HW 8

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Problem 11 page 303 A company buys 100 lightbulbs, each of which has an exponential lifetime of 1000 hours. What is the expected time for the first of these bulbs to burn out? (See Exercise 10.)

Ex - 10 Let X1, X2, . . . , Xn be n independent random variables each of which has an exponential density with mean µ. Let M be the minimum value of the Xj . Show that the density for M is exponential with mean µ/n. Hint: Use cumulative distribution functions.

Answer

As the hint suggests, using Exercise 10, the 100 lightbulbs are the $X_1, X_2, ..., X_n$ meaning $n = 100$ independent random variables. Each has an exponential density with mean μ. M will be the minimum value of X_j. The density for M is exponential with mean μ/n.

$n = 100$

$\mu = 1000$

As $E(M) = \mu/n$, then 1000/100

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n <- 100
mu <- 1000
M <- mu / n
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Answer: $E(M) = 10$ hours is the expected time for the first of these bulbs to burn out

Problem 14 page 303

Assume that X_1 and X_2 are independent random variables, each having an exponential density with parameter λ. Show that $Z = X_1 - X_2$ has density $f_Z(z) = (1/2)\lambda e^{-\lambda|z|}$.

Answer

Sum of Two Independent Exponential Random Variables

$Z = X_1 - X_2$ change for $Z = X - Y$

$$f_X(x) = f_Y(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

$$f_Z(z) = \int_0^z \lambda e^{-\lambda x} * e^{-\lambda x - z} dx$$

Thus

$$f_Z(z) = f_Z(-z)$$
$$= \frac{\lambda}{2} e^{-\lambda z},\ for\ z \leq 0$$

Symmetric distribution

$$f_Z(1) = f_Z(-1)$$