ME352 QUBE Motor Lab 1:

DC motor modeling
first principles and model identification

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(last updated: 8/24/23)

1 Overview

In this course, we will perform a series of labs that address the control of a DC motor. In the first lab, we identify input-output models for this system from (1) first principles, and (2) using identification on experimental step response data. The input to the motor is voltage, and the output is angular speed. Once these relationships are known, we can use them for open-and closed-loop control of the DC motor system.

2 Goals

Our hands-on goals for today are:

- Familiarize yourself with the lab equipment (MATLAB/Simulink software and Quanser QUBE- Servo 2).
- Derive input-output model from first principles.
- Identify input-output model using experimental step response data.
- Understand the concepts of time constant and DC gain.
- Compare the first-order modeled response to the experimental response and observe the response behavior over a range of input voltages

3 Modeling of the Motor

The motor can be modeled as the sum of an electrical and a mechanical component, see Fig. 1. For the electrical model, we have according to Kirchoff's voltage law:

$$L\frac{di}{dt} + R\,i + e = v \tag{1}$$

here e is the voltage generated as a result of the rotation of the motor (electromotive force, or back EMF):

$$e = K_b\,\omega \tag{2}$$

This is the "law of the generator" which relates the angular speed ω to the voltage induced by the motor e, where K_b is a constant, the back EMF constant or motor electrical constant.

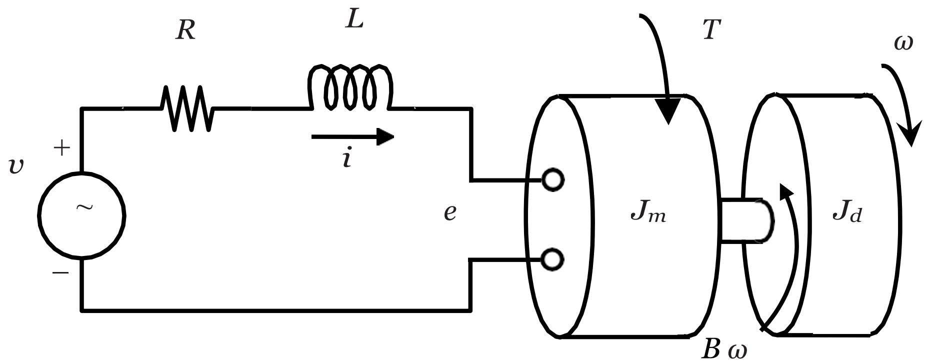


Figure 1: Schematic for the electrical and mechanical model of an electrical motor with disc (load) connected to the motor hub

The mechanical equations follow from Newton's law for the rotational moment of inertia assuming a rigid connection between motor hub and disc (load):

$$J\dot{\omega} + B\omega = T \tag{3}$$

Where J is the total motor inertia (includes both J_m , the motor and motor hub inertia, and J_d , the disc inertia). B is the viscous damping constant, and the driving torque T is given by:

$$T = K_m\, i \tag{4}$$

This is the "law of the motor" and relates the motor torque T to the current i through the motor, where K_m is a constant.

For simplicity we introduce two assumptions that result in a first-order model:

1. Neglect the influence of inductance, L
2. Neglect linear viscous friction, B

Note that internal friction of the motor, stick-slip friction due to the brushes and other couplings present in the motor test stand are not modeled.

The combined set of equations simplify then to

$$R\,i + K_b\,\omega = v \tag{5}$$

$$J\dot{\omega} = K_m\, i \tag{6}$$

where $J = J_m + J_d$.

Pre-Lab Assignment

1. Write the system as a second-order model with non-negligible inductance and linear viscous damping in **transfer function** form $Y(s) = G(s)\,U(s)$ where Y is the motor angular velocity Ω and U is the applied voltage V.

2. Write the second-order model in **state-space** form where $x = \begin{bmatrix} \omega \\ i \end{bmatrix}$:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du \tag{7}$$

3. Now assume the inductance and linear viscous friction is negligible (ie., L=0, B=0) and write the system as a first-order model in transfer function form.

4. Write the first-order model in **state-space** form where $x = \omega$, $u = v$, and $y = \omega$.

5. The following numerical values for the motor were found in the Quanser User Manual: R = 8.4 Ω, L = 0.00116 H, J_m = 4.65x10^{-6} kg-m^2, K_m = 0.042 N-m/A, and K_b = 0.042 V/(rad/s). To calculate the disc inertia, J_d, assume the disc has a mass of 0.053 kg and radius of 0.0248 m. Use both J_m and J_d to find the total motor inertia, J. Obtain the time constant (τ) and the DC gain (K) in terms of the system parameters. What are the numerical values of τ and K? What do these values '*tell*' you about the system?

6. Find the numerical values of the first-order and second-order poles of the system. What can you say about the second-order poles relative to the first-order pole? What can you conclude about the system comparing the first-order and second-order poles?

4 Laboratory Setup

Motor test stand (QUBE-Servo 2)

The Quanser QUBE-Servo 2, pictured in Figure 2, is a compact rotary servo system that can be used to perform a variety of classic servo control and inverted pendulum based experiments. It is controlled by a computer via USB connection. We quote from the Quanser User Manual: "The system is driven using a direct-drive 18V brushed DC motor. The motor is powered by a built-in PWM (pulse width modulation) amplifier with integrated current sense. Single-ended rotary encoders are used to measure the angular position of the DC motor and pendulum."

Figure 2: Quanser QUBE-Servo 2

Each station consists of a QUBE, a flywheel, USB A to USB B cable, and power adapter. Before starting anything, make sure that all the cables are plugged in properly. Attach the flywheel disc to the top of the QUBE. Initially, the light on the QUBE should be red before any program starts running.

Open Matlab. From the Matlab directory, open the lab1_open_loop_voltage_params.m and open_looper_voltage.sltx files. The diagram in Fig. 3 should appear on your screen.

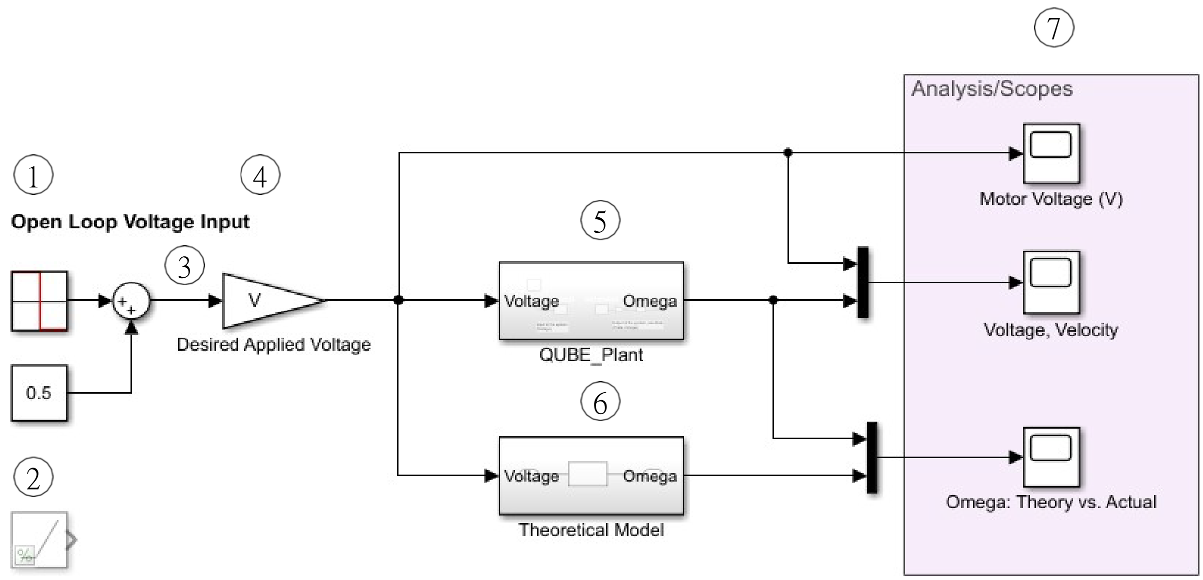


Figure 3: Simulink Block Diagram of DC motor setup

Each block serves the following purpose:

1 is Signal Generator. In this case, a periodic square wave taking the values 0.5 and -0.5.

2 is Offset (constant). It is used to shift the square wave upwards by 0.5.

3 is Sum. Its inputs are added together to create an output.

4 is Gain. Its output is its input multiplied by a constant.

5 is the Plant (DC motor system). Inside this block are Quanser provided blocks to actuate the motor and read its sensor (see below).

6 is the (theoretical) Plant Model. It is a model (transfer function or state-space representation) of the DC motor with the parameters you calculated in the pre-lab.

7 are Scopes. They display relevant data.

The contents of the Plant block are shown in Fig. 4.

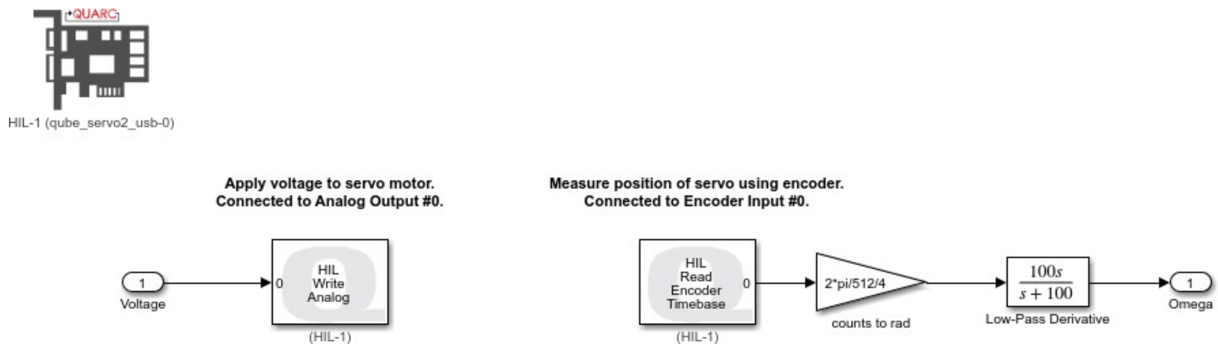


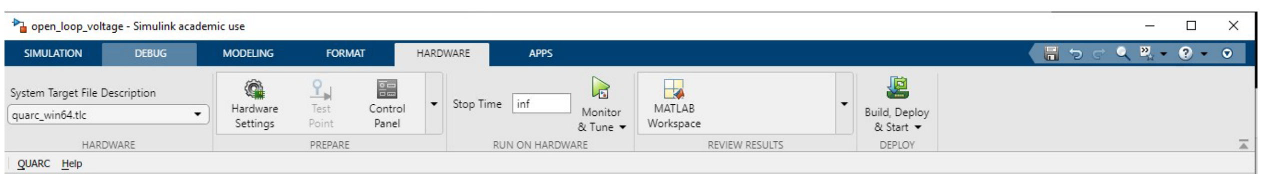
Figure 4: Plant Simulink block

Before you proceed, make sure you understand the purpose of the block diagram.

Running Simulink and the QUBE motor

Run the lab1_param.m file to load the motor model parameters. Check the Command Window and Workspace to ensure that the parameters are loaded. To run programs through the QUBE and related software, parameters are assigned values and compiled beforehand. For this lab, you will change the parameter DC Gain K and time constant τ values.

The following menu bar lets you run the motor:



1. Under the "Hardware" tab, press the "Build, Deploy & Start" button, then,
2. Press the "Monitor & Tune" button.

Verify that the QUBE light is green. Press start to begin the experiment. The motor should start rotating and the scopes should display a total of 3 graphs. Run the QUBE motor with the initial 6V square wave input. Stop the "Monitor and Tune" and change the desired applied voltage in the Matlab parameters and run the motor at twice the angular speed.

Lab Assignment

1. Input the time constant and DC gain using the equations and manufacturer specified values in the pre-lab. Compare the simulated transient and steady-state response to the experimental response. Estimate the experimental time constant τ and the DC gain K.

2. Sketch or take screenshots of the step responses for three different input voltages (see the appropriate scope) noting differences in the step response versus free response and how the motor behaves for different input voltages.

3. Compare your experimental estimates of the time constant and DC gain with the calculated values. Discuss any differences and how these differences compare with your expectations.

5 Deliverables

Turn in all pre-lab and lab assignment calculations and answers to questions. Clearly show your calculated and experimental values of the constants τ and K.