Final Project for MATH 2103: Calculus III

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1 Introduction

For my project, I designed an animation in MATLAB of a curve, with each point's unit tangent and normal vectors being plotted as a function of time.

First, I had to figure out what curve I wanted to use, and then decide what bounds and step size to work with. I used $\mathbf{r}(t) = \langle \cos(t), \sin(t), t \rangle$ as my curve, because it would be easy see the entire curve when graphed, and the circular nature of it made it a great choice. I defined t from $0 \leq t \leq 6\pi$, so the curve would revolve around the origin three times. Finally, I chose a step size of 0.1, because I wanted a smooth curve, and eventually, the unit tangent and normal vectors to plot close together.

Next, I defined variables x, y, and z to be $\cos(t)$, $\sin(t)$, and t, respectively. Then I was able to plot variable rt with the *plot3* function. After that, I manipulated some graph properties to set labels, title, etc.

2 Calculating Vectors

For each point in t, I wanted the **unit tangent vector** ($\hat{\mathbf{T}}$), and the **unit normal vector** ($\hat{\mathbf{N}}$). This means I wanted to find the *vector functions* in regard to $\mathbf{r}(t)$: ($\hat{\mathbf{T}}(t)$ and $\hat{\mathbf{N}}(t)$).

2.1 Finding $\hat{\mathbf{T}}(t)$

The formula for the unit tangent vector looks like this:

$$\hat{\mathbf{T}}(t) = \frac{\mathbf{r}\,'(t)}{\|\mathbf{r}\,'(t)\|}$$

$\mathbf{r}\,'(t)$ can be found by taking the derivative of each of the components of $\mathbf{r}(t)$. Knowing that $x(t) = \cos(t)$, $y(t) = \sin(t)$, and $z(t) = t$, I could calculate each derivative: $x\,'(t) = -\sin(t)$,$y\,'(t) = \cos(t)$, and $z\,'(t) = 1$. Therefore:

$$\mathbf{r}\,'(t) = \langle -\sin(t), \cos(t), 1 \rangle$$

$\|\mathbf{r}\,'(t)\|$ can be found by finding the vector length of $\mathbf{r}\,'(t)$, which we can do using the formula:

$$\|\mathbf{r}\,'(t)\| = \sqrt{(x\,'(t))^2 + (y\,'(t))^2 + (z\,'(t))^2}$$

So, using $\mathbf{r}\,'(t)$ and our equation, we know that

$$\|\mathbf{r}\,'(t)\| = \sqrt{(-\sin(t))^2 + (\cos(t))^2 + (1)^2} = \sqrt{\sin^2(t) + \cos^2(t) + 1} = \sqrt{1+1} = \sqrt{2}$$

So, plugging those two values back into our original equation yields our vector function:

$$\hat{\mathbf{T}}(t) = \frac{\mathbf{r}\,'(t)}{\|\mathbf{r}\,'(t)\|} = \frac{\langle -\sin(t), \cos(t), 1\rangle}{\sqrt{2}} = \langle \frac{-\sqrt{2}\sin(t)}{2}, \frac{\sqrt{2}\cos(t)}{2}, \frac{\sqrt{2}}{2}\rangle$$

2.2 Finding $\hat{\mathbf{N}}(t)$

The formula for the unit tangent vector looks like this:

$$\hat{\mathbf{N}}(t) = \frac{\hat{\mathbf{T}}\,'(t)}{\|\hat{\mathbf{T}}\,'(t)\|}$$

$\hat{\mathbf{T}}\,'(t)$ can be found by taking the derivative of each of the components of $\hat{\mathbf{T}}(t)$

$$\hat{\mathbf{T}}\,'(t) = \langle \frac{-\sqrt{2}\cos(t)}{2}, \frac{-\sqrt{2}\sin(t)}{2}, 0\rangle$$

$\|\hat{\mathbf{T}}\,'(\mathbf{t})\|$ can be found by finding the vector length of $\hat{\mathbf{T}}\,'(t)$, which we can do using the formula:

$$\|\hat{\mathbf{T}}\,'(t)\| = \sqrt{(\frac{-\sqrt{2}\cos(t)}{2})^2 + (\frac{-\sqrt{2}\sin(t)}{2})^2 + (0)^2} = \sqrt{\frac{1}{2}\cos^2(t) + \frac{1}{2}\sin^2(t)} = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$$

So, plugging those two values back into our original equation yields our vector function:

$$\hat{\mathbf{N}}(t) = \frac{\hat{\mathbf{T}}\,'(t)}{\|\hat{\mathbf{T}}\,'(t)\|} = \frac{\langle \frac{-\sqrt{2}\cos(t)}{2}, \frac{-\sqrt{2}\sin(t)}{2}, 0\rangle}{\frac{\sqrt{2}}{2}} = \langle -\cos(t), -\sin(t), 0\rangle$$

Now we have both $\hat{\mathbf{T}}(t)$ and $\hat{\mathbf{N}}(t)$, which are the unit tangent vectors and the unit normal vectors as vector functions in terms of t. With these variables, I plotted the two vectors at each value of t using a for loop and the *quiver3* function, along with the original curve, then animated each vector using the *getframe* function.