Solution to Problem 1:

The conditional distribution for mean (Θ) is a normal with parameters:

$$\tau^* = \left(\frac{1}{\tau^2} + n\rho\right)^{-1/2} \qquad \mu^* = \frac{\mu/_{\tau^2} + \rho \sum_i x_i}{1/_{\tau^2} + n\rho}$$

Therefore, for the surface distribution we have:

$$\tau^*{}_s = \left(\frac{1}{1.5^2} + 10\rho_s\right)^{-1/2} \qquad \mu^*{}_s = \frac{6/_{1.5^2} + 48.04\rho_s}{1/_{1.5^2} + 10\rho_s}$$

Both τ_s and μ_s are functions of ρ_s.

Also, for the bottom distribution we have:

$$\tau^*{}_b = \left(\frac{1}{1.5^2} + 10\rho_b\right)^{-1/2} \qquad \mu^*{}_b = \frac{6/_{1.5^2} + 58.39\rho_b}{1/_{1.5^2} + 10\rho_b}$$

Both τ_b and μ_b are functions of ρ_b.

The conditional distribution for precision (P) is a gamma with parameters:

$$\alpha^* = \alpha_0 + \frac{n}{2} \qquad \beta^* = \left(\frac{1}{\beta} + \frac{1}{2}\sum_{i=1}^{n}(x_i - \theta)^2\right)^{-1}$$

Therefore, for the Surface distribution we have:

$$\alpha^*{}_s = 4.5 + \frac{10}{2} = 9.5 \qquad \beta^*{}_s = \left(\frac{1}{0.19} + \frac{1}{2}\sum_{i=1}^{10}\left(x_{s_i} - \theta_s\right)^2\right)^{-1}$$

β_s is a function of θ_s, in which, x_{s_i} is the ith observation in the surface sample data.

Also, for the bottom distribution we have:

$$\alpha^*{}_b = 4.5 + \frac{10}{2} = 9.5 \qquad \beta^*{}_b = \left(\frac{1}{0.19} + \frac{1}{2}\sum_{i=1}^{10}\left(x_{b_i} - \theta_b\right)^2\right)^{-1}$$

β_b is a function of θ_b, in which, x_{b_i} is the ith observation in the surface sample data.

Solution to Problem 2:

Using 10000 Gibbs samples of Θ_s, Θ_b, P_s and P_b, the estimated 90% credible intervals are reported in the following table:

	5% Quantile	95% Quantile
Θ_s	4.373	5.320
Θ_b	5.304	6.407
Σ_s	0.6973	1.2204
Σ_b	0.8299	1.4344
Θ_b- Θ_s	0.2734	1.7270

The estimated distributions are illustrated in figures 1 to 3.

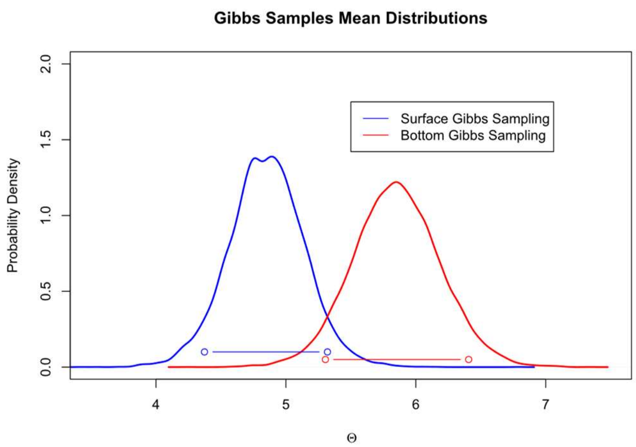

Figure 1

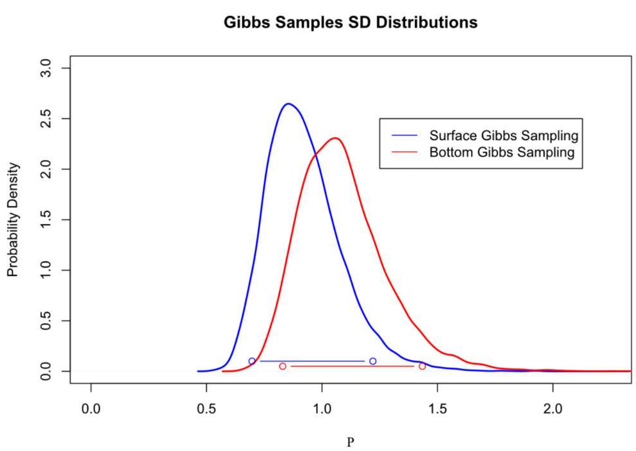

Figure 2

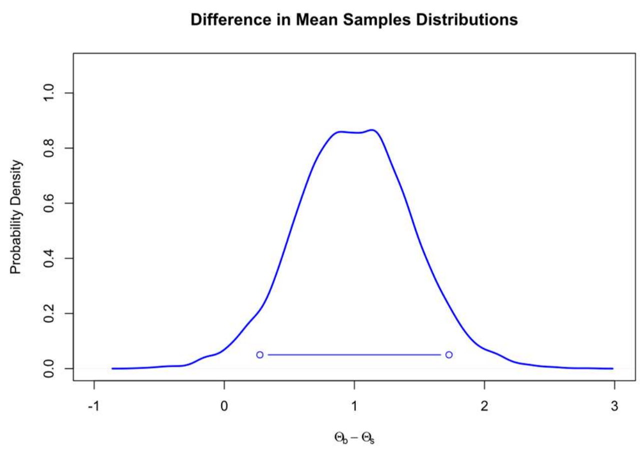

Figure 3

Solution to Problem 3:

The effective sample size was acquired using "effectivesize" function in R and it is equal to 10000, showing that the sampling is meaningful.

The traceplot and autocorrelation function are shown in figure 4 and figure 5.

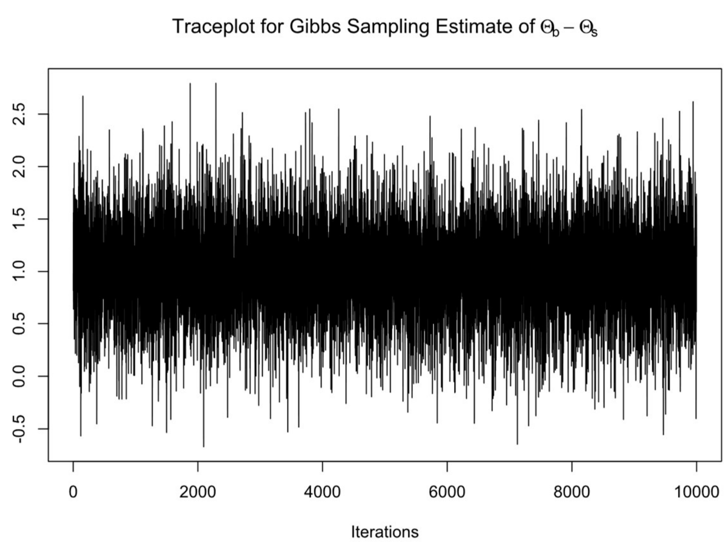

Figure 4

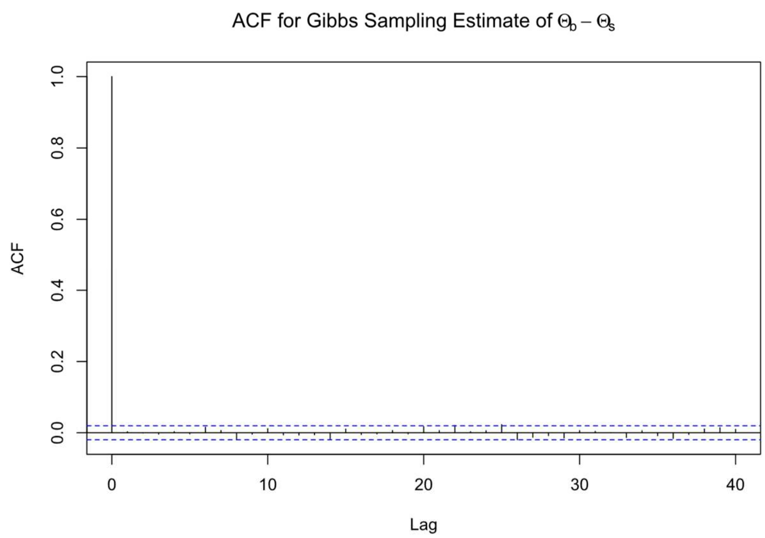

Figure 5

Solution to Problem 4:

The traceplot shows samples are oscillating around 1, and it means that the sampler has reached the stationary distribution.

The auto correlation function shows correlation inside the limits; therefore, the Gibbs samples are reliable.

Visual investigation shows that figure 1 and figure 2, are compatible with results from problem 3 of homework 6, which say there is a 99% probability that Θ_b is greater than Θ_s, and there is a 91.6% probability that Σ_b is greater than Σ_s.

A comparison of results from problem 1 to results from problems 1 and 2 from homework 6 is presented in the following table:

	HW8 P1		HW6 P1		HW6 P2	
	5% Quantile	95% Quantile	5% Quantile	95% Quantile	5% Quantile	95% Quantile
Θ_s	4.373	5.320	4.44	5.17	4.44	5.16
Θ_b	5.304	6.407	5.25	6.43	5.26	6.43
Σ_s	0.6973	1.2204	0.46	1.04	0.46	1.04
Σ_b	0.8299	1.4344	0.74	1.67	0.74	1.68

The results are close to those from homework 6. Given the uncertainty about the posterior in Gibbs sampling, it seems to be a reliable method in case of semi-conjugate distributions.