Solution to Problem 1:

a)

To answer this question, both Q-Q plot (Figure 1) and exploratory data analysis (Figure 2) were investigated. The Q-Q plot shows a good match between the theoretical and observed data, the data analysis shows that the data almost match a data set inferred from exponential distribution. Therefore, I think exponential distribution is can be a candidate theoretical model to explain intervals between observation in this experiment.

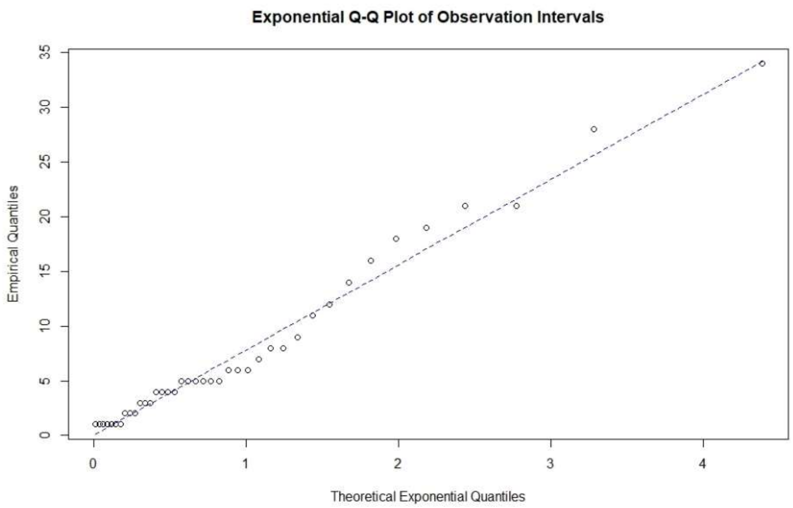

Figure 1

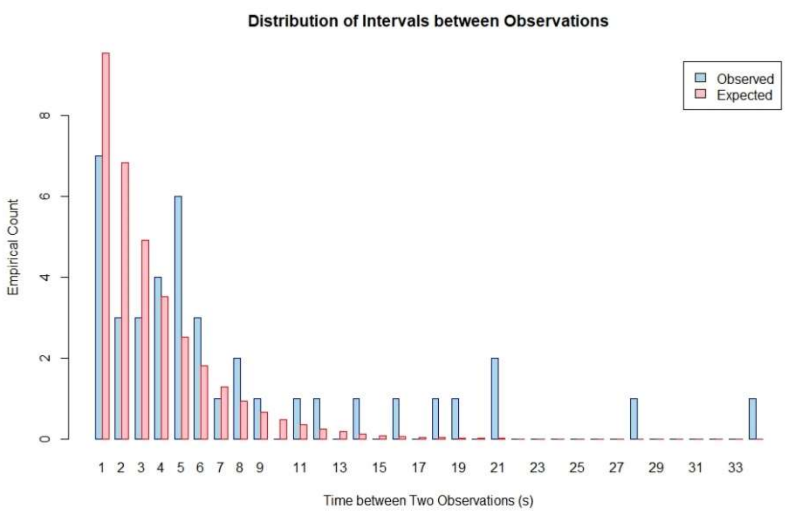

Figure 2

b)

The observed rates were calculated in R, and the same investigations were performed on the resulting dataset. Based on the results (Figures 3 and 4), Poisson distribution seems to be a good mathematical distribution to model the rate of observations in 15-second blocks.

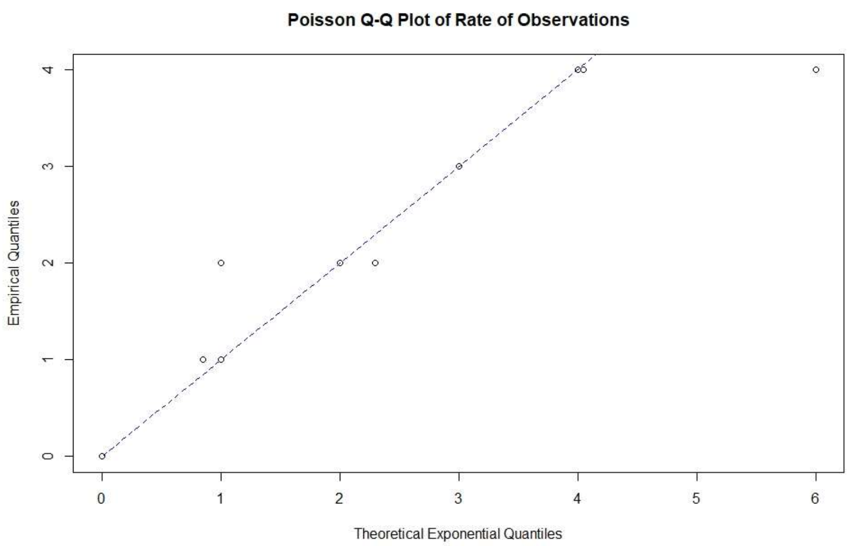

Figure 3

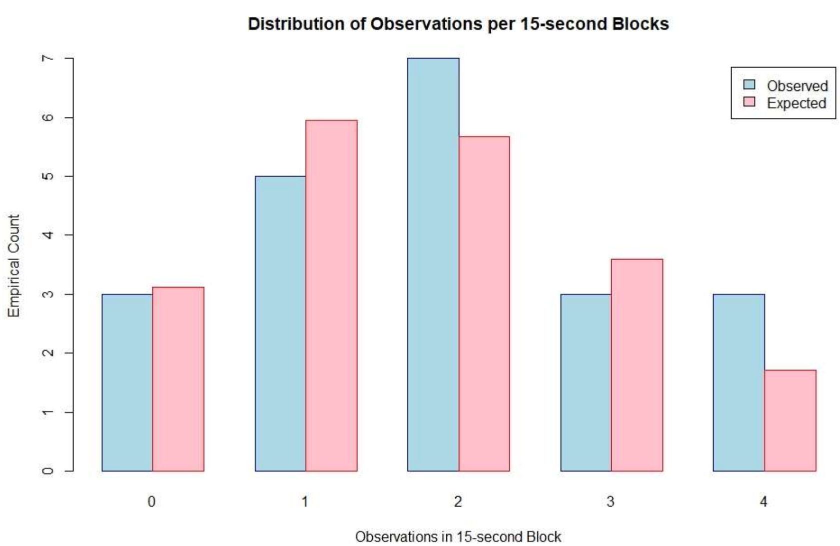

Figure 4

c)

The prior distribution of parameter λ is shown in the Figure 5.

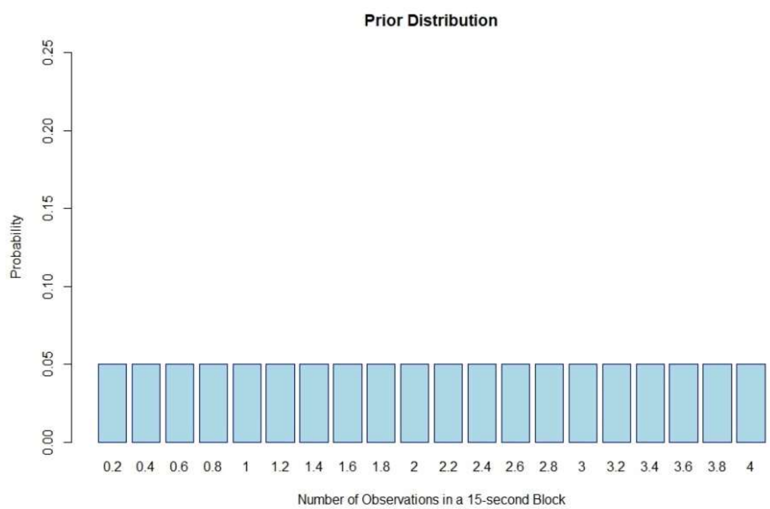

Figure 5

The prior was updated using the first ten rates of observation in 15-second blocks by applying the Bayes rule (R is the observed rate):

$$P(\lambda|R) = \frac{\prod_i P(R_i|\lambda)}{\sum_\lambda \prod_i P(R_i|\lambda)} \times P(\lambda)$$

Using Poisson distribution, we have:

$$P(R_i|\lambda) = \frac{\lambda^{R_i} e^{-\lambda}}{R!}$$

After coding those equations into R, the resulting posterior distribution for λ is illustrated in the Figure 6.

The Parameters of the posterior distribution of λ are as follows:

Mean	1.70
Standard Deviation	0.41
Median	1.40
95% Quantile	2.20

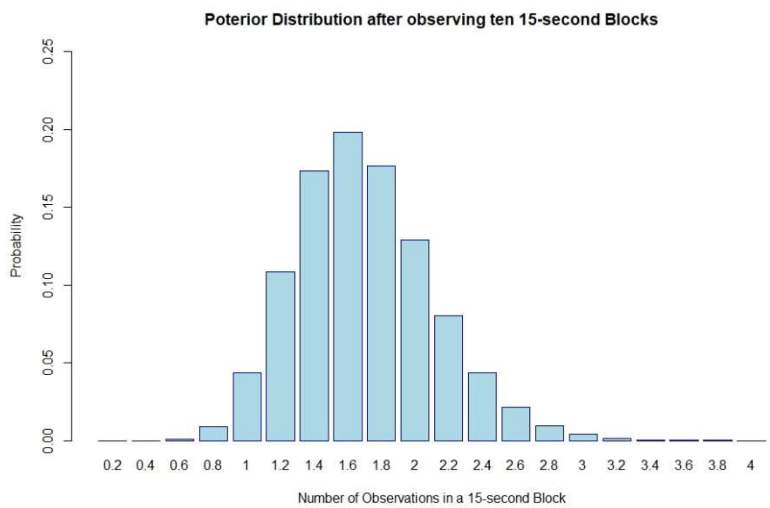

Figure 6

d)

After applying the same procedure on the posterior distribution from last part (Figure 6), using the rest 11 collected rates of observations, the final posterior distribution of λ was resulted, which is shown in Figure 7. The final distribution parameters are as follows:

Mean	1.95
Standard Deviation	0.30
Median	1.80
95% Quantile	2.20

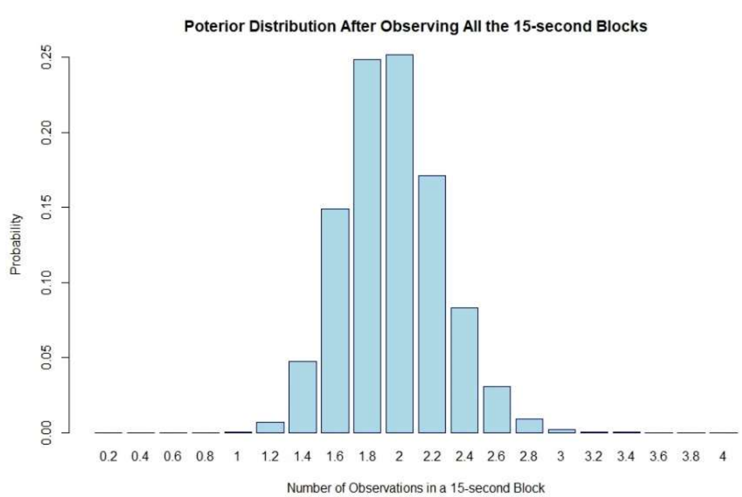

Figure 7

e)

To find the degree of belief on different numbers as the rate of observation in the 22nd 15-second block (the PMF), the Poisson distribution was summed over different values for λ:

$$P(x) = \sum_{\lambda} P(x|\lambda)P(\lambda), \qquad x \in \{0,1,2,3,4\}$$

PMF= {0.15, 0.28, 0.26, 0.17, 0.09}

The resulting PMF is shown in the Figure 8

To find the probability of the next rate being greater than 4, a cumulative scheme was used:

$$P(x > 4) = 1 - \sum_{i=1}^{4} P(x_i) = 0.052$$

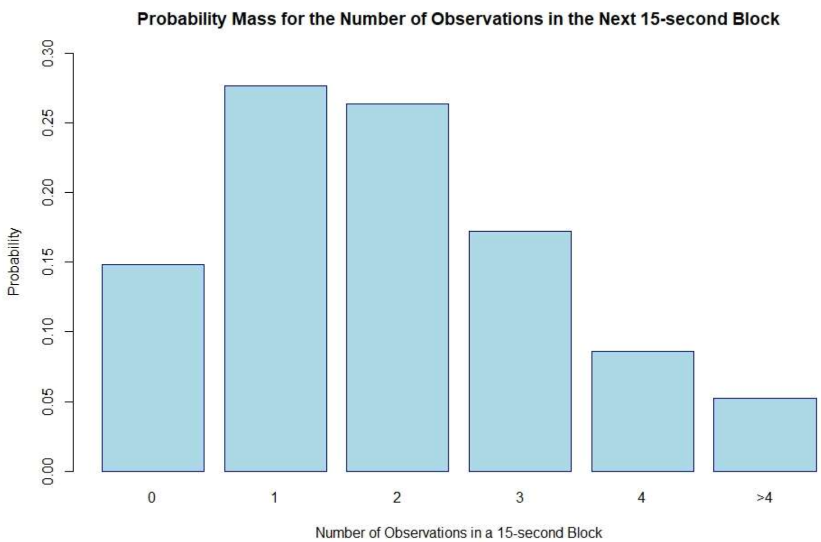

Figure 8

f)

The expected value of the PMF from last part (ignoring the 0.052 probability of seeing more than four cars) is 1.67. It means, on average, the vehicles are 8.98 seconds apart.

On high speed, a truck should maintain at least a distance of 660 feet from the vehicle in front of it. 660 feet is 0.125 miles. 0.125 miles per 8.98 seconds is equal to 50 mph. Based on that I guess the motorway under investigation is a surface road.