Ginorio_Assignement_2

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Problem Set 1

1. Show that $A^T A \neq AA^T$ in general. (Proof and demonstration.)
2. For a special type of square matrix A, we get $A^T A = AA^T$. Under what conditions could this be true? (Hint: The Identity matrix I is an example of such a matrix).

Part 1

Proof of $AA^T \neq A^T A$ in general.

Proof

Now to prove $A^T A \neq AA^T$ in general for a square matrix.

Given a matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

With a transpose as

$$A^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

Matrix multiply AA^T

$$AA^T = \begin{bmatrix} a^2 + b^2 & ac + bd \\ ac + bd & c^2 + d^2 \end{bmatrix}$$

Matrix multiply $A^T A$

$$A^T A = \begin{bmatrix} a^2 + c^2 & ab + cd \\ ab + cd & b^2 + d^2 \end{bmatrix}$$

Thus

$$\begin{bmatrix} a^2 + b^2 & ac + bd \\ ac + bd & c^2 + d^2 \end{bmatrix} \neq \begin{bmatrix} a^2 + c^2 & ab + cd \\ ab + cd & b^2 + d^2 \end{bmatrix}$$

Demonstration

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

With a transpose as

$$A^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Matrix multiply AA^T

$$AA^T = \begin{bmatrix} 10 & 14 \\ 14 & 20 \end{bmatrix}$$

Matrix multiply $A^T A$

$$A^T A = \begin{bmatrix} 5 & 11 \\ 11 & 25 \end{bmatrix}$$

Thus

$$\begin{bmatrix} 10 & 14 \\ 14 & 20 \end{bmatrix} \neq \begin{bmatrix} 5 & 11 \\ 11 & 25 \end{bmatrix}$$

Part 2

Under what conditions is $AA^T = A^T A$?

Answer: When matrix A is **symmetric**. Thus $A = A^T$. And then, with substitution $AA = AA$.

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

With a transpose as

$$A^T = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

Matrix multiply AA^T

$$AA^T = \begin{bmatrix} 5 & 10 \\ 10 & 20 \end{bmatrix}$$

Matrix multiply $A^T A$

$$A^T A = \begin{bmatrix} 5 & 10 \\ 10 & 20 \end{bmatrix}$$

Thus

$$\begin{bmatrix} 5 & 10 \\ 10 & 20 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 10 & 20 \end{bmatrix}$$

Problem Set 2

Write an R function to factorize a square matrix A into LU or LDU, whichever you prefer. Please submit your response in an R Markdown document.

You don't have to worry about permuting rows of A and you can assume that A is less than 5×5, if you need to hard-code any variables in your code.

Solution

LU Decomposition Gaussian Elimination L = Lower Trialgular Matrix U = Upper Triangular Matrix

Conditions

- A must be able to be reduced to row-echelon form
- L and U are not unique
- Build L using the opposites multiplies

```
library(matrixcalc)
```

```
## Warning: package 'matrixcalc' was built under R version 4.0.5
```

```
A <- matrix( c ( 1, 2, 2, 1 ), nrow=2, byrow=TRUE)
luA <- lu.decomposition( A )
L <- luA$L
U <- luA$U

print( L )
```

```
##      [,1] [,2]
## [1,]    1    0
## [2,]    2    1
```

```
print( U )
```

```
##      [,1] [,2]
## [1,]    1    2
## [2,]    0   -3
```

```
print( L %*% U )
```

```
##      [,1] [,2]
## [1,]    1    2
## [2,]    2    1
```

```
print( A )
```

```
##      [,1] [,2]
## [1,]    1    2
## [2,]    2    1
```

```r
B <- matrix( c( 2, -1, -2, -4, 6, 3, -4, -2, 8 ), nrow=3, byrow=TRUE )
luB <- lu.decomposition( B )
L <- luB$L
U <- luB$U
print( L )
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    0
## [2,]   -2    1    0
## [3,]   -2   -1    1
```

```r
print( U )
```

```
##      [,1] [,2] [,3]
## [1,]    2   -1   -2
## [2,]    0    4   -1
## [3,]    0    0    3
```

```r
print( L %*% U )
```

```
##      [,1] [,2] [,3]
## [1,]    2   -1   -2
## [2,]   -4    6    3
## [3,]   -4   -2    8
```

```r
print( B )
```

```
##      [,1] [,2] [,3]
## [1,]    2   -1   -2
## [2,]   -4    6    3
## [3,]   -4   -2    8
```