Solution to Problem 1:

This a Poisson process with rate Λ. The prior distribution of Λ is a uniform gamma, hence α=1 and β=∞. Using data from observations, the updated gamma distribution (posterior) parameters would be $\alpha^* = \alpha_0 + \sum x_i = 41$ and $\beta^* = \frac{1}{1/\beta_0 + n} = 0.048$.

In the next step, a predictive negative binomial distribution is used with parameters size=α^*=41 and $prob = \frac{1}{1+\beta^*} = 0.954$. The resulting predictive distribution for number of cars in the next 15-second block is illustrated in the Figure 1 and compared with the results from assignment 3.

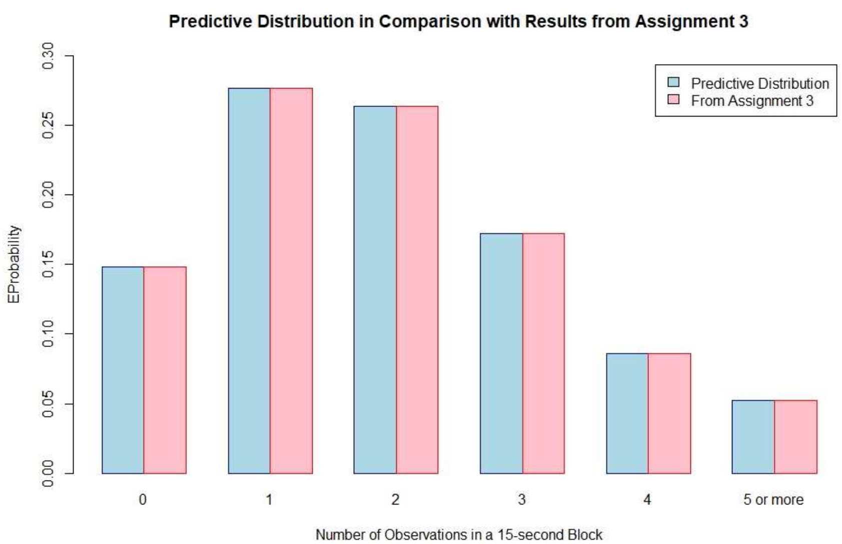

Figure 1

Results from this part and results from assignment 3 match up to 5 decimal places. This shows that the assumption that this process is a Poisson process was right.

Solution to Problem 2:

a)

Finding mean and standard deviation of Beta(1,3):

$$Mean = \frac{\alpha}{(\alpha + \beta)} = 0.25$$

$$Standard\ Deviation = \sqrt{\frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}} = 0.194$$

95% symmetric tail area credible interval is: [0.008 0.708]

A plot of beta distribution for π (prior) is shown in the Figure 2.

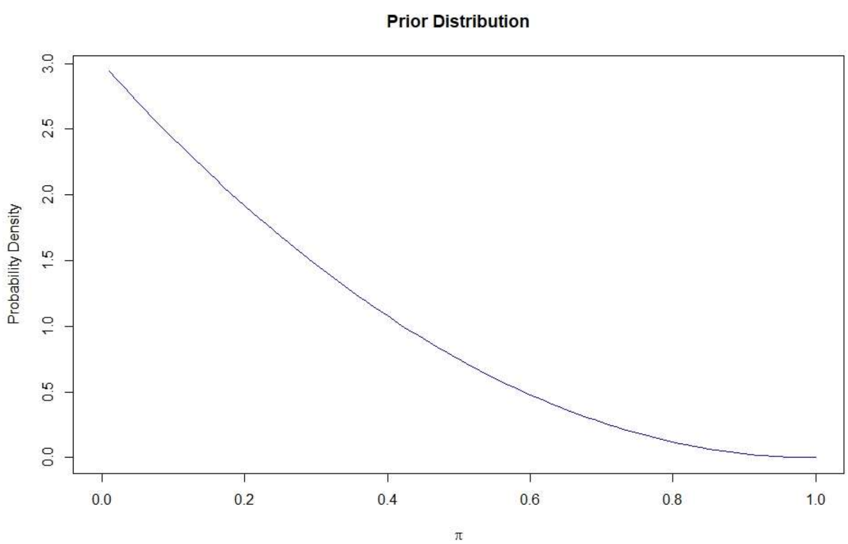

Figure 2

This prior seems to be reasonable to me, because, as it was discussed in assignment 2, people who do not know about statistics and decision theory (ordinary people) wouldn't choose B and C at the same time. As it can be seen in Figure 2, the lower values of π have a higher density and the mode of this distribution (the value with highest density) is zero.

b)

In this part, number of trials is (n) is 47 and number of successes (x) is 19. The posterior is a beta distribution with $\alpha^* = \alpha_0 + x = 20$ and $\beta^* = \beta + n - x = 31$. Consequently:

$$Mean = \frac{\alpha}{(\alpha+\beta)} = 0.392$$

$$Standard\ Deviation = \sqrt{\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}} = 0.0.68$$

95% symmetric tail area credible interval is: [0.264 0.528]

The triplot is illustrated in the Figure 3.

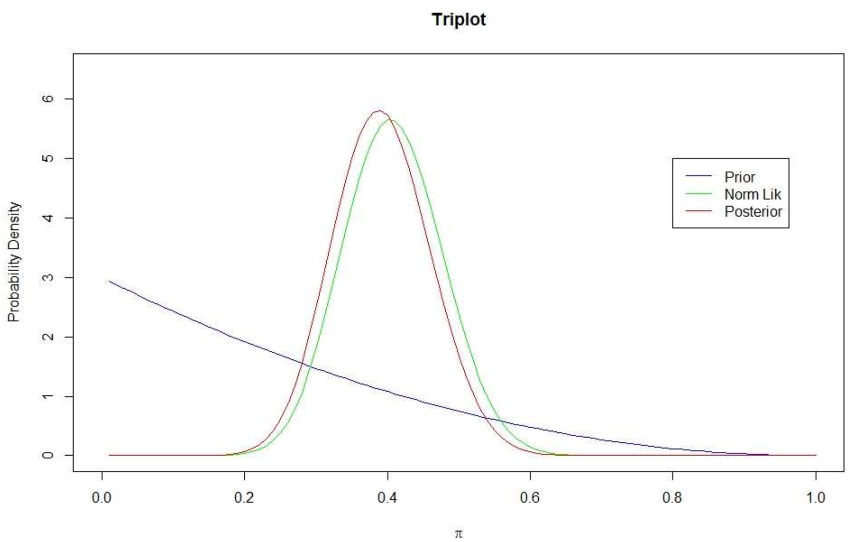

Figure 3

c)

The predictive distribution is a beta-binomial with $\alpha = \frac{\alpha^*}{(\alpha^*+\beta^*)} = 0.392$ and $\beta = \alpha^* + \beta^* = 51$. The distribution is presented in Figure 4.

In Figure 5, the predictive distribution is plotted next to a binomial distribution with probability of success θ=19/47=0.404 (sample frequency). The binomial distribution is taller and less spread. I think that is because the binomial distribution is from only one-point estimate of π, so it shows more certainty about the value of π. Figure 6 compares the predictive distribution to a binomial distribution of π with probability of success θ=0.392 (posterior mean). The same pattern is observable here as well.

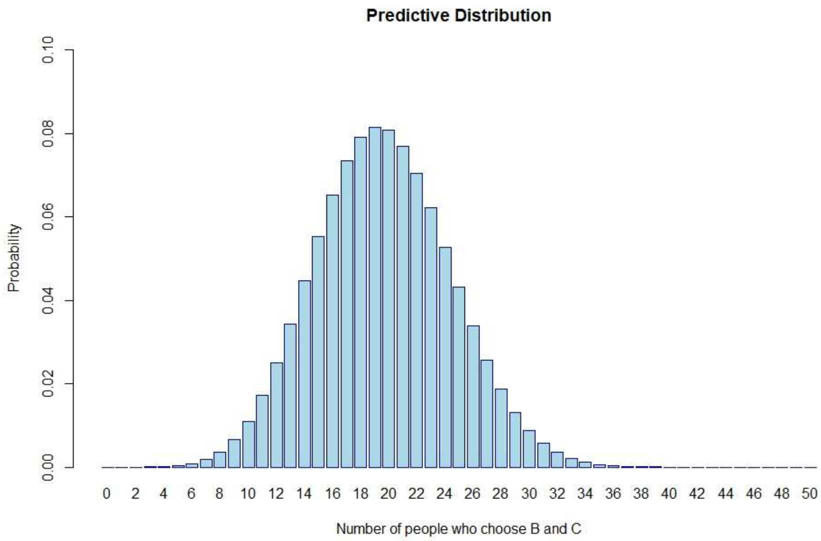

Figure 4

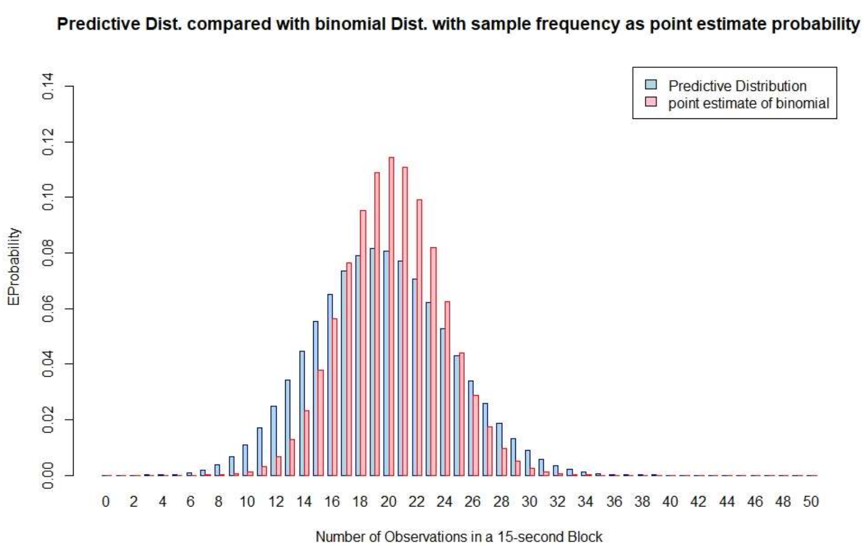

Figure 5

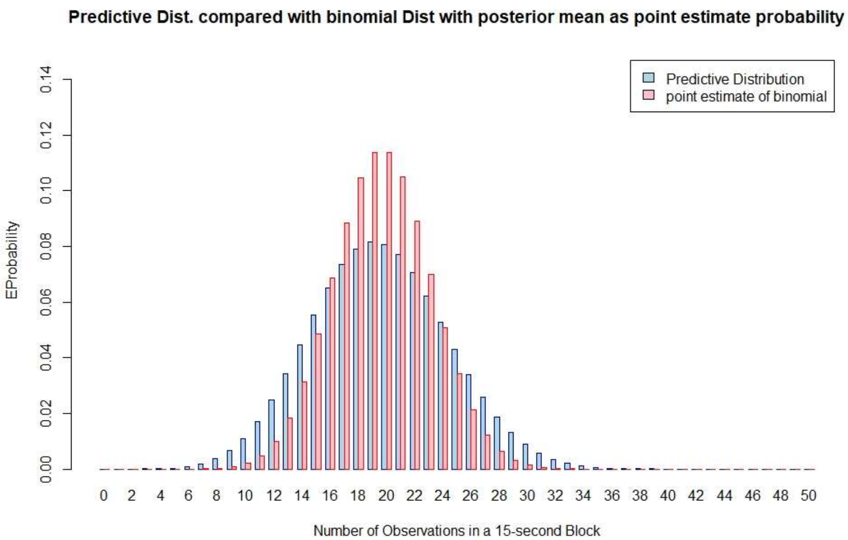

Figure 6

d)

The analysis began with reasonably assuming that probability that a random person selects option B and C (π), has a Beta(1,3) distribution. The experiment with a randomly selected sample of 47 people changed our belief about π from Beta(1,3) to Beta(20,31). It means it is more probable that a random person chooses B and C than we initially assumed. This can be inferred from the resulting triplot (Figure 3). Based on the new beta distribution for π, a beta-binomial distribution was used as a predictive distribution for the number of people in a group of 50, who choose B and C. Since the predictive distribution includes the uncertainty about π, it is more spread, than binomial distributions with certain probabilities of success (Figures 5 and 6).