Solution to Problem 1:

From the given data, σ, the average of SD of every sample is equal to 0.157, and $\bar{\theta}$ the average of the whole dataset is 5.718.

Given $(\theta_{1:11}, \tau), \mu$ is normally distributed with

$$SD = \left(\frac{1}{0.22^2} + \frac{11}{\tau^2}\right)^{-1/2} = \left(20.66 + \frac{11}{\tau^2}\right)^{-1/2}$$

$$mean = \frac{\frac{5.25}{0.22^2} + \frac{11 \times \bar{\theta}}{\tau^2}}{\frac{1}{0.22^2} + \frac{11}{\tau^2}} = \frac{108.47 + \frac{62.898}{\tau^2}}{20.66 + \frac{11}{\tau^2}}$$

Given $(\theta_{1:11}, \mu)$, ${}^{1}/_{\tau^2}$ has a gamma distribution with (subscript i stands for individual)

$$shape = \frac{1}{2} + \frac{11}{2} = 6$$

$$scale = \left(\frac{1}{50} + \frac{1}{2}\sum_{i=1}^{11}(\theta_i - \mu)^2\right)^{-1} = \left(\frac{1}{50} + \frac{1}{2}\sum_{i=1}^{11}\left(\theta_i{}^2 + \mu^2 - 2\theta_i\mu\right)\right)^{-1}$$

$$= \left(\frac{1}{50} + \frac{1}{2}(359.77 + 11\mu^2 - 2 \times 62.9\mu)\right)^{-1} = \frac{1}{11\mu^2 - 125.811\mu + 179.9}$$

Given $(\mu, \tau, y_{1:11})$, each θ_s is normally distributed with (subscript i stands for individual and subscript s stands for sample)

$$SD = \left(\frac{1}{\tau^2} + \frac{30}{0.157^2}\right)^{-1/2} = \left(\frac{1}{\tau^2} + 1217.09\right)^{-1/2}$$

$$mean_i = \frac{\frac{\sum_{s=1}^{30} y_{si}}{0.157^2} + \frac{\mu}{\tau^2}}{\frac{30}{0.157^2} + \frac{1}{\tau^2}} = \frac{1217.09 \sum_{i=1}^{30} y_{si} + \frac{\mu}{\tau^2}}{1217.09 + \frac{1}{\tau^2}}$$

Solution to Problem 2:

A JAGS model was used to perform Gibbs sampling in this part. The resulting estimated posterior distribution for μ and τ are plotted in figures 1 and 2 respectively.

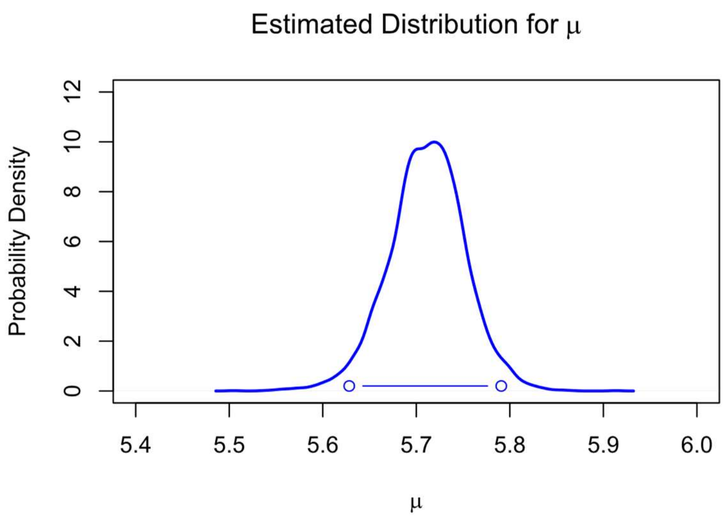


Figure 1

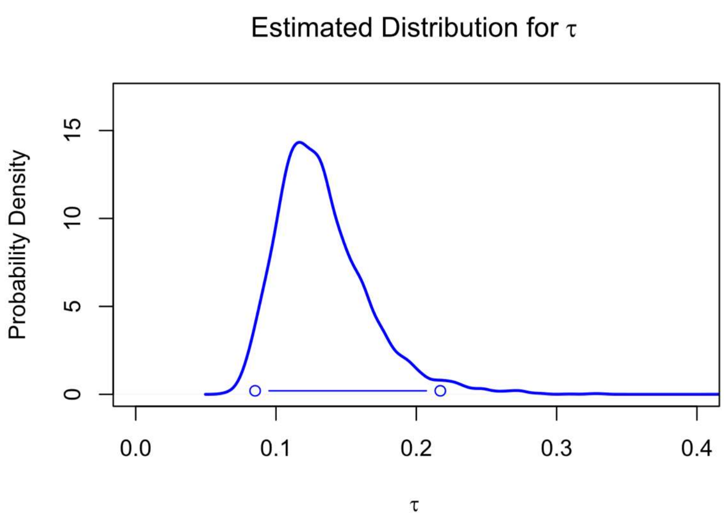


Figure 2

The 95% confidence interval for μ, τ, and θ for each individual are presented in the following tables.

	2.5% quantile	97.5% quantile
μ	5.63	5.79
τ	0.085	0.217

2.5%	5.675	5.830	5.658	5.649	5.521	5.740	5.803	5.533	5.493	5.718	5.661
97.5%	5.786	5.940	5.763	5.762	5.632	5.849	5.914	5.642	5.607	5.829	5.772

Solution to Problem 3:

Using the following equations from part 1, posterior distributions for mean of results of each of 11 individuals were calculated.

$$SD = \left(\frac{1}{\tau^2} + \frac{30}{0.157^2}\right)^{-1/2} = \left(\frac{1}{0.117^2} + 1217.09\right)^{-1/2} = 0.02783$$

$$mean_s = \frac{\frac{\sum_{i=1}^{30} y_{si}}{0.157^2} + \frac{\mu}{\tau^2}}{\frac{30}{0.157^2} + \frac{1}{\tau^2}} = \frac{\frac{\sum_{i=1}^{30} y_{si}}{0.157^2} + \frac{5.718}{0.117^2}}{\frac{30}{0.157^2} + \frac{1}{0.117^2}} = 0.031 \sum_{i=1}^{30} y_{si} + 0.324$$

11 means and 95% confidence intervals are presented in the following table.

mean	5.731	5.886	5.711	5.706	5.579	5.794	5.859	5.588	5.551	5.774	5.716
2.5%	5.677	5.832	5.656	5.651	5.524	5.740	5.804	5.534	5.497	5.720	5.662
97.5%	5.786	5.941	5.765	5.760	5.633	5.845	5.913	5.643	5.606	5.829	5.771

Solution to Problem 4:

A Q-Q plot in figure 3 of the entire dataset shows normal distribution is a good assumption for the dataset except for about 10 datapoints.

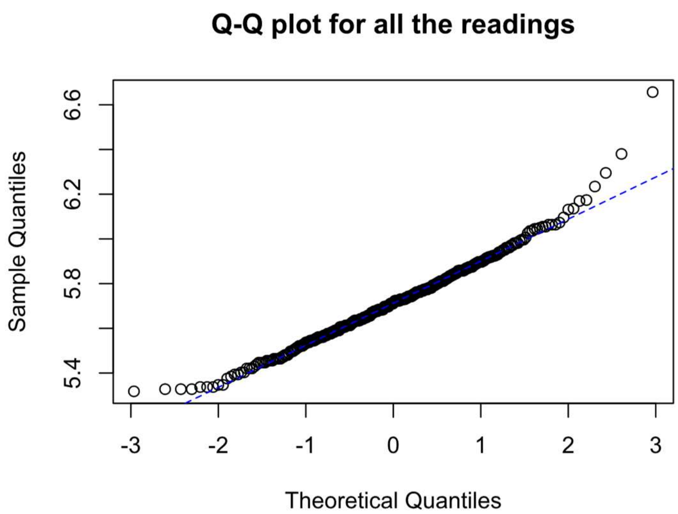


Figure 3

Traceplots and auto correlation functions for μ and τ are illustrated in figures 4 and 5

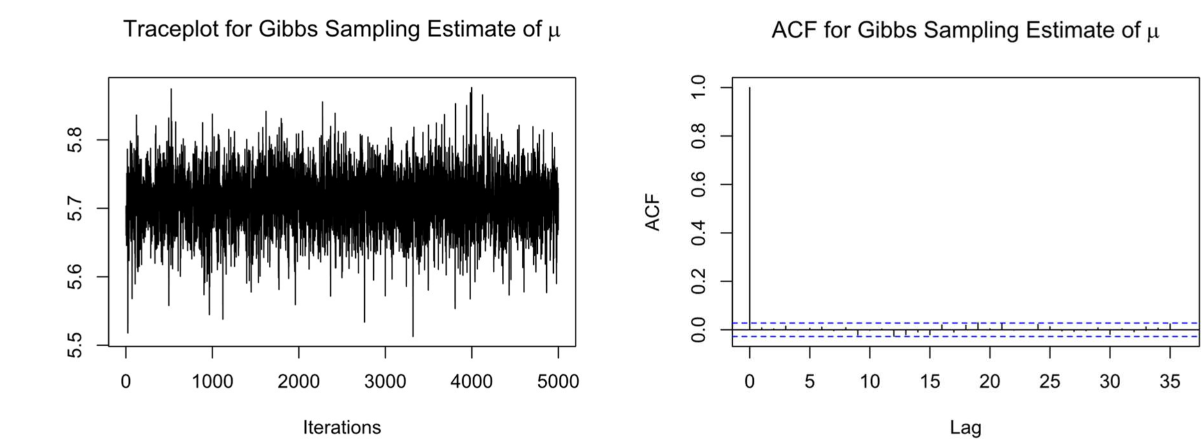


Figure 4

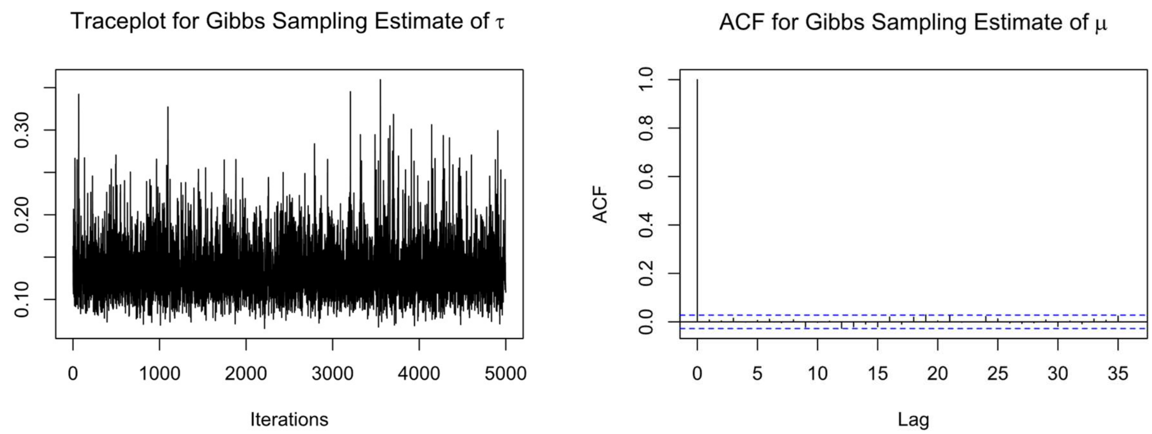


Figure 5

Shrinkage plots for sample means are presented in figure 6, showing a good estimate.

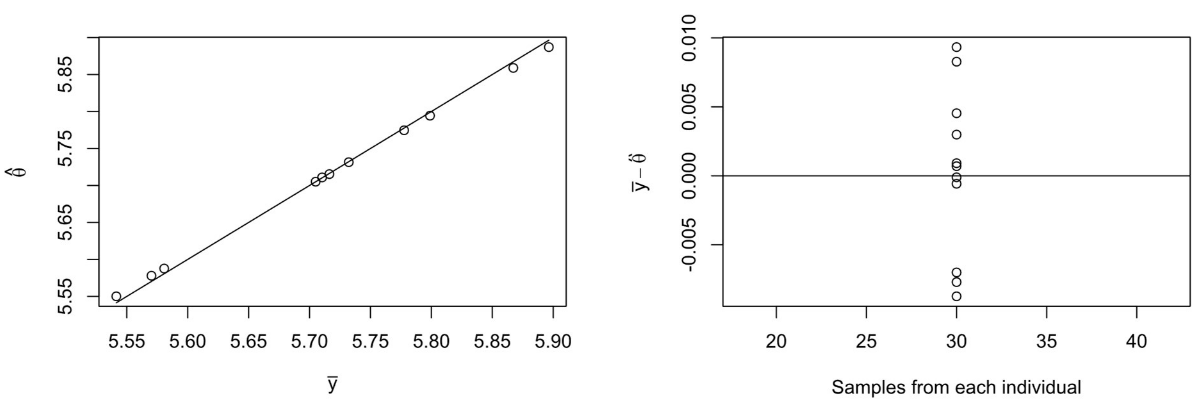
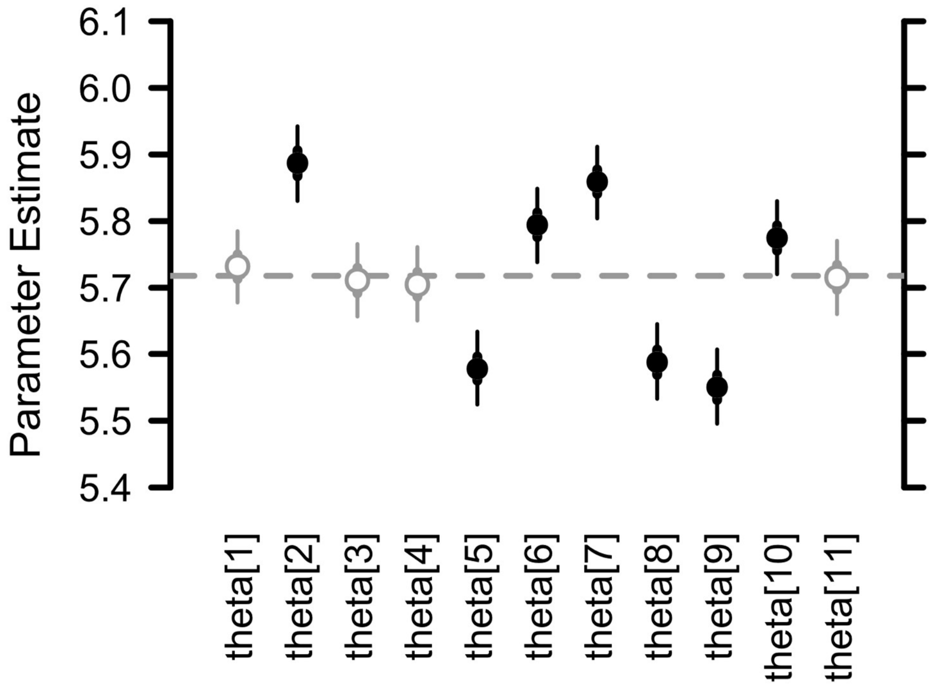
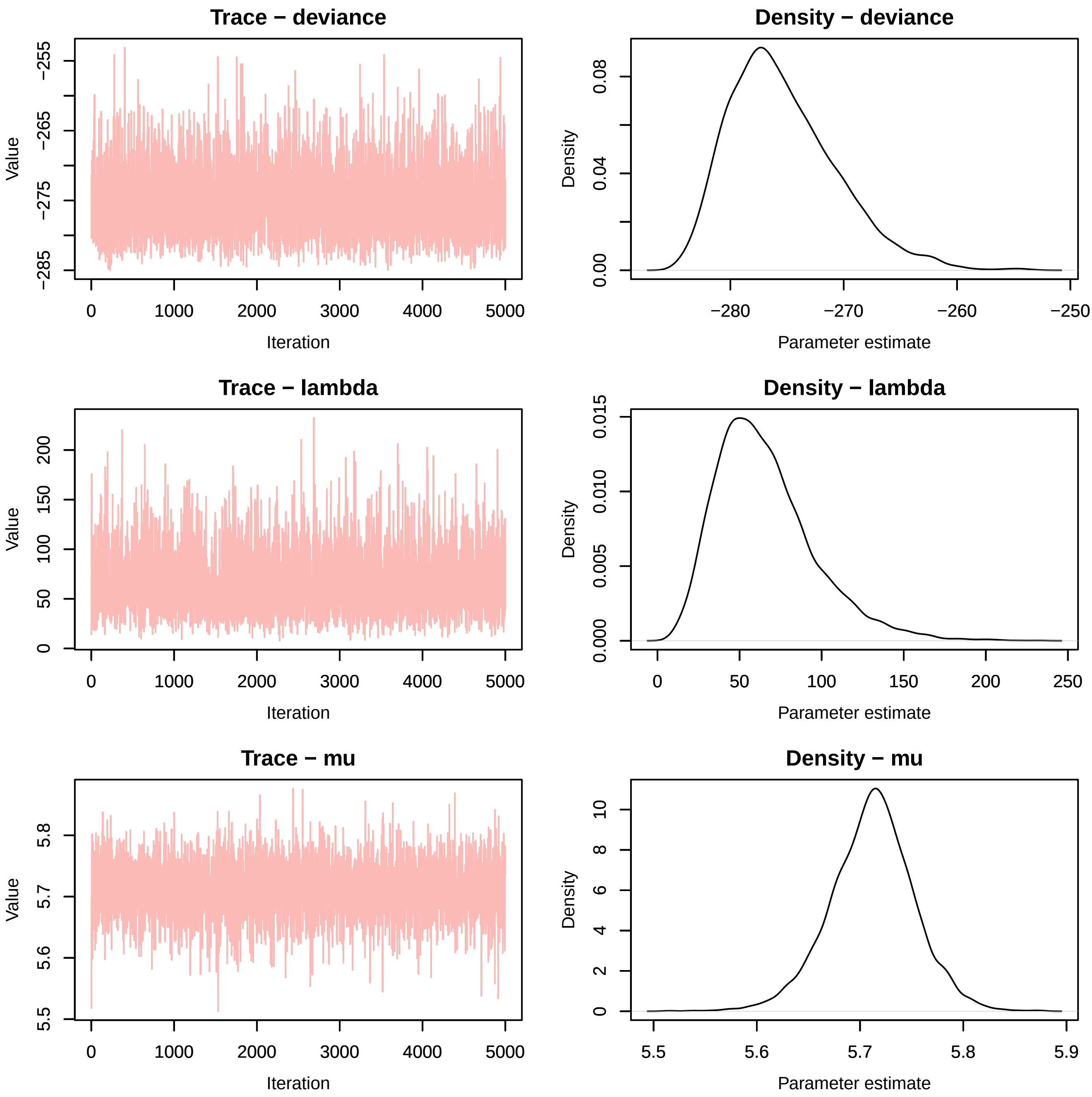


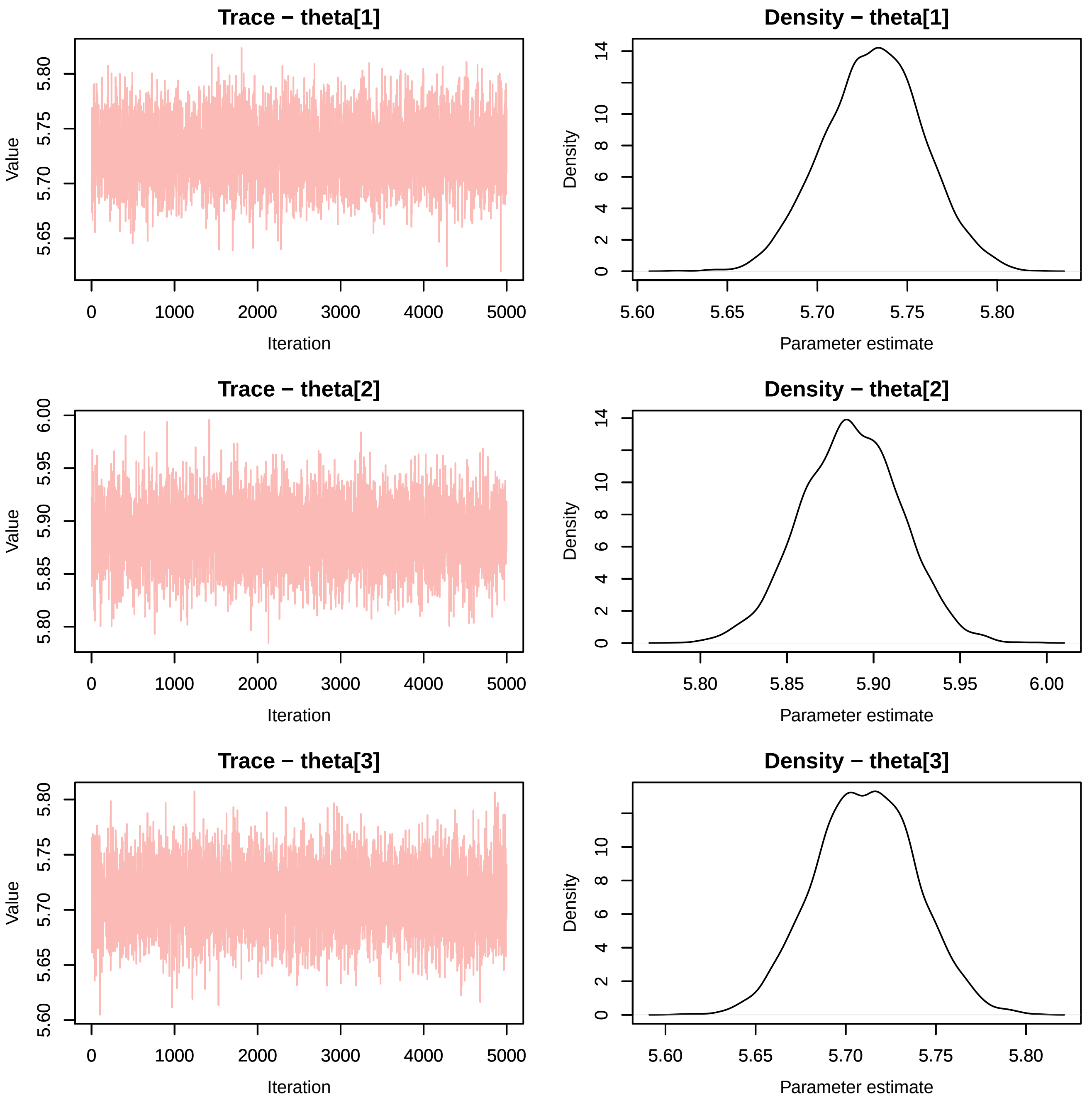
Figure 6

The caterpillar plot in figure 7 shows, for 4 individuals the mean is inside the 50% intervals, while for the others the mean is outside their 95% interval.

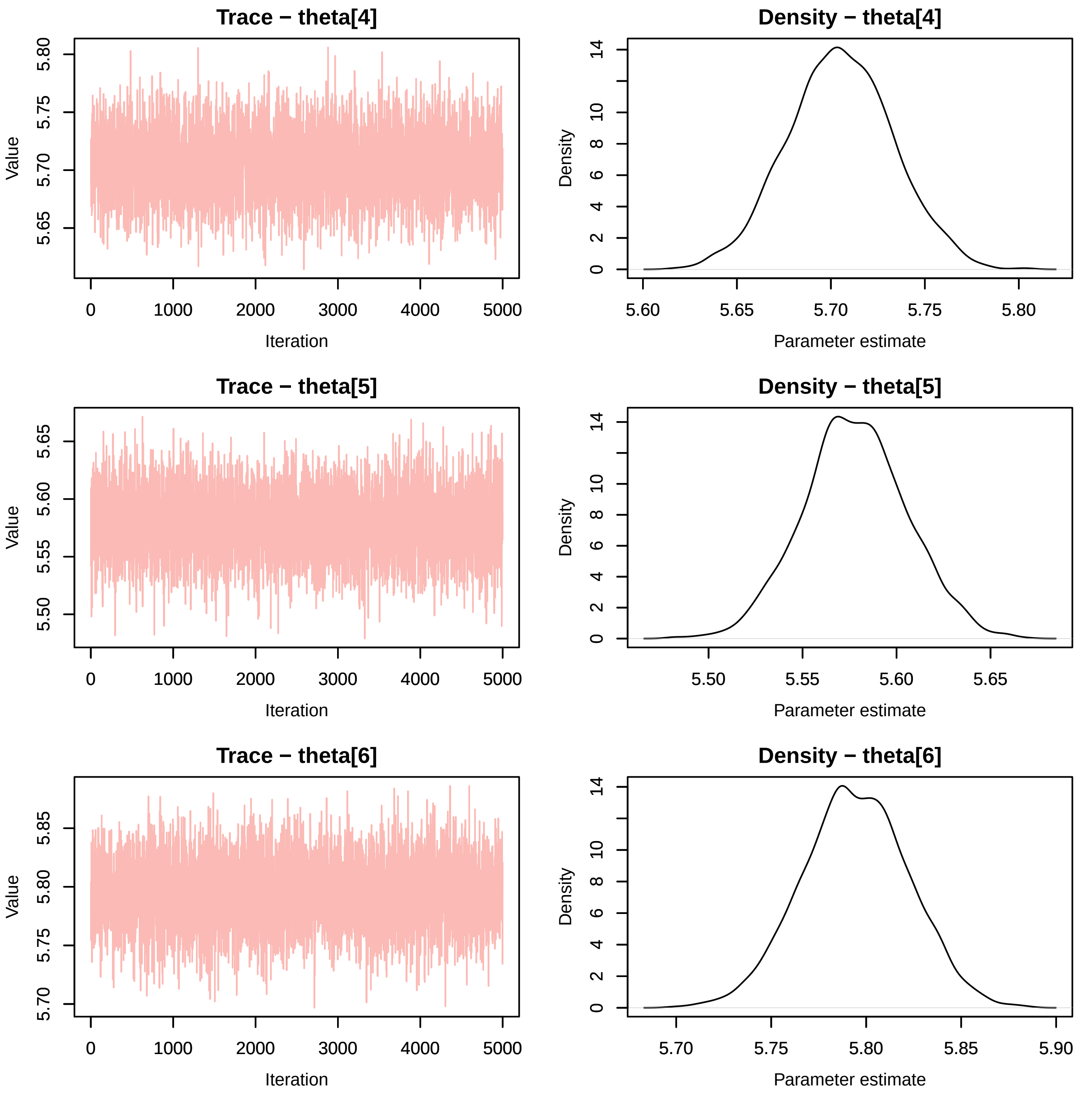


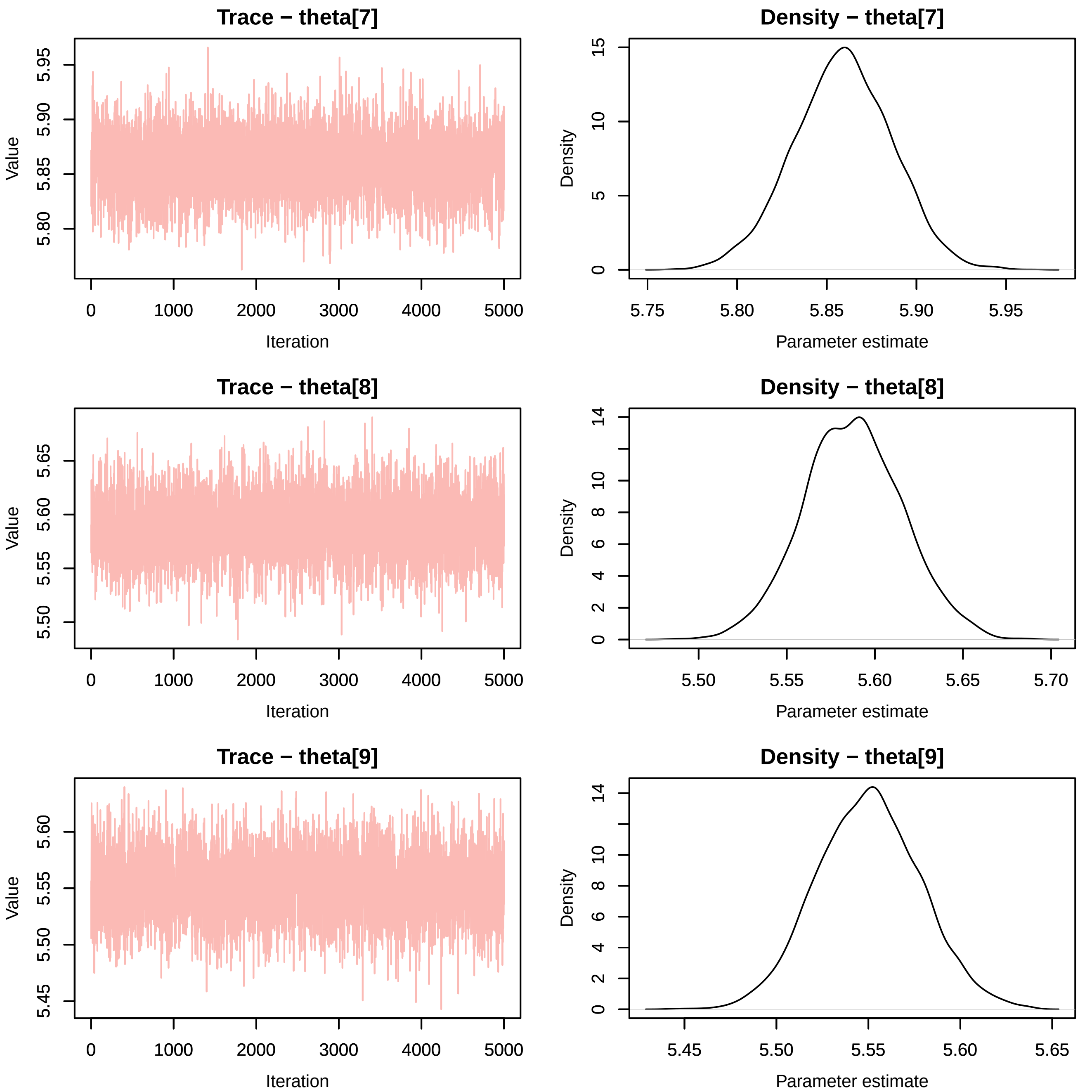
The rest of traceplots and estimate density distributions are presented in the following 5 pages. All the evidence shows the assumptions were sound and the sampling process provided us with acceptable estimates for posterior distributions.


Trace − deviance
Value
Iteration
Density − deviance
Density
Parameter estimate
Trace − lambda
Value
Iteration
Density − lambda
Density
Parameter estimate
Trace − mu
Value
Iteration
Density − mu
Density
Parameter estimate

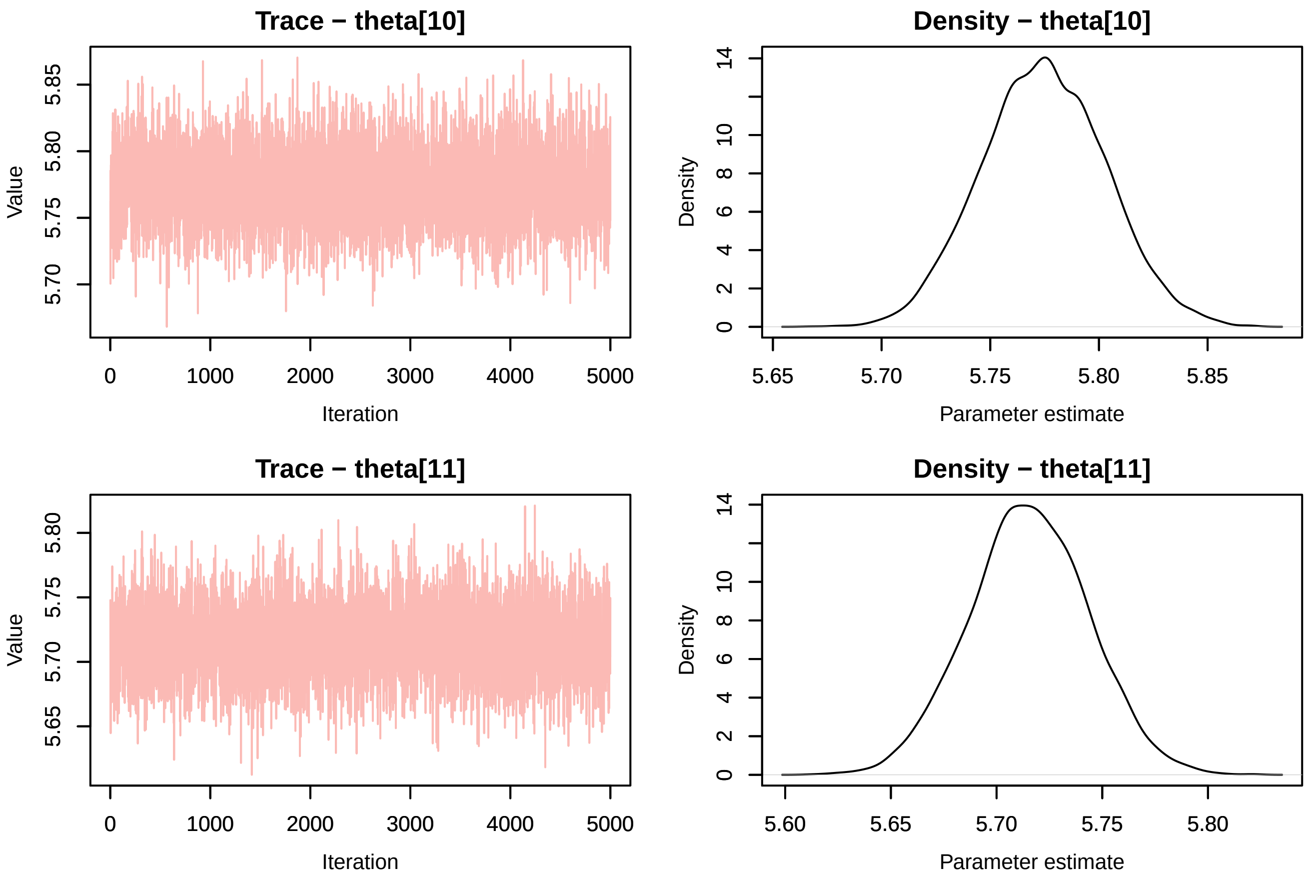


Trace − theta[1]
Value
5.65
5.70
5.75
5.80
0
1000
2000
3000
4000
5000
Iteration
Density − theta[1]
Density
0
2
4
6
8
10
14
5.60
5.65
5.70
5.75
5.80
Parameter estimate
Trace − theta[2]
Value
5.80
5.85
5.90
5.95
6.00
0
1000
2000
3000
4000
5000
Iteration
Density − theta[2]
Density
0
2
4
6
8
10
14
5.80
5.85
5.90
5.95
6.00
Parameter estimate
Trace − theta[3]
Value
5.60
5.65
5.70
5.75
5.80
0
1000
2000
3000
4000
5000
Iteration
Density − theta[3]
Density
0
2
4
6
8
10
5.60
5.65
5.70
5.75
5.80
Parameter estimate


Trace − theta[4]
Value
5.65
5.70
5.75
5.80
0
1000
2000
3000
4000
5000
Iteration
Density − theta[4]
Density
0
2
4
6
8
10
14
5.60
5.65
5.70
5.75
5.80
Parameter estimate
Trace − theta[5]
Value
5.50
5.55
5.60
5.65
0
1000
2000
3000
4000
5000
Iteration
Density − theta[5]
Density
0
2
4
6
8
10
14
5.50
5.55
5.60
5.65
Parameter estimate
Trace − theta[6]
Value
5.70
5.75
5.80
5.85
0
1000
2000
3000
4000
5000
Iteration
Density − theta[6]
Density
0
2
4
6
8
10
14
5.70
5.75
5.80
5.85
5.90
Parameter estimate



Trace − theta[7]
Value
5.80
5.85
5.90
5.95
0
1000
2000
3000
4000
5000
Iteration
Density − theta[7]
Density
0
5
10
15
5.75
5.80
5.85
5.90
5.95
Parameter estimate
Trace − theta[8]
Value
5.50
5.55
5.60
5.65
0
1000
2000
3000
4000
5000
Iteration
Density − theta[8]
Density
0
2
4
6
8
10
14
5.50
5.55
5.60
5.65
5.70
Parameter estimate
Trace − theta[9]
Value
5.45
5.50
5.55
5.60
0
1000
2000
3000
4000
5000
Iteration
Density − theta[9]
Density
0
2
4
6
8
10
14
5.45
5.50
5.55
5.60
5.65
Parameter estimate


Trace − theta[10]
Value
5.70
5.75
5.80
5.85
0
1000
2000
3000
4000
5000
Iteration
Density − theta[10]
Density
0
2
4
6
8
10
14
5.65
5.70
5.75
5.80
5.85
Parameter estimate
Trace − theta[11]
Value
5.65
5.70
5.75
5.80
0
1000
2000
3000
4000
5000
Iteration
Density − theta[11]
Density
0
2
4
6
8
10
14
5.60
5.65
5.70
5.75
5.80
Parameter estimate