Solution to Problem 1:

A uniform distribution was simulated in R, using a gamma distribution with α=1 and β=Inf, and was used as prior for Λ. The resulting posterior after 21, 15-second observations (Figure 1), is also a gamma distribution with $\alpha = \alpha_0 + \sum x_i = 41$ and $\beta = \frac{1}{1/\beta_0 + n} = 0.048$.

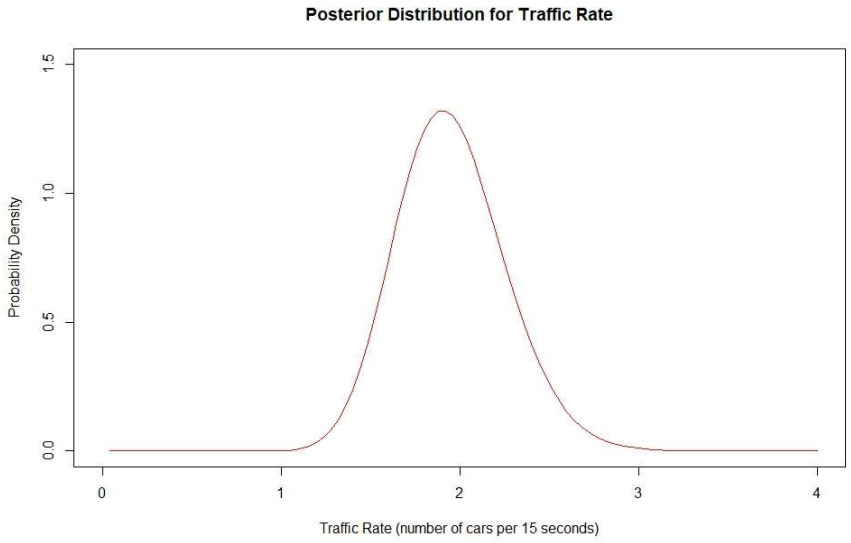

Figure 1

The resulting parameters are listed in the following table along with the same parameters from the previous assignment. The 95% symmetric tail area posterior credible interval for Λ is [1.40 2.59].

	From Assignment 3	Results from Continuous Distribution
Mean	1.95	1.95
Standard Deviation	0.30	0.30
Median	2.00	1.94
95% Quantile	2.40	2.59
5% Quantile	1.40	1.40
Mode	2.00	1.90

The results from using continuous distributions are very close to ones from discrete distributions.

Solution to Problem 2:

R package "rriskDistributions" was installed and used to solve this problem. Function "get.gamma.par" was used with the given data as input. The following table shows the resulting parameters.

α	5.22
β	1/1.31=0.763

To check the accuracy of the results, 10%, 50% and 90% quantiles were extracted from a gamma distribution with the resulted parameters. The following table shows that these parameters are a fairly good fit to the given data.

	10% Quantile	50% Quantile	90% Quantile
Goal	1.50	3.75	7.00
Achieved	1.97	3.73	6.31

Solution to Problem 3:

The fitting distribution from problem 2 was used as a prior, and the procedure from problem 1 was repeated. The resulting posterior after 21, 15-second observations (Figure 2), is also a gamma distribution with $\alpha = \alpha_0 + \sum x_i = 45.22$ and $\beta = \frac{1}{1/\beta_0 + n} = 0.045$. The 95% symmetric tail area posterior credible interval for Λ is [1.48 2.66]. The results are presented in the following table:

	From Assignment 3	Results from Fitting Distribution	Results from Problem1
Mean	1.95	2.03	1.95
Standard Deviation	0.30	0.30	0.30
Median	2.00	2.01	1.94
95% Quantile	2.40	2.66	2.59
5% Quantile	1.40	1.48	1.40
Mode	2.00	1.98	1.90

Results in all three columns are pretty close to each other. Since the variable in this experiment is a discrete one, it seems to be safe to say all three methods give the same set of results.

Solution to Problem 4:

The triplot of the prior distribution, normalized likelihood and posterior distribution for Problem 3 is presented in the Figure 2.

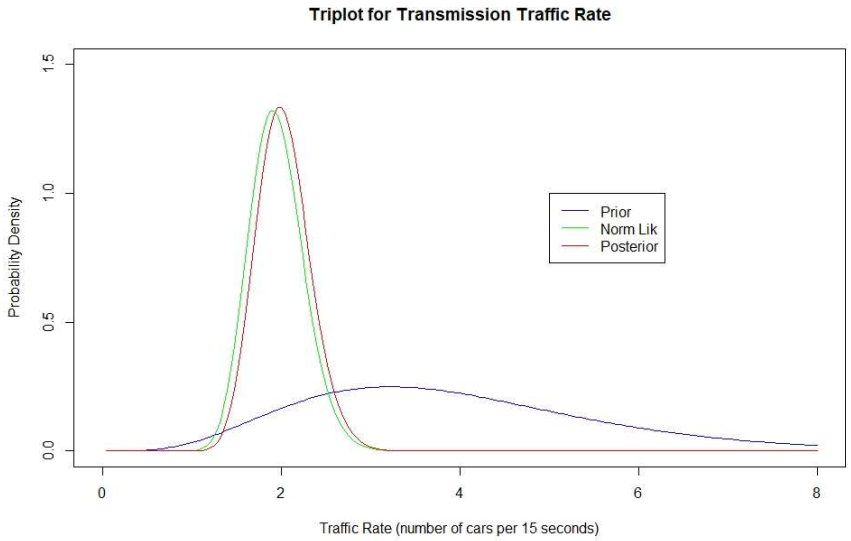

Figure 2

The posterior mode is between likelihood norm and prior mode; however, it is much closer to the likelihood mode. All three plots are gamma distributions with different scale and shape parameters, summarized in the following table.

	α	β
Prior	5.22	0.763
Normalized Likelihood	41.00	0.048
Posterior	45.22	0.045