2 f(1+f\mu^2)$$

$$\Rightarrow \boxed{\frac{\Delta P_s}{P} \simeq -\frac{4}{5}\hat{R}^2 f(1+f\mu^2)\mathcal{L}_2(\mu)}$$

For $l=0$ this leads to,

$$\boxed{\left.\frac{\Delta P_s}{P}\right|_{l=0} = -\frac{4}{5}\hat{R}^2 f^2\frac{2}{15} = -\frac{8}{75}f^2\hat{R}^2}$$

or

$$\frac{\Delta P_s^{(l=0)}}{P_s^{(l=0)}} = \frac{-\frac{8}{75}f^2}{1+\frac{2}{3}f+\frac{f^2}{5}}\hat{R}^2 \underset{\substack{f=1/2 \\ \hat{R}\simeq 1/2}}{=} -\frac{2}{415} \simeq -0.0048 \simeq -0.5\%$$

For $l=2$, we get

$$\frac{\Delta P_s^{(l=2)}}{P_s^{(l=2)}} = -\frac{4}{105}\frac{f(21+11f)\hat{R}^2}{\frac{4}{3}f+\frac{4}{7}f^2} \simeq -0.156 = -15.6\% \;!$$

However, this is NOT the estimator we use for the quadrupole, that is, we do NOT calculate quadrupole by doing integral with

$\int_{-1}^{1}\frac{5}{2}\mathcal{L}_2(\mu)\,d\mu\,P_s(k,\mu)$ for $\mu = \hat{k}\cdot\hat{d}$. We have a different estimator for $l=2$ for radial distortions.

For $\ell>0$ we need to construct the radial-distortion estimator of multipoles, following

$$\hat{P}_\ell(k) = (2\ell+1)\int \frac{d\Omega_k}{4\pi}\, \delta_\ell(\vec{k})\, \delta_0(-\vec{k}) \qquad P_\ell(k) = \langle \hat{P}_\ell(k)\rangle$$

LOS unit vector

where $\delta_\ell(\vec{k}) = \frac{1}{N_g^3}\sum_{\vec{x}} e^{-i\vec{k}\cdot\vec{x}}\, \delta_s(\vec{x})\, \mathcal{L}_\ell(\hat{k}\cdot\hat{x})$

i.e. neglecting $\frac{2\hat{r}\cdot\vec{u}}{|\vec{x}+\vec{d}|}$ vs. $\hat{r}_i\hat{r}_j\nabla_i u_j$

using $\delta_s(\vec{x}) = \delta(\vec{x}) + f\, \nabla\cdot[\hat{x}(\hat{x}\cdot\vec{u})]$

and dropping again "boundary term" we have,

$$\delta_\ell(\vec{k}) = \frac{1}{N_g^3}\sum_{\vec{x}} e^{-i\vec{k}\cdot\vec{x}}\, \delta(\vec{x})\, \mathcal{L}_\ell(\hat{k}\cdot\hat{x}) + f\,\frac{1}{N_g^3}\sum_{\vec{x}} e^{-i\vec{k}\cdot\vec{x}}\, \mathcal{L}_\ell(\hat{k}\cdot\hat{x})\, \hat{r}_i\hat{r}_j\nabla_i u_j$$

$$\delta_{\ell}(\vec{k}) = \sum_{\vec{q}} \delta(\vec{q}) \underbrace{\left[\frac{1}{N_g^3}\sum_{\vec{x}} e^{-i(\vec{k}-\vec{q})\cdot\vec{x}}\, \mathcal{L}_\ell(\hat{k}\cdot\hat{x})\right]}_{I_\ell(\vec{k},\vec{q})} + \sum_{\vec{q}} \theta(\vec{q})\frac{q_iq_j}{q^2} \underbrace{\left[\frac{1}{N_g^3}\sum_{\vec{x}} e^{-i(\vec{k}-\vec{q})\cdot\vec{x}}\, \mathcal{L}_\ell(\hat{k}\cdot\hat{x})\, \hat{r}_i\hat{r}_j\right]}_{J^{(\ell)}_{ij}(\vec{k},\vec{q})}$$

Now, the calculation of $\ell=2$ for example follows same path as before, only that now we have to calculate two more integrals -

$$\delta_\ell(\vec{k}) = \sum_{\vec{q}} \delta(\vec{q})\, I_\ell(\vec{k},\vec{q}) + \sum_{\vec{q}} \theta(\vec{q})\frac{q_iq_j}{q^2} J^{(\ell)}_{ij}(\vec{k},\vec{q})$$

$\delta(\vec{k}) + f\sum_{\vec{p}}\theta_{\vec{p}}\,\frac{p_ip_j}{p^2} J^{(\ell')}_{ij}(\vec{k},\vec{p})$

$$P_\ell(k) = (2\ell+1)\int\frac{d\Omega_k}{4\pi}\left\langle \sum_{\vec{q}}\left[\delta(\vec{q})\, I_\ell(\vec{k},\vec{q}) + f\theta(\vec{q})\frac{q_iq_j}{q^2} J^{(\ell)}_{ij}(\vec{k},\vec{q})\right]\left[\sum_{\vec{p}}[\ \ ]_{\ell'}\right]\right\rangle \qquad (\ell'=0)$$

$$= \cancel{(2\ell+1)\int\frac{d\Omega_k}{4\pi}\sum_{\vec{q}} P(\vec{q})\, I_\ell(\vec{k},\vec{q})}$$

$$= (2\ell+1)\int\frac{d\Omega_k}{4\pi}\Big\{P(\vec{k})\, I_\ell(\vec{k},+\vec{k}) + f\,\frac{k_ik_j}{k^2} J^{(\ell)}_{ij}(\vec{k},+\vec{k})\, P(k) +$$

$$+ f\sum_{\vec{q}} P(\vec{q})\, I_\ell(\vec{k},\vec{q})\frac{q_iq_j}{q^2} J_{ij}(-\vec{k}+\vec{q}) + f^2\sum_{\vec{q}} P(\vec{q})\left[\frac{q_iq_j}{q^2} J^{(\ell)}_{ij}(\vec{k},\vec{q})\right]\left[\frac{q_iq_j}{q^2} J_{ij}(-\vec{k}+\vec{q})\right]\Big\}$$

Neglecting $\vec{q} \neq \vec{k}$ terms (this has to be checked but we expect such contributions to decay quickly)

we have, ④

$$\frac{P_\ell(k)}{P(k)} = (2\ell+1)\int \frac{d\Omega_k}{4\pi}\left[I_\ell(\vec{k},\vec{k}) + f\,\frac{1}{N_g^3}\sum_{\vec{x}} \mathcal{L}_\ell(\hat{k}\cdot\hat{r})\,(\hat{k}\cdot\hat{r})^2 \right.$$

$$+ f\, I_\ell(\vec{k},\vec{k})\,\frac{1}{N_g^3}\sum_{\vec{x}} (\hat{k}\cdot\hat{r})^2 +$$

$$\left. + f^2\,\frac{1}{N_g^3}\sum_{\vec{x}} \mathcal{L}_\ell(\hat{k}\cdot\hat{r})\,(\hat{k}\cdot\hat{r})^2\,\frac{1}{N_g^3}\sum_{\vec{x}'}(\hat{k}\cdot\hat{r}')^2 \right]$$

with $I_\ell(\vec{k},\vec{k}) = \frac{1}{N_g^3}\sum_{\vec{x}} \mathcal{L}_\ell(\hat{k}\cdot\hat{r})$

① So, first term is just:

$$(2\ell+1)\int\frac{d\Omega_k}{4\pi}\,\frac{1}{N_g^3}\sum_{\vec{x}}\mathcal{L}_\ell(\hat{k}\cdot\hat{r}) = (2\ell+1)\,\frac{1}{N_g^3}\sum_{\vec{x}}\underbrace{\int\frac{d\Omega_k}{4\pi}\,\mathcal{L}_\ell(\hat{k}\cdot\hat{r})}_{\delta_{\ell 0}} = \delta_{\ell 0}$$

② Next term:

$$(2\ell+1)\int\frac{d\Omega_k}{4\pi}\,\frac{1}{N_g^3}\sum_{\vec{x}}\mathcal{L}_\ell(\hat{k}\cdot\hat{r})\,(\hat{k}\cdot\hat{r})^2 = \frac{1}{N_g^3}\sum_{\vec{x}}\,(2\ell+1)\int\frac{d\Omega_k}{4\pi}\,\mathcal{L}_\ell(\hat{k}\cdot\hat{r})\,(\hat{k}\cdot\hat{r})^2$$

$$= \frac{1}{N_g^3}\sum_{\vec{x}}\,(2\ell+1)\int\frac{d\Omega_k}{4\pi}\,\mathcal{L}_\ell(\hat{k}\cdot\hat{r})\left[\frac{2}{3}\mathcal{L}_2(\hat{k}\cdot\hat{r}) + \frac{1}{3}\mathcal{L}_0(\hat{k}\cdot\hat{r})\right]$$

$$= \frac{1}{N_g^3}\sum_{\vec{x}}\left[\frac{2}{3}\delta_{\ell 2} + \frac{1}{3}\delta_{\ell 0}\right] = \frac{2}{3}\delta_{\ell 2} + \frac{1}{3}\delta_{\ell 0}$$

③ The third term is:

$$(2\ell+1)\int\frac{d\Omega_k}{4\pi}\,\frac{1}{N_g^3}\sum_{\vec{x}}\mathcal{L}_\ell(\hat{k}\cdot\hat{r})\,\frac{1}{N_g^3}\sum_{\vec{x}'}(\hat{k}\cdot\hat{r}')^2$$

$$= \frac{1}{N_g^3}\sum_{\vec{x}}\,\frac{1}{N_g^3}\sum_{\vec{x}'}\,(2\ell+1)\int\frac{d\Omega_k}{4\pi}\,\mathcal{L}_\ell(\hat{k}\cdot\hat{r})\,(\hat{k}\cdot\hat{r}')^2$$

Now:

$$(2l+1)\int \frac{d\Omega k}{4\pi}\, \mathcal{L}_l(\hat{k}\cdot\hat{r})\,(\hat{k}\cdot\hat{r}')^2 = \hat{r}'_i\,\hat{r}'_j\,(2l+1)\int\frac{d\Omega k}{4\pi}\,\mathcal{L}_l(\hat{k}\cdot\hat{r})\,\hat{k}_i\,\hat{k}_j$$

And using tensorial decomposition,

$$(2l+1)\int\frac{d\Omega k}{4\pi}\,\mathcal{L}_l(\hat{k}\cdot\hat{r})\,\hat{k}_i\,\hat{k}_j = \delta_{l0}\,\frac{\delta_{ij}}{3} + \delta_{l2}\left(\hat{r}_i\,\hat{r}_j - \frac{\delta_{ij}}{3}\right)$$

$$\Rightarrow (2l+1)\int\frac{d\Omega k}{4\pi}\,\mathcal{L}_l(\hat{k}\cdot\hat{r})\,(\hat{k}\cdot\hat{r}')^2 = \frac{1}{3}\delta_{l0} + \delta_{l2}\left[(\hat{r}\cdot\hat{r}')^2 - \frac{1}{3}\right]$$

and thus the third term is,

$$\frac{1}{N_g^3}\sum_{x}\frac{1}{N_g^3}\sum_{x'}\left\{\frac{1}{3}\delta_{l0} + \delta_{l2}\left[(\hat{r}\cdot\hat{r}')^2 - \frac{1}{3}\right]\right\} =$$

$$= \frac{1}{3}\left(\delta_{l0} - \delta_{l2}\right) + \delta_{l2}\,\frac{1}{N_g^3}\sum_{x}\frac{1}{N_g^3}\sum_{x'}(\hat{r}\cdot\hat{r}')^2$$

Now we need to calculate the double sum

$$\frac{1}{N_g^3}\sum_{x}\frac{1}{N_g^3}\sum_{x'}(\hat{r}\cdot\hat{r}')^2$$

but $\frac{1}{N_g^3}\sum_{x'}(\hat{r}\cdot\hat{r}')^2$ is the same calculation as $\hat{J}(\hat{r},-\hat{r})$

$$= (\hat{r}\cdot\hat{a})^2 + \mathcal{L}_2(\hat{r}\cdot\hat{a})\left[\frac{1}{8\hat{R}^2} - \frac{1}{3} - \frac{\hat{R}^2}{8} + \frac{(\hat{R}^2-1)^3}{32\hat{R}^3}\ln\left(\frac{\hat{R}+1}{\hat{R}-1}\right)^2\right]$$

So:

$$\frac{1}{N_g^3}\sum_{x}\frac{1}{N_g^3}\sum_{x'}(\hat{r}\cdot\hat{r}')^2 = \frac{1}{N_g^3}\sum_{x}(\hat{r}\cdot\hat{a})^2 + [\;]\left(\frac{3}{2}\,\frac{1}{N_g^3}\sum_{x}(\hat{r}\cdot\hat{a})^2 - \frac{1}{2}\right)$$

but $\frac{1}{N_g^3}\sum_{x}(\hat{r}\cdot\hat{a})^2$ is $\hat{J}(\hat{a},-\hat{a})$ so:

$$= 1 + [\;] + [\;]\left(\frac{3}{2}[\;] + \frac{3}{2} - \frac{1}{2}\right) = 1 + 2[\;] + \frac{3}{2}[\;]^2$$

So

$$\frac{1}{N_g^3}\sum_{\vec{x}} \frac{1}{N_g^3}\sum_{\vec{x}} (\hat{r}\cdot\hat{r}')^2 = 1 + 2[\ ] + \frac{3}{2}[\ ]^2 \equiv \mathcal{D}$$

$$[\ ] \equiv \frac{1}{8\hat{R}^2} - \frac{1}{3} - \frac{\hat{R}^2}{8} + \frac{(\hat{R}^2-1)^3}{32\hat{R}^3}\ln\left(\frac{\hat{R}+1}{\hat{R}-1}\right)^2$$

with expansion $\mathcal{D} \simeq 1 + 2\left(-\frac{2}{5}\right)\left(\frac{R}{d}\right)^2 + \cdots$

$$= 1 - \frac{4}{5}\hat{R}^2$$

Thus, the third term is then,

$$\frac{1}{3}(\delta_{\ell 0} - \delta_{\ell 2}) + \delta_{\ell 2}\,\mathcal{D}$$

(4) The last $\mathcal{O}(f^2)$ term is,

$$\frac{1}{N_g^3}\sum_{\vec{x}} \frac{1}{N_g^3}\sum_{\vec{x}'} (2\ell+1)\int \frac{d^2k}{4\pi}\, \mathcal{L}_\ell(\hat{k}\cdot\hat{r})\,(\hat{k}\cdot\hat{r})^2\,(\hat{k}\cdot\hat{r}')^2$$

Now we need tensorial decomposition with 4 indices:

$$\int f(\hat{k}\cdot\hat{r})\,\hat{k}_i\hat{k}_j\hat{k}_k\hat{k}_\ell \frac{d^2k}{4\pi} = \left\{\hat{r}_i\hat{r}_j\hat{r}_k\hat{r}_\ell - \frac{1}{7}[\delta_{ij}\hat{r}_k\hat{r}_\ell + \text{cyc.}] + \frac{1}{35}[\delta_{ij}\delta_{k\ell} + \text{cyc.}]\right\}\int f\,\mathcal{L}_4(\hat{k}\cdot\hat{r})\frac{d^2k}{4\pi}$$

$$+ \left\{\frac{1}{7}[\delta_{ij}\hat{r}_k\hat{r}_\ell + \text{cyc.}] - \frac{2}{21}[\delta_{ij}\delta_{k\ell} + \text{cyc.}]\right\}\int f\,\mathcal{L}_2(\hat{k}\cdot\hat{r})\frac{d^2k}{4\pi}$$

$$+ \frac{1}{15}[\delta_{ij}\delta_{k\ell} + \text{cyc.}]\int f\,\mathcal{L}_0(\hat{k}\cdot\hat{r})\frac{d^2k}{4\pi}$$

where

$$\delta_{ij}\hat{r}_k\hat{r}_\ell + \text{cyc.} = \delta_{ij}\hat{r}_k\hat{r}_\ell + \delta_{jk}\hat{r}_\ell\hat{r}_i + \delta_{k\ell}\hat{r}_i\hat{r}_j + \delta_{\ell i}\hat{r}_j\hat{r}_k + \delta_{ik}\hat{r}_j\hat{r}_\ell + \delta_{j\ell}\hat{r}_i\hat{r}_k$$

$$\delta_{ij}\delta_{k\ell} + \text{cyc.} = \delta_{ij}\delta_{k\ell} + \delta_{jk}\delta_{\ell i} + \delta_{ik}\delta_{j\ell}$$

That expression can be checked by contracting with $\delta_{ij}\delta_{k\ell}$, $\delta_{ij}\hat{r}_k\hat{r}_\ell$ and $\hat{r}_i\hat{r}_j\hat{r}_k\hat{r}_\ell$ both sides and verifying it works. Now, in our case $f = (2\ell+1)\mathcal{P}_\ell$, so using orthogonality we have,

$$(2\ell+1)\int\frac{d^2k}{4\pi}\,\mathcal{P}_\ell(\hat{k}\cdot\hat{r})\,\hat{k}_i\hat{k}_j\hat{k}_k\hat{k}_\ell =$$

$$= \left\{\hat{r}_i\hat{r}_j\hat{r}_k\hat{r}_\ell - \frac{1}{7}\left[\delta_{ij}\hat{r}_k\hat{r}_\ell + \text{cyc.}\right] + \frac{1}{35}\left[\delta_{ij}\delta_{k\ell} + \text{cyc.}\right]\right\}\delta_{\ell 4}$$

$$+ \left\{\frac{1}{7}\left[\delta_{ij}\hat{r}_k\hat{r}_\ell + \text{cyc.}\right] - \frac{2}{21}\left[\delta_{ij}\delta_{k\ell} + \text{cyc.}\right]\right\}\delta_{\ell 2}$$

$$+ \frac{1}{15}\left[\delta_{ij}\delta_{k\ell} + \text{cyc.}\right]\delta_{\ell 0}$$

Thus the fourth term reads:

$$\frac{1}{N_g^3}\sum_{x}\frac{1}{N_g^3}\sum_{x'}\Big\{\Big[(\hat{r}\cdot\hat{r}')^2 - \frac{1}{7}\left(5(\hat{r}\cdot\hat{r}')^2+1\right) + \frac{1}{35}\left(1+2(\hat{r}\cdot\hat{r}')^2\right)\Big]\delta_{\ell 4}$$

$$+\Big[\frac{1}{7}\left(5(\hat{r}\cdot\hat{r}')^2+1\right) - \frac{2}{21}\left(1+2(\hat{r}\cdot\hat{r}')^2\right)\Big]\delta_{\ell 2}$$

$$+ \frac{1}{15}\left(1+2(\hat{r}\cdot\hat{r}')^2\right)\delta_{\ell 0}\Big\}$$

$$= \mathcal{D}\left\{\Big[\overbrace{1 - \frac{5}{7} + \frac{2}{35}}^{12/35}\Big]\delta_{\ell 4} + \Big(\overbrace{\frac{5}{7} - \frac{4}{21}}^{11/21}\Big)\delta_{\ell 2} + \frac{2}{15}\delta_{\ell 0}\right\}$$

$$+ \underbrace{\Big[-\frac{1}{7} + \frac{1}{35}\Big]}_{-4/35}\delta_{\ell 4} + \underbrace{\Big(\frac{1}{7} - \frac{2}{21}\Big)}_{1/21}\delta_{\ell 2} + \frac{1}{15}\delta_{\ell 0}$$

$$= \boxed{\left(\frac{1}{15}\delta_{\ell 0} + \frac{1}{21}\delta_{\ell 2} - \frac{4}{35}\delta_{\ell 4}\right) + \left(\frac{2}{15}\delta_{\ell 0} + \frac{11}{21}\delta_{\ell 2} + \frac{12}{35}\delta_{\ell 4}\right)\mathcal{D}}$$

We can now put things together:

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$$\boxed{\frac{P_\ell(k)}{P(k)}} = \delta_{\ell 0} + f\left(\tfrac{1}{3}\delta_{\ell 0} + \tfrac{2}{3}\delta_{\ell 2}\right) + f\left(\tfrac{1}{3}(\delta_{\ell 0}-\delta_{\ell 2}) + \mathcal{D}\,\delta_{\ell 2}\right)$$

$$+ f^2\left[\left(\tfrac{1}{15}\delta_{\ell 0} + \tfrac{1}{21}\delta_{\ell 2} - \tfrac{4}{35}\delta_{\ell 4}\right) + \left(\tfrac{2}{15}\delta_{\ell 0} + \tfrac{11}{21}\delta_{\ell 2} + \tfrac{12}{35}\delta_{\ell 4}\right)\mathcal{D}\right]$$

$$\boxed{= \delta_{\ell 0} + f\left[\tfrac{2}{3}\delta_{\ell 0} + \left(\tfrac{1}{3}+\mathcal{D}\right)\delta_{\ell 2}\right] + f^2\left[\frac{1+2\mathcal{D}}{15}\delta_{\ell 0} + \frac{1+11\mathcal{D}}{21}\delta_{\ell 2} + \frac{12\mathcal{D}-4}{35}\delta_{\ell 4}\right]}$$

Now we can double check multipole by multipole:

$\ell=0$: $$\boxed{\frac{\Delta P_0}{P_0}} = \frac{2f^2}{15}\frac{(\mathcal{D}-1)}{1+\frac{2}{3}f+\frac{f^2}{5}} \underset{\mathcal{D}-1 \simeq -\frac{4}{5}\hat{R}^2}{\simeq} \boxed{-\frac{8f^2}{75}\frac{\hat{R}^2}{1+\frac{2}{3}f+\frac{f^2}{5}}}$$ (same result as before)

$\ell=2$: $$\frac{\Delta P_2}{P_2} = \frac{f(\mathcal{D}-1) + f^2\frac{11}{21}(\mathcal{D}-1)}{\frac{4}{3}f + \frac{4}{7}f^2} \simeq -\frac{4}{5}\hat{R}^2\,\frac{\cancel{f}(1+\frac{11}{21}f)}{\frac{4\cancel{f}}{3}(1+\frac{3}{7}f)}$$

$$\Rightarrow \boxed{\frac{\Delta P_2}{P_2} \simeq -\frac{3}{5}\hat{R}^2\,\frac{1+\frac{11}{21}f}{1+\frac{3}{7}f}} \underset{\substack{f\sim 1/2 \\ \hat{R}\sim 1/2}}{\approx} -\frac{53}{340} \simeq -0.156$$

15.6% HUGE!

Note it gives same result as doing std fixed-LOS ~~[illegible]~~ quadrupole, so something is wrong obviously!

$$\boxed{\frac{\Delta P_4}{P_4}} = \frac{\frac{12\cancel{f^2}}{\cancel{35}}(\mathcal{D}-1)}{\frac{8\cancel{f^2}}{\cancel{35}}} = \frac{3}{2}(\mathcal{D}-1) \simeq \boxed{-\frac{6}{5}\hat{R}^2}$$