DICE 2013R:

Introduction and User's Manual

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Table of Contents (New materials are with *)

I. Preface (*) [1]

The present manual combines a discussion of the subject of integrated assessment models (IAMs) of climate-change economics, a detailed description of the DICE model as an example of an IAM, and the results of the latest projections and analysis using the DICE-2013R model.

The main focus here is an introduction to the DICE-2013R model (which is an acronym for the Dynamic Integrated model of Climate and the Economy). The 2013 version is a major update from the last fully documented version, which was the DICE-2007 model (Nordhaus 2008). The purpose of this manual is to explain in a self-contained publication the structure, calculations, algorithmics, and results of the current version. Some of the materials has been published in earlier documents, but this manual attempts to combine the earlier materials in a convenient fashion.

The author would like to thank the many co-authors and collaborators who have contributed to this project over the many decades of its development. More than any single person, my colleague and co-author Tjalling Koopmans was an intellectual and personal inspiration for this line of research. I will mention particularly his emphatic recommendation for using mathematical programming rather than econometric modeling for energy and environmental economics.

Other important contributors have been George Akerlof, Lint Barrage, Scott Barrett, Joseph Boyer, William Brainard, William Cline, Jae Edmonds, Ken Gillingham, Charles Kolstad, Tom Lovejoy, Alan Manne, Robert Mendelsohn, Nebojsa Nakicenovic, David Popp, John Reilly, Richard Richels, John Roemer, Tom Rutherford, Jeffrey Sachs, Leo Schrattenholzer, Herbert Scarf, Robert Stavins, Nick Stern, Richard Tol, David Victor, Martin Weitzman, John Weyant, Zili Yang, Janet Yellen, and Gary Yohe, as well as many anonymous referees and reviewers.

We have denoted sections or chapters that are largely new materials with asterisks. This will be helpful for those familiar with earlier versions or writings who would like to move quickly to the new material.

Those who would like access to the model and material can find it at dicemodel.net.

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II. DICE and RICE Models as Integrated Assessment Models

A. Introduction to the models

The DICE model (Dynamic Integrated model of Climate and the Economy) is a simplified analytical and empirical model that represents the economics, policy, and scientific aspects of climate change. Along with its more detailed regional version, the RICE model (Regional Integrated model of Climate and the Economy), the models have gone through several revisions since their first development around 1990.

The prior fully documented versions are the RICE-2010 and DICE-2007 model. The present version is an update of those earlier models, with several changes in structure and a full updating of the underlying data. This section draws heavily on earlier expositions Nordhaus (1994, 2008, 2010, 2012), along with Nordhaus and Yang (1996) and Nordhaus and Boyer (2000).

The DICE-2013R model is a globally aggregated model. The RICE-2010 model is essentially the same except that output, population, emissions, damages, and abatement have regional structures for 12 regions. The discussion in this manual will focus on the DICE model, and the analysis applies equally to the RICE model for most modules. The differences will be described later.

The DICE model views the economics of climate change from the perspective of neoclassical economic growth theory (see particularly Solow 1970). In this approach, economies make investments in capital, education, and technologies, thereby reducing consumption today, in order to increase consumption in the future. The DICE model extends this approach by including the "natural capital" of the climate system. In other words, it views concentrations of GHGs as negative natural capital, and emissions reductions as investments that raise the quantity of natural capital (or reduce the negative capital). By devoting output to emissions reductions, economies reduce consumption today but prevent economically harmful climate change and thereby increase consumption possibilities in the future.

Figure 1 shows a schematic flow chart of the major modules and logical structure of the DICE and RICE models.

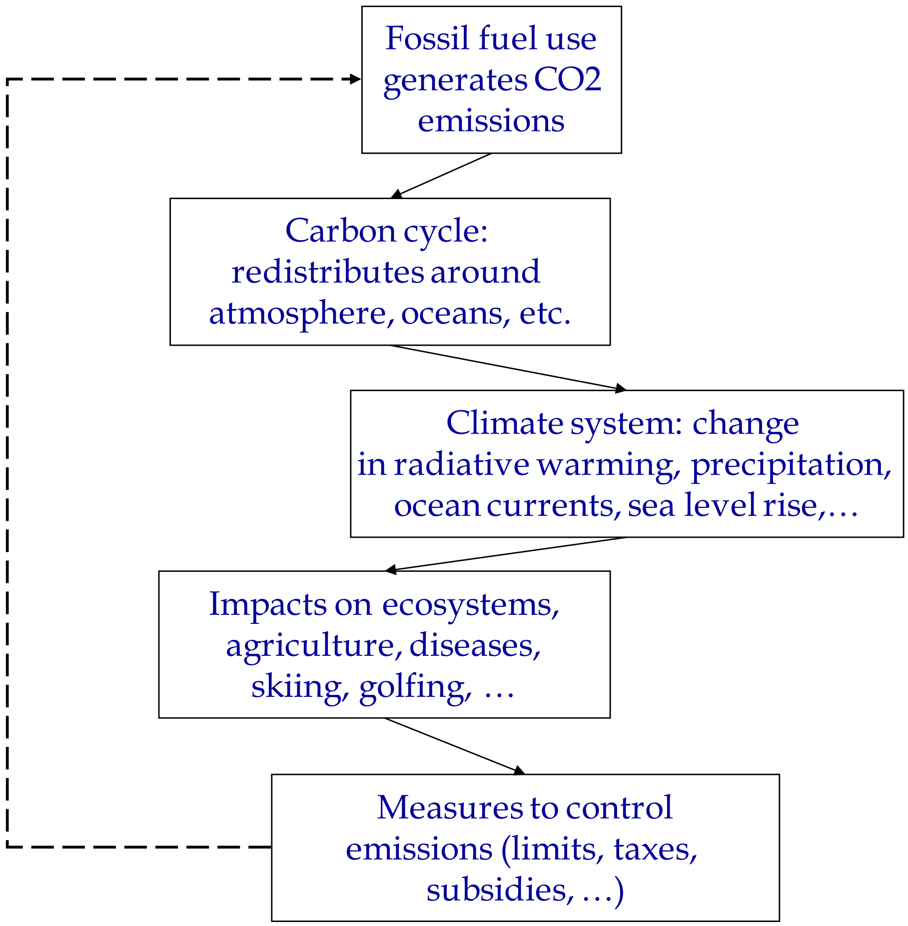

Figure 1. Schematic flow chart of a full integrated assessment model for climate change science, economics, and policy

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B. Objectives of Integrated Assessment Models (IAMs)

IAMs can be divided into two general classes – policy optimization and policy evaluation models (this distinction was emphasized in an excellent chapter of the IPCC report by Weyant et al. 1996). Policy evaluation model generally are recursive or equilibrium models that generate paths of important variables but do not optimize an economic or environmental outcome.

Policy optimization models have an objective function or welfare function that is maximized and can be used to evaluate alternative paths or policies. In models that have an economic structure, the objective function is generally a measure of economic welfare. This would typically be a set of utility functions in general equilibrium models or consumer and producer surplus in partial equilibrium models.

These two approaches are not as different as might be supposed, as policy optimization models can be run in a non-policy mode, while policy evaluation models can compare different policies. However, there are often differences in the solution algorithms as recursive models are often much simpler to solve computationally than are optimization models.

The DICE/RICE models are primarily designed as policy optimization models, although they can be run as simple projection models as well. In both modes, the approach is to maximize an economic objective function. The objective function represents the goal implicit in the problem. For the DICE/RICE models, the objective function refers to the economic well-being (or utility) associated with a path of consumption.

As will be emphasized below, the use of optimization can be interpreted in two ways: First, from a positive point of view, optimization is a means of simulating the behavior of a system of competitive markets; and, second, from a normative point of view, it is a possible approach to comparing the impact of alternative paths or policies on economic welfare. The models are available at dicemodel.net.

III. Detailed Equations of the DICE-2013R Model

A. Preferences and the Objective Function

In the DICE and RICE models, the world or individual regions are assumed to have well-defined preferences, represented by a social welfare function, which ranks different paths of consumption. The social welfare function is increasing in the number of people and in the per capita consumption of each generation, with diminishing marginal utility of consumption.

The importance of a generation's per capita consumption depends on the size of the population. The relative importance of different generations is affected by two central normative parameters, the pure rate of social time preference ("generational discounting") and the elasticity of the marginal utility of consumption (the "consumption elasticity"). These two parameters interact to determine the discount rate on goods, which is critical for intertemporal economic choices. In the modeling, we set the preference parameters to be consistent with observed economic outcomes as reflected by interest rates and rates of return on capital, a choice that will be central to the results and is further discussed in the section on discounting below.

The DICE model assumes that economic and climate policies should be designed to optimize the flow of consumption over time. It is important to emphasize that consumption should be interpreted as "generalized consumption," which includes not only traditional market goods and services like food and shelter but also non-market items such as leisure, health status, and environmental services.

The mathematical representation of this assumption is that policies are chosen to maximize a social welfare function, *W,* that is the discounted sum of the population-weighted utility of per capita consumption. The notation is that *c(t)* is per capita consumption, *L(t)* is population as well as labor inputs, and *R(t)* is the discount factor, all of which are discussed as we proceed. Equation (1) is the mathematical statement of the objective function. This representation is a standard one in modern theories of optimal economic growth (see Ramsey 1928, Koopmans 1965, Cass 1965).

$$(1) \quad W = \sum_{t=1}^{T\,max} U[c(t), L(t)]R(t)$$

There are a number of further assumptions underlying this choice of an objective function. First, it involves a specific representation of the value or "utility" of consumption. The DICE/RICE models assume that utility is represented by a constant elasticity utility function, as shown in equation (2).

$$(2) \qquad U[c(t),L(t)] = L(t)[c(t)^{1-\alpha} / (1-\alpha)]$$

This form assumes a constant elasticity of the marginal utility of consumption, α. (In the limiting case where $\alpha = 1$, the utility function is logarithmic.) The elasticity parameter is best thought of as aversion to generational inequality. Put differently, the elasticity represents the diminishing social valuations of consumption of different generations. If α is close to zero, then the consumptions of different generations are close substitutes, with low aversion to inequality; if α is high, then the consumptions are highly differentiated, and this reflects high inequality aversion. Often, α will also be used to represent risk aversion, but these are strictly speaking quite distinct concepts and should not be confused (see Epstein and Zin 1989, 1991). Additionally, the elasticity is distinct from the *personal* behavioral characteristics. We calibrate α in conjunction with the pure rate of time preference, as is discussed below.

Second, this specification assumes that the value of consumption in a period is proportional to the population. In the RICE model, the presence of multiple agents will lead to major issues of interpretation and computation, but this is not relevant in the DICE model and will be largely ignored in this manual.

Third, this approach applies a discount on the economic well-being of future generations, as is defined in Equation (3).

$$(3) \qquad R(t) = (1+\rho)^{-t}$$

In this specification, *R(t)* is the discount factor, while the pure rate of social time preference, ρ , is the discount rate which provides the welfare weights on the utilities of different generations.

We should add a note of interpretation of the equilibrium in the DICE model. We have specified the baseline case so that, from a conceptual point of view, it represents the *outcome of market and policy factors as they currently exist*. In other words, the baseline model is an attempt to project from a positive perspective the levels and growth of major economic and environmental variables as would occur with current climate-change policies. It does not make any case for the social desirability of the distribution of incomes over space or time of existing conditions, any more than a marine biologist makes a moral judgment on the equity of the eating habits of sharks or guppies.

We can put this point differently in terms of welfare improvements. The calculations of the potential improvements in world welfare from efficient climate-change policies examine potential improvements within the context of the existing distribution of income and investments across space and time. There may be other improvements – in local pollution policies, in tax or transfer programs, or in international aid programs – that would improve the human condition, and might improve it even more than the policies we consider, but these are outside the scope of this analysis. This point is discussed at length in Nordhaus (2012).

B. Economic Variables

The economic sectors of the DICE model are standard to the economic growth literature. The main difference from standard analysis is the very long time frame that is required for climate-change modeling. While most macroeconomic models run for a few years, or in the development context a few decades, climate-change projects necessarily must encompass more than a century. The result is that many of the projections and assumptions are based on very thin evidence.

We begin with the standard neoclassical decisions about capital accumulation and then consider the geophysical constraints. The DICE/RICE models are simplified relative to many models because they assume a single commodity, which can be used for consumption, investment, or abatement. Consumption should be viewed broadly to include not only food and shelter but also non-market environmental amenities and services.

The output, population, and emissions variables are built up from national data. They are generally aggregated into major regions (United States, China, EU, India, and so forth). They are then projected separately. The regional aggregates are used in the RICE model. For the DICE model, they are simply aggregated together for the world total.

Each region is endowed with an initial stock of capital and labor and an initial and region-specific level of technology. Population growth and technological change are region-specific and exogenous, while capital accumulation is determined by optimizing the flow of consumption over time for each region. Regional outputs and capital stocks are aggregated using purchasing power parity (PPP) exchange rates (although this has been controversial, see IPCC Fourth Assessment, Mitigation 2007 and Nordhaus 2007a).

We next describe the equations for the different economic variables in the DICE-2013R model. The first set of equations determines the evolution of world output over time. Population and the labor force are exogenous. These are simplified to be logistic-type equations of the form $L(t) = L(t-1)\left[1+g_L(t)\right]$, where $g_L(t) = g_L(t-1)/(1+\delta_L)$. The initial population in 2010 is given, and the growth rate declines so that total world population approaches a limit of 10.5 billion in 2100. The initial growth rate of population, $g_L(2015)$, of 13.4% per period (5 years) is set so that population equals the UN projection for 2050. These numbers have been revised upward in line with the most recent UN projections and are about 20 percent higher than the 2007 DICE/RICE model estimates. (A fine recent review is Lee 2011 and other articles in the same issue.)

Output is produced with a Cobb-Douglas production function in capital, labor, and energy. Energy takes the form of either carbon-based fuels (such as coal) or non-carbon-based technologies (such as solar or geothermal energy or nuclear power).

Technological change takes two forms: economy-wide technological change and carbon-saving technological change. The level of total factor productivity [TFP, represented by A(t)] is a logistic equation similar to that of population. It takes the form $A(t) = A(t-1)\left[1+g_A(t)\right]$, where $g_A(t) = g_A(t-1)/(1+\delta_A)$. In this specification, TFP growth declines over time. In the current specification, $A(2010)$ is set to calibrate the model to gross world product in 2010; $g_A(2015)$ = 7.9 % per five years; and δ_A = 0.6% per five years. This specification leads to growth in consumption per capita of 1.9% per year from 2010 to 2100 and 0.9% per year from 2100 to 2200.

Carbon-saving technological change is modeled as reducing the ratio of CO_2 emissions to output (described below). Carbon fuels are limited in supply, with a total limit of 6000 billion tons of carbon content. In the current version, the carbon constraint is not binding in the base case. Substitution from carbon to non-carbon fuels takes place over time as carbon-based fuels become more expensive, either because of resource exhaustion or because policies are taken to limit carbon emissions.

The underlying population and output estimates are aggregated up from a twelve-region model. Outputs are measured in purchasing power parity (PPP) exchange rates using the IMF estimates (Nordhaus 2007a). Total output for each region is projected using a partial convergence model, and the outputs are then aggregated to the world total. The regional and global production functions are assumed to be constant-returns-to-scale Cobb-Douglas production functions in capital, labor, and Hicks-neutral technological change. Global output is shown in Equation (4):

$$(4) \quad Q(t) = [1 - \Lambda(t)] A(t) K(t)^{\gamma} L(t)^{1-\gamma} / [1 + \Omega(t)]$$

In this specification, *Q(t)* is output net of damages and abatement, *A(t)* is total factor productivity (of the Hicks-neutral variety), and *K(t)* is capital stock and services. The additional variables in the production function are $\Omega(t)$ and $\Lambda(t)$, which represent climate damages and abatement costs, shown in Equations (5) and (6).

$$(5) \quad \Omega(t) = \psi_1 T_{AT}(t) + \psi_1 [T_{AT}(t)]^2$$

Equation (5) involves the economic damages or impacts of climate change, which is the thorniest issue in climate-change economics. These estimates are indispensable for making sensible decisions about the appropriate balance between costly emissions reductions and climate damages. However, providing reliable estimates of the damages from climate change over the long run has proven extremely difficult.

The damage function in (5) has been greatly simplified from earlier DICE/RICE versions. Earlier versions relied on detailed sectoral estimates from Nordhaus and

Boyer (2000). However, further work indicated that those estimates were increasingly outdated and unreliable.

The 2013 model instead uses a highly simplified damage function that relies on current estimates of the damage function. More precisely, DICE-2013R uses estimates of monetized damages from the Tol (2009) survey as the starting point. However, current studies generally omit several important factors (the economic value of losses from biodiversity, ocean acidification, and political reactions), extreme events (sea-level rise, changes in ocean circulation, and accelerated climate change), impacts that are inherently difficult to model (catastrophic events and very long term warming), and uncertainty (of virtually all components from economic growth to damages). I have added an adjustment of 25 percent of the monetized damages to reflect these non-monetized impacts. While this is consistent with the estimates from other studies (see Hope 2011, Anthoff and Tol 2010, and FUND 2013), it is recognized that this is largely a judgmental adjustment. The current version assumes that damages are a quadratic function of temperature change and does not include sharp thresholds or tipping points, but this is consistent with the survey by Lenton et al. (2008).

Figure 2 shows the results of the Tol (2009) survey on damages, the IPCC assessment from the Third and Fourth Assessment Reports, and the assumption in the DICE-2013R model as a function of global mean temperature increase.

I would note an important warning about the functional form in equation (5) when using for large temperature increases. The damage function has been calibrated for damage estimates in the range of 0 to 3 °C. In reality, estimates of damage functions are virtually non-existent for temperature increases above 3 °C. Note also that the functional form in (5), which puts the damage ratio in the denominator, is designed to ensure that damages do not exceed 100% of output, and this limits the usefulness of this approach for catastrophic climate change. The damage function needs to be examined carefully or re-specified in cases of higher warming or catastrophic damages.

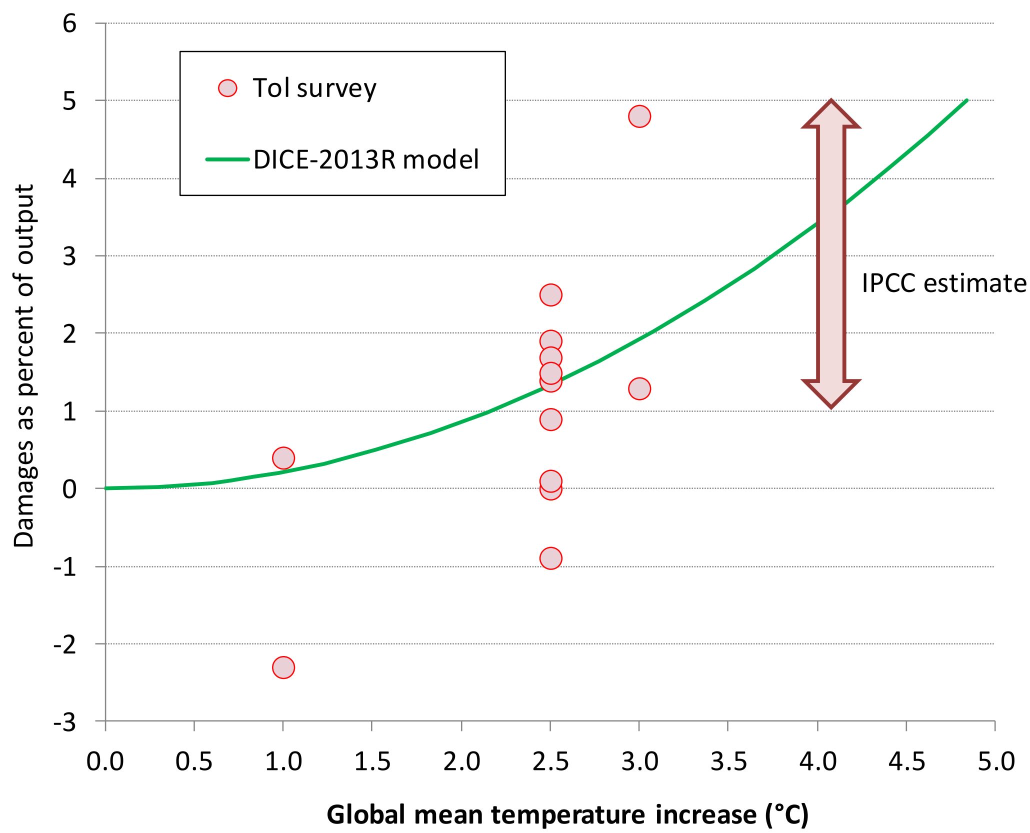

Figure 2. Estimates of the Impact of Climate Change on the Global Economy
This figure shows a compilation of studies of the aggregate impacts or damages of global warming for each level of temperature increase (dots are from Tol 2009). The solid line is the estimate from the DICE-2013R model. The arrow is from the IPCC (2007a). [impacts_survey.xlsx]

The abatement cost function in equation (6) shows the determinants of $\Lambda(t)$, which is the ratio of abatement cost to output.

$$(6) \quad \Lambda(t) = \theta_1(t)\mu(t)^{\theta_2}$$

The abatement cost equation in (6) is a reduced-form type model in which the costs of emissions reductions are a function of the emissions reduction rate, $\mu(t)$. The abatement cost function assumes that abatement costs are proportional to output and to a power function of the reduction rate. The cost function is estimated to be highly convex, indicating that the marginal cost of reductions rises from zero more than linearly with the reductions rate.

The DICE-2013R model explicitly includes a backstop technology, which is a technology that can replace all fossil fuels. The backstop technology could be one that removes carbon from the atmosphere or an all-purpose environmentally benign zero-carbon energy technology. It might be solar power, or carbon-eating trees or windmills, or some as-yet undiscovered source. The backstop price is assumed to be initially high and to decline over time with carbon-saving technological change.

In the full regional model, the backstop technology replaces 100 percent of carbon emissions at a cost of between \$230 and \$540 per ton of CO_2 depending upon the region in 2005 prices. For the global DICE-2013R model, the 2010 cost of the backstop technology is \$344 per ton CO_2 at 100% removal. The cost of the backstop technology is assumed to decline at 0.5% per year. The backstop technology is introduced into the model by setting the time path of the parameters in the abatement-cost equation (6) so that the marginal cost of abatement at a control rate of 100 percent is equal to the backstop price for a given year.

The next three equations are standard economic accounting equations. Equation (7) states that output includes consumption plus gross investment. Equation (8) defines per capita consumption. Equation (9) states that the capital stock dynamics follows a perpetual inventory method with an exponential depreciation rate.

$$(7) \quad Q(t) = C(t) + I(t)$$

$$(8) \quad c(t) = C(t) / L(t)$$

$$(9) \quad K(t) = I(t) - \delta_K K(t-1)$$

CO_2 emissions are projected as a function of total output, a time-varying emissions-output ratio, and the emissions-control rate. The emissions-output ratio is estimated for individual regions and is then aggregated to the global ratio. The emissions-control rate is determined by the climate-change policy under examination. The cost of emissions reductions is parameterized by a log-linear function, which is calibrated to the EMF-22 report and the models contained in that (Clarke et al. 2010).

Early versions of the DICE and RICE models used the emissions control rate as the control variable in the optimization because it is most easily used in linear-program algorithms. In recent versions, we have also incorporated a carbon tax as a control variable. This can be accomplished using an Excel SOLVER version with a modified Newton method to find the optimum. It can also be used in the GAMS version if the carbon price is solved explicitly (which can be done in the current version). The carbon price is determined by assuming that the price is equal to the marginal cost of emissions. The marginal cost is easily calculated from the abatement cost equation in (6) and by substituting the output equations.

The final two equations in the economic module are the emissions equation and the resource constraint on carbon fuels. Baseline industrial CO_2 emissions in Equation (10) are given by a level of carbon intensity, *σ(t),* times output. (See the change in the definition of the baseline below.) Actual emissions are then reduced by one minus the emissions-reduction rate, *[1-μ(t)].*

$$(10) \quad E_{Ind}(t) = \sigma(t)[1-\mu(t)]A(t)\,K(t)^{\gamma}L(t)^{1-\gamma}$$

The carbon intensity is taken to be exogenous and is built up from emissions estimates of the twelve regions, whereas the emissions-reduction rate is the control variable in the different experiments. Estimates of baseline carbon intensity are a logistics-type equation similar to that of total factor productivity. It takes the form $\sigma(t)=\sigma(t-1)\left[1+g_{\sigma}(t)\right]$, where $g_{\sigma}(t)=g_{\sigma}(t-1)/(1+\delta_{\sigma})$. In the current specification, *σ(2010)* is set to equal the carbon intensity in 2010, 0.549 tons of CO_2 per \$1000 of GDP; $g_{\sigma}(2015)$ = -1.0% per year; and δ_{σ} = -0.1% per five years. This specification leads to rate of change of carbon intensity (with no climate change policies) of -0.95%per year from 2010 to 2100 and -0.87% per year from 2100 to 2200.

Equation (11) is a limitation on total resources of carbon fuels, given by *CCum.* In earlier versions, the carbon constraint was binding, but it is not in the current version. The model assumes that incremental extraction costs are zero and that carbon fuels are efficiently allocated over time by the market. We have simplified the current version by incorporating the complicated Hotelling procedure in the program rather than in a subroutine. This shortens the code length but uses the same algorithm. Note that the procedure can overestimate Hotelling rents because they are estimated in a no-damage calculation. The limit in the DICE-2013R model has not changed from earlier versions and is 6000 tons of carbon content.

$$(11) \quad CCum \geq \sum_{t=1}^{T\,max} E_{Ind}(t)$$

Cumulative carbon emissions from 2010 to 2100 in the baseline DICE-2013R model are projected to be 1870 GtC, and for the entire period 4800 GtC. Estimates for 2100 are slightly higher than the models surveyed in the IPCC Fifth Assessment Report, Science (2013), Figure 6.25.

C. Geophysical sectors

The DICE-2013R model includes several geophysical relationships that link the economy with the different forces affecting climate change. These relationships include the carbon cycle, a radiative forcing equation, climate-change equations, and a climate-damage relationship. A key feature of IAMs is that the modules operate in an integrated fashion rather than taking variables as exogenous inputs from other models or assumptions.

The structure of the geophysical sectors is largely unchanged from the last versions, although the parameters and initial conditions are updated. Equations (12) to (18) below link economic activity and greenhouse-gas emissions to the carbon cycle, radiative forcings, and climate change. These relationships were developed for early versions of the DICE model and have remained relatively stable over recent revisions. They need to simplify what are inherently complex dynamics into a small number of equations that can be used in an integrated economic-geophysical model. As with the economics, the modeling philosophy for the geophysical relationships has been to use parsimonious specifications so that the theoretical model is transparent and so that the optimization model is empirically and computationally tractable.

In the DICE-2013R model, the only GHG that is subject to controls is industrial CO_2. This reflects the fact that CO_2 is the major contributor to global warming and that other GHGs are likely to be controlled in different ways (the case of the chlorofluorocarbons through the Montreal Protocol being a useful example). Other GHGs are included as exogenous trends in radiative forcing; these include primarily CO_2 emissions from land-use changes, other well-mixed GHGs, and aerosols.

Recall that equation (10) generated industrial emissions of CO_2. Equation (12) then generates total CO_2 emissions as the sum of industrial and land-use emissions. CO_2 arising from land-use changes are exogenous and are projected based on studies by other modeling groups and results from the Fifth Assessment of the IPCC. Current estimates are that land-use changes contribute about 3 $GtCO_2$ per year (IPCC Fifth Assessment, Science, 2013, Chapter 6).

(12) $E(t) = E_{Ind}(t) + E_{Land}(t)$

The carbon cycle is based upon a three-reservoir model calibrated to existing carbon-cycle models and historical data. We assume that there are three reservoirs for carbon. The variables $M_{AT}(t)$, $M_{UP}(t)$, and $M_{LO}(t)$ represent carbon in the atmosphere, carbon in a quickly mixing reservoir in the upper oceans and the biosphere, and carbon in the deep oceans. Carbon flows in both directions between adjacent reservoirs. The mixing between the deep oceans and other reservoirs is extremely slow. The deep oceans provide a large sink for carbon in the long run. Each of the three reservoirs is assumed to be well-mixed in the short run. Equations (13) through (15) represent the equations of the carbon cycle.

(13) $M_{AT}(t) = E(t) + \phi_{11} M_{AT}(t-1) + \phi_{21} M_{UP}(t-1)$

(14) $M_{UP}(t) = \phi_{12} M_{AT}(t-1) + \phi_{22} M_{UP}(t-1) + \phi_{32} M_{LO}(t-1)$

(15) $M_{LO}(t) = \phi_{23} M_{UP}(t-1) + \phi_{33} M_{LO}(t-1)$

The parameters ϕ_{ij} represent the flow parameters between reservoirs. Note that emissions flow into the atmosphere.

The carbon cycle is limited because it cannot represent the complex interactions of ocean chemistry and carbon absorption. We have adjusted the carbon flow parameters to reflect carbon-cycle modeling for the 21st century, which show lower ocean absorption than for earlier periods. This implies that the model overpredicts atmospheric absorption during historical periods. The impact of a 100 GtC pulse is that 35% remains in the atmosphere after 100 years. It is useful to compare this with the results from the Fifth Assessment Report of the IPCC. The DICE model atmospheric concentrations 100 years after a pulse are lower than the average of models, which is around 40% IPCC (IPCC Fifth Assessment, Science, 2013, Box 6.1, Fig. 1, p. 6-122).

The next step concerns the relationship between the accumulation of GHGs and climate change. The climate equations are a simplified representation that includes an equation for radiative forcing and two equations for the climate system. The radiative forcing equation calculates the impact of the accumulation of GHGs on the radiation balance of the globe. The climate equations calculate the mean surface temperature of the globe and the average temperature of the deep oceans for each time-step.

Accumulations of GHGs lead to warming at the earth's surface through increases in radiative forcing. The relationship between GHG accumulations and increased radiative forcing is derived from empirical measurements and climate models, as shown in Equation (16).

$$(16) \quad F(t) = \eta\{log_2[M_{AT}(t) / M_{AT}(1750)]\} + F_{EX}(t)$$

F(t) is the change in total radiative forcings of greenhouse gases since 1750 from anthropogenic sources such as CO_2. $F_{EX}(t)$ is exogenous forcings, and the first term is the forcings due to CO_2.

The equation uses estimated carbon in different reservoirs in the year 1750 as the pre-industrial equilibrium. The major part of future warming is projected to come from to CO_2, while the balance is exogenous forcing from other long-lived greenhouse gases, aerosols, ozone, albedo changes, and other factors. The DICE model treats other greenhouse gases and forcing components as exogenous either because these are relatively small, or their control is exogenous (as the case of CFCs), or because they are poorly understood (as with cloud albedo effects).

Estimates of future impacts of aerosols have proven challenging, and the current model uses estimates from the scenarios prepared for the Fifth Assessment of the IPCC. The estimates in DICE-2013R are drawn from the guidance for the "Representative Concentration Pathways" (RCPs, see http://tntcat.iiasa.ac.at:8787/RcpDb/ dsd?Action=htmlpage&page=compare). The high path has exceptionally high and unreasonable estimates of methane forcings. The estimates here use the RCP 6.0 W/ m^2 representative scenario, which is more consistent with the other scenarios and with historical trends. These estimate non-CO_2 forcings of 0.25 W/m^2 in 2010 and 0.7 W/m^2 in 2100. Non-CO_2 forcings are small relative to estimated CO_2 forcings, with 6.5 W/m^2 of forcings from CO_2 in 2100 in the DICE baseline projection.

Higher radiative forcing warms the atmospheric layer, which then warms the upper ocean, gradually warming the deep ocean. The lags in the system are primarily due to the diffusive inertia of the different layers.

$$(17) \quad T_{AT}(t) = T_{AT}(t-1) + \xi_1\{F(t) - \xi_2 T_{AT}(t-1) - \xi_3[T_{AT}(t-1) - T_{LO}(t-1)]\}$$

$$(18) \quad T_{LO}(t) = T_{LO}(t-1) + \xi_4\{T_{AT}(t-1) - T_{LO}(t-1)]\}$$

$T_{AT}(t)$ and $T_{LO}(t)$ represent respectively the mean surface temperature and the temperature of the deep oceans. Note that the equilibrium temperature sensitivity is given by $\Delta T_{AT} = \Delta F(t) / \xi_2$.

A critical parameter is the equilibrium climate sensitivity (°C per equilibrium CO_2 doubling). The method for determining that parameter has been changed in the most recent model. The precise procedure for the new estimates and the calibration is the following: Earlier estimates of the climate sensitivity in the DICE model relied exclusively on estimates from GCMs. However, there is increasing evidence from other sources, such as the historical record. In the DICE 2013 model, we used a synthesis of the different sources, and took a weighted average of the estimates. The revised estimate is a climate sensitivity of 2.9 °C for an equilibrium CO_2 doubling.

This revision is based largely on data from a systematic survey of recent evidence in Knutti and Hegerl (2008). The procedure used for the estimate is to take a weighted average of the estimates of temperature sensitivity from different estimation techniques. The current version combines estimates from instrumental records, the current mean climate state, GCMs, the last millennium, volcanic eruptions, the last glacial maximum (data and models), long-term proxy records, and expert assessments. The weights are from the author, with most of the weights on the model results and the instrumental and historical record. Because the historical record provides a lower estimate, this combined procedure lowers the equilibrium sensitivity slightly (to 2.9 °C for an equilibrium CO_2 doubling).

Note that this reduction is paralleled by a reduction of the lower bound estimate of the TSC in the IPCC's Fifth Assessment Report from 2 °C to 1.5 °C. Additionally, a visual inspection of the summary of different probabilistic assessments in the Fifth Assessment Report indicates that the range of estimates using different techniques is in the range of 1.8 °C to 3.0 °C. An interesting feature is that the climate models are at the high end of the different techniques, with a mean of the ensemble of 3.2 °C in the Fifth Report (see IPCC Fifth Assessment, Science, 2013, Chapter 9, especially Table 9-2).

A further change is to adjust the parameters of the model to match the transient temperature sensitivity for models with an equilibrium sensitivity of 2.9 °C. The relationship between equilibrium and transient climate sensitivity uses the estimates of those two parameters from the IPCC Fifth Assessment Report, which provided both transient and equilibrium temperature sensitivities for several models. We used regression analyses to estimate the transient sensitivity at 2.9 °C equilibrium. The parameterized transient sensitivity from the regressions is set at 1.70 °C. This is done by changing the diffusion parameter ξ_1 (similar to the standard calibrating parameter in simple energy-balance models of the vertical diffusivity) to 0.98. There is also an adjustment of the diffusion parameter in the model if the equilibrium parameter changes.

This completes the description of the DICE model. We now turn to describe the difference between the DICE and RICE models.

D. The RICE-2010 Model

The RICE model (Regional Integrated model of Climate and the Economy) is a regionalized version of the DICE model. It has the same basic economic and geophysical structure, but contains a regional elaboration. The last full version is described in Nordhaus (2010), with detailed in the Supplemental Information to Nordhaus (2010).

The general structure of the RICE model is similar to the DICE model with disaggregation into regions. However, the specification of preferences is different because it must encompass multiple agents (regions). The general preference function is a Bergson-Samuelson social welfare function over regions of the form $W = \mathscr{W}(U^1, \ldots, U^N)$, where U^I is the preference function of the I^{th} region. The model is specified using the Negishi approach in which regions are aggregated using time- and region-specific weights subject to budget constraints, yielding

$$(19) \quad W = \sum_{t=1}^{T\,max} \sum_{I=1}^{N} \psi_{I,t} U^I [c^I(t), L^I(t)] R^I(t)$$

In this specification, the $\psi_{I,t}$ are the "Negishi weights" on each region and each time period. Each region has individual consumption and population. In principle, they may have different rates of time preference, although in practice the RICE model assumes that they are all equal. The Negishi algorithm in the RICE model sets each of the weights so that the marginal utility of consumption is equal in each region and each period, which ensures that the requirement for maximization as market simulation principle holds. We elaborate below on the Negishi approach, which is widely used in IAMs for climate change, in the section on "Computational and algorithmic aspects."

The RICE-2010 model divides the world into 12 regions. These are US, EU, Japan, Russia, Eurasia (Eastern Europe and several former Soviet Republics), China, India, Middle East, Sub-Saharan Africa, Latin America, Other high income countries, and Other developing countries. Note that some of the regions are large countries such as the United States or China; others are large multi-country regions such as the European Union or Latin America.

Each region is assumed to produce a single commodity, which can be used for consumption, investment, or emissions reductions. Each region is endowed with an initial stock of capital and labor and with an initial and region-specific level of technology. Population data are from the United Nations, updated with more recent estimates through 2009, with projections using the United Nations' estimates to 2300. Output is measured as standard gross domestic product (GDP) in constant prices, and the GDPs of different countries are converted into constant U.S. international prices using purchasing-power-parity exchange rates. Output data through 2009 are from the World Bank and the International Monetary Fund (IMF), with projections to 2014 from the IMF. CO_2 emissions data are from the U.S. Energy Information Administration and Carbon Dioxide Information Analysis Center and are available in preliminary form through 2008.

The population, technology, and production structure is the same as in the DICE model. However, each region has its own levels and trends for each variable. The major long-run variable is region-specific technological change, which is projected for a frontier region (the United States), and other countries are assumed to converge partially to the frontier.

The geophysical equations are basically the same as the DICE model as of 2010, but they differ slightly from the current version. The major difference is that there are region-specific land-use CO_2 emissions, but these are exogenous and have little effect on the outcomes.

The objective function used in the RICE model differs from that in the DICE model. Each region is assumed to have a social welfare function, and each region optimizes its consumption, GHG policies, and investment over time. The parameters for each region are calibrated to ensure that the real interest rate in the model is close to the average real interest rate and the average real return on capital in real-world markets in the specific region. We interpret the output and calibration of optimization models as "markets as maximization algorithms" (see Nordhaus 2012 for a discussion). We do not view the solution as one in which a world central planner is allocating resources in an optimal fashion. Rather, output and consumption is determined according to the initial endowments of technology. "Dollar votes" in the RICE model may not correspond to any ethical norms but instead reflects the laws of supply and demand. To put this in terms of standard welfare economics, the outcome is optimal in the sense of both efficient and fair if the initial endowments are ethically appropriate, but without that assumption we can only label the outcome as Pareto efficient.

E. Interpretation of Positive and Normative Models (*)

One of the issues that pervades the use of IAMs is whether they should be interpreted as normative or positive.[2] In other words, should they be seen as the recommendations of a central planner, a world environmental agency, or a disinterested observer incorporating a social welfare function? Or are they meant to be a description of how economies and real-world decision makers (consumers, firms, and governments) actually behave? This issue also arises in the analysis of the discount rate.

For most simulation models, such as general circulation climate models, the interpretation is clearly that these are meant to be descriptive. The interpretation of optimization models is more complex, however. In some cases, the purpose is clearly normative. For example, the *Stern Review* represented an attempt to provide normative guidance on how to cope with the dangers raised by climate change. In other cases, such as baseline projections, these are clearly meant to be descriptive.

The ambiguity arises particularly because many models use optimization as a technique for calibrating market outcomes in a positive approach. This is the interpretation of "market mechanisms as maximization or minimization devices." The question was addressed in one of the earliest energy-model comparisons, chaired by Tjalling Koopmans, "The use of optimization in these models should be seen as a means of simulating, as a first approximation, the behavior of a system of interacting competitive markets." (MRG 1978, p. 5, emphasis added.)

This point was elaborated at length in the integrated assessment study of copper by Gordon, Koopmans, Nordhaus, and Skinner (1987, with minor edits to simplify and emphasis added):

> We can apply this result to our problem of exhaustible resources as follows: if each firm is faced with the same market prices for its inputs and outputs, and if each firm chooses its activities so as to maximize the firm's discounted profits, then the outcome will be economically efficient. In more precise language, such an equilibrium will be economically efficient in the sense that (1) each firm will provide its share of the market at minimum discounted cost; and (2) the requirements of the market will be met by

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[2] This section draws heavily on Nordhaus (2012).

producers in a manner that satisfies total demand at minimum discounted total cost to society.

Examining these two conditions, we see that our competitive equilibrium has indeed solved a minimization problem of sorts – it has found a way of providing the appropriate array of services at lowest possible costs. But this minimization is exactly the objective of a linear-programming problem as well. Consequently, we can mimic the outcome of the economic equilibrium by solving the LP problem that minimizes the same set of cost functions subject to the same set of technical constraints. Put differently, given the appropriate quantities of resources available and the proper demand requirements, by solving a cost-minimizing LP problem we can determine the equilibrium market prices and quantities for all future periods. We call this lucky analytical coincidence the correspondence principle: *determining the prices and quantities in a general economic equilibrium and solving the embedded cost-minimization problem by linear programming are mathematically equivalent.*

This discussion implies that we can interpret optimization models as a device for estimating the equilibrium of a market economy. As such, it does not necessarily have a normative interpretation. Rather, the maximization is an algorithm for finding the outcome of efficient competitive markets.

F. Consistency with the IPCC Fifth Assessment Report (*)

The DICE-2013R was developed over the 2012-13 period and launched shortly after the release of the Working Group I report of the IPCC, or "AR5" (IPCC, Fifth Assessment Report, Science, 2013). Most of the results of AR5 were available before the release, and as a result the geophysical modules were largely consistent with the final report.

Among the major findings of AR5 that relate to the DICE-2013R model, here are the major ones:

- The range of estimates of the climate sensitivity was increased from 2.0 – 4.5 to 1.5 – 4.5 °C, which is assessed to be the likely range of equilibrium climate sensitivity. (The IPCC uses the term "likely" to represent 66–100% probability.) There were no major changes in the average climate sensitivity of the ensemble of models.
- AR5 contained an extended discussion of alternative estimates of the climate sensitivity using different approaches. The summary statistics of the

alternatives was between 1.8 °C and 3.0 °C. The new approach to climate sensitivity is consistent with the trend toward looking at a broader array of sources for that parameter. (p. TS-113)

- The results of the carbon cycle models were largely unchanged from the Fourth Report. See the discussion of the carbon cycle above. (AR5, Chapter 6)
- AR5 used a completely different approach to scenario modeling. It relied on ““Representative Concentration Pathways" (RCPs) as a replacement for the SRES approach to scenarios. As the report states, “These RCPs represent a larger set of mitigation scenarios and were selected to have different targets in terms of radiative forcing at 2100 (about 2.6, 4.5, 6.0 and 8.5 W m−2; see Figure TS.15). The scenarios should be considered plausible and illustrative, and do not have probabilities attached to them. “ (p. TS-44) As noted below, this opens up a large gap between economic analysis and global-warming science.
- The DICE-2013R baseline radiative forcings is close to the RCP 8.5 forcing estimates through 2100, then midway between the RCP 8.5 and RCP 6.0 after 2150. The DICE temperature projection for the baseline scenario is very close to the model ensemble for the RCP 8.5 through 2200. This suggests that the DICE-2015R has a short-run temperature sensitivity that is slightly higher than the AR5 model ensemble. (Figures 12.4, 12.5)
- Emissions in the baseline are close to those of the RCP 8.5 scenario. Total CO_2 emissions in the DICE baseline total 103 $GtCO_2$ compared to 106 $GtCO_2$ in RCP 8.5. Cumulative CO_2 emissions in the DICE baseline are 1889 GtC compared to 1750- 1900 GtC in the models used for RCP 8.5. (p. I-60, I-61)

I close with a final word on the limitations of the RCPs. They have the strong advantage of providing a coherent set of inputs for the calculations of climate and ecological models. However, the RCP are only weakly linked back to the economic drivers of emissions. The models that produce the concentrations and forcings are based on economic and energy models. However, there is no attempt to harmonize the output, population, emissions, and other driving variables across different scenarios. Putting this differently, the IPCC RCPs have very little value in integrating the economic policies and variables with the geophysical calculations and projections.

IV. Results from the DICE-2013R Model (*)

A. Scenarios

Integrated assessment models have a wide variety of applications. Among the most important applications are the following:

- Making consistent projections, i.e., ones that have consistent inputs and outputs of the different components of the system (for example, so that the world output projections are consistent with the emissions projections).
- Calculating the impacts of alternative assumptions on important variables such as output, emissions, temperature change, and impacts.
- Tracing through the effects of alternative policies on all variables in a consistent manner, as well as estimating the costs and benefits of alternative strategies.
- Estimating the uncertainties associated with alternative variables and strategies.
- Calculating the effects of reducing uncertainties about key parameters or variables, as well as estimating the value of research and new technologies.

With these objectives in mind, this section presents illustrative results for different scenarios using the DICE-2013R model. We present the results of five scenarios.

- *Baseline:* Current policies as of 2010 are extended indefinitely. The conceptual definition of the baseline scenario has changed from earlier versions. In earlier runs, "baseline" meant "no policies." In the current version, base is existing policies as of 2010. This approach is standard for forecasting, say of government budgets, and is more appropriate for a world of evolving climate policies. Estimates from Nordhaus (2010) indicate that 2010 policies were the equivalent of $1 per ton of CO_2 global emissions reductions. Note that is requires calculating baseline emissions intensities as reflecting this level of emissions reductions.
- *Optimal:* Climate-change policies maximize economic welfare, with full participation by all nations starting in 2015 and without climatic constraints. The "optimal" scenario assumes the most efficient climate-change policies; in this context, efficiency involves a balancing of the present value of the costs of abatement and the present value of the benefits of reduced climate damages. Although unrealistic, this scenario provides an efficiency benchmark against which other policies can be measured.

- *Temperature-limited:* The optimal policies are undertaken subject to a further constraint that global temperature does not exceed 2 °C above the 1900 average. The "temperature-limited" scenario is a variant of the optimal scenario that builds in a precautionary constraint that a specific temperature increase is not exceeded. This scenario is also consistent with the goals adopted under the "Copenhagen Accord," although countries have not adopted national targets that would reach this limit.
- *Low discounting according to Stern Review. The Stern Review* advocated using very low discount rates for climate-change policy. This was implemented using a time discount rate of 0.1 percent per year and a consumption elasticity of 1. This leads to low real interest rates and generally to higher carbon prices and emissions control rates.
- *Low time preference with calibrated interest rates.* Because the Stern Review run leads to real interest rates that are below the assumed level, we adjust the parameters of the preference function to match the calibrated real interest rates. This run draws on the Ramsey equation; it keeps the near-zero time discount rate and calibrates the consumption elasticity to match observable variables on average through 2040. The calibration keeps the rate of time preference at 0.1 percent per year but raises the consumption elasticity to 2.1.
- *Copenhagen Accord.* In this scenario, high-income countries are assumed to implement deep emissions reductions over the next four decades, with developing countries following gradually. It is assumed that implementation is through system of national emission caps with full emissions trading within and among countries (although a harmonized carbon tax would lead to the same results). We note that most countries are not on target to achieving these goals.

B. Major Results (*)

We present a limited set of results for the different scenarios. The full results are available in a spreadsheet on request from the author.

Table 1 and Figures 3 - 5 show the major economic variables in the different scenarios. These show rapid projected economic growth. The real interest rate is a critical variable for determining climate policy.

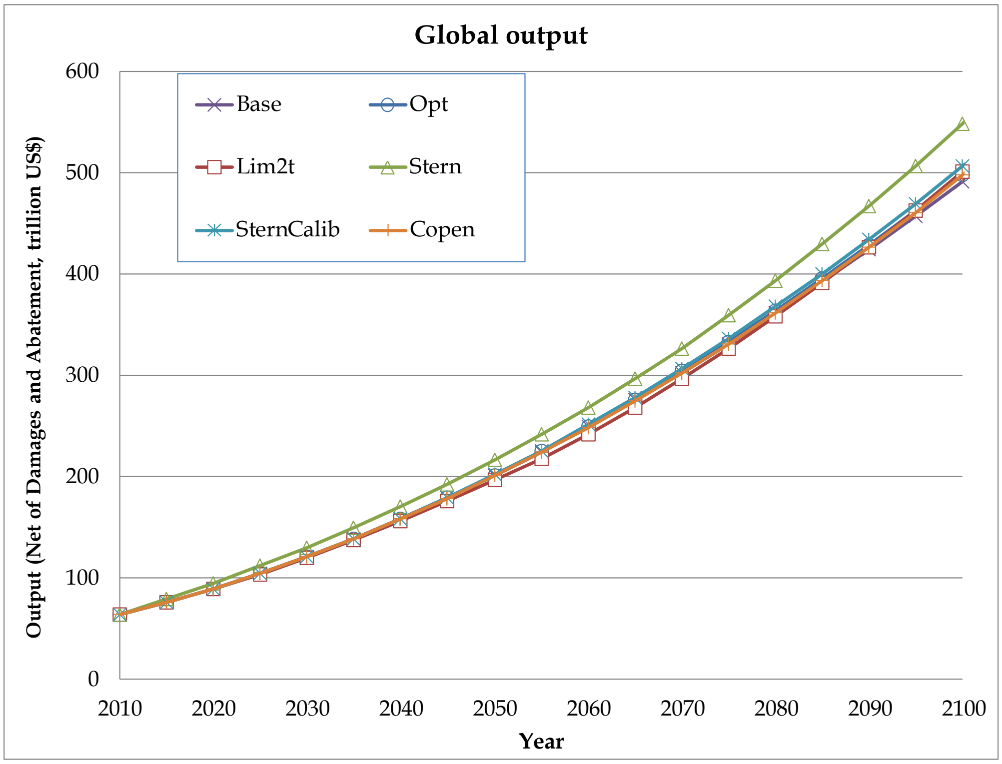

Figure 3. Global output 2010-2100 under alternative policies, DICE-2013R model

[Sources for Figures 3 – 10: Graphicsv5_manual_051713.xlsm]

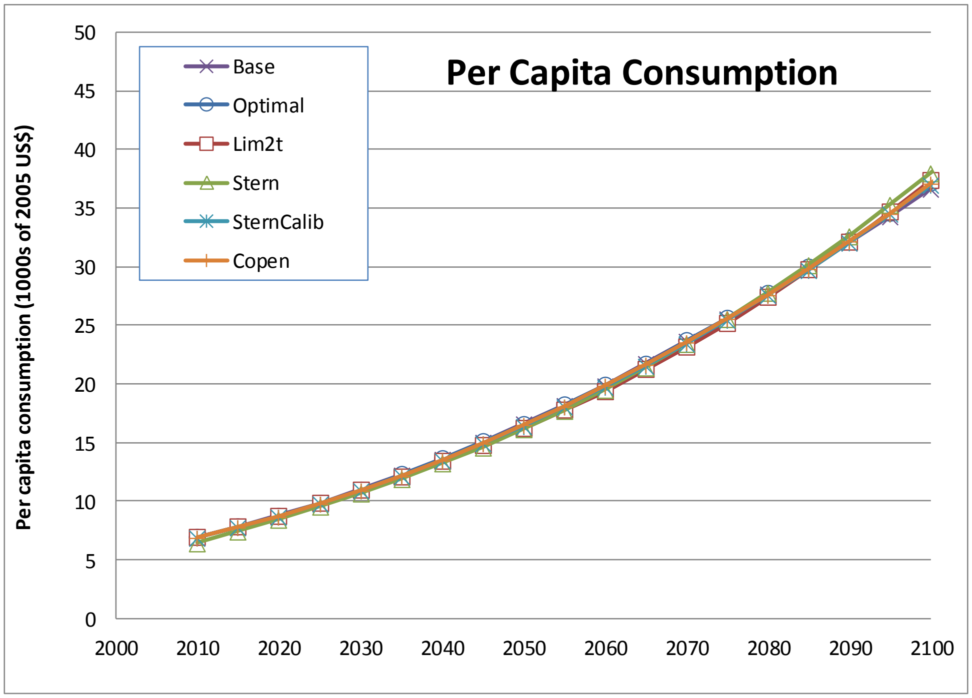

Figure 4. Per capita consumption 2010-2100 under alternative policies, DICE-2013R model

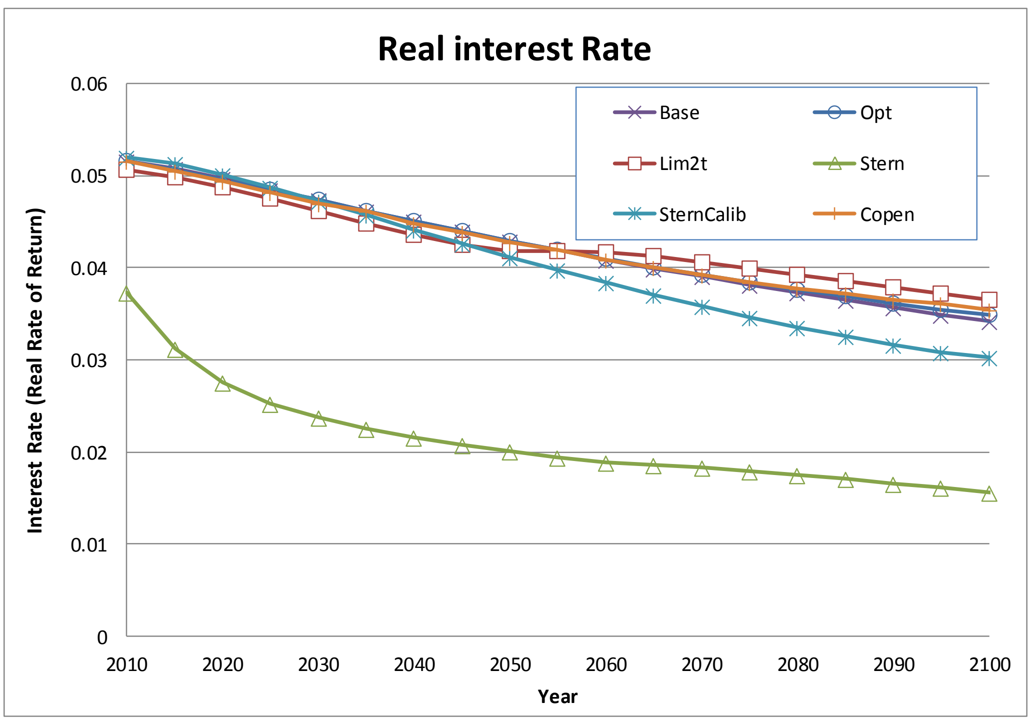

Figure 5. Real interest rate in alternative runs

Note that the real interest rates are similar except for the Stern run, in which case real interest rates are much lower.

Gross World Output (trillions 2005 US$)	2010	2020	2030	2050	2100	2150	2200
Base	63.58	89.59	121.47	203.19	511.56	951.42	1,487.13
Optimal	63.58	89.66	121.61	203.68	516.42	974.47	1,547.66
Limit T < 2 °C	63.58	89.43	121.15	203.14	515.76	981.90	1,555.38
Stern Discounting	63.58	95.87	132.24	222.75	565.11	1,070.41	1,689.24
Stern Recalibrated	63.58	89.39	121.36	204.46	526.81	1,007.44	1,612.62
Copenhagen	63.58	89.65	121.61	203.57	515.25	972.67	1,538.18
Per Capita Consumption (1000 2005 US$)	**2010**	**2020**	**2030**	**2050**	**2100**	**2150**	**2200**
Base	6.886	8.768	11.011	16.600	36.819	64.123	95.981
Optimal	6.878	8.756	10.992	16.567	37.063	67.609	108.390
Limit T < 2 °C	6.897	8.728	10.891	16.112	37.292	69.588	110.419
Stern Discounting	6.103	8.432	10.812	16.361	37.740	70.284	111.996
Stern Recalibrated	6.911	8.743	10.950	16.489	36.984	68.445	109.817
Copenhagen	6.881	8.755	10.974	16.504	37.053	67.951	106.734
Real Interest Rate (% per year)	**2010**	**2020**	**2030**	**2050**	**2100**	**2150**	**2200**
Base	5.16%	4.97%	4.74%	4.29%	3.42%	2.86%	2.52%
Optimal	5.16%	4.96%	4.73%	4.30%	3.49%	3.02%	2.71%
Limit T < 2 °C	5.07%	4.87%	4.62%	4.18%	3.65%	3.02%	2.68%
Stern Discounting	3.73%	2.76%	2.37%	2.01%	1.56%	1.14%	0.91%
Stern Recalibrated	5.21%	5.01%	4.73%	4.12%	3.02%	2.32%	1.82%
Copenhagen	5.16%	4.94%	4.70%	4.28%	3.54%	3.03%	2.62%

Table 1. Major economic variables in different scenarios
[Source: Excel_Handbook_TandG_110711a_092713.xlsx, tab 1-3]

Table 2 and Figures 6 - 8 show the major environment variables: industrial CO_2 emissions, atmospheric CO_2 concentrations, and global mean temperature increase. The model projects substantial warming over the next century and beyond if no controls are taken. Baseline temperature is projected to be around 3.8°C above 1900 levels by 2100, and continuing to rise after that. Atmospheric concentrations of CO_2 are estimated to be 858 ppm in 2100. Total radiative forcings in 2100 for the baseline case are 6.9 W/m^2, which is about 1 W/m^2 below the high case in the IPCC runs for the Fifth Assessment Report.

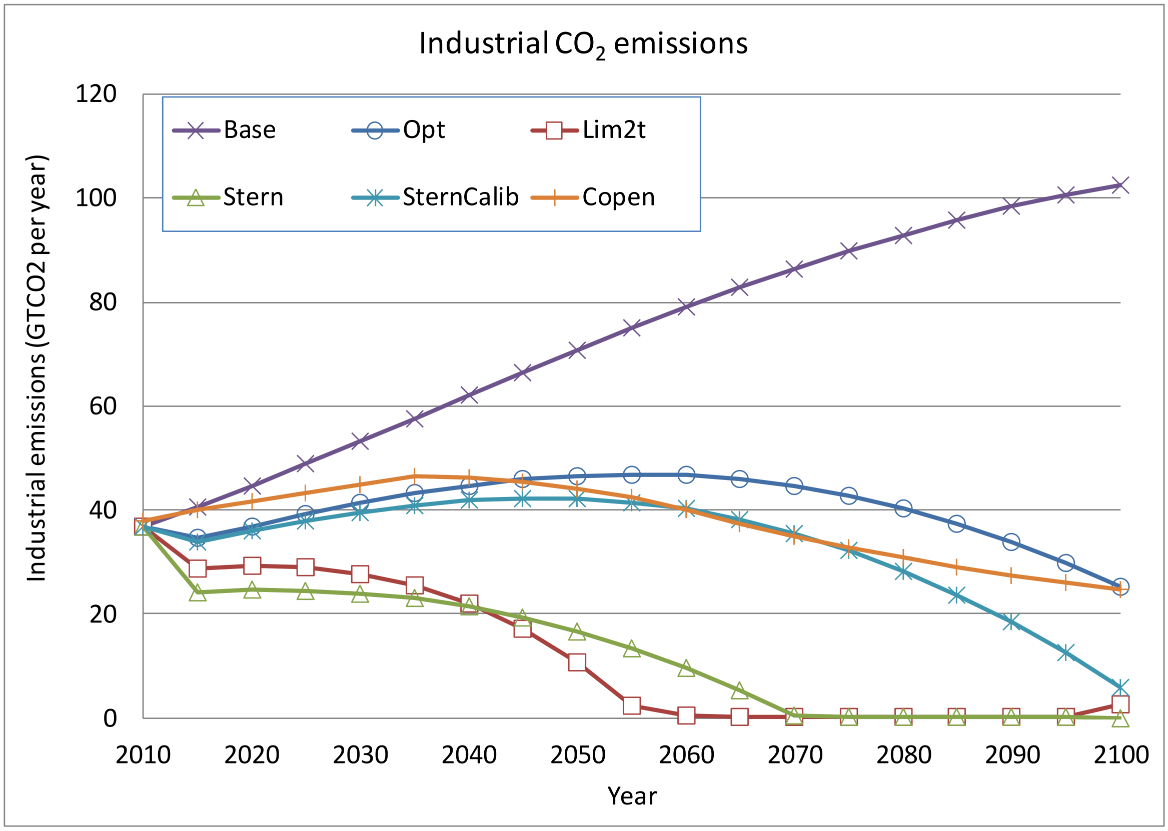

Figure 6. Projected emissions of CO_2 under alternative policies, DICE-2013R model

Note that other GHGs are taken to be exogenous in the projections.

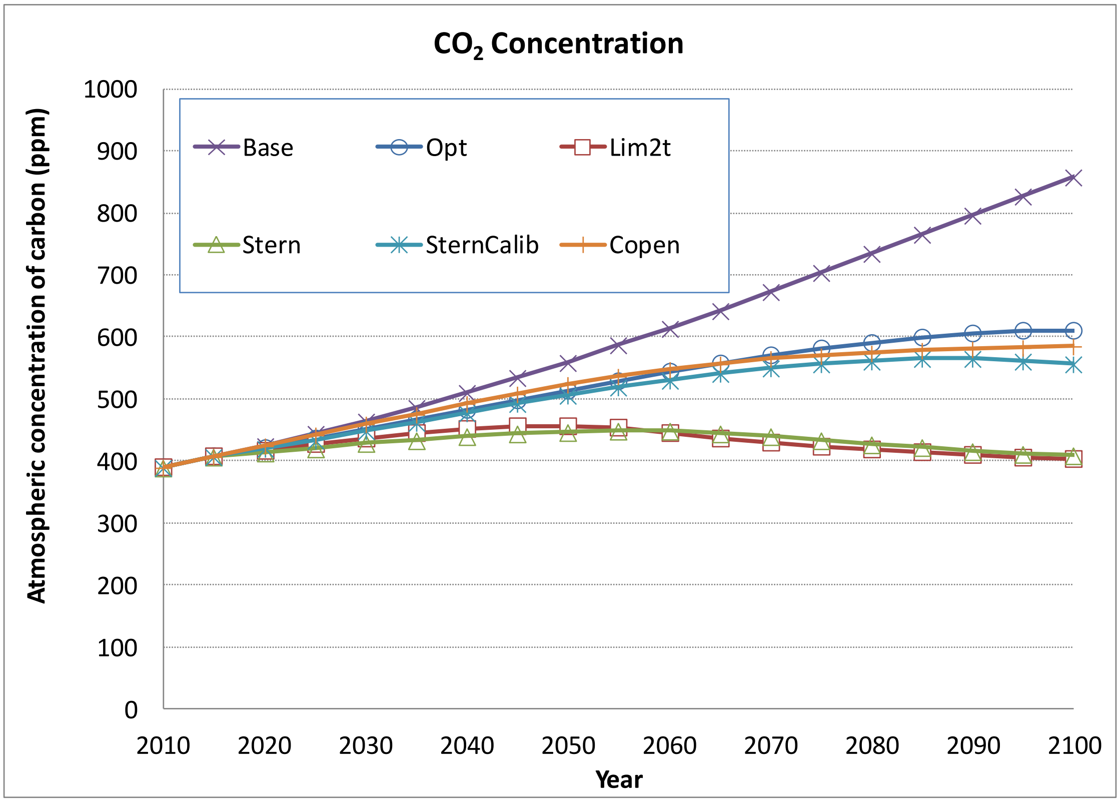

Figure 7. Atmospheric concentrations of CO_2 under alternative policies, DICE-2013R model

Projected atmospheric concentrations of CO2 associated with different policies. The concentrations include emissions from land-use changes. Policies are explained in text.

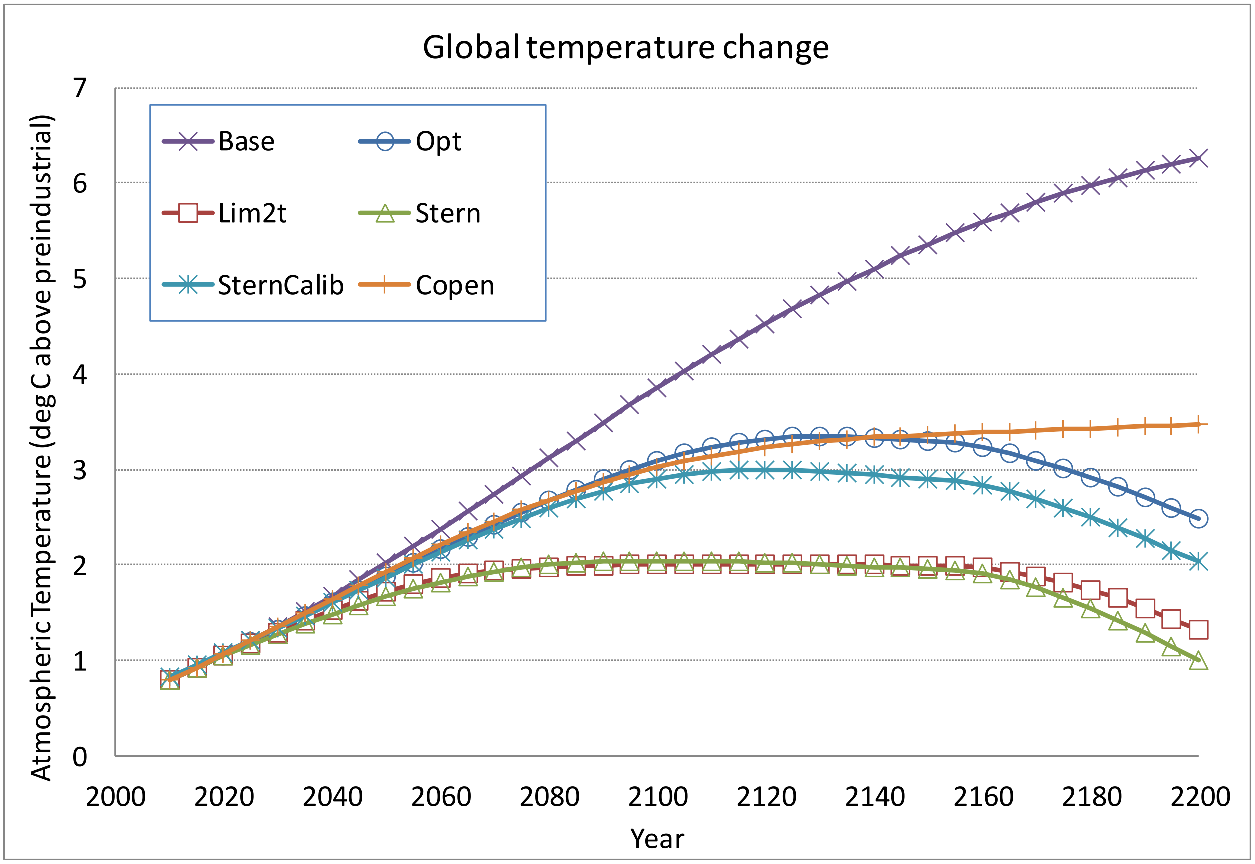

Figure 8. Global temperature increase (°C from 1900) under alternative policies, DICE-2013R model

Projected global mean temperature paths associated with different policies.

Industrial CO_2 Emissions ($GtCO_2$/yr)	2010	2020	2030	2050	2100	2150	2200
Base	33.6	42.5	51.9	70.2	102.5	103.0	66.6
Optimal	33.6	34.8	40.0	46.1	25.1	0.0	-30.0
Limit T < 2 ^oC	33.6	27.1	26.4	10.2	2.7	0.0	-26.8
Stern Discounting	33.6	22.5	22.7	16.1	0.0	0.0	-32.7
Stern Recalibrated	33.6	33.8	38.2	41.6	5.9	0.0	-31.3
Copenhagen	34.6	39.6	43.5	43.5	24.6	14.4	14.9
CO2 concentrations (ppm)	**2010**	**2020**	**2030**	**2050**	**2100**	**2150**	**2200**
Base	390	425	464	560	858	1,134	1,270
Optimal	390	421	451	513	610	535	372
Limit T < 2 ^oC	390	417	436	456	402	395	276
Stern Discounting	390	414	428	447	408	388	243
Stern Recalibrated	390	420	449	506	556	482	328
Copenhagen	390	425	459	524	585	574	584
Temperature Increase (^oC from 1900)	**2010**	**2020**	**2030**	**2050**	**2100**	**2150**	**2200**
Base	0.80	1.06	1.35	2.01	3.85	5.36	6.26
Optimal	0.80	1.05	1.32	1.88	3.09	3.30	2.48
Limit T < 2 ^oC	0.80	1.05	1.29	1.72	2.00	1.99	1.32
Stern Discounting	0.80	1.05	1.27	1.67	2.04	1.96	1.00
Stern Recalibrated	0.83	1.08	1.34	1.87	2.91	2.90	2.04
Copenhagen	0.80	1.06	1.34	1.93	3.02	3.37	3.47

Table 2. Major geophysical variables in different scenarios

Perhaps the most important outputs of integrated economic models of climate change are the near-term "carbon prices." This is a concept that measures the market price of emissions of GHGs. In a market environment, such as a cap-and-trade regime, the carbon prices would be the trading price of carbon emission permits. In a carbon-tax regime, these would be the harmonized carbon tax among participating regions. If the policy is optimized, then the carbon price is also the social cost of carbon.

Table 3 and Figures 9 - 10 show policies in the different scenarios. The optimal carbon price in 2015 is estimated to be \$18 per ton of CO2, rising to \$52 per ton in 2050 and \$143 per ton in 2100. The average increase in the real price is 3.1% from 2015 to 2050 and 2.1% per year from 2050 to 2100. The emissions control rate in the optimal case starts at 21%, rises to around 39% in 2050, and reaches 79% in 2100. Industrial emissions in the optimal case peak around 2050. I emphasize that these are global figures, not just those for rich countries.

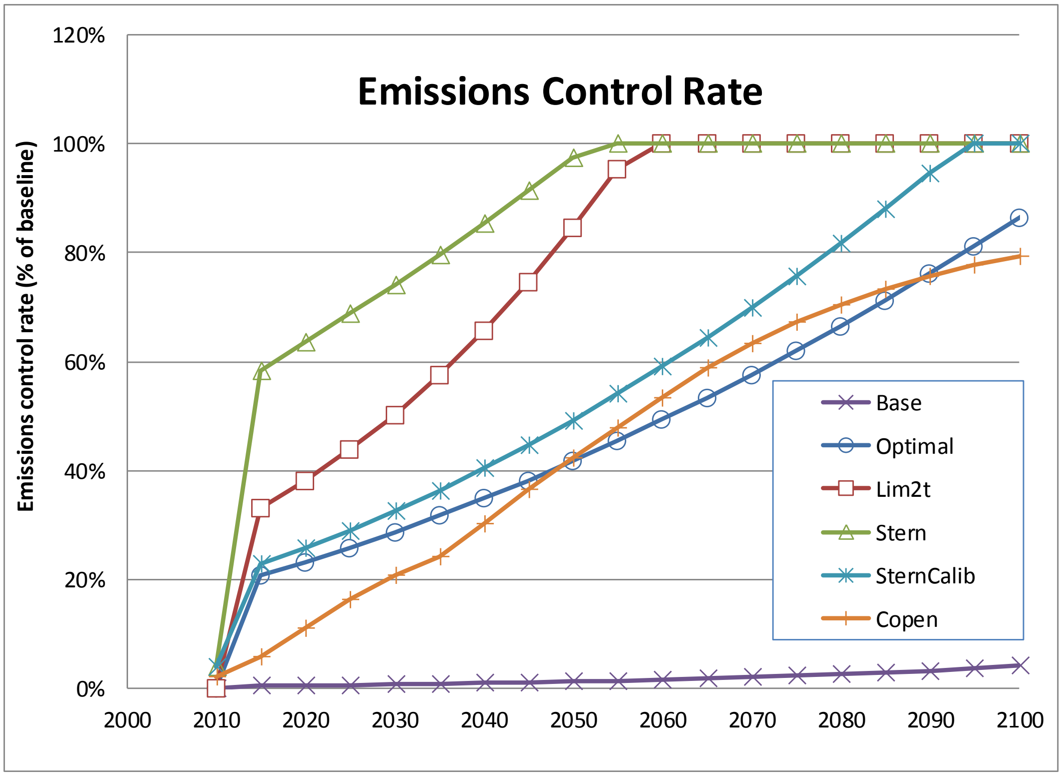

Figure 9. Emissions control rates, alternative scenarios, DICE-2013R model

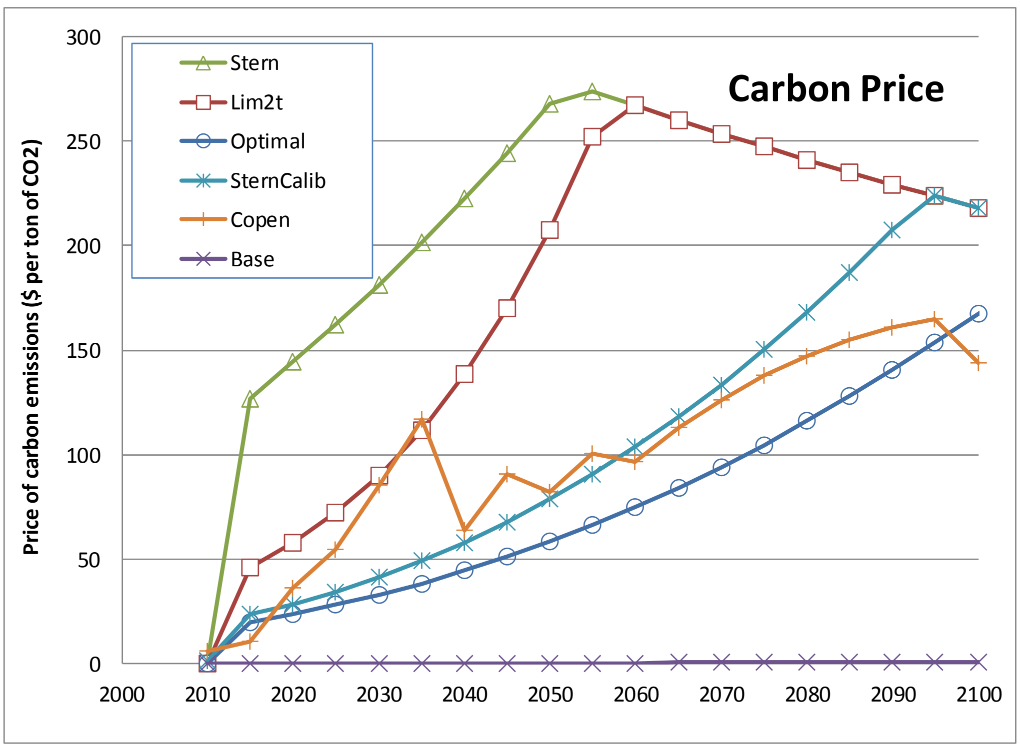

Figure 10. Globally averaged carbon prices, alternative scenarios, DICE-2013R model

Emissions Control Rate (%)	2010	2020	2030	2050	2100	2150	2200
Base	4%	4%	5%	7%	14%	27%	54%
Optimal	4%	22%	27%	39%	79%	100%	120%
Limit T < 2 °C	4%	39%	52%	87%	98%	100%	118%
Stern Discounting	4%	53%	62%	80%	100%	100%	120%
Stern Recalibrated	4%	24%	30%	45%	95%	100%	120%
Copenhagen	1%	11%	21%	42%	79%	90%	90%
Carbon Price (2005$ per ton CO_2)	**2010**	**2020**	**2030**	**2050**	**2100**	**2150**	**2200**
Base	1.0	1.2	1.5	2.2	5.9	16.0	43.1
Optimal	1.0	21.2	29.3	51.5	142.8	169.3	182.5
Limit T < 2 °C	1.0	60.1	94.4	216.4	209.4	169.3	176.5
Stern Discounting	1.0	103.7	131.3	190.0	218.1	169.3	182.5
Stern Recalibrated	1.0	25.0	35.9	66.9	199.4	169.3	182.5
Copenhagen	1.0	35.8	85.4	81.9	144.0	140.1	108.7

Table 3. Major climate-policy variables in different scenarios

The requirement for achieving the 2 °C target is ambitious. The global emissions control rate would be 50% by 2030, and industrial emissions would need to reach zero by 2060.

Table 4 shows the large stakes involved in climate-change policies as measured by aggregate costs and benefits. Using the model discount rates, the optimal scenario raises the present value of world income by $21 trillion, or 0.83% of discounted income. This is equivalent to an annuity of $904 billion per year at a 4% annual discount rate. Imposing the 2 °C temperature constraint has a significant economic penalty, reducing the net benefit by almost half, because of the difficulty of attaining that target with so much inertia in the climate system. The Copenhagen Accord with phased-in participation of developing countries has substantial net benefits, but lack of participation in the "rich only" case reduces the benefits below the optimal level.

Policy Scenario	Level	Level	Difference from base	Difference from base	Difference from base	Difference from base
	Present value of utility	Present value of consumption	Present value of utility	Present value of consumption	Difference from base	Annuity (at 4% per year)
	Trillions of 2005$	Trillions of 2005$	Trillions of 2005$		% of Base	Billions of $ per year
Base	2685.0	2685.0	0.00	0.00	0.00%	0
Optimal	2706.1	2707.6	21.05	22.60	0.83%	904
Limit T < 2 °C	2686.7	2689.7	1.71	4.63	0.17%	185
Stern Discounting	na	2658.0	na	-27.00	-1.02%	-1,080
Stern Recalibrated	na	2706.3	na	21.28	0.79%	851
Copenhagen	2700.1	2701.3	15.03	16.27	0.60%	651

Table 4. Present value of global consumption, different policies, DICE-2013R model (US international dollars, 2005 prices) [source: Tables_092513.xlsx]

The estimates are the present value of global consumption equivalent for the entire period. This is equivalent to the present value of utility in consumption units. The difference in numerical column 3 shows the difference between the control run and the baseline run. The last column is the constant consumption annuity that would be generated by different policies.

There are many conclusions that can be drawn from the present modeling effort. One important result is that, even if countries meet their ambitious objectives under the Copenhagen Accord, global temperatures are unlikely to keep within the 2 °C objective. This conclusion is reinforced if developing countries delay their full participation beyond the 2030-2050 timeframe.

V. The Recommendation for a Cumulative Emissions Limit (*)

Scientists in the Fifth Assessment Report suggested a new approach to climate change targets. They noted that a limit of 2 °C would imply that cumulative emissions should be limited to 1 trillion (1000 billion) tons of carbon. Since cumulative emissions to 2010 are, by their calculation, 531 billion, this would allow 469 billion tons of carbon of additional emissions in the future.

This proposal is easily implemented in the DICE-2013R model. It involves putting constrains on future emissions by the recommended amount. The result is temperature and emissions paths that are slightly higher than the 2 °C limit path, but with those variables lower than in the optimal path. The path is significantly less efficient than the optimal plan because it targets an intermediate variable (cumulative emissions) rather than an ultimate variable (economic welfare). There are also serious issues involved in negotiating cumulative emissions limits, similar to those of annual emissions limits. However, this is another useful idea to consider among alternative architectures. (For a discussion of alternative architectures and the limitations of quantitative targets, see Nordhaus 2013.)

VI. Revisions from earlier vintages (*)

A. Data and structural revisions (*)

There are several large and small changes in the DICE-2013R model compared to earlier versions. The prior complete documented version of the DICE model is Nordhaus (2008), while the last complete version of the regional (RICE) model is in Nordhaus (2010).

The first revision is that the time step has been changed to five years. This change is taken because improvements in computational capacities allow the model to be easily solved with a finer time resolution. The change in the time step also allows removing several ad hoc procedures designed to calibrate actual dynamic processes.

A second change is the projection of future output growth. Earlier versions of the DICE and other IAMs tended to have a stagnationist bias, with the growth rate of total factor productivity declining rapidly in the coming decades. The current version assumes continued rapid total factor productivity growth over the next century, particularly for developing countries.

A third revision incorporates a less rapid decline in the CO_2-output ratio in several regions and for the world, which reflects the last decade's observations. Earlier trends (through 2004) showed rapid global decarbonization, at a rate between 1½ and 2 percent per year. Data through 2010 indicate that decarbonization has been closer to 1 percent per year. The new version assumes that, conditional on output growth, uncontrolled CO_2 emissions will grow at ½ percent per year faster than earlier model assumptions.

A fourth assumption involves the damage function. This change was discussed above and will not be repeated here.

A fifth revision recalibrates the carbon-cycle and climate models to recent earth system models. The equilibrium and transient temperature impacts of CO_2 accumulation have been revised to include a wider range of estimates. Earlier versions relied entirely on the estimates from general circulation models (for example, the ensemble of models used in the IPCC Fourth Assessment Report IPCC Fourth Assessment, Science 2007). The present version uses estimates from sources such as the instrumental record and estimates based on the paleoclimatic reconstructions. The carbon cycle has been adjusted to reflect the saturation of ocean absorption with higher temperatures and carbon content.

A sixth set of changes are updates to incorporate the latest output, population, and emissions data and projections. Output histories and projections come from the IMF World Economic Outlook database. Population projections through 2100 are from the United Nations. CO_2 emissions are from the Carbon Dioxide Information Analysis Center (CDIAC). Non-CO_2 radiative forcings for 2010 and projections to 2100 are also from projections prepared for the IPCC Fifth Assessment (see above). The definition of regions (particularly the EU and developing countries) has changed to reflect changing compositions and reflects the structure as of 2012.

A seventh revision is to change the convention for measurement from tons of carbon to tons of CO_2 or CO_2-equivalent, this being to reflect the current conventions in most price and economic data.

B. Revisions to the discount rate (*)

A final question concerns calibration of the model for rates of return on capital. The philosophy behind the DICE model is that the capital structure and rate of return should reflect actual economic outcomes. This implies that the parameters should generate savings rates and rates of return on capital that are consistent with observations (this is sometimes called the "descriptive approach" to discounting after Arrow et al. 1995).

The data on rates of return used in calibration are as follows. (a) The normal risk-free real return, generally taken to be U.S. or other prime sovereign debt, is in the range of 0 – 1 % per year depending upon period, concept, and tax status; (b) the rate of return on risky capital of large corporations in mature markets, after company taxes but before individual taxes, is in the range of 5 to 8 % per year depending on period, concept, and tax status; (c) the rate of return on risky investments in illiquid or immature markets, as well as for poorly capitalized individuals, is generally much higher than for corporations and ranges from 0 to 100 % per year depending on the circumstances.

The major question in the present context is the interpretation of the difference between risky and the risk-free returns. The following explains the approach taken in the DICE and RICE models. (d) It is unclear how much of the difference between the return on risky capital and the risk-free return (this can be called the "capital premium" as an extension of the "equity premium) is compensation for non-diversifiable risk. For the present study, I assume that the capital premium reflects non-diversifiable risks; (e) the extent to which climate investments are correlated with systematic consumption risk is an open question, although preliminary results from Nordhaus (2008) and Gollier (2013) suggest a relatively high consumption beta. For the present study, I assume that the consumption beta on climate investments is 1. This assumption implies that climate investments share the same risk structure of the average investment portfolio.

Based on these assumptions, I assume that the appropriate rate of return for IAMs like the DICE and RICE models is 5% per year in the near term and 4¼ % per year over the period to 2100. Note that in a multi-region model, this would represent a lower figure for the U.S. and a higher figure for other countries, and it is therefore consistent with estimates in other studies, such as the U.S. Interagency Working Group discussed later, that uses U.S. data.

With this calibration, we choose the pure rate of social time preference (ρ) to be 1.5% per year and the consumption elasticity (α) to be 1.45. These parameters, along with the assumed rates of growth of total factor productivity and population, produce average rates of return on capital equal to the numbers just cited.

VII. Impacts of the Revisions (*)

A. Last round of revisions (*)

We have calculated the effects of the different revisions by moving step by step from DICE-2013R back to DICE-2007.

Tables 5 and 6 show a step-by-step decomposition between the different vintages of the DICE model. These tables show the different steps for five output variables from 2010 to 2055: the optimal carbon price, the optimal emissions control rate, the real interest rate, and the temperature in the baseline and optimal cases. The steps are somewhat arbitrary because there may be interactions. However, some experiments indicate that the changes are roughly additive, so the general effects can be easily seen.

Optimal carbon price (2005$ per ton of CO2)

Row		2010	2015	2020	2025	2030	2035	2040	2045	2050
1	DICE 2013R		17.73	21.16	25.02	29.35	34.15	39.44	45.22	51.52
1a	DICE 2013		19.58	23.41	27.78	32.69	38.17	44.24	50.93	58.24
2	Ex1 (pop)		16.96	20.15	23.77	27.83	32.36	37.36	42.86	48.88
3	Ex2 (Y0)		16.67	19.19	22.23	25.76	29.77	34.27	39.24	44.71
4	Ex3 (growth)		16.40	18.66	21.05	23.58	26.29	29.19	32.29	35.61
5	Ex4 (TSC)		15.18	17.25	19.45	21.78	24.27	26.94	29.78	32.83
6	Ex5 (damages)		16.13	18.33	20.66	23.14	25.78	28.61	31.63	34.86
7	Ex6 (carbon intensity)		16.38	18.64	21.02	23.55	26.25	29.13	32.22	35.51
8	Ex7 (calibration for interest rate)		10.56	11.98	13.53	15.23	17.09	19.10	21.27	23.62
9	DICE 2007	9.31	11.36	13.10	14.83	16.84	18.85	21.13	23.41	25.96
10	DICE 2007 (DICE 2013 interest rates)	12.28	14.66	16.73	18.81	21.20	23.60	26.30	29.00	32.01

Optimal control rate (fraction of baseline emissions)

Row		2010	2015	2020	2025	2030	2035	2040	2045	2050
1	DICE 2013R		0.195	0.218	0.243	0.269	0.297	0.327	0.357	0.390
1a	DICE 2013		0.206	0.231	0.258	0.286	0.316	0.348	0.382	0.417
2	Ex1 (pop)	-	0.190	0.213	0.236	0.262	0.289	0.317	0.347	0.378
3	Ex2 (Y0)	-	0.189	0.207	0.228	0.251	0.276	0.302	0.330	0.360
4	Ex3 (growth)	-	0.187	0.204	0.221	0.239	0.257	0.276	0.296	0.317
5	Ex4 (TSC)	-	0.179	0.195	0.211	0.228	0.246	0.264	0.283	0.303
6	Ex5 (damages)	-	0.185	0.202	0.219	0.236	0.254	0.273	0.293	0.314
7	Ex6 (carbon intensity)	-	0.187	0.204	0.221	0.238	0.257	0.276	0.296	0.317
8	Ex7 (calibration for interest rate)	-	0.146	0.159	0.173	0.187	0.202	0.218	0.235	0.253
9	DICE 2007	0.172	0.185	0.198	0.212	0.226	0.240	0.254	0.269	0.285

Interest rate (percent per year, real)

Row		2010	2015	2020	2025	2030	2035	2040	2045	2050
1	DICE 2013R	5.16%	5.07%	4.97%	4.85%	4.74%	4.62%	4.51%	4.40%	4.29%
1a	DICE 2013	5.16%	5.07%	4.96%	4.85%	4.74%	4.62%	4.51%	4.40%	4.30%
2	Ex1 (pop)	5.13%	5.03%	4.92%	4.81%	4.70%	4.59%	4.49%	4.38%	4.28%
3	Ex2 (Y0)	3.97%	4.35%	4.53%	4.58%	4.57%	4.52%	4.45%	4.36%	4.27%
4	Ex3 (growth)	4.19%	3.99%	3.86%	3.77%	3.71%	3.67%	3.63%	3.60%	3.58%
5	Ex4 (TSC)	4.20%	3.99%	3.86%	3.78%	3.72%	3.67%	3.64%	3.61%	3.58%
6	Ex5 (damages)	4.19%	3.99%	3.86%	3.77%	3.71%	3.67%	3.63%	3.60%	3.58%
7	Ex6 (carbon intensity)	4.19%	3.99%	3.86%	3.77%	3.71%	3.67%	3.63%	3.60%	3.57%
8	Ex7 (calibration for interest rate)	4.56%	4.55%	4.54%	4.51%	4.49%	4.46%	4.43%	4.40%	4.36%
9	DICE 2007	4.37%	4.46%	4.46%	4.46%	4.43%	4.40%	4.32%	4.25%	4.22%
10	DICE 2007 (DICE 2013 interest rates)	3.57%	3.64%	3.64%	3.64%	3.61%	3.59%	3.54%	3.49%	3.47%

Table 5. Decomposition of changes from DICE-2007 to DICE-2013R model, policy variables and real interest rate

Temperature increase, reference case (°C from 1900)

		2010	2015	2020	2025	2030	2035	2040	2045	2050
1	DICE 2013R	0.80	0.93	1.06	1.20	1.35	1.51	1.67	1.84	2.01
1a	DICE 2013	0.83	0.93	1.05	1.18	1.32	1.47	1.63	1.80	1.98
2	Ex1 (pop)	0.83	0.93	1.05	1.18	1.31	1.46	1.61	1.77	1.93
3	Ex2 (Y0)	0.83	0.93	1.05	1.17	1.30	1.44	1.59	1.74	1.89
4	Ex3 (growth)	0.83	0.93	1.04	1.16	1.29	1.42	1.55	1.68	1.82
5	Ex4 (TSC)	0.83	0.93	1.03	1.14	1.26	1.38	1.51	1.63	1.76
6	Ex5 (damages)	0.83	0.93	1.03	1.14	1.26	1.38	1.51	1.63	1.76
7	Ex6 (carbon intensity)	0.83	0.93	1.03	1.14	1.26	1.38	1.51	1.64	1.78
8	Ex7 (calibration for interest rate)	0.83	0.93	1.03	1.14	1.26	1.38	1.50	1.63	1.77
9	DICE 2007	0.85	0.96	1.08	1.20	1.33	1.46	1.58	1.71	1.84

Temperature increase, optimal case (°C from 1900)

		2010	2015	2020	2025	2030	2035	2040	2045	2050
1	DICE 2013R	0.80	0.93	1.05	1.19	1.32	1.46	1.60	1.74	1.88
1a	DICE 2013	0.83	0.93	1.04	1.16	1.28	1.41	1.54	1.67	1.80
2	Ex1 (pop)	0.83	0.93	1.04	1.16	1.28	1.40	1.53	1.65	1.78
3	Ex2 (Y0)	0.83	0.93	1.04	1.16	1.27	1.39	1.51	1.63	1.76
4	Ex3 (growth)	0.83	0.93	1.04	1.15	1.26	1.37	1.48	1.59	1.70
5	Ex4 (TSC)	0.83	0.93	1.03	1.13	1.23	1.34	1.44	1.55	1.65
6	Ex5 (damages)	0.83	0.93	1.03	1.13	1.23	1.34	1.44	1.55	1.65
7	Ex6 (carbon intensity)	0.83	0.93	1.02	1.13	1.23	1.34	1.44	1.55	1.66
8	Ex7 (calibration for interest rate)	0.83	0.93	1.02	1.13	1.23	1.34	1.45	1.56	1.68
9	DICE 2007	0.84	0.95	1.06	1.17	1.27	1.38	1.48	1.58	1.68

Table 6. Decomposition of changes from DICE-2007 to DICE-2013R model, temperature increase [source: compare dices 092613.xlsx]

Key:

Ex1 (pop) = population revised.

Ex2 (Y0) = Ex1 + initial output revised.

Ex3 (growth) = Ex2 + growth of output revised.

Ex4 (TSC) = Ex3 + TSC revised.

Ex5 (damages) = Ex4 + damage function revised.

Ex6 (carbon intensity) = Ex5 + carbon intensity revised.

Ex7 (calibration for interest rate) = Ex6 + elasticity of marginal utility of consumption revised.

Here is the series of steps. The steps move backwards from the 2013R model to the 2007 model.

- Row 1. This is the optimized run for the DICE-2013R model.
- Row 1a. This is the optimized run for DICE-2013 from April 2013. There are several small differences from DICE-2013R, particularly initial conditions, lowering the temperature sensitivity coefficient, and raising exogenous non-CO_2 forcings. The change in the social cost of carbon is largely due to the lowering of the temperature sensitivity.
- Row 2. The population projection changes from the IIASA model to the UN projections. This made little difference in the near term but increased the asymptotic population from 8,600 to 10,500 million.
- Row 3. The next correction is for the output level in 2010. The 2007 model overpredicted output by about 6 percent, primarily because of failure to see the Great Recession. We have reduced output to put the new path on the current starting point.
- Row 4. We have revised downward slightly the rate of growth of total factor productivity over the coming decades. For example, projected world GDP for 2105 was revised upward from 1.55% per year to 2.23% per year. The projection after 2105 was essentially unchanged.
- Row 5. The temperature sensitivity coefficient (effect of doubling CO_2 concentrations on equilibrium temperature) was increased from 3.0 °C to 3.2 °C in the 2013 version. Row 5 shows the effect of reducing it back to the 2007 assumption.
- Row 6. The damage function was recalibrated to correspond to the Tol survey, which had slightly lower damages than the earlier DICE/RICE models. In row 6, we increase the damage function to correspond to the DICE-2007 assumption.
- Row 7. Emissions in 2010 were about 2.5% higher than were projected in the 2007 model. We therefore lowered the initial emissions intensity to correspond to the 2007 estimate. Carbon intensity was projected to grow about 0.3% per year more rapidly than the new projection. Row 7 shows the effect of going back to the higher carbon intensity and the difference in initial emissions.
- Row 8. The parameters of the utility function are set at the DICE-2007 level. This increases the elasticity of the utility function from 1.45 to 2.
- Row 9. Shows the results of the DICE-2007 model.
- Row 10. This takes the DICE-2007 model and changes the consumption elasticity from 2 to 1.45 to determine the effect of this parameter alone.

The results show that the optimal carbon price (and the social cost of carbon) increased sharply from the 2007 model to the 2013R model, from $11.36 per ton of CO_2 to $17.73 per ton of CO_2 in 2015, an increase of approximately 60%. Two variables contributed most to this change: the calibration of interest rates and the higher population growth. These contributed approximately equally to the increase in the SCC.

All other variables individually made a modest difference, sometimes positive and sometimes negative, but the other variables netted out to approximately zero. If the two key variables are changed at the same time, the SCC is very close to the 2007 calculation ($11.53 v. $11.36). As noted above, the population revision reflects moving from the IIASA projection to the UN projection. The change in the interest rate is primarily due to a more careful calibration in the 2013 model with a specific target real rate of interest explicitly targeted.

B. Revisions in the DICE model over the last two decades (*)

We have reviewed the revisions since the first published version of the DICE model in Nordhaus (1992) as well as carbon prices since the earliest related model. This review shows that model developments and revisions have had substantial effects on the outcomes.

Figure 11 compares the projections of atmospheric CO_2 concentrations of the DICE-2013R model with several models of the EMF-22 comparison. The DICE-2013R is higher in part because of projections of higher CO_2 emissions and in part because of higher atmospheric retention (see above).

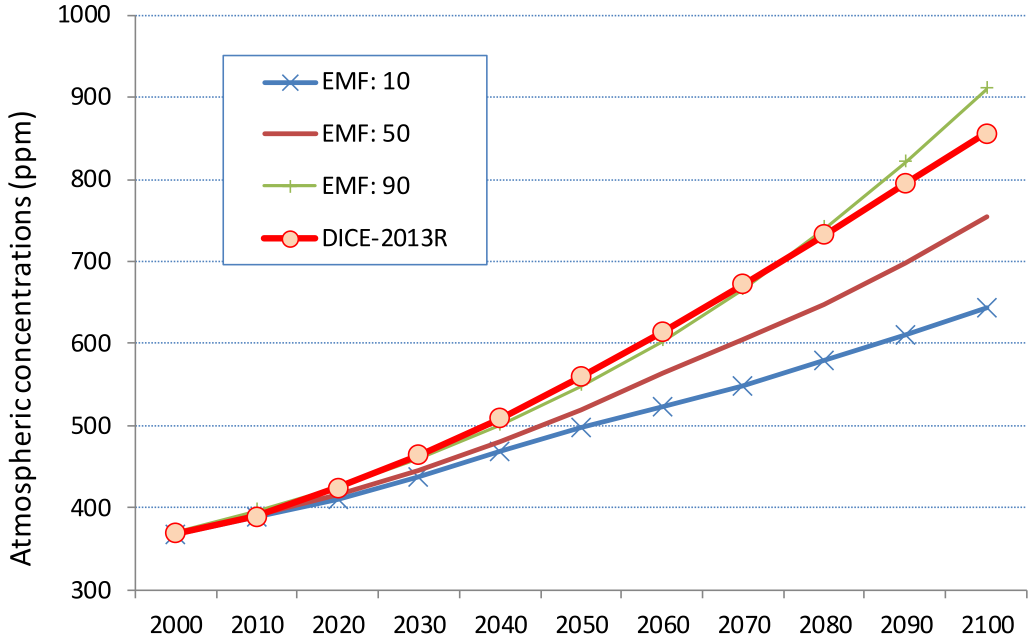

Figure 11. Comparison of CO_2 concentrations between DICE-2013R and EMF-22 models

[Source: Clarke et al. (2009) and spreadsheet with results provided by Leon Clarke. Manual_Figs_092713.xlsx, F8]

Figure 12 shows the projections of temperature increase. The earliest vintages had an intermediate temperature projection. The projected change to 2100 decreased and then increased for the latest vintage. The earliest versions projected 2100 temperature increase of a little above 3 °C, whereas the latest version is a full degree C above that. The increase is a combination of more rapid output growth, higher temperature sensitivity, and a higher fraction of emissions retained in the atmosphere.

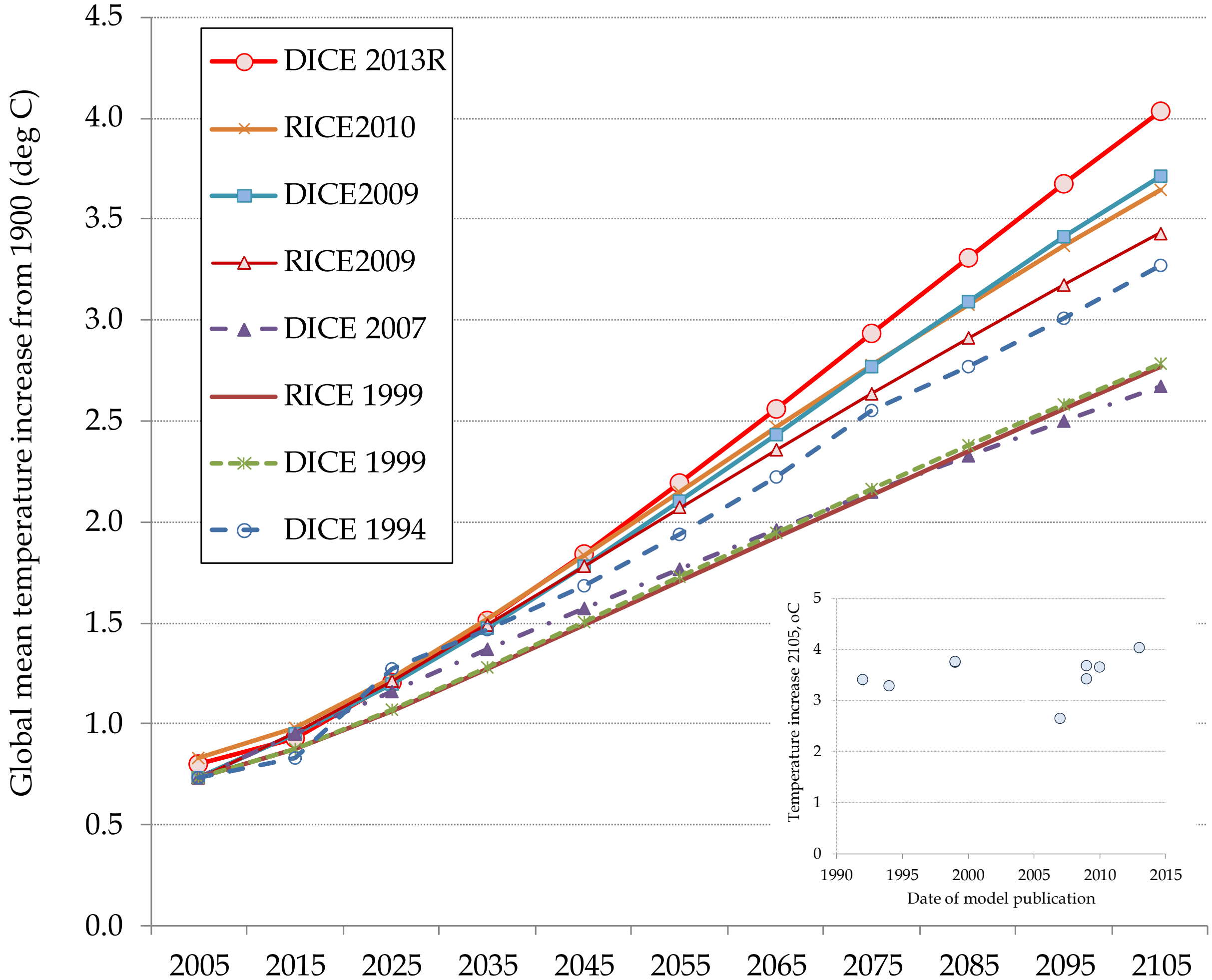

Figure 12. Projected baseline temperature increase, different vintages DICE/RICE models

The small window at the bottom right shows the projected temperature change in 2105 for different vintages [source: Manual_Figs_092713.xlsx, F7]

Figure 13 shows the damage-output ratio over the vintages. The estimated damage-output has risen by about one-third over the different vintages. This is primarily due to higher temperature projections .

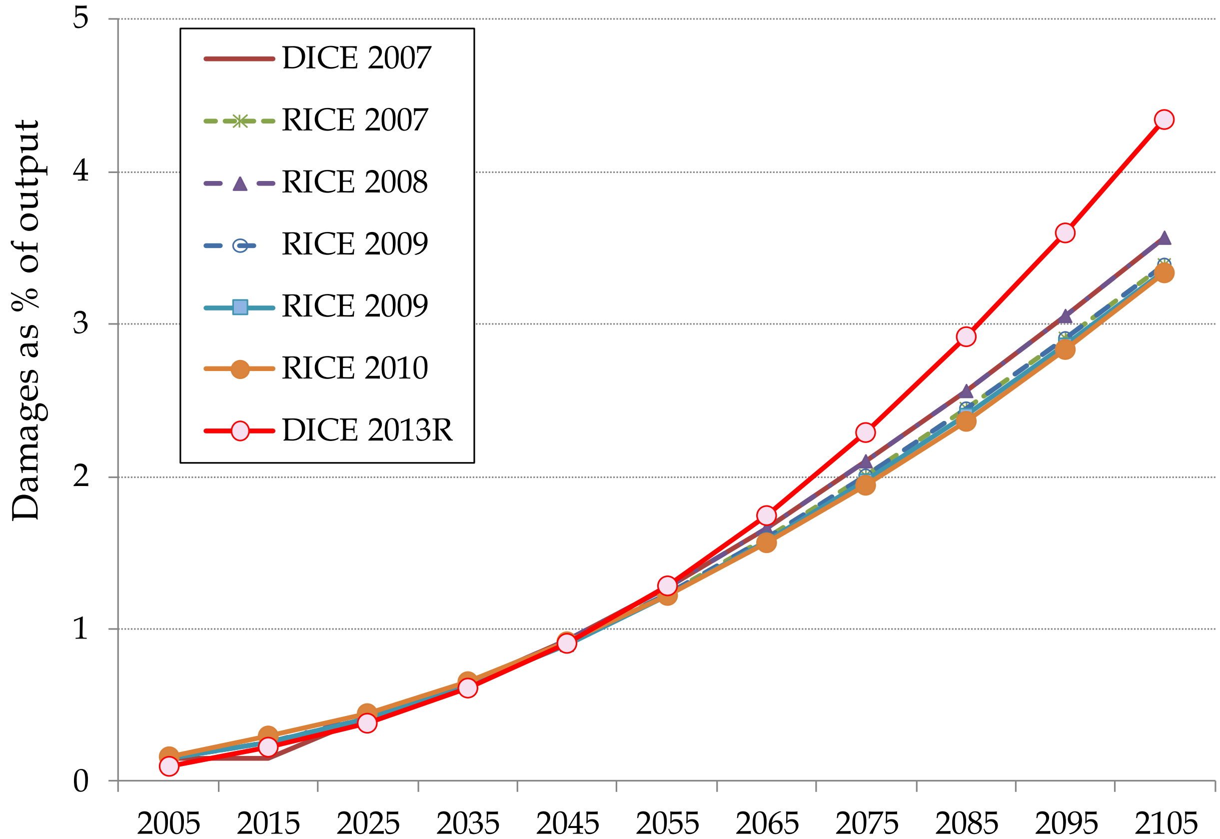

Figure 13. Damages as percent of output, baseline, different vintages DICE/RICE models
[Source: Manual_Figs_092713.xlsx; SF4n]

Figure 15 shows the calculated optimal carbon prices for vintages of models going back to the first unpublished study in 1975, looking at the estimated prices for three different years. The prices have been put in comparable prices and concepts, although some differences remain. The optimal price in 2013 is surprisingly close to the first calculations even though the models have changed dramatically.[3] This

[3] The earliest calculation of an optimal carbon tax was in Nordhaus (1975). This calculated the price of keeping CO_2 concentrations to a doubling, which is approximately the optimal path.

picture also shows the extent to which model uncertainty is a profound issue in integrated assessment models.

Figure 14 shows the trajectories of calculated optimal carbon price. The optimal carbon price has also increased sharply over the last two decades. This is due to a combination of all the major factors (output, damage function, temperature increase, and discounting) working in the same direction.

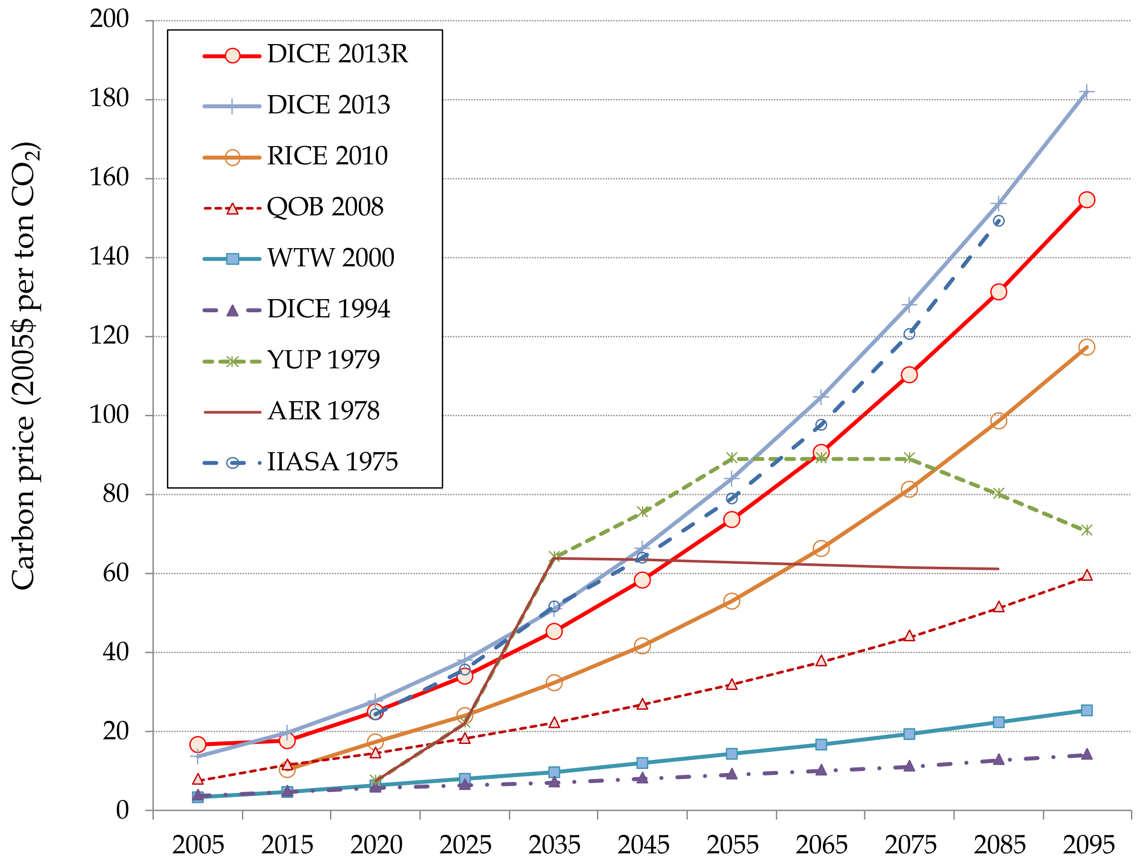

Figure 14. Optimal carbon price, different vintages DICE/RICE models, 2005 $ per ton carbon
[Source: Manual_Figs_092613.xlsx; SF5n]

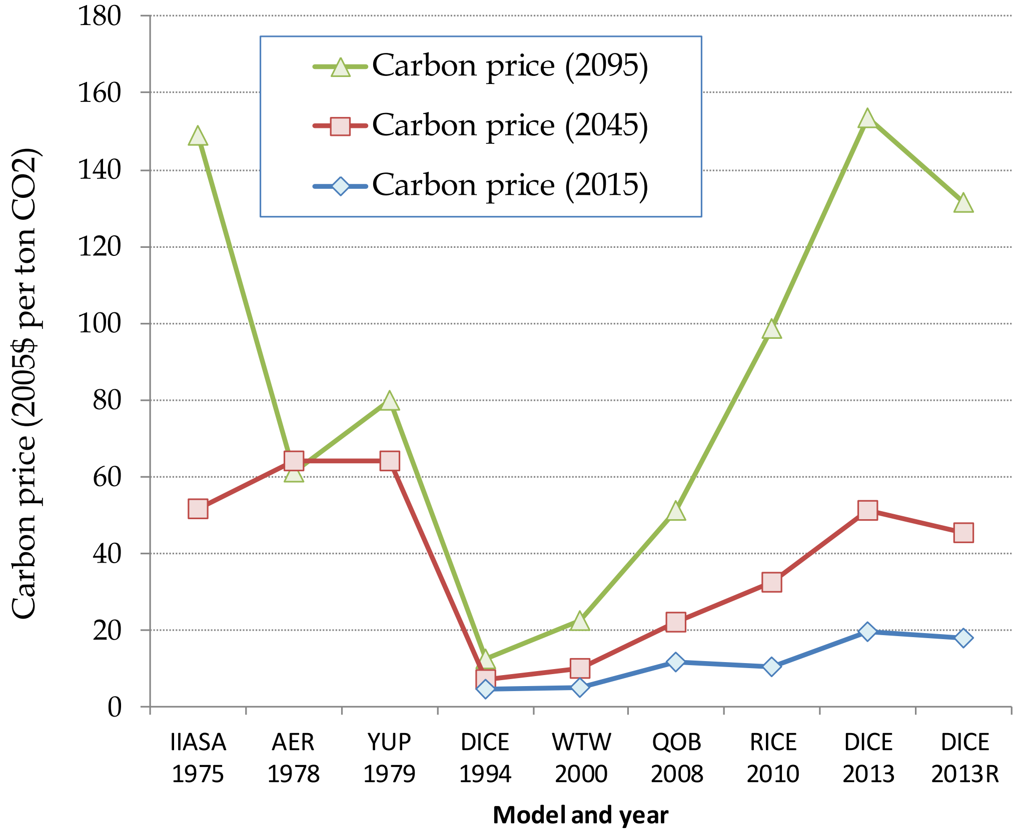

Figure 15. Optimal carbon price, different vintages DICE/RICE models, 2005 $ per ton CO_2 . Prices are for the indicated years.
[Source: Manual_Figs_092613.xlsx; SF5n]

Legend:

IIASA 1975	"Can we control carbon dioxide," IIASA WP-75-63
AER 1978	"Economic Growth and Climate: The Carbon Dioxide Problem," *AER*, Feb 1977, 341-346.
YUP 1979	*The Efficient Use of Energy Resources*, Yale Press
DICE 1994	*Managing the Global Commons*, MIT Press
WTW 2000	*Warming the World*, with Joseph Boyer, MIT Press
QOB 2008	A *Question of Balance*, Yale Press
RICE 2010	"Economic aspects of global warming," *PNAS*, June 14, 2010.
DICE 2013	Earlier version of DICE-2013
DICE 2013R	Current version.

VIII. Computational and algorithmic aspects (*)

A. Analytical background (*)

As we discuss in the next section, IAMs are generally computationally complex compared to physical science models, such as climate models, that use recursive time-stepped algorithms. Among IAMs, the DICE model is relatively simple because it is a straightforward non-linear optimization problem. The DICE model has generally been solved using the CONOPT or NLP solver in the GAMS modeling system (see Brooke et al. 2005). This is based on the generalized reduced gradient (GRG) algorithm. The details of the algorithm are available in the user manual for the CONOPT solver.

CONOPT is generally a local solver and cannot ensure that the solution is a global optimum. However, the DICE-2013R version has also been solved using the BARON solver, which determines whether the solution is a global optimum. The BARON solver is slower than the CONOPT or other local solvers, but can solve most examples within a minute or so. A number of runs of different versions (Base, Optimal, as well as a version with a highly concave-convex damage function) indicates that the solutions with CONOPT are in all examined cases also the global optima.

Over the last decade, we have also used the EXCEL Solver (using the Risk Solver Platform or other premium product). This has the major advantage that optimization can be performed over prices, which is a natural approach for global warming economics. It is very difficult to implement a solution using prices as a decision variable in a standard linear programming algorithm. If the prices can be solved analytically, as is the case with the current DICE model, then prices can be used as a solution variable, but only as an implicit variable. Using EXCEL Solver is also much easier to understand and to detect programming errors. It is also easier to use Excel when introducing new variables and models as the graphics can be employed to find problems. Recent versions of GAMS using auxiliary software such as "R" make graphics easier but still cumbersome. EXCEL has the shortcoming of having much longer and more complex coding.

By contrast with the DICE model, the RICE model (with multiple optimizing agents in equilibrium) is conceptually a fixed point problem. Many integrated assessment models today use a Negishi algorithm to solve this, and this is the approach followed in the RICE solutions. The origins of the Negishi approach date from work of Takashi Negishi, Alan Manne, Peter Dixon, Victor Ginsberg and Jean Waelbroeck, and Thomas Rutherford. The Negishi theorem is essentially an application of the second theorem of welfare economics. Several authors implemented this in the mid-1990s, particularly Nordhaus and Yang (1996) in the first version of the RICE model, although the actual implementations were and continue to differ among IAMs.

The RICE-2010 model has been implemented only in the Excel format. The baseline RICE-2010 model can be used by researchers and students in the Excel format and need not rely upon Solver. However, the full optimization requires the advanced proprietary versions of Solver.[4] We hope to develop a new version of the RICE model in the near future.

It should also be noted that Excel Solver is unable to solve the largest version of the RICE model in a reliable fashion, and errors sometimes occur when using Solver. Even for the DICE model, Excel Solver is unreliable. For example, when using the Solver to optimize the solution for reaching a global optimum for limiting temperature to 2 °C, different starting points yield optimal carbon prices that differ substantially for the first few periods when tolerances are set at their maximum. In some circumstances, Solver simply balks and cannot find a solution, and sometimes it finds a wildly incorrect solution.

B. Solution concepts (*)

We can summarize the points in this section in one paragraph: The DICE model is relatively simple compared to many integrated assessment models. Nonetheless, solving the model—particularly when optimizing emissions reductions – requires modern algorithms for solving non-linear optimization problems. The current DICE model is available in two different platforms. The simplest one is Excel Risk Solver Platform (available for $640 to academic users). The second platform is the GAMS software system (General Algebraic Modeling System). This can be accessed only with proprietary software (available to academics for around $1000). For those who have limited research budgets, the Excel version is the most convenient platform.

Here is a more complete discussion: Optimization IAMs are generally computationally complex compared to physical science models, such as climate models, that use recursive time-stepped algorithms. Optimization problems are computationally complex because (from a mathematical point of view) they require solving a set of equilibrium conditions, such as first-order conditions. While some optimization problems can be solved quickly and efficiently, in general the computational costs rise as a polynomial or exponential function of the number of

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[4] Those who wish to use the simple (free) version of Solver can use the Excel versions by simplifying the constraints and optimizing variables. For example, if a small number of variables are optimized, the free version can be used. While the present author has used that on occasions, there is no available version at present.

variables. By contrast, recursive problems (such as climate models) are linear in the number of variables.

The DICE model traditionally was solved using the GAMS modeling system (see Brooke et al. 2005). GAMS is a high-level modeling system for mathematical programming and optimization. It contains a high-level language and several high-performance solvers. We usually employ NLP solver in solving the DICE-RICE models. This is based on the generalized reduced gradient (GRG) algorithm. This is an algorithm that in practice has proven very efficient at solving large non-linear optimization problems where the constraints are smooth. If the model is "almost linear," it can use inner linear-programming-like iterations to achieve a rapid solution.

In the latest round of models, we have used the EXCEL Solver (using the Risk Solver Platform or other premium product). This has the several advantages. One is that optimization can be performed over prices, which is a natural approach for global warming economics. (It is difficult to implement a solution using prices as a decision variable in a standard linear programming algorithm.) Using EXCEL Solver is also much easier to understand and to detect programming errors. Note that if the price can be solved analytically, it can be used as a control variable in GAMS (which is the case in the most recent version of GAMS).

By contrast, the RICE model (with multiple optimizing agents in equilibrium) is conceptually a fixed point problem. Most integrated assessment models today use a Negishi algorithm to solve this, and this is the approach followed in the RICE solutions. The origins of the Negishi approach date from work of Takashi Negishi, Peter Dixon, Victor Ginsberg and Jean Waelbroeck, Thomas Rutherford, and Rutherford and Manne. The Negishi theorem is essentially an application of the second theorem of welfare economics. Several authors implemented this in the mid-1990s, particularly Nordhaus and Yang (1996) in the first version of the RICE model, although the actual implementations were and continue to differ among IAMs.

One disadvantage of the Excel solvers is that they have trouble with large problems and are often unreliable. For example, when using the Solver to optimize the solution for reaching a global optimum for limiting temperature to 2 °C, different starting points yield optimal carbon prices that differ substantially. In some circumstances, Solver simply stops and cannot find a solution, and sometimes it finds a wildly incorrect solution.

A compendium of studies in several areas with many illuminating articles of CGE modeling is contained in Dixon and Jorgenson (2012).

C. Software architecture (*)

A major issue in design of integrated assessment models is the proper design of software. This issue has been largely ignored in the IAM community.

The GAMS code is very small, although it depends upon a largely invisible translation into actual computational steps. The GAMS code, particularly for the DICE model, is easy to read over and check for mistakes. However, it requires great care in examining the equations to make sure they perform correctly. By contrast, all the Excel programs are huge, although many of the cells are duplicates. A major difficulty in all versions is to assure that there are no mistakes arising from interactions across the equations.

Table 7 shows the size of the source code for different versions of the DICE/RICE models.

Model	Cells or line of code
DICE-2013R: GAMS	
Total lines of code:	262
Total removing comments	222
RICE-2005: GAMS	
Total lines of code (approx.)	2000
DICE-2013R: EXCEL	
Total cells of code:	28,657
RICE-2010: EXCEL	
Total cells of code:	104,795

Table 7. Size of code for different DICE/RICE models
Note the huge size of code for Excel versions. These are largely copied cells because each time period is a cell. GAMS is much smaller but uses a high-level idiosyncratic language.

One of the thorny issues in developing IAMs is their computational complexity. This concern arises because of the increasing size and complexity of computerized modeling in environmental sciences and economics. Specialists in software architecture have studied the issues involved in developing large programs and emphasize the difficulties of ensuring that software is reliable and well-tested. A rule of thumb is that well-developed software contains in the order of 1 error per source line of code (SLOC). Since many computerized climate and integrated assessment models contain between 10,000 and 1 million SLOC, there is the prospect of many bugs contained in our code. [5]

This proposition is not just theoretical. There are many examples of catastrophically bad software, such as the errors that led to the crashing of a spacecraft because of insertion of a period instead of a comma in a FORTRAN statement; or inappropriate shutdown of five nuclear power reactors because of an incorrect formula programmed. Current luxury automobiles have millions of lines of code and probably contain untold thousands of bugs.

I take it as a given that large IAMs have a variety of errors, some consequential, some not. I did a routine check of one of my large models (the RICE 2011 in development at the time). I found a high error rate in terms of stranded code, poor definitions, and mistaken references. For the lines I examined, there were no substantive mistakes, but I suspect that had I gone further some would have turned up. In the RICE-2010 version, there was a mistaken reference in the sea-level rise module that led to small errors in the numerical projections. This was discovered by an interested user and corrected. Another example of coding issues was from the OECD Green model (discussed in Nordhaus (2012).

I will explore one error in depth because it is so subtle that it was found only after an intensive examination. The FUND model is one of the leading models used by researchers and governments to understand the economics of global warming. It has been used to calculate the social cost of carbon for the U.S., which calculation affects tens of billions of dollars of regulations.

The problem with the FUND model arose because of a formula for one of the components of the damage function in an early version (since corrected). The specification had agricultural damages, which were calculated with a formula having a normal variable in both the numerator and the denominator. This looks unnecessarily complex but innocuous. In fact, it turned out to be a serious error. This was pointed out in an article by Ackerman and Munitz (2012), which made the following statement: "The manner in which the optimum temperature effect is modeled in FUND 3.5 could cause division by zero for a plausible value of a Monte Carlo parameter."

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[5] This section draws on a lecture presented at the Prague meetings of the EAREA in June 2012 and the debate about software design that ensued after that. References are contained in that discussion.

It will be useful to examine the issue from a statistical point of view. The details are the following: In FUND 3.5, according to the model description, the damages for the level of temperature on agriculture have two terms. The first term of the damage component can be written as $y = az/(b-z)T$, where y = damages, T = temperature (an endogenous variable); a and b are parameters ; and z is a random variable, which in the FUND model is $(T-T^{opt,r})$. The variable $T^{opt,r}$ represents the optimal temperature in region r and is a normally distributed random variable, so y is the ratio of two normal variables.

The ratio of two normal distributions with non-zero means is a non-central Cauchy distribution. A non-central Cauchy distribution has a standard Cauchy term and another complicated term, but we can focus on the Cauchy term. This distribution is "fat tailed" and has both infinite mean and infinite variance. So the damages from agriculture in FUND 3.5 (from a statistical point of view) will dominate both the mean and dispersion of the estimated damages. Taken literally, the expected value of damages to agriculture are infinite at every temperature increase. This is subject to sampling error in finite samples of any size, but the sampling error is infinite since the moments do not exist, so any numerical calculations with finite samples are (infinitely) inaccurate. There is also a coding issue because it is not possible to get an accurate estimate of the distribution of a variable with infinite mean and variance in finite samples. The most troubling impact of this specification is the estimate of the distribution of outcomes (such as the social cost of carbon or SCC). If the damages are a fat tailed distribution, then the SCC is also fat- tailed. In finite samples, of course, all the moments are finite, but the estimates are unreliable or fragile and depend upon the sample.

I assume that this strange distribution was not intended, and in any case is easily corrected. My point was not to dwell on the shortcomings of our models. Rather, we need to recognize that most economists and environmental scientists are amateurs at software design and architecture. As computers get faster, as software packages get more capable, as our theories get more elaborate – there is a tendency to develop models that increase in parallel with the rapidly expanding frontier of computational abilities. This leads to increasingly large and complex models. We need also to ask, do we fully understand the implication of our assumptions? Is disaggregation really helping or hurting?

There is another lesson here about uncertainty analyses. Deterministic IAMs are already complex non-linear systems. Introducing uncertainty through a set of complicated functions of random variables adds yet another layer of complexity. Modelers need to be especially careful that they have not changed the properties and outcomes of the models because of strange behavior or interactions of the added random variables. The properties of linear stochastic systems are moderately well-understood, but that is not the case for all non-linear stochastic systems.

The conclusions here are four and apply to the DICE/RICE and other large IAMs. First, we modelers need to recognize the importance of good software architecture. Second, we should restrain the urge to develop ever larger and more complex computational models unless there is a clear and convincing case that they will improve our understanding or are necessary to understand the phenomena at hand. Third, we need to undertake special scrutiny when we add random elements to non-linear dynamic models. Finally, we need to take the extra time and effort to examine, re-examine, and test our software.

IX. Conclusion (*)

The present manual is intended to provide users as well as those interested in integrated assessment modeling a self-contained document for understanding and using the DICE/RICE family of models.

As with all large-scale models of this kind, they must be continuously updated. Additionally, they are prone to errors in the software and structure, data and scientific views evolve, and users must be attentive to the potential for large and small changes in the economics and natural sciences. Problems arise particularly when modifications are made to models (such as alternative parameterizations) and the model is not carefully tested to make sure that the changes do not alter the behavior or introduce instabilities.

The DICE model has evolved significantly over the years since its development. The vast changes in the projections of different variables might lead some to conclude that these undermine the credibility of the modeling approach.

My response would be different, however, and can best be summarized by a remark made in another context. The economist John Maynard Keynes was criticized for changing his views on monetary policy during the Great Depression. His response is reported to be, "When the facts change, I change my mind. Pray, sir, what do you do?" This is a reminder of the need to be constantly attentive to changing economic and scientific findings. Even more important is to resist getting dug into a Maginot Line defense of particular views or projections. To err is human, so to be humble is divine.

X. References

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XI. Appendix A. Nuts and Bolts of Running the Models

This appendix has been prepared primarily by Paul Sztorc of the Yale Department of Economics.

Overview

This appendix is designed to help new users get started. Comments and suggestions for improvement are welcome.

Where Do I Find the Models?

The models are on the DICE web page at DICEmodel.net.

Required Software

The latest DICE-2013R comes in two formats: a GAMS software program and an EXCEL spreadsheet. Previous versions of DICE/RICE were either excel-only or GAMS-only:

DICE Version	Software Required
2013, 2008, 1999, 1994, DICE123	GAMS (General Algebraic Modeling System)[6]
2013, 2010	Microsoft Excel (in a macro-enabled workbook).

[6] The GAMS software codes for DICE versions 2013R, 2008, 1999, and 1994 are available in Appendix B of this manual.

Basics for running GAMS

- A GAMS license is needed to run a model the size and complexity of DICE 2013. With an academic discount, such a license is available for around $1000 from the GAMS site.
- After opening the *.gms file, press F9 to solve the model. You can also click on an icon of a man running.
- Gams takes a *.gms file, which contains the model, and uses it to produce a *.lst file, which contains information about the model solution.
- Use the "display" command to ensure that the variables you are interested in learning about appear in the *.lst file. You can see examples of display statements in the GAMS files in Appendix B.

Basics for running the EXCEL version

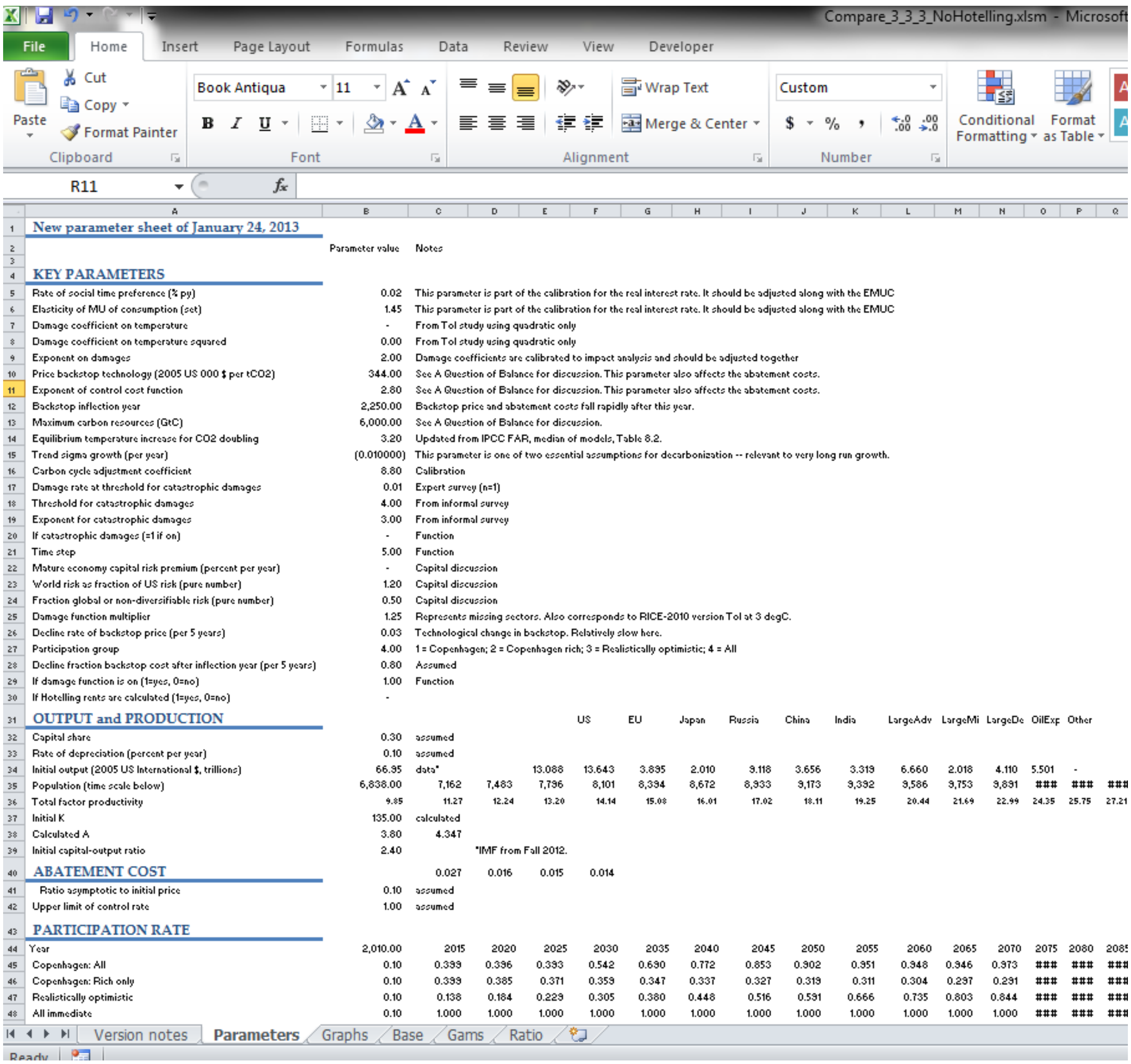
The basic EXCEL version can be used without any further software with standard Excel software. To do the optimization, we used proprietary software called "Risk Solver Platform," developed by FrontlineSolvers. With an academic discount, this costs about $1000. Again, our Excel sheet ships with optimized numbers, but Risk Solver (or GAMS, or another solver) is required for doing new optimizations, should you choose to alter the model.

Viewing the Model Inputs

An important set of variables are the input parameters (rate of growth of population, climate parameters, etc.). These can be varied, although care must be taken to make sure that they do not change the structure in an inappropriate way.

DICE 2010 [Excel]

In Excel parameters are entered in standard spreadsheet format in "Parameters" sheet.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	New parameter sheet of January 24, 2013																
2		Parameter value	Notes														
3																	
4	KEY PARAMETERS																
5	Rate of social time preference (% py)	0.02	This parameter is part of the calibration for the real interest rate. It should be adjusted along with the EMUC														
6	Elasticity of MU of consumption (set)	1.45	This parameter is part of the calibration for the real interest rate. It should be adjusted along with the EMUC														
7	Damage coefficient on temperature	-	From Tol study using quadratic only														
8	Damage coefficient on temperature squared	0.00	From Tol study using quadratic only														
9	Exponent on damages	2.00	Damage coefficients are calibrated to impact analysis and should be adjusted together														
10	Price backstop technology (2005 US 000 $ per tCO2)	344.00	See A Question of Balance for discussion. This parameter also affects the abatement costs.														
11	Exponent of control cost function	2.80	See A Question of Balance for discussion. This parameter also affects the abatement costs.														
12	Backstop inflection year	2,250.00	Backstop price and abatement costs fall rapidly after this year.														
13	Maximum carbon resources (GtC)	6,000.00	See A Question of Balance for discussion.														
14	Equilibrium temperature increase for CO2 doubling	3.20	Updated from IPCC FAR, median of models, Table 8.2.														
15	Trend sigma growth (per year)	(0.010000)	This parameter is one of two essential assumptions for decarbonization -- relevant to very long run growth.														
16	Carbon cycle adjustment coefficient	8.80	Calibration														
17	Damage rate at threshold for catastrophic damages	0.01	Expert survey (n=1)														
18	Threshold for catastrophic damages	4.00	From informal survey														
19	Exponent for catastrophic damages	3.00	From informal survey														
20	If catastrophic damages (=1 if on)	-	Function														
21	Time step	5.00	Function														
22	Mature economy capital risk premium (percent per year)	-	Capital discussion														
23	World risk as fraction of US risk (pure number)	1.20	Capital discussion														
24	Fraction global or non-diversifiable risk (pure number)	0.50	Capital discussion														
25	Damage function multiplier	1.25	Represents missing sectors. Also corresponds to RICE-2010 version Tol at 3 degC.														
26	Decline rate of backstop price (per 5 years)	0.03	Technological change in backstop. Relatively slow here.														
27	Participation group	4.00	1 = Copenhagen; 2 = Copenhagen rich; 3 = Realistically optimistic; 4 = All														
28	Decline fraction backstop cost after inflection year (per 5 years)	0.80	Assumed														
29	If damage function is on (1=yes, 0=no)	1.00	Function														
30	If Hotelling rents are calculated (1=yes, 0=no)	-															
31	OUTPUT and PRODUCTION					US	EU	Japan	Russia	China	India	LargeAdv	LargeMi	LargeDe	OilExp	Other	
32	Capital share	0.30	assumed														
33	Rate of depreciation (percent per year)	0.10	assumed														
34	Initial output (2005 US International $, trillions)	66.95	data*		13.088	13.643	3.895	2.010	9.118	3.656	3.319	6.660	2.018	4.110	5.501	-	
35	Population (time scale below)	6,838.00	7,162	7,483	7,796	8,101	8,394	8,672	8,933	9,173	9,392	9,586	9,753	9,891	###	###	###
36	Total factor productivity	9.85	11.27	12.24	13.20	14.14	15.08	16.01	17.02	18.11	19.25	20.44	21.69	22.99	24.35	25.75	27.21
37	Initial K	135.00	calculated														
38	Calculated A	3.80	4.347														
39	Initial capital-output ratio	2.40		*IMF from Fall 2012.													
40	ABATEMENT COST		0.027	0.016	0.015	0.014											
41	Ratio asymptotic to initial price	0.10	assumed														
42	Upper limit of control rate	1.00	assumed														
43	PARTICIPATION RATE																
44	Year	2,010.00	2015	2020	2025	2030	2035	2040	2045	2050	2055	2060	2065	2070	2075	2080	2085
45	Copenhagen: All	0.10	0.399	0.396	0.393	0.542	0.690	0.772	0.853	0.902	0.951	0.948	0.946	0.973	###	###	###
46	Copenhagen: Rich only	0.10	0.399	0.385	0.371	0.359	0.347	0.337	0.327	0.319	0.311	0.304	0.297	0.291	###	###	###
47	Realistically optimistic	0.10	0.138	0.184	0.229	0.305	0.380	0.448	0.516	0.591	0.666	0.735	0.803	0.844	###	###	###
48	All immediate	0.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	###	###	###

Dice 2013R [GAMS]

In GAMS, parameters are entered as in-line software code.

DICE2013_050313_mup_base.gms

```
$ontext
This is the DICE-2013 model. It is matched Excel version DICE_NR_032813.xlsm.
This has been revised from SCC version.
This version is DICE2013_032813.gms
It needs the include files.
$offtext

$title        DICE-2013        April 26, 2013 TFR Version

set         t  Time periods (5 years per period) /1*60/   ;


parameters

**Time Step
        tstep Years per Period                                   /5/

** Preferences
        elasmu   Elasticity of marginal utility of consumption     /  1.45    /
        prstp    Initial rate of social time preference per year  / .015    /

** Population and technology
        gama     Capital elasticity in production function        /.300 /
        pop0     Initial world population (millions)              /6838/
        popadj   Growth rate to calibrate to 2050 pop projection  /0.134490 /
        popasym  Asymptotic population                            /10500/
        dk       Depreciation rate on capital (per year)          /.100/
        q0       Initial world gross output (trill 2005 USD)      /66.950/
        k0       Initial capital value (trill 2005 USD)           /135/
        a0       Initial level of total factor productivity       /3.7976214 /
        ga0      Initial growth rate for TFP per 5 years          /0.079/
        dela     Decline rate of TFP per 5 years                  /0.006/

** Emissions parameters
        sig0     Initial Sigma (industrial MTCO2 per thous 2005 USD )     /0.488982524/
        gsigma1  Initial growth of sigma (coninuous per year )            /-0.01 /
        dsig     Decline rate of decarbonization per period               /-0.001/
        eland0   Initial Carbon emissions from land (GtCO2 per period)    / 1.53972/
        deland   Decline rate of land emissions (per period)              / .2 /

** Carbon cycle
* Initial Conditions
        mat0   Initial Concentration in atmosphere 2010 (GtC)        /818.985/
        mu0    Initial Concentration in upper strata 2010 (GtC)      /1527.000/
```

Viewing the Model Structure

DICE 2013R [Excel]

The model structure of DICE 2013R Excel is contained within the formulas entered on each sheet. For example, the atmospheric concentrations of CO_2 are shown as a formula below. There are about 28,000 'lines of code' (discrete cells) in the Excel version, so the model is possibly subject to errors. Users should be very careful when changing parameters to make sure that they have checked the results.

	A	B	C	D	E	F
1	Global	1.000	2.000	3.000	4.000	5.000
2		2,010.000	2,015.000	2,020.000	2,025.000	2,030.000
81	CLIMATE MODULE TRANSITION PARAMETERS (per 5 years)					
82	Speed of adjustment parameter for atmospheric temperature	**0.104**				
83	Coefficient of heat loss from atmosphere to oceans	**0.088**				
84	Coefficient of heat gain by deep oceans	**0.025**				
85	**Global Environmental Variables**					
86	**CARBON CYCLE (beginning of period)**					
87	Atmospheric concentration of carbon (GTC)	818.985	849.920	=Base!$B65*C87/100+Base!$B66*C89/100+C113*5/3.666		
88	Atmospheric concentration of carbon (ppm)	384.500	399.023	414.989	432.459	451.454
89	Concentration in biosphere and upper oceans (GTC)	1,527.000	1,540.103	1,555.394	1,573.053	1,593.266
90	Concentration in deep oceans (GTC)	10,010.000	10,010.439	10,010.911	10,011.421	10,011.974
91	**CUMULATIVE EMISSIONS/HOTELLING RENTS**					
92	Cumulative Emissions to date	90.000	132.377	180.468	234.504	294.653
93	Ratio to Max	0.000	0.000	0.000	0.000	0.000

Dice 2013R [GAMS]

The GAMS code is written in a high-level language and is much more succinct. For the base and optimal runs, it consists of about 330 lines of code (of which about half are comments, blank, or otherwise trivial). The following shows the way the model structure (equation set) is defined and written.

DICE2013_050313_mup_base.gms

```
EQUATIONS
        CCACCA(t)         Cumulative carbon emissions
        UTIL              Objective function
        YY(t)             Output net equation
        YNETEQ(t)         Output net of damages equation
        YGROSSEQ(t)       Output gross equation
        DAMEQ(t)          Damage equation
        ABATEEQ(t)        Cost of emissions reductions equation
        CC(t)             Consumption equation
        KK(t)             Capital balance equation
        CPCE(t)           Per capita consumption definition
        EEQ(t)            Emissions equation
        EINDEQ(t)         Industrial emissions
        SEQ(t)            Savings rate equation
        RIEQ(t)           Interest rate equation
        FORCE(t)          Radiative forcing equation
        MMAT(t)           Atmospheric concentration equation
        MMU(t)            Shallow ocean concentration
        MML(t)            Lower ocean concentration
        TATMEQ(t)         Temperature-climate equation for atmosphere
        TOCEANEQ(t)       Temperature-climate equation for lower oceans
        DAMFRACEQ(t)      Equation for damage fraction
        MCABATEEQ(t)      Equation for MC abatement
        CARBPRICEEQ(t)    Carbon price equation from abatement
        CEMUTOTPEREQ(t)   Period utility
        PERIODUEQ(t)      Instantaneous utility function equation;
*        MIUPART(t)        MIU Adjusted for participation fraction;
*        CPRICEPARTICIPANTS(t) Carbon price Adjusted for participation fraction;

** Equations of the model

ccacca(t+1)..           CCA(t+1)       =E=  CCA(t)+ EIND(t)*5/3.666;
kk(t+1)..               K(t+1)         =L=  (1-dk)**tstep * K(t) + tstep * I(t);
eindeq(t)..             EIND(t)        =E=  sigma(t) * YGROSS(t) * (1-(MIU(t)));
eeq(t)..                E(t)           =E=  EIND(t) + etree(t);
force(t)..              FORC(t)        =E=  fco22x * ((log((MAT(t)/588.000))/log(2))) + forcoth(t);
mmat(t+1)..             MAT(t+1)       =E= MAT(t)*b11 + MU(t)*b21 + (E(t)*(5/3.666));
mml(t+1)..              ML(t+1)        =E= ML(t)*b33  + MU(t)*b23;
mmu(t+1)..              MU(t+1)        =E= MAT(t)*b12 + MU(t)*b22 + ML(t)*b32;
tatmeq(t+1)..           TATM(t+1)      =E=  TATM(t) + c1 * ((FORC(t+1)-(fco22x/t2xco2)*TATM(t))-(c3*(TATM(t)-TO
toceaneq(t+1)..         TOCEAN(t+1)    =E= TOCEAN(t) + c4*(TATM(t)-TOCEAN(t));
ygrosseq(t)..           YGROSS(t)      =E= (al(t)*(L(t)/1000)**(1-GAMA))*(K(t)**GAMA);
dameq(t)..              DAMAGES(t)     =E= YGROSS(t) * DAMFRAC(t);
```

Definitions of variables

This section answers the questions: What does this variable mean? Where are these terms defined?

DICE 2013R [Excel]

In Excel, the explanations are written in column A.

2		2,010.000	2,015.000	2,020
99	CLIMATE MODULE			
100	Atmospheric temperature (degrees Celsius above preindustrial)	0.830	0.934	1
101	Total increase in radiative forcing since preindustrial (Watts per square meter)	1.824	2.062	2
102	Lower ocean temperature (degrees Celsius above preindustrial)	0.007	0.027	0
103	Economic Endogenous Variables			
104	OUTPUT			
105	Output gross of abatement cost and climate damage ($trill)	63.542	75.806	89

DICE 2013R [GAMS]

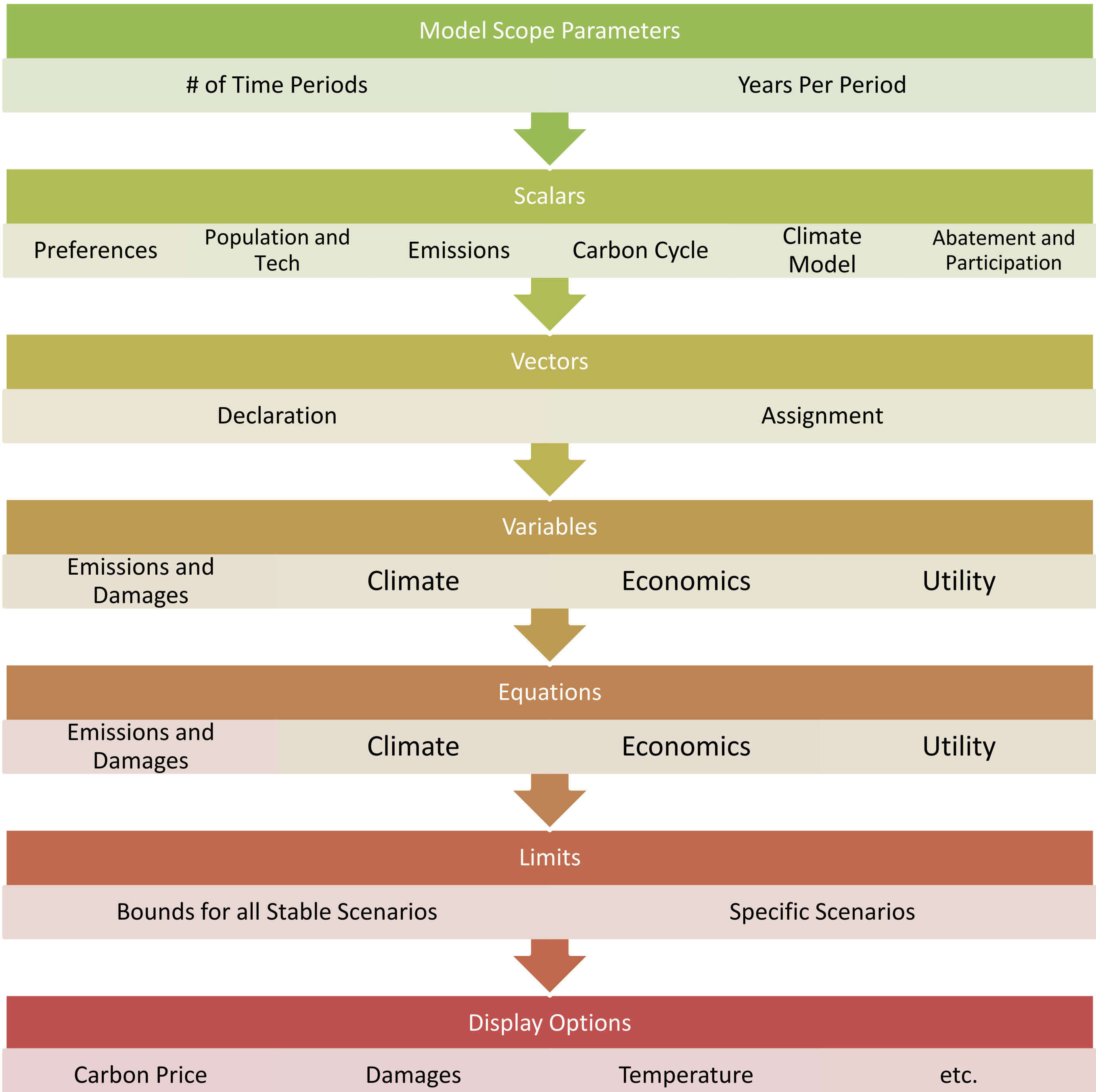
In GAMS, the definitions are given when a variable is declared:

DICE2013_050313_mup_base.gms

```
PARAMETERS
        L(t)            Level of population and labor
        al(t)           Level of total factor productivity
        sigma(t)        CO2-equivalent-emissions output ratio
        rr(t)           Average utility social discount rate
        ga(t)           Growth rate of productivity from 0 to T
        forcoth(t)      Exogenous forcing for other greenhouse gases
        gl(t)           Growth rate of labor (0 to T)
        gcost1          Growth of cost factor
        gsig(t)         Change in sigma (cumulative improvement of energy efficiency)
        etree(t)        Emissions from deforestation
        cost1(t)        Adjusted cost for backstop
        partfract(t)    Fraction of emissions in control regime
        lam             Climate model parameter
```

Structure of DICE 2013R Software Code

The flow of information through the DICE model can be visualized as follows:



Time Periods

The current version of DICE-2013R has a five-year time step, compared to ten years in the earlier versions.

```
$ontext
This is the DICE-2013R model, version DICE2013R_090713.gms, revised from April version.
This version just includes the optimal and base depending upon the "Ifopt" control.
This version does not have any output statements.
$offtext

$title        DICE-2013R September 2013

set        t  Time periods (5 years per period)                    /1*60/ ;

parameters

**Time Step
        tstep    Years per Period                                   /5/

** If optimal control
        ifopt    If optimized 1 and if base is 0                    /1/

```

In the Excel version, the years are given and obvious. In using the models, it is not obvious what the year is in GAMS. DICE-2013R starts in year 2010, and runs in 5 year periods for 60 periods, spanning the timeframe from 2010 to 2305. The notation /1*60/ is interpreted as “1, 2,..., 59, 60 .” This corresponds to the years “2010, 2015, 2020, ..., 2305.”

Changing the time periods is a common and useful edit. However, the only number than can be reasonably changed is the ending time period (60 in this case), as the data inputs have been configured for initial conditions in 2010, and for time step of 5 years (for example, for population growth).

Scalars

Scalars are the easiest to change. Simply replace the number in between "/ /" with its desired value. For example, replace "/6838/" with "/7000/" to start off the world with a higher world population.

```
parameters

**Time Step
        tstep Years per Period                                     /5/

** Preferences
        elasmu   Elasticity of marginal utility of consumption      /  1.45    /
        prstp    Initial rate of social time preference per year   / .015    /

** Population and technology
        gama     Capital elasticity in production function         /.300 /
        pop0     Initial world population (millions)               /6838/
```

Vectors

Vectors are slightly more complicated to change, as they are not directly observed until the model runs. Vectors are declared as parameters which are indexed along a set, for example:

```
PARAMETERS
        L(t)          Level of population and labor
        al(t)         Level of total factor productivity
```

After being declared, they are assigned a value using simple equality statements, typically within a loop function over the indexed set (t 'time period' in this example).

```
L("1") = pop0;
loop(t, L(t+1)=L(t););
loop(t, L(t+1)=L(t)*(popasym/L(t))**popadj ;);
```

To change a vector, it highly advisable to test the declaration in a new GAMS window. What follows is an example of a simple program with the sole purpose of calculating and displaying the parameter L (the population level):

```
$title        DICE-2013       April 26, 2013 TFR Version

set         t  Time periods (5 years per period) /1*60/   ;
parameters

** Population and technology
        gama     Capital elasticity in production function        /.300 /
        pop0     Initial world population (millions)              /6838/
        popadj   Growth rate to calibrate to 2050 pop projection  /0.134490 /
        popasym  Asymptotic population                            /10500/


PARAMETERS
        L(t)          Level of population and labor;

L("1") = pop0;
loop(t, L(t+1)=L(t););
loop(t, L(t+1)=L(t)*(popasym/L(t))**popadj ;);

display L;
```

You might run the program and display the answer to test that the change is correctly made, as in the following.

temppp.lst

.lst file

```
GAMS Rev 239  WEX-WEI 23.9.5 x86_64/MS Windows
DICE-2013        April 26, 2013 TFR Version
C o m p i l a t i o n




COMPILATION TIME     =        0.000 SECONDS      3 Mb  WEX239-239 Nov  9,
GAMS Rev 239  WEX-WEI 23.9.5 x86_64/MS Windows
DICE-2013        April 26, 2013 TFR Version
E x e c u t i o n


----     42 PARAMETER L  Level of population and labor

1   6838.000,    2   7244.013,    3   7614.833,    4   7951.071,    5
8   8982.998,    9   9173.507,    10  9341.654,    11  9489.673,    12
15  9920.485,    16  9996.523,    17 10062.804,    18 10120.526,    19 1
22 10285.335,    23 10313.948,    24 10338.777,    25 10360.315,    26 1
29 10421.384,    30 10431.922,    31 10441.052,    32 10448.961,    33 1
36 10471.328,    37 10475.180,    38 10478.514,    39 10481.401,    40 1
43 10489.559,    44 10490.963,    45 10492.178,    46 10493.229,    47 1
50 10496.200,    51 10496.711,    52 10497.153,    53 10497.536,    54 1
57 10498.617,    58 10498.803,    59 10498.964,    60 10499.103
```

Output

Variables and Equations

With knowledge of GAMS syntax, it is easy to edit the variables and equations. However, editing variables/equations is by far the most perilous change because one runs the risk of unintentionally producing a model with an entirely new structure, or one with unstable or unrealistic solutions, or an infeasible model with no valid solutions whatsoever.

With that caveat, here is the setup: Variables are first declared. Secondly, they may be constrained as positive. Thirdly, the variables are used in a list of equations: Equalities are set using “=E=” (for “equal to”); or as inequalities using “=L=” for “less than” or =G= for “greater than”). Notably, this list of equations appears twice: There is literally a list of equations (and their descriptions); and then a second list in which each equation is defined mathematically.

```
EQUATIONS
*Emissions and Damages
        EEQ(t)              Emissions equation
        EINDEQ(t)           Industrial emissions
        CCACCA(t)           Cumulative carbon emissions

        FORCE(t)            Radiative forcing equation
        DAMFRACEQ(t)        Equation for damage fraction
        DAMEQ(t)            Damage equation

        ABATEEQ(t)          Cost of emissions reductions equation
        MCABATEEQ(t)        Equation for MC abatement
        CARBPRICEEQ(t)      Carbon price equation from abatement

*Climate
        MMAT(t)             Atmospheric concentration equation
        MMU(t)              Shallow ocean concentration
        MML(t)              Lower ocean concentration
        TATMEQ(t)           Temperature-climate equation for atmosphere
        TOCEANEQ(t)         Temperature-climate equation for lower oceans

*Economics
        YGROSSEQ(t)         Output gross equation
        YNETEQ(t)           Output net of damages equation
        YY(t)               Output net equation

        CC(t)               Consumption equation
        CPCE(t)             Per capita consumption definition

        SEQ(t)              Savings rate equation
        KK(t)               Capital balance equation
        RIEQ(t)             Interest rate equation

* Utility
        CEMUTOTPEREQ(t)     Period utility
        PERIODUEQ(t)        Instantaneous utility function equation
        UTIL                Objective function      ;

** Equations of the model
*Emissions and Damages
eeq(t)..                 E(t)          =E= EIND(t) + etree(t);
```

Notice, again, the equations are first declared under the header "EQUATIONS" and then secondarily defined in a lower list. For example, the EEQ or emissions equation is declared at the top of the list above. Then the actual equation is given directly at the bottom of the listing as "eeq(t)..   E(t) =E= EIND(t) + etree(t) ;"

Limits and Scenarios

Finally, the model requires certain bounds for reasons of stability, logical coherence, or to calibrate a specific scenario. For example, to limit temperature increase to 2 °C, you would add the following limit constraint:

```
TATM.up(t)   = 2 ;
```

The GAMS version of DICE 2013R is available in two 'flavors': simple and complex. The simple flavor toggles between two modes ('base' vs 'optimal'), and produces a small sample output file, with the intention that users customize the output to their needs. The complex flavor includes several optional on/off switches, not mutually exclusive, referring to a wide array of climate change policy scenarios. Moreover, the complex flavor produces a large output file for each scenario activated, and this output file is in exactly the same format as the corresponding sheet of DICE 2013R Excel model (and should produce exactly the same results).

```
$title        DICE-2013R October 2013

parameters
*These parameters govern the inclusion/exclusion of different model scenarios:

** Scenarios
        BaseRun              /1/
        OptRun               /1/
        L2Run                /1/
        SternRun             /1/
        SternCalibRun        /1/
        CopenRun             /1/

** Comparison Assistance
*          FixSavingsAtTwo   /0/

set         t  Time periods (5 years per period)                    /1*60/ ;

parameters

**Time Step
        tstep    Years per Period                                     /5/
```

Notice that, later, these options trigger the GAMS code related to each scenario:

```
*Base Run
If (BaseRun eq 1,
    put results4;
$include Include\def_base.gms
$include Include\PutOutputAllT.gms
$include Include\def_base_cleanup.gms
    putclose;
);
```

To these meaningful constraints, we add a group of simpler constraints that work to speed up the solution.

```
K.LO(t)             = 1;
MAT.LO(t)           = 10;
MU.LO(t)            = 100;
ML.LO(t)            = 1000;
C.LO(t)             = 2;
TOCEAN.UP(t)        = 20;
TOCEAN.LO(t)        = -1;
TATM.UP(t)          = 40;
CPC.LO(t) = .01;
```

Display Options

Model outputs can be viewed in two ways: as put files or as displayed in the *.lst file.
The simplest way is to use the display command, followed by a list of relevant outputs.

```
display aa1,L,e.l,eeq.m;
```

Parameters need no suffix, but variables (e), and equations (eeq), require suffixes (of ".l" for "level", and ".m" for "marginal", in this example).

The above statement produces the following results in the .lst file:

```
DICE2013_041213.lst

E x e c u t i o n


----   2034 PARAMETER aa1                   =        0.000  Damage intercept

----   2034 PARAMETER L  Level of population and labor

1   6838.000,    2   7244.013,    3   7614.833,    4   7951.071,    5   8254.051,    6
8   8982.998,    9   9173.507,    10  9341.654,    11  9489.673,    12  9619.676,    1
15  9920.485,    16  9996.523,    17 10062.804,    18 10120.526,    19 10170.752,    2
22 10285.335,    23 10313.948,    24 10338.777,    25 10360.315,    26 10378.992,    2
29 10421.384,    30 10431.922,    31 10441.052,    32 10448.961,    33 10455.811,    3
36 10471.328,    37 10475.180,    38 10478.514,    39 10481.401,    40 10483.901,    4
43 10489.559,    44 10490.963,    45 10492.178,    46 10493.229,    47 10494.140,    4
50 10496.200,    51 10496.711,    52 10497.153,    53 10497.536,    54 10497.867,    5
57 10498.617,    58 10498.803,    59 10498.964,    60 10499.103


----   2034 VARIABLE E.L  CO2-equivalent emissions (GtC)

1        32.611,    2        29.237,    3        31.481,    4        33.569,    5       35
7        38.222,    8        39.048,    9        39.438,    10       39.355,    11      38
13       36.021,    14       33.826,    15       31.074,    16       27.765,    17      23
19       14.552,    20        9.088,    21        3.125,    22        0.014,    23       0
25        0.007,    26        0.006,    27        0.005,    28        0.004,    29       0
31        0.002,    32        0.002,    33        0.001,    34 9.759135E-4,    35 7.80730
37 4.996677E-4,    38 3.997342E-4,    39 3.197873E-4,    40 2.558299E-4,    41 2.04663
43 1.309849E-4,    44 1.047879E-4,    45 8.383033E-5,    46 6.706426E-5,    47 5.36514
49 3.433690E-5,    50 2.746952E-5,    51 2.197562E-5,    52 1.758049E-5,    53 1.40644
55 9.001213E-6,    56 7.200971E-6,    57 5.760776E-6,    58       10.875,    59      51


----   2034 EQUATION EEQ.M  Emissions equation

1        -0.081,    2        -0.076,    3        -0.071,    4        -0.066,    5       -0
7        -0.053,    8        -0.049,    9        -0.045,    10       -0.041,    11      -0
13       -0.032,    14       -0.030,    15       -0.027,    16       -0.025,    17      -0
19       -0.019,    20       -0.017,    21       -0.016,    22       -0.015,    23      -0
25       -0.011,    26       -0.010,    27       -0.009,    28       -0.008,    29      -0
31       -0.006,    32       -0.006,    33       -0.005,    34       -0.004,    35      -0
37       -0.003,    38       -0.003,    39       -0.003,    40       -0.002,    41      -0
```

"Parameter L" shows population, where "1" is 2010 is 6838 million, and so forth. "E.L" is the emissions of CO_2 in billions (G or giga) of tons of CO_2. The "CPRICE.L" is the market price of CO_2 emissions in the optimized runs, also the social cost of carbon. Note that these numbers are not necessarily correct and were created to serve as an example.

Further readings

Further readings on the GAMS model can be found on the GAMS site at http://www.gams.com/docs/document.htm.

Manuals for Solver can be found at http://www.solver.com/user-guides-frontline-systems-excel-solvers.

One of the leading scholars who has developed GAMS for energy and economic modeling is Tom Rutherford. See for example his "Solution Software for Computable General Equilibrium Modeling," with Mark Horridge, Alex Meeraus and Ken Pearson in *Handbook of Computable General Equilibrium Modeling* (ISBN: 9780444595683), Peter B. Dixon and Dale W. Jorgenson (eds.), Elsevier, 1331–1381, 2013.

XII. Appendix B. GAMS Code for Different Vintages of the DICE Model

This appendix contains the GAMS codes for four vintages of models: 1992-94, 1999, 2007, and 2013R. There were intermediate vintages as well, but these were the most thoroughly documented and form the basis of most of the publications. The RICE model program is published in Nordhaus and Boyer (2000) as well. These can be run by simply creating a *.gms file, copying the text in, and running. There may be some formatting problems, but these should be easily corrected.

A. 1992-1994 version of DICE model

```
* DICE123a
* July 28, 1994

*   This is an optimal growth model to calculate the optimal control
* rate and timing for the abatement of CO2 and other Greenhouse Gases.
*   This is the standard model used for Science (Nov. 1992) and for
* the base model in W. Nordhaus, Managing the Global Commons,
* MIT Press, 1994, forthcoming.

SETS   T        Time periods        /1*40/
       TFIRST(T) First period
       TLAST(T)  Last period

SCALARS  BET    Elasticity of marginal utility            /0/
       R      Rate of social time pref per year          /.03/
       GL0    Growth rate of population per decade        /.223/
       DLAB   Decline rate of pop growth per dec          /.195/
       DELTAM  Removal rate carbon per decade             /.0833/
       GA0    Initial growth rate for technology per dec   /.15/
       DELA   Decline rate of technology per dec          /.11  /
       SIG0   CO2-equiv-GNP ratio                        /.519/
       GSIGMA  Growth of sigma per decade                 /-.1168/
       DK     Depreciation rate on capital per year       /.10/
       GAMA   Capital elasticity in output               /.25/
       M0     CO2-equiv concentrations 1965 bill t C      /677/
       TL0    Lower stratum temperature (C) 1965          /.10/
       T0     Atmospheric temperature (C) 1965            /.2/
       ATRET  Marginal atmospheric retention rate         /.64/
       Q0     1965 world gross output trillions 89 US dol  /8.519/
       LL0    1965 world population millions              /3369/
       K0     1965 value capital billions 1989 US dollars  /16.03/
       C1     Coefficient for upper level                /.226/
       LAM    Climate feedback factor                    /1.41/
       C3     Coefficient trans upper to lower stratum     /.440/
       C4     Coeff of transfer for lower level           /.02/
       A0     Initial level of total factor productivity   /.00963/
       A1     Damage coeff for co2 doubling (frac GWP)     /.0133/
       B1     Intercept control cost function            /.0686/
       B2     Exponent of control cost function          /2.887/
       PHIK   transversality coeff capital              /140 /
       PHIM   Transversality coeff carbon ($ per ton)     /-9/
```

```
      PHITE     Transversalit coeff temper (bill $ per deg C)  /-7000 /

PARAMETERS   L(T)         Level of population and labor
          AL(T)        Level of Total factor productivity
          SIGMA(T)     Emissions-output ratio
          RR(T)        Discount factor
          GA(T)        Growth rate of T. F. P. from 0 to T
          FORCOTH(T)    Exogenous forcing other greenhouse gases
          GL(T)        Growth rate of labor 0 to T
          GSIG(T)      Cumulative improvement of energy efficiency
          DUM(T)       dummy variable 0 except 1 in last period;

TFIRST(T) = YES$(ORD(T) EQ 1);
TLAST(T)  = YES$(ORD(T) EQ CARD(T));
DISPLAY TFIRST, TLAST;

GL(T) = (GL0/DLAB)*(1-exp(-DLAB*(ord(t)-1)));
L(T)=LL0*exp(GL(t));
GA(T)  = (GA0/DELA)*(1-exp(-DELA*(ord(t)-1)));
AL(T) =  a0*exp(GA(t));
GSIG(T) = (GSIGMA/DELA)*(1-exp(-DELA*(ord(t)-1)));
SIGMA(T)=SIG0*exp(GSIG(t));
DUM(T)=1$(ord(T)  eq card(T));

RR(T) = (1+R)**(10*(1-ord(t)));
FORCOTH(T) = 1.42;
FORCOTH(T)$(ord(t) lt 15) = .2604+.125*ord(T)-.0034*ord(t)**2;

VARIABLES            MIU(T)        Emission control rate GHGs
                 FORC(T)       Radiative forcing, W per m2
                 TE(T)         Temperature, atmosphere C
                 TL(T)         Temperature, lower ocean C
                 M(T)          CO2-equiv concentration bill t
                 E(T)          CO2-equiv emissions bill t
                 C(T)          Consumption trill US dollars
                 K(T)          Capital stock trill US dollars
                 CPC(T)         Per capita consumption thousands US dol
                 PCY(t)        Per capita income thousands US dol
                 I(T)          Investment trill US dollars
                 S(T)          Savings rate as fraction of GWP
                 RI(T)         Interest rate per annum
                 TRANS(T)      transversality variable last period
                 Y(T)          OUTPUT
                 UTILITY;

POSITIVE VARIABLES MIU, E, TE, M, Y, C, K, I;

EQUATIONS     UTIL      Objective function
           YY(T)     Output
           CC(T)     Cconsumption
           KK(T)     Capital balance
           KK0(T)    Initial condition of K
           KC(T)     Terminal condition of K
           CPCE(t)   Per capita consumption
           PCYE(T)   Per capita income equation
           EE(T)     Emissions process
           SEQ(T)    Savings rate equation
           RIEQ(T)   Interest rate equation
           FORCE(T)  Radiative forcing equation
           MM(T)     CO2 distribution equation
           MM0(T)    Initial condition for M
```

```
        TTE(T)    Temperature-climate equation for atmosphere
        TTE0(T)   Initial condition for atmospheric temp
        TLE(T)    Temperature-climate equation for lower oceans
        TRANSE(t) Transversality condition
        TLE0(T)   Initial condition for lower ocean;

KK(T)..       K(T+1) =L= (1-DK)**10 *K(T)+10*I(T);
KK0(TFIRST)..  K(TFIRST) =E= K0;
KC(TLAST)..    R*K(TLAST) =L= I(TLAST);

EE(T)..       E(T)=G=10*SIGMA(T)*(1-MIU(T))*AL(T)*L(T)**(1-GAMA)*K(T)**GAMA;
FORCE(T)..     FORC(T) =E=  4.1*(log(M(T)/590)/log(2))+FORCOTH(T);
MM0(TFIRST)..  M(TFIRST) =E= M0;
MM(T+1)..      M(T+1) =E= 590+ATRET*E(T)+(1 - DELTAM)*(M(T)-590);

TTE0(TFIRST).. TE(TFIRST) =E= T0;
TTE(T+1)..     TE(T+1) =E= TE(t)+C1*(FORC(t)-LAM*TE(t)-C3*(TE(t)-TL(t)));
TLE0(TFIRST).. TL(TFIRST) =E= TL0;
TLE(T+1)..     TL(T+1) =E= TL(T)+C4*(TE(T)-TL(T));

YY(T)..       Y(T) =E= AL(T)*L(T)**(1-GAMA)*K(T)**GAMA*(1-B1*(MIU(T)**B2))
              /(1+(A1/9)*SQR(TE(T)));
SEQ(T)..      S(T) =e= I(T)/(.001+Y(T));
RIEQ(T)..     RI(T) =E= GAMA*Y(T)/K(T)- (1-(1-DK)**10)/10  ;

CC(T)..       C(T) =E= Y(T)-I(T);
CPCE(T)..     CPC(T) =e= C(T)*1000/L(T);
PCYE(T)..     PCY(T) =e= Y(T)*1000/L(T);

TRANSE(TLAST).. TRANS(TLAST)=E=RR(TLAST)
  *(PHIK*K(TLAST)+PHIM*M(TLAST)+PHITE*TE(TLAST));

UTIL..        UTILITY =E=
    SUM(T, 10 *RR(T)*L(T)*LOG(C(T)/L(T))/.55 +TRANS(T)*DUM(T));

*  Upper and Lower Bounds: General for stability
MIU.up(T) = 0.99;
MIU.lo(T) = 0.01;
K.lo(T) = 1;
TE.up(t) = 20;
M.lo(T) = 600;
C.LO(T) = 2;

* Upper and lower bounds for historical constraints

MIU.fx('1')=0.;
MIU.fx('2')=0.;
MIU.fx('3')=0.;

* Solution options

option iterlim = 99900;
option reslim = 99999;
option solprint = off;
option limrow = 0;
option limcol = 0;
model CO2 /all/;
solve CO2 maximizing UTILITY using nlp ;
display Y.l, C.l, S.l, K.l, MIU.l, E.l, M.l, TE.l, FORC.l, RI.l,
 CC.m, EE.m, KK.m, MM.m, TTE.m, CPC.l, TL.l, PCY.l, i.l;
display SIGMA, RR, L, AL, DUM, FORCOTH;
```

B. 1999 version of DICE model

```
** DICE 1999. Optimal carbon policy
** New optimal DICE as of 5/5/99
** Calibrated to RICE99 of 5/3/99

SETS  T           Time periods              /1*40/
     TFIRST(T)       First period
     TLAST(T)        Last period
     tearly(T)       First 20 periods
     TLATE(T)  Second 20 periods;

SCALARS
 SRTP    Initial rate of social time preference per year /.03/
 DR      Decline rate of social time preference per year /.0025719/
 GL0     Growth rate of population per decade          /.157/
 DLAB    Decline rate of pop growth per decade         /.2220/
 A0      Initial level of total factor productivity     /.01685/
 GA0     Initial growth rate for technology per decade   /.038/
 DELA    Decline rate of technol. change per decade     /.00000001/
 SIG0    CO2-equivalent emissions-GNP ratio            /.274/
 GSIGMA   Growth of sigma per decade                  /-.158854/
 desig    Decline rate of decarbonization              /.02358711/
 desig2   Quadratic term in decarbonization             /-.00085/
 DK      Depreciation rate on capital per year         /.10/
 GAMA    Capital elasticity in production function      /.30/
 MAT1990  Concentration in atmosphere 1990 (b.t.c.)      /735/
 MU1990   Concentration in upper strata 1990 (b.t.c)     /781/
 ML1990   Concentration in lower strata 1990 (b.t.c)     /19230/
 b11      Carbon cycle transition matrix               /0.66616/
 b12      Carbon cycle transition matrix               /0.33384/
 b21      Carbon cycle transition matrix               /0.27607/
 b22      Carbon cycle transition matrix               /0.60897/
 b23      Carbon cycle transition matrix               /0.11496/
 b32      Carbon cycle transition matrix               /0.00422/
 b33      Carbon cycle transition matrix               /0.99578/
 TL0      1985 lower strat. temp change (C) from 1900     /.06/
 T0       1985 atmospheric temp change (C)from 1900      /.43/
 Q0       1990 world gross output trill 90 US dollars     /21.08/
 LL0      1990 world population millions               /5632.7/
 K0       1990 value capital trill 1990 US dollars       /47/
 C1       Climate-equation coefficient for upper level    /.226/
 LAM      Climate feedback factor                     /1.41/
 C3       Transfer coeffic. upper to lower stratum       /.440/
 C4       Transfer coeffic for lower level              /.02/
 A1       Damage coeff linear term                    /-.0045/
 A2       Damage coeff quadratic term                  /.0035/
 COST10   Intercept control cost function              /.03/
 COST2    Exponent of control cost function            /2.15/
 ET0      C Emiss from deforest (bill tons per dec)       /11.28/
 dmiufunc Decline in cost of abatement function (per decade) /-.08/
 decmiu   Change in decline of cost function              /.005/
 coefopt1     Scaling coefficient in the objective function  /.333187/
 coefopt2     Scaling coefficient in the objective function  /5135680.6/  ;


PARAMETERS
  L(T)        Level of population and labor
  AL(T)        Level of total factor productivity
  SIGMA(T)      CO2-equivalent-emissions output ratio
```

```
 R(T)        Instantaeous rate of social time preference
 RR(T)        Average utility social discount rate
 GA(T)        Growth rate of productivity from 0 to T
 FORCOTH(T)    Exogenous forcing for other greenhouse gases
 GL(T)        Growth rate of labor 0 to T
 gcost1
 GSIG(T)      Cumulative improvement of energy efficiency
 ETREE(T)      Emissions from deforestation
 cost1(t)       cost function for abatement ;

TFIRST(T) = YES$(ORD(T) EQ 1);
TLAST(T)  = YES$(ORD(T) EQ CARD(T));
TEARLY(T) = YES$(ORD(T) LE 20);
TLATE(T)  = YES$(ORD(T) GE 21);
DISPLAY TFIRST, TLAST;

GL(T) = (GL0/DLAB)*(1-exp(-DLAB*(ord(t)-1)));
L(T)=LL0*exp(GL(t));


ga(T)=ga0*EXP(-dela*10*(ORD(T)-1));
al("1") = a0;
LOOP(T,
al(T+1)=al(T)/((1-ga(T)));
);

gsig(T)=gsigma*EXP (  -desig*10*(ORD(T)-1) - desig2*10* ((ord(t)-1)**2)  )   ;
sigma("1")=sig0;
LOOP(T,
sigma(T+1)=(sigma(T)/((1-gsig(T+1))));
);
gcost1(T)=dmiufunc*EXP(-decmiu*10*(ORD(T)-1));
cost1("1")=cost10;
LOOP(T,
cost1(T+1)=cost1(T)/((1+gcost1(T+1)));
);

ETREE(T) = ET0*(1-0.1)**(ord(T)-1);

R(T)=srtp*EXP(-DR*10*(ORD(T)-1));
RR("1")=1;
LOOP(T,
RR(T+1)=RR(T)/((1+R(T))**10);
);
FORCOTH(T)=(-0.1965+.149*(ORD(T)-1)-.0019*(ORD(T)-1)**2)$
    (ORD(T) LT 12) + 1.15$(ORD(T) GE 12);

VARIABLES
 MIU(T)   Emission control rate GHGs
 FORC(T)  Radiative forcing, W per m2
 TE(T)    Temperature, atmosphere C
 TL(T)    Temperature, lower ocean C
 MAT(T)   Carbon concentration in atmosphere (b.t.c.)
 MU(T)    Carbon concentration in shallow oceans (b.t.c.)
 ML(T)    Carbon concentration in lower oceans (b.t.c.)
 E(T)     CO2-equivalent emissions bill t
 C(T)     Consumption trill US dollars
 K(T)     Capital stock trill US dollars
 CPC(T)   Per capita consumption thousands US dol
 PCY(t)   Per capita income thousands US dol
 I(T)     Investment trill US dollars
```

```
 S(T)     Savings rate as fraction of GWP
 RI(T)    Real interest rate per annum
 Y(T)     Output

 UTILITY;

POSITIVE VARIABLES MIU, TE, E, Mat, mu, ml, Y, C, K, I ;

EQUATIONS
 UTIL      Objective function
 YY(T)     Output equation
 CC(T)     Consumption equation
 KK(T)     Capital balance equation
 KK0(T)    Initial condition for K
 KC(T)     Terminal condition for K
 CPCE(t)   Per capita consumption definition
 PCYE(T)   Per capita income definition
 EE(T)     Emissions process
 SEQ(T)    Savings rate equation
 RIEQ(T)   Interest rate equation
 FORCE(T)  Radiative forcing equation
 MMAT0(T)  Starting atmospheric concentration
 MMAT(T)   Atmospheric concentration equation
 MMU0(T)   Initial shallow ocean concentration
 MMU(T) Shallow ocean concentration
 MML0(T)   Initial lower ocean concentration
 ML(T)    Lower ocean concentration
 TTE(T)    Temperature-climate equation for atmosphere
 TTE0(T)   Initial condition for atmospheric temperature
 TLE(T)    Temperature-climate equation for lower oceans
 TLE0(T)   Initial condition for lower ocean
 ;

** Equations of the model

KK(T)..        K(T+1) =L= (1-DK)**10 *K(T)+10*I(T);
KK0(TFIRST)..   K(TFIRST) =E= K0;
KC(TLAST)..     .02*K(TLAST) =L= I(TLAST);

EE(T)..        E(T)=G=10*SIGMA(T)*(1-MIU(T))*AL(T)*L(T)**(1-GAMA)*K(T)**GAMA + ETREE(T);
FORCE(T)..     FORC(T) =E=  4.1*((log(Mat(T)/596.4)/log(2)))+FORCOTH(T);

MMAT0(TFIRST).. MAT(TFIRST) =E= MAT1990;
MMU0(TFIRST)..  MU(TFIRST) =E= MU1990;
MML0(TFIRST)..  ML(TFIRST) =E= ML1990;
MMAT(T+1)..     MAT(T+1) =E= MAT(T)*b11+E(T)+MU(T)*b21;
MML(T+1)..      ML(T+1) =E= ML(T)*b33+b23*MU(T);
MMU(T+1)..      MU(T+1) =E= MAT(T)*b12+MU(T)*b22+ML(T)*b32;

TTE0(TFIRST)..  TE(TFIRST) =E= T0;
TTE(T+1)..      TE(T+1) =E= TE(t)+C1*(FORC(t)-LAM*TE(t)-C3*(TE(t)-TL(t)));
TLE0(TFIRST)..  TL(TFIRST) =E= TL0;
TLE(T+1)..      TL(T+1) =E= TL(T)+C4*(TE(T)-TL(T));

YY(T)..        Y(T) =E= AL(T)*L(T)**(1-GAMA)*K(T)**GAMA*(1-cost1(t)*(MIU(T)**cost2))
          /(1+a1*TE(T)+ a2*TE(T)**2);

SEQ(T)..       S(T) =e= I(T)/(.001+Y(T));
RIEQ(T)..      RI(T) =E= GAMA*Y(T)/K(T)- (1-(1-DK)**10)/10  ;
CC(T)..        C(T) =E= Y(T)-I(T);
CPCE(T)..      CPC(T) =e= C(T)*1000/L(T);
```

```
PCYE(T)..      PCY(T) =e= Y(T)*1000/L(T);

UTIL..         UTILITY =E= SUM(T, 10 *RR(T)*L(T)*LOG(C(T)/L(T))/coefopt1)+ coefopt2 ;

**  Upper and Lower Bounds: General conditions imposed for stability

MIU.up(T)      = 1.0;
MIU.lo(T)      = 0.000001;
K.lo(T)        = 1;
TE.up(t)       = 12;
MAT.lo(T)      = 10;
MU.lo(t)       = 100;
ML.lo(t)       = 1000;
C.lo(T)        = 2;

** Emissions control policy. Current setting is for optimal policy.
** Reinstate equation "Miu.fx(t) = .0" for no-control run.

* For base, remove the * from the following to get zero controls
*MIU.fx(t)=.0;

** Solution options

option iterlim = 99900;
option reslim = 99999;
option solprint = on;
option limrow = 0;
option limcol = 0;

model CO2 /all/;

solve CO2 maximizing UTILITY using nlp ;
solve CO2 maximizing UTILITY using nlp ;
solve CO2 maximizing UTILITY using nlp ;

** Display of results

display y.l, c.l, s.l, k.l, miu.l, e.l, te.l, forc.l, ri.l;
display cc.m, ee.m, kk.m, tte.m, cpc.l, tl.l, pcy.l, i.l;
display sigma, rr, l, al, forcoth, etree;
display mat.l,mu.l,ml.l;


Parameters
Year(t)        Date
Indem(t)       Industrial emissions (b.t.c. per year)
Wem(t)         Total emissions (b.t.c. per year);
Year(t)        = 1995 +10*(ord(t)-1);
Indem(t)       = e.l(t)-etree(t);
Wem(t)         = e.l(t);Parameters
 Tax(t)        Carbon tax ($ per ton)
 damtax(t)     Concentration tax ($ per ton)
 dam(t)        Damages
 cost(t)       Abatement costs;
tax(t)  = -1*ee.m(t)*1000/(kk.m(t));
damtax(t)      = -1*mmat.m(t)*1000/kk.m(t);
dam(t)  = y.l(t)*(1-1/(1+a1*te.l(t)+ a2*te.l(t)**2));
cost(t)        = y.l(t)*(cost1(t)*(miu.l(t)**cost2));

display gsig, sigma;
display ga, al, cost1, gcost1, tax,miu.l,e.l,te.l;
```

C. 2008 version of DICE model

```
$ontext
DICE delta version 8
July 17, 2008.
This version is used for the DICE book, A Question of Balance (YUP, 2008).
We have included only the base, Hotelling, and optimal runs.
Exclude statements are removed so that it can run as a self-contained program.
$offtext

SETS  T              Time periods                  /1*60/ ;

SCALARS

** Preferences
 B_ELASMU   Elasticity of marginal utility of consumption     /  2.0    /
 B_PRSTP    Initial rate of social time preference per year   / .015    /

** Population and technology
 POP0     2005 world population millions              /6514     /
 GPOP0    Growth rate of population per decade          /.35      /
 POPASYM  Asymptotic population                      / 8600    /
 A0       Initial level of total factor productivity      /.02722   /
 GA0      Initial growth rate for technology per decade   /.092      /
 DELA     Decline rate of technol change per decade      /.001     /
 DK       Depreciation rate on capital per year          /.100     /
 GAMA     Capital elasticity in production function      /.300     /
 Q0       2005 world gross output trill 2005 US dollars   /61.1     /
 K0       2005 value capital trill 2005 US dollars       /137.     /

** Emissions
 SIG0     CO2-equivalent emissions-GNP ratio 2005        /.13418    /
 GSIGMA   Initial growth of sigma per decade            /-.0730    /
 DSIG     Decline rate of decarbonization per decade     /.003    /
 DSIG2    Quadratic term in decarbonization             / .000    /
 ELAND0   Carbon emissions from land 2005(GtC per decade) / 11.000  /

** Carbon cycle
 MAT2000  Concentration in atmosphere 2005 (GtC)         /808.9   /
 MU2000   Concentration in upper strata 2005 (GtC)       /1255     /
 ML2000   Concentration in lower strata 2005 (GtC)       /18365     /
 b11      Carbon cycle transition matrix                /0.810712 /
 b12      Carbon cycle transition matrix                /0.189288 /
 b21      Carbon cycle transition matrix                /0.097213 /
 b22      Carbon cycle transition matrix                /0.852787 /
 b23      Carbon cycle transition matrix                /0.05     /
 b32      Carbon cycle transition matrix                /0.003119 /
 b33      Carbon cycle transition matrix                /0.996881 /

** Climate model
 T2XCO2   Equilibrium temp impact of CO2 doubling oC     / 3    /
 FEX0     Estimate of 2000 forcings of non-CO2 GHG       / -.06   /
 FEX1     Estimate of 2100 forcings of non-CO2 GHG       / 0.30   /
 TOCEAN0  2000 lower strat. temp change (C) from 1900     /.0068   /
 TATM0    2000 atmospheric temp change (C)from 1900      /.7307   /
 C1       Climate-equation coefficient for upper level    /.220    /
 C3       Transfer coeffic upper to lower stratum        /.300    /
 C4       Transfer coeffic for lower level              /.050    /
 FCO22X   Estimated forcings of equilibrium co2 doubling  /3.8     /
```

```
** Climate damage parameters calibrated for quadratic at 2.5 C for 2105
 A1      Damage intercept                       / 0.00000    /
 A2      Damage quadratic term                   /  0.0028388 /
 A3      Damage exponent                        / 2.00      /

** Abatement cost
 EXPCOST2   Exponent of control cost function            /2.8   /
 PBACK     Cost of backstop 2005 000$ per tC 2005        /1.17  /
 BACKRAT    Ratio initial to final backstop cost          / 2    /
 GBACK     Initial cost decline backstop pc per decade    /.05   /
 LIMMIU     Upper limit on control rate                 / 1    /

** Participation
 PARTFRACT1  Fraction of emissions under control regime 2005 /1      /
 PARTFRACT2  Fraction of emissions under control regime 2015 /1      /
 PARTFRACT21 Fraction of emissions under control regime 2205 /1      /
 DPARTFRACT  Decline rate of participation                 /0      /

** Availability of fossil fuels
 FOSSLIM  Maximum cumulative extraction fossil fuels        / 6000  /

** Scaling and inessential parameters
  scale1 Scaling coefficient in the objective function      /194    /
  scale2 Scaling coefficient in the objective function      /381800 / ;

* Definitions for outputs of no economic interest
SETS
     TFIRST(T)
     TLAST(T)
     TEARLY(T)
     TLATE(T);

PARAMETERS
  L(T)         Level of population and labor
  AL(T)        Level of total factor productivity
  SIGMA(T)     CO2-equivalent-emissions output ratio
  R(T)         Instantaeous rate of social time preference
  RR(T)        Average utility social discount rate
  GA(T)        Growth rate of productivity from 0 to T
  FORCOTH(T)    Exogenous forcing for other greenhouse gases
  GL(T)        Growth rate of labor 0 to T
  GCOST1       Growth of cost factor
  GSIG(T)      Cumulative improvement of energy efficiency
  ETREE(T)     Emissions from deforestation
  COST1(t)     Adjusted cost for backstop
  PARTFRACT(T)  Fraction of emissions in control regime
  AA1          Variable A1
  AA2          Variable A2
  AA3          Variable A3
  ELASMU       Variable elasticity of marginal utility of consumption
  PRSTP        Variable nitial rate of social time preference per year
  LAM          Climate model parameter
  Gfacpop(T)    Growth factor population ;

PARAMETERS
  L(T)         Level of population and labor
  AL(T)        Level of total factor productivity
  SIGMA(T)     CO2-equivalent-emissions output ratio
  RR(T)        Average utility social discount factor
  GA(T)        Growth rate of productivity from 0 to T
  FORCOTH(T)    Exogenous forcing for other greenhouse gases
```

```
 GL(T)        Growth rate of labor 0 to T
 GCOST1       Growth of cost factor
 GSIG(T)      Cumulative improvement of energy efficiency
 ETREE(T)     Emissions from deforestation
 COST1(t)     Adjusted cost for backstop
 PARTFRACT(T)  Fraction of emissions in control regime
 AA1          Variable A1
 AA2          Variable A2
 AA3          Variable A3
 ELASMU       Variable elasticity of marginal utility of consumption
 PRSTP        Variable nitial rate of social time preference per year
 LAM          Climate model parameter
 Gfacpop(T)    Growth factor population ;

* Unimportant definitions to reset runs
TFIRST(T) = YES$(ORD(T) EQ 1);
TLAST(T)  = YES$(ORD(T) EQ CARD(T));
TEARLY(T) = YES$(ORD(T) LE 20);
TLATE(T)  = YES$(ORD(T) GE 21);
AA1 = A1;
AA2 = A2;
AA3 = A3;
ELASMU = B_ELASMU;
PRSTP  = B_PRSTP;

b11 = 1 - b12;
b21 = 587.473*B12/1143.894;
b22 = 1 - b21 - b23;
b32 = 1143.894*b23/18340;
b33 = 1 - b32 ;


* Important parameters for the model
LAM     = FCO22X/ T2XCO2;
Gfacpop(T) =   (exp(gpop0*(ORD(T)-1))-1)/exp(gpop0*(ORD(T)-1));
L(T)=POP0* (1- Gfacpop(T))+Gfacpop(T)*popasym;
ga(T)=ga0*EXP(-dela*10*(ORD(T)-1));
al("1") = a0;
LOOP(T, al(T+1)=al(T)/((1-ga(T))););
gsig(T)=gsigma*EXP(-dsig*10*(ORD(T)-1)-dsig2*10*((ord(t)-1)**2));sigma("1")=sig0;LOOP(T,sigma(T+1)=(sigma(T)/((1-
gsig(T+1))));); 
cost1(T) = (PBACK*SIGMA(T)/EXPCOST2)* ( (BACKRAT-1+ EXP (-gback* (ORD(T)-1) ) )/BACKRAT);
ETREE(T) = ELAND0*(1-0.1)**(ord(T)-1);
RR(t)=1/((1+prstp)**(10*(ord(T)-1)));
FORCOTH(T)= FEX0+ .1*(FEX1-FEX0)*(ORD(T)-1)$(ORD(T) LT 12)+ 0.36$(ORD(T) GE 12);
partfract(t) = partfract21;
PARTFRACT(T)$(ord(T)<25) = Partfract21 + (PARTFRACT2-Partfract21)*exp(-DPARTFRACT*(ORD(T)-2));
partfract("1")= PARTFRACT1;


VARIABLES
 MIU(T)         Emission control rate GHGs
 FORC(T)        Radiative forcing in watts per m2
 TATM(T)        Temperature of atmosphere in degrees C
 TOCEAN(T)      Temperatureof lower oceans degrees C
 MAT(T)         Carbon concentration in atmosphere GtC
 MATAV(T)       Average concentrations
 MU(T)          Carbon concentration in shallow oceans Gtc
 ML(T)          Carbon concentration in lower oceans GtC
 E(T)           CO2-equivalent emissions GtC
 C(T)           Consumption trillions US dollars
```

```
 K(T)          Capital stock trillions US dollars
 CPC(T)        Per capita consumption thousands US dollars
 PCY(t)        Per capita income thousands US dollars
 I(T)          Investment trillions US dollars
 S(T)          Gross savings rate as fraction of gross world product
 RI(T)         Real interest rate per annum
 Y(T)          Gross world product net of abatement and damages
 YGROSS(T)     Gross world product GROSS of abatement and damages
 YNET(T)       Output net of damages equation
 DAMAGES(T)    Damages
 ABATECOST(T)  Cost of emissions reductions
 CCA(T)        Cumulative industrial carbon emissions GTC
 PERIODU(t)    One period utility function
 UTILITY;

POSITIVE VARIABLES MIU, TATM, TOCE, E, MAT, MATAV, MU, ML, Y, YGROSS, C, K, I, CCA ;

EQUATIONS

 CCTFIRST(T)     First period cumulative carbon
 CCACCA(T)       Cumulative carbon emissions
 UTIL            Objective function
 YY(T)           Output net equation
 YNETEQ(T)       Output net of damages equation
 YGROSSEQ(T)     Output gross equation
 DAMEQ(T)        Damage equation
 ABATEEQ(T)      Cost of emissions reductions equation
 CC(T)           Consumption equation
 KK(T)           Capital balance equation
 KK0(T)          Initial condition for capital
 KC(T)           Terminal condition for capital
 CPCE(t)         Per capita consumption definition
 PCYE(T)         Per capita income definition
 EE(T)           Emissions equation
 SEQ(T)          Savings rate equation
 RIEQ(T)         Interest rate equation
 FORCE(T)        Radiative forcing equation
 MMAT0(T)        Starting atmospheric concentration
 MMAT(T)         Atmospheric concentration equation
 MMATAVEQ(t)     Average concentrations equation
 MMU0(T)         Initial shallow ocean concentration
 MMU(T)          Shallow ocean concentration
 MML0(T)         Initial lower ocean concentration
 MML(T)          Lower ocean concentration
 TATMEQ(T)       Temperature-climate equation for atmosphere
 TATM0EQ(T)      Initial condition for atmospheric temperature
 TOCEANEQ(T)     Temperature-climate equation for lower oceans
 TOCEAN0EQ(T)    Initial condition for lower ocean temperature
 PERIODUEQ(t)    Instantaneous utility function equation  ;

** Equations of the model

CCTFIRST(TFIRST).. CCA(TFIRST)=E=0;
CCACCA(T+1)..     CCA(T+1)=E=CCA(T)+ E(T);
KK(T)..           K(T+1) =L= (1-DK)**10 *K(T)+10*I(T);
KK0(TFIRST)..     K(TFIRST) =E= K0;
KC(TLAST)..       .02*K(TLAST) =L= I(TLAST);
EE(T)..           E(T)=E=10*SIGMA(T)*(1-MIU(T))*AL(T)*L(T)**(1-GAMA)*K(T)**GAMA + ETREE(T);
FORCE(T)..        FORC(T) =E=  FCO22X*((log((Matav(T)+.000001)/596.4)/log(2)))+FORCOTH(T);
MMAT0(TFIRST)..   MAT(TFIRST) =E= MAT2000;
MMU0(TFIRST)..    MU(TFIRST)  =E= MU2000;
```

```
MML0(TFIRST)..    ML(TFIRST)  =E= ML2000;
MMAT(T+1)..       MAT(T+1)    =E= MAT(T)*b11+MU(T)*b21 + E(T);
MMATAVEQ(t)..     MATAV(T)    =e= (MAT(T)+MAT(T+1))/2 ;
MML(T+1)..        ML(T+1)     =E= ML(T)*b33+b23*MU(T);
MMU(T+1)..        MU(T+1)     =E= MAT(T)*b12+MU(T)*b22+ML(T)*b32;
TATM0EQ(TFIRST).. TATM(TFIRST) =E= TATM0;
TATMEQ(T+1)..     TATM(T+1) =E= TATM(t)+C1*(FORC(t+1)-LAM*TATM(t)-C3*(TATM(t)-TOCEAN(t)));
TOCEAN0EQ(TFIRST).. TOCEAN(TFIRST) =E= TOCEAN0;
TOCEANEQ(T+1)..   TOCEAN(T+1) =E= TOCEAN(T)+C4*(TATM(T)-TOCEAN(T));
YGROSSEQ(T)..  YGROSS(T) =e= AL(T)*L(T)**(1-GAMA)*K(T)**GAMA;
DAMEQ(T)..     DAMAGES(t) =E= YGROSS(T)- YGROSS(T)/(1+aa1*TATM(T)+ aa2*TATM(T)**aa3);
YNETEQ(T)..    YNET(T) =E=  YGROSS(T)/(1+aa1*TATM(T)+ aa2*TATM(T)**aa3);
ABATEEQ(T)..   ABATECOST(T) =E= (PARTFRACT(T)**(1-expcost2))*YGROSS(T)*(cost1(t)*(MIU(T)**EXPcost2));
YY(T)..        Y(T) =E= YGROSS(T)*((1-(PARTFRACT(T)**(1-expcost2))*cost1(t)*(MIU(T)**EXPcost2)))/(1+aa1*TATM(T)+
aa2*TATM(T)**aa3);
SEQ(T)..       S(T)    =E= I(T)/(.001+Y(T));
RIEQ(T)..      RI(T)   =E= GAMA*Y(T)/K(T)- (1-(1-DK)**10)/10  ;
CC(T)..        C(T)    =E= Y(T)-I(T);
CPCE(T)..      CPC(T)  =E= C(T)*1000/L(T);
PCYE(T)..      PCY(T)  =E= Y(T)*1000/L(T);
PERIODUEQ(T).. PERIODU(T)  =E=   ((C(T)/L(T))**(1-ELASMU)-1)/(1-ELASMU);
UTIL..         UTILITY =E= SUM(T, 10 *RR(T)*L(T)*(PERIODU(T))/scale1)+ scale2 ;

**  Upper and Lower Bounds: General conditions for stability

K.lo(T)        = 100;
MAT.lo(T)      = 10;
MU.lo(t)       = 100;
ML.lo(t)       = 1000;
C.lo(T)        = 20;
TOCEAN.up(T)    = 20;
TOCEAN.lo(T)    = -1;
TATM.up(t)      = 20;
miu.up(t)       = LIMMIU;
partfract("1")= 0.25372;

* First period predetermined by Kyoto Protocol
miu.fx("1")     = 0.005;

** Fix savings assumption for standardization if needed
*s.fx(t)=.22;

** Cumulative limits on carbon use at 6000 GtC
CCA.up(T) = FOSSLIM;

** Solution options
option iterlim = 99900;
option reslim = 99999;
option solprint = on;
option limrow = 0;
option limcol = 0;
model CO2 /all/;

* Optimal run
* Solution for optimal run

solve CO2 maximizing UTILITY using nlp ;
solve CO2 maximizing UTILITY using nlp ;
solve CO2 maximizing UTILITY using nlp ;
solve CO2 maximizing UTILITY using nlp ;
solve CO2 maximizing UTILITY using nlp ;
```

```
solve CO2 maximizing UTILITY using nlp ;

* Definition of opt results

Parameters
Year(t)         Date
opt_y(t)
opt_cpc(t)
opt_s(t)
opt_indem(t)
opt_sigma(t)
opt_tatm(t)
opt_mat(t)
opt_tax(t)
opt_ri(t)
opt_rr(t)
opt_al(t)
opt_forcoth(t)
opt_l(t)
opt_etree(t)
opt_yy(t)
opt_cc(t)
opt_miu(t)
opt_wem(t)
opt_ri(t)
opt_dam(t)
opt_abate(t)
opt_mcemis(t)
opt_utility
opt_scc(t) ;

Year(t)         = 2005 +10*(ord(t)-1);
opt_y(t)=y.l(t);
opt_cpc(t)=cpc.l(t);
opt_s(t)=s.l(t)     ;
opt_indem(t)= e.l(t)-etree(t);;
opt_sigma(t)=sigma(t) ;
opt_tatm(t)=tatm.l(t)  ;
opt_mat(t)=mat.l(t)     ;
opt_tax(t)=-1*ee.m(t)*1000/(kk.m(t)+.00000000001)       ;
opt_ri(t)=ri.l(t);
opt_rr(t)=rr(t)   ;
opt_al(t)=al(t)    ;
opt_forcoth(t)=forcoth(t);
opt_l(t)=l(t);
opt_etree(t)=etree(t);
opt_yy(t)=yy.m(t)     ;
opt_cc(t)=cc.m(t)      ;
opt_miu(t)=miu.l(t)     ;
opt_wem(t)= e.l(t);
opt_ri(t)=ri.l(t)         ;
opt_dam(t)= damages.l(t);
opt_abate(t) = abatecost.l(t);
opt_mcemis(t)= expcost2*cost1(t)*miu.l(t)**(expcost2-1)/sigma(t)*1000;
opt_utility=utility.l  ;
opt_scc(t)=-ee.m(t)/cc.m(t)*(1000)    ;

option decimals=6;
display opt_scc,y.l,opt_miu,ee.m, cc.m, yy.m;
```

D. 2013R version of DICE model (*)

This is the central file for the version of October 2013. The files for the different options are available online on the DICE-2013R website. There are two versions available. (1) The version below is the "Vanilla" version. It is self-contained and can be run without any further subroutines. It contains a full set of output statements. (2) The "Rockyroad" version has all scenarios, with a full set of outputs. It requires the subroutines ("include" programs). The results of this manual are from the Rockyroad version of October 4, 2013.

For the models, see [dicemodel.net.](dicemodel.net)

```
$ontext
This is the DICE-2013R model, version DICE2013Rv2_102213_vanilla_v24b.gms, revised from April version.
The vanilla version includes only the optimal and baseline scenarios.
These are determined by setting the "ifopt" control at 1 (optimal) or 0 (baseline).
This version has write ("put") output but does not have subroutines ("include").
A full discussion is included in the "DICE 2013R Manual" on the web at dicemodel.net.
$offtext

$title      DICE-2013R October 2013

set      t  Time periods (5 years per period)              /1*60/ ;

parameters

**Time Step
     tstep    Years per Period                           /5/

** If optimal control
     ifopt    If optimized 1 and if base is 0                 /1/

** Preferences
     elasmu   Elasticity of marginal utility of consumption     /  1.45 /
     prstp    Initial rate of social time preference per year   / .015  /

** Population and technology
     gama     Capital elasticity in production function        /.300    /
     pop0     Initial world population (millions)             /6838    /
     popadj   Growth rate to calibrate to 2050 pop projection  /0.134   /
     popasym  Asymptotic population (millions)                /10500   /
     dk       Depreciation rate on capital (per year)         /.100    /
     q0       Initial world gross output (trill 2005 USD)     /63.69   /
     k0       Initial capital value (trill 2005 USD)          /135     /
     a0       Initial level of total factor productivity      /3.80    /
     ga0      Initial growth rate for TFP per 5 years         /0.079   /
     dela     Decline rate of TFP per 5 years                 /0.006   /

** Emissions parameters
     gsigma1  Initial growth of sigma (per year)              /-0.01   /
     dsig     Decline rate of decarbonization (per period)    /-0.001  /
     eland0   Carbon emissions from land 2010 (GtCO2 per year)   / 3.3    /
     deland   Decline rate of land emissions (per period)     / .2     /
     e0       Industrial emissions 2010 (GtCO2 per year)      /33.61   /
     miu0     Initial emissions control rate for base case 2010   /.039    /

** Carbon cycle
* Initial Conditions
     mat0   Initial Concentration in atmosphere 2010 (GtC)     /830.4   /
     mu0    Initial Concentration in upper strata 2010 (GtC)   /1527.   /
     ml0    Initial Concentration in lower strata 2010 (GtC)   /10010.  /
```

```
      mateq  Equilibrium concentration atmosphere  (GtC)         /588     /
      mueq   Equilibrium concentration in upper strata (GtC)      /1350    /
      mleq   Equilibrium concentration in lower strata (GtC)      /10000   /

* Flow paramaters
      b12      Carbon cycle transition matrix                  /.088   /
      b23      Carbon cycle transition matrix                  /0.00250/

* These are for declaration and are defined later
      b11      Carbon cycle transition matrix
      b21      Carbon cycle transition matrix
      b22      Carbon cycle transition matrix
      b32      Carbon cycle transition matrix
      b33      Carbon cycle transition matrix
      sig0     Carbon intensity 2010 (kgCO2 per output 2005 USD 2010)

** Climate model parameters
      t2xco2   Equilibrium temp impact (oC per doubling CO2)    / 2.9   /
      fex0     2010 forcings of non-CO2 GHG (Wm-2)           / 0.25  /
      fex1     2100 forcings of non-CO2 GHG (Wm-2)           / 0.70  /
      tocean0  Initial lower stratum temp change (C from 1900)  /.0068  /
      tatm0    Initial atmospheric temp change (C from 1900)    /0.80   /

      c10      Initial climate equation coefficient for upper level /0.098  /
      c1beta   Regression slope coefficient(SoA~Equil TSC)        /0.01243/

      c1       Climate equation coefficient for upper level     /0.098  /
      c3       Transfer coefficient upper to lower stratum      /0.088  /
      c4       Transfer coefficient for lower level           /0.025  /
      fco22x   Forcings of equilibrium CO2 doubling (Wm-2)      /3.8    /

** Climate damage parameters
      a10      Initial damage intercept                    /0      /
      a20      Initial damage quadratic term                 /0.00267 /
      a1       Damage intercept                          /0      /
      a2       Damage quadratic term                       /0.00267 /
      a3       Damage exponent                            /2.00    /

** Abatement cost
      expcost2  Exponent of control cost function            / 2.8  /
      pback     Cost of backstop 2005$ per tCO2 2010          / 344  /
      gback     Initial cost decline backstop cost per period   / .025 /
      limmiu    Upper limit on control rate after 2150         / 1.2  /
      tnopol    Period before which no emissions controls base  / 45   /
      cprice0   Initial base carbon price (2005$ per tCO2)      / 1.0  /
      gcprice   Growth rate of base carbon price per year       /.02   /

** Participation parameters
      periodfullpart Period at which have full participation          /21  /
      partfract2010  Fraction of emissions under control in 2010       / 1  /
      partfractfull  Fraction of emissions under control at full time  / 1  /

** Availability of fossil fuels
      fosslim        Maximum cumulative extraction fossil fuels (GtC) /6000/

** Scaling and inessential parameters
* Note that these are unnecessary for the calculations but are for convenience
      scale1      Multiplicative scaling coefficient           /0.016408662 /
      scale2      Additive scaling coefficient                /-3855.106895/ ;

* Program control variables
```

```
sets     tfirst(t), tlast(t), tearly(t), tlate(t);

PARAMETERS
      L(t)          Level of population and labor
      al(t)         Level of total factor productivity
      sigma(t)      CO2-equivalent-emissions output ratio
      rr(t)         Average utility social discount rate
      ga(t)         Growth rate of productivity from
      forcoth(t)    Exogenous forcing for other greenhouse gases
      gl(t)         Growth rate of labor
      gcost1        Growth of cost factor
      gsig(t)       Change in sigma (cumulative improvement of energy efficiency)
      etree(t)      Emissions from deforestation
      cost1(t)      Adjusted cost for backstop
      partfract(t)  Fraction of emissions in control regime
      lam           Climate model parameter
      gfacpop(t)    Growth factor population
      pbacktime(t)  Backstop price
      optlrsav      Optimal long-run savings rate used for transversality
      scc(t)        Social cost of carbon
      cpricebase(t) Carbon price in base case
      photel(t)     Carbon Price under no damages (Hotelling rent condition);

* Program control definitions
      tfirst(t) = yes$(t.val eq 1);
      tlast(t)  = yes$(t.val eq card(t));

* Parameters for long-run consistency of carbon cycle
      b11 = 1 - b12;
      b21 = b12*MATEQ/MUEQ;
      b22 = 1 - b21 - b23;
      b32 = b23*mueq/mleq;
      b33 = 1 - b32 ;

* Further definitions of parameters
      sig0 = e0/(q0*(1-miu0));
      lam = fco22x/ t2xco2;
      L("1") = pop0;
      loop(t, L(t+1)=L(t););
      loop(t, L(t+1)=L(t)*(popasym/L(t))**popadj ;);

      ga(t)=ga0*exp(-dela*5*((t.val-1)));
      al("1") = a0; loop(t, al(t+1)=al(t)/((1-ga(t))););

      gsig("1")=gsigma1; loop(t,gsig(t+1)=gsig(t)*((1+dsig)**tstep) ;);
      sigma("1")=sig0;   loop(t,sigma(t+1)=(sigma(t)*exp(gsig(t)*tstep)););

      pbacktime(t)=pback*(1-gback)**(t.val-1);
      cost1(t) = pbacktime(t)*sigma(t)/expcost2/1000;

      etree(t) = eland0*(1-deland)**(t.val-1);
      rr(t) = 1/((1+prstp)**(tstep*(t.val-1)));
      forcoth(t) = fex0+ (1/18)*(fex1-fex0)*(t.val-1)$(t.val lt 19)+ (fex1-fex0)$(t.val ge 19);
      optlrsav = (dk + .004)/(dk + .004*elasmu + prstp)*gama;

      partfract(t)$(ord(T)>periodfullpart) = partfractfull;
      partfract(t)$(ord(T)<periodfullpart+1) = partfract2010+(partfractfull-partfract2010)*(ord(t)-1)/periodfullpart;

      partfract("1")= partfract2010;

*Transient TSC Correction ("Speed of Adjustment Parameter")
```

```
      c1 =  c10 + c1beta*(t2xco2-2.9);

*Base Case      Carbon Price
      cpricebase(t)= cprice0*(1+gcprice)**(5*(t.val-1));

VARIABLES
      MIU(t)         Emission control rate GHGs
      FORC(t)        Increase in radiative forcing (watts per m2 from 1900)
      TATM(t)        Increase temperature of atmosphere (degrees C from 1900)
      TOCEAN(t)      Increase temperatureof lower oceans (degrees C from 1900)
      MAT(t)         Carbon concentration increase in atmosphere (GtC from 1750)
      MU(t)          Carbon concentration increase in shallow oceans (GtC from 1750)
      ML(t)          Carbon concentration increase in lower oceans (GtC from 1750)
      E(t)           Total CO2 emissions (GtCO2 per year)
      EIND(t)        Industrial emissions (GtCO2 per year)
      C(t)           Consumption (trillions 2005 US dollars per year)
      K(t)           Capital stock (trillions 2005 US dollars)
      CPC(t)         Per capita consumption (thousands 2005 USD per year)
      I(t)           Investment (trillions 2005 USD per year)
      S(t)           Gross savings rate as fraction of gross world product
      RI(t)          Real interest rate (per annum)
      Y(t)           Gross world product net of abatement and damages (trillions 2005 USD per year)
      YGROSS(t)      Gross world product GROSS of abatement and damages (trillions 2005 USD per year)
      YNET(t)        Output net of damages equation (trillions 2005 USD per year)
      DAMAGES(t)     Damages (trillions 2005 USD per year)
      DAMFRAC(t)     Damages as fraction of gross output
      ABATECOST(t)   Cost of emissions reductions  (trillions 2005 USD per year)
      MCABATE(t)     Marginal cost of abatement (2005$ per ton CO2)
      CCA(t)         Cumulative industrial carbon emissions (GTC)
      PERIODU(t)     One period utility function
      CPRICE(t)      Carbon price (2005$ per ton of CO2)
      CEMUTOTPER(t)  Period utility
      UTILITY        Welfare function
;

NONNEGATIVE VARIABLES  MIU, TATM, MAT, MU, ML, Y, YGROSS, C, K, I;

EQUATIONS
*Emissions and Damages
      EEQ(t)         Emissions equation
      EINDEQ(t)      Industrial emissions
      CCACCA(t)      Cumulative carbon emissions

      FORCE(t)       Radiative forcing equation
      DAMFRACEQ(t)   Equation for damage fraction
      DAMEQ(t)       Damage equation

      ABATEEQ(t)     Cost of emissions reductions equation
      MCABATEEQ(t)   Equation for MC abatement
      CARBPRICEEQ(t) Carbon price equation from abatement

*Climate and carbon cycle
      MMAT(t)        Atmospheric concentration equation
      MMU(t)         Shallow ocean concentration
      MML(t)         Lower ocean concentration
      TATMEQ(t)      Temperature-climate equation for atmosphere
      TOCEANEQ(t)    Temperature-climate equation for lower oceans

*Economic variables
      YGROSSEQ(t)    Output gross equation
      YNETEQ(t)      Output net of damages equation
```

```
      YY(t)          Output net equation
      CC(t)          Consumption equation
      CPCE(t)        Per capita consumption definition
      SEQ(t)         Savings rate equation
      KK(t)          Capital balance equation
      RIEQ(t)        Interest rate equation

* Utility
      CEMUTOTPEREQ(t)  Period utility
      PERIODUEQ(t)     Instantaneous utility function equation
      UTIL             Objective function      ;

** Equations of the model
*Emissions and Damages
 eeq(t)..            E(t)          =E= EIND(t) + etree(t);
 eindeq(t)..         EIND(t)       =E= sigma(t) * YGROSS(t) * (1-(MIU(t)));
 ccacca(t+1)..       CCA(t+1)      =E= CCA(t)+ EIND(t)*5/3.666;
 force(t)..          FORC(t)       =E= fco22x * ((log((MAT(t)/588.000))/log(2))) + forcoth(t);
 damfraceq(t) ..     DAMFRAC(t)    =E= (a1*TATM(t))+(a2*TATM(t)**a3) ;
 dameq(t)..          DAMAGES(t)    =E= YGROSS(t) * DAMFRAC(t);
 abateeq(t)..        ABATECOST(t)  =E= YGROSS(t) * cost1(t) * (MIU(t)**expcost2) * (partfract(t)**(1-expcost2));
 mcabateeq(t)..      MCABATE(t)    =E= pbacktime(t) * MIU(t)**(expcost2-1);
 carbpriceeq(t)..    CPRICE(t)     =E= pbacktime(t) * (MIU(t)/partfract(t))**(expcost2-1);

*Climate and carbon cycle
 mmat(t+1)..         MAT(t+1)      =E= MAT(t)*b11 + MU(t)*b21 + (E(t)*(5/3.666));
 mml(t+1)..          ML(t+1)       =E= ML(t)*b33  + MU(t)*b23;
 mmu(t+1)..          MU(t+1)       =E= MAT(t)*b12 + MU(t)*b22 + ML(t)*b32;
 tatmeq(t+1)..       TATM(t+1)     =E= TATM(t) + c1 * ((FORC(t+1)-(fco22x/t2xco2)*TATM(t))-(c3*(TATM(t)-TOCEAN(t))));
 toceaneq(t+1)..     TOCEAN(t+1)   =E= TOCEAN(t) + c4*(TATM(t)-TOCEAN(t));

*Economic variables
 ygrosseq(t)..       YGROSS(t)     =E= (al(t)*(L(t)/1000)**(1-GAMA))*(K(t)**GAMA);
 yneteq(t)..         YNET(t)       =E= YGROSS(t)*(1-damfrac(t));
 yy(t)..             Y(t)          =E= YNET(t) - ABATECOST(t);
 cc(t)..             C(t)          =E= Y(t) - I(t);
 cpce(t)..           CPC(t)        =E= 1000 * C(t) / L(t);
 seq(t)..            I(t)          =E= S(t) * Y(t);
 kk(t+1)..           K(t+1)        =L= (1-dk)**tstep * K(t) + tstep * I(t);
 rieq(t+1)..         RI(t)         =E= (1+prstp) * (CPC(t+1)/CPC(t))**(elasmu/tstep) - 1;

*Utility
 cemutotpereq(t)..   CEMUTOTPER(t)  =E= PERIODU(t) * L(t) * rr(t);
 periodueq(t)..      PERIODU(t)     =E= ((C(T)*1000/L(T))**(1-elasmu)-1)/(1-elasmu)-1;
 util..              UTILITY        =E= tstep * scale1 * sum(t,  CEMUTOTPER(t)) + scale2 ;

*Resource limit
CCA.up(t)       = fosslim;

* Control rate limits
MIU.up(t)            = limmiu*partfract(t);
MIU.up(t)$(t.val<30) = 1;

**  Upper and lower bounds for stability
K.LO(t)         = 1;
MAT.LO(t)       = 10;
MU.LO(t)        = 100;
ML.LO(t)        = 1000;
C.LO(t)         = 2;
TOCEAN.UP(t)    = 20;
TOCEAN.LO(t)    = -1;
```

```
TATM.UP(t)      = 40;
CPC.LO(t)       = .01;

* Control variables
* Set savings rate for steady state for last 10 periods
set lag10(t) ;
lag10(t) =  yes$(t.val gt card(t)-10);
S.FX(lag10(t)) = optlrsav;

* Initial conditions
CCA.FX(tfirst)    = 90;
K.FX(tfirst)      = k0;
MAT.FX(tfirst)    = mat0;
MU.FX(tfirst)     = mu0;
ML.FX(tfirst)     = ml0;
TATM.FX(tfirst)   = tatm0;
TOCEAN.FX(tfirst) = tocean0;

** Solution options
option iterlim = 99900;
option reslim = 99999;
option solprint = on;
option limrow = 0;
option limcol = 0;
model  CO2 /all/;

* For base run, this subroutine calculates Hotelling rents
* Carbon price is maximum of Hotelling rent or baseline price
If (ifopt eq 0,
      a2 = 0;
      solve CO2 maximizing UTILITY using nlp;
      photel(t)=cprice.l(t);
      a2 = a20;
      cprice.fx(t)$(t.val<tnopol+1) = max(photel(t),cpricebase(t));
);

miu.fx('1')$(ifopt=1) = miu0;
solve co2 maximizing utility using nlp;
solve co2 maximizing utility using nlp;
solve co2 maximizing utility using nlp;

** POST-SOLVE
* Calculate social cost of carbon
scc(t) = -1000*eeq.m(t)/cc.m(t);

** Display at bottom of output for visual inspection
option decimals=4;
display tatm.l,cpc.l,scc,y.l,s.l;
display ri.l,miu.l,cca.l,photel,cpricebase,cprice.l,utility.l,t2xco2 ,a2,partfract;

* Some sample results.
* Produces a file "DiceResults.csv" in the base directory
* For aLL relevant information, see 'PutOutputAllT.gms' in the Include folder.
* The statement at the end of the *.lst file "Output..." will tell you where to find the file.

file results /DiceResults.csv/;     results.nd = 10 ; results.nw = 0 ; results.pw=1200; results.pc=5;
put results;
put /"Results of DICE model run using model DICE2013R_103113_vanilla_v2b4.gms";
put /"This is optimal if ifopt = 1 and baseline if ifopt = 0";
put /"ifopt =" ifopt;
put // "Period";
```

```
Loop (T, put T.val);
put / "Year" ;
Loop (T, put (2005+(TSTEP*T.val) ));
put / "Industrial Emissions (GTCO2 per year)" ;
Loop (T, put EIND.l(T));
put / "Atmospheric concentration of carbon (ppm)" ;
Loop (T, put (MAT.l(T)/2.13));
put / "Atmospheric Temperature (deg C above preindustrial)" ;
Loop (T, put TATM.l(T));
put / "Output (Net of Damages and Abatement, trillion USD pa) " ;
Loop (T, put Y.l(T));
put / "Climate Damages (fraction of gross output)" ;
Loop (T, put DAMFRAC.l(T));
put / "Consumption Per Capita (thousand USD per year)" ;
Loop (T, put CPC.l(T));
put / "Carbon Price (per t CO2)" ;
Loop (T, put cprice.l(T));
put / "Emissions Control Rate (total)" ;
Loop (T, put MIU.l(T));
put / "Social cost of carbon" ;
Loop (T, put scc(T));
put / "Interest Rate (Real Rate of Return)" ;
Loop (T, put RI.l(T));

putclose;
```