DC motor modeling:
first principles and model identification
ME352 Fall 2022
Lab 1

Prof. Baglione, Prof. Luchtenburg
Teaching Assistant: Aiden Bowman, Jonathan Lerner, Kameron Wang
Technical staff: Michael Giglia

The Cooper Union

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1 Overview

For this part of the course, we will perform a series of labs that address the control of a DC motor. In the first lab, we identify input-output models for this system from (1) first principles, and (2) using identification on experimental step response data. The input to the motor is voltage, and the output is angular speed. Once these relationships are known, we can use them for open-and closed-loop control of the DC motor system.

2 Goals

Our hands-on goals for today are:

- Familiarize yourself with the lab equipment (MATLAB/Simulink software and Quanser QUBE-Servo 2).
- Derive input-output model from first principles.
- Identify input-output model using experimental step response data.
- Understand the concepts of time constant and DC gain.

There is a one-page lab report that you will hand in before leaving the lab. **Save your simulink models and data. The transient data will be used in a future problem set.**

3 Modeling of the Motor

The motor can be modeled as the sum of an electrical and a mechanical component, see Fig. 1. For the electrical model, we have according to Kirchoff's voltage law

$$L\,\frac{di_a}{dt} + R\,i_a + e = v_a, \tag{1}$$

where e is the voltage generated as a result of the rotation of the motor (electromotive force, or EMF).

$$e = K_e\,\omega. \tag{2}$$

This is the "law of the generator" which relates the angular speed ω to the voltage induced by the motor e, where K_e is a constant.

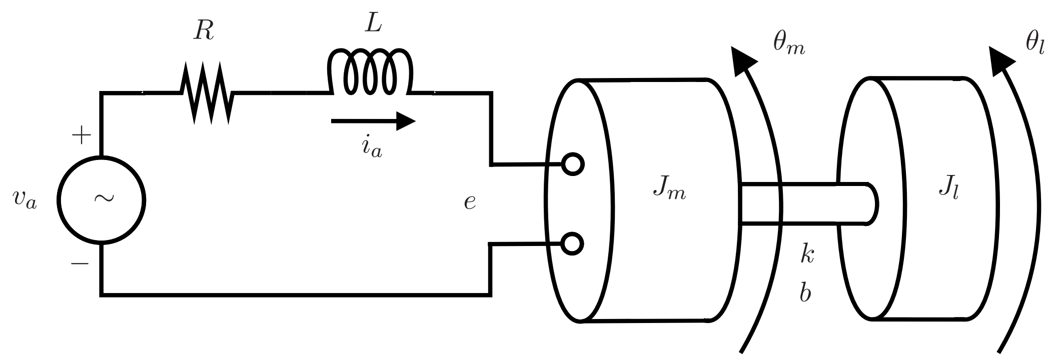


Figure 1: Schematic for the electrical and mechanical model of an electrical motor.

The mechanical equations follow from Newton's law for the rotational moment of inertia

$$J_m\,\ddot{\theta}_m + b(\dot{\theta}_m - \dot{\theta}_l) + k(\theta_m - \theta_l) = T, \tag{3a}$$
$$J_l\,\ddot{\theta}_l + b(\dot{\theta}_l - \dot{\theta}_m) + k(\theta_l - \theta_m) = 0, \tag{3b}$$

where k, b are the rotational spring and damping constant respectively, and the driving torque T is given by

$$T = K_t\,i_a. \tag{4}$$

This is the "law of the motor" and relates the motor torque T to the current i_a through the motor, where K_t is a constant.

Note that internal friction of the motor, stick-slip friction due to the brushes and other couplings present in the motor test stand are not modeled. For simplicity we introduce two more assumptions:

1. Assume a rigid connection between motor and load (flywheel)
2. Neglect the influece of induction

The combined set of equations simplify then to

$$i_a\,R + K_e\,\omega = v_a, \tag{5a}$$
$$J\,\dot{\omega} = K_t\,i_a, \tag{5b}$$

where $J = J_m + J_l$.

Pre-Lab Assignment

1. Consult the "Quanser USER MANUAL, QUBE-Servo 2 Experiment" or Homework #1 and write down the numerical values for R, K_t, K_e, and J.

2. Write the model in **state space** form:

$$\dot{x} = Ax + Bu \tag{6}$$
$$y = Cx + Du, \tag{7}$$

   where $x = \omega$, $u = v_a$, $y = \omega$. Make sure your expresssions for A, B, C, D are correct before you proceed with numerical calculation.

3. Write the model also in **transfer function** form:

$$Y(s) = G(s)\, U(s). \tag{8}$$

4. Obtain the time constant (τ) and the DC gain (K) in terms of the system parameters. What are the numerical values? What do these values 'tell' you?

5. **Bonus** Here we do not neglect the motor inductance. Write down the (full) state space form. Use MATLAB to compute the poles. How many do you get? Which one did we neglect above?

4 Laboratory Setup

Motor test stand (QUBE-Servo 2)

The Quanser QUBE-Servo 2, pictured in Figure 2, is a compact rotary servo system that can be used to perform a variety of classic servo control and inverted pendulum based experiments. It is controlled by a computer via USB connection. We quote from the Quanser User Manual: "The system is driven using a direct-drive 18V brushed DC motor. The motor is powered by a built-in PWM amplifier with integrated current sense. Single-ended rotary encoders are used to measure the angular position of the DC motor and pendulum."

Figure 2: Quanser QUBE-Servo 2.

Each station consists of a QUBE, a flywheel, USB A to USB B cable, and power adapter. Before starting anything, make sure that all the cables are plugged in properly. Attach the flywheel to the top of the QUBE. Initially, the light on the QUBE should be red before any program starts running.

From the Matlab directory, open in the Documents -> DC_MOTOR_LABS_DOCS folder the lab1_open_loop_voltage_params.m and open_looper_voltage.sltx files. The diagram below should appear on your screen.

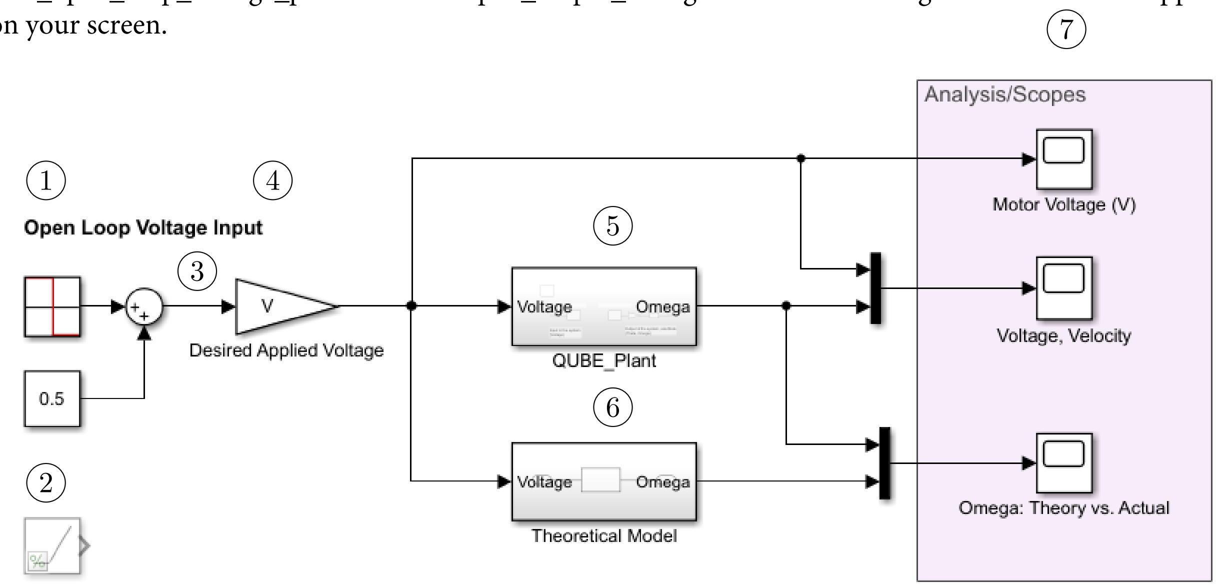


Figure 3: Simulink Block Diagram of DC motor setup.

Each block serves the following purpose:

1 is Signal Generator. In this case, a periodic square wave taking the values 0.5 and -0.5.

2 is Offset (constant). It is used to shift the square wave upwards by 0.5.

3 is Sum. Its inputs are added together to create an output.

4 is Gain. Its output is its input multiplied by a constant.

5 is the Plant (DC motor system). Inside this block are Quanser provided blocks to actuate the motor and read its sensor (see below).

6 is the (theoretical) Plant Model. It is a model (transfer function or state space representation) of the DC motor with the the parameters you looked up above.

7 are Scopes. They display relevant data.

The contents of the Plant block are shown below:

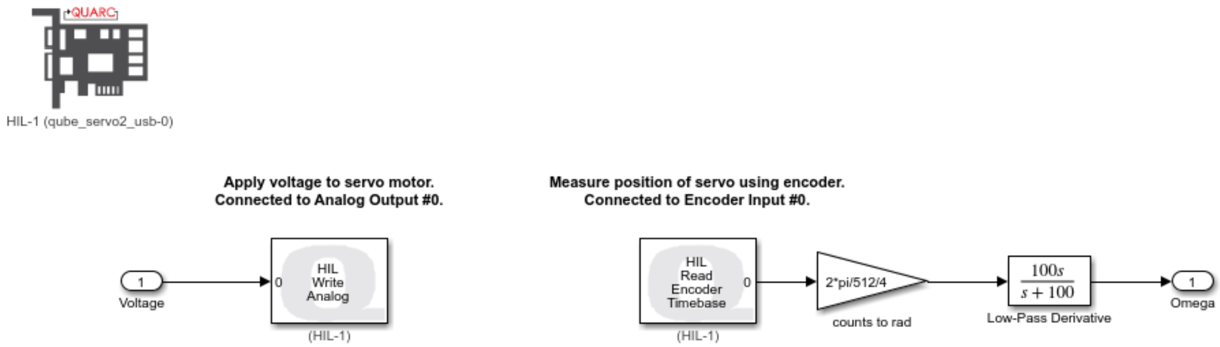


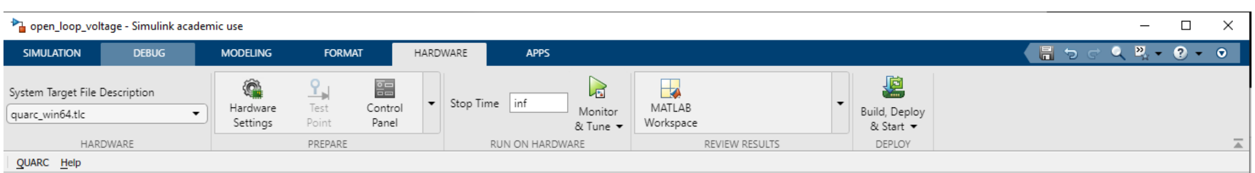
Figure 4: Contents of Plant block.

Before you proceed, make sure you understand what the purpose of the block diagram is.

Running programs

Run lab1...`param.m` and check the Command Window to ensure it is running. To run programs through the QUBE and related software, parameters are assigned values and compiled beforehand. For this lab, you will change the parameter DC Gain K and time constant τ values.

The following menu bar lets you run the motor:



1. Press the Build, Deploy & Start button and Monitor & Tune button.

2. Double click the "Analysis/Scope" blocks to view the graphs. Under "View" toggle "Legend".

Verify that the QUBE light is green. The motor should start rotating and the scopes should display a total of 3 graphs. Run the QUBE motor with an initial 6V square wave input. Stop Simulink and change the desired applied voltages in the parameter file. Re-run the motor by clicking Monitor & Tune.

Lab Assignment

1. Input the time constant and DC gain calculated using the equations and manufacturer specified values. Compare the simulated transient and steady-state response to the experimental response. Estimate the experimental time constant τ and the DC gain K.

2. Sketch or take screenshots of the step responses for three different input voltages (see the appropriate scope) noting differences in the step response versus free response and at different input voltages.

3. Compare your experimental estimates of the time constant and DC gain with the calculated values. Discuss any differences and how these differences compare with your expectations.

5 Deliverables

- Turn in all pre-lab and lab assignment calculations and answers to questions. (Make sure that the calculated and experimental values of your constants are clearly noted.)

Quanser Qube-SERVO-2 manufacturer specs:

$R = 8.4\ \Omega$, $L = 0.00116$ H, $J_m = 4.65\text{x}10^{-6}$ kg-m^2, $K_t = 0.042$ N-m/A, and $K_e = 0.042$ V/(rad/s)

To calculate the load of the disc, J_l, assume the disc has a mass of 0.053 kg and radius of 0.0248 m.