Integrate-and-Fire models of neuronal excitability

Computational Neuroscience

Plan for the day(s)

- Simplification of AP generation: the Integrate-&-Fire model
- The Frequency-Current formula for the Integrate-&-Fire
- The Integrate-&-Fire with spike-frequency adaptation
- Models of Synaptic transmission and their simplification

So far: adding biological **realism** and grounding mathematical descriptions into biophysics

Now: **simplify** the detailed models of neurons into reduced descriptions

Stripping down a complex model to its bare essential may provide an **explanatory model** (easier to *understand*)

Ultimate goal of studying emergence of non-intuitive phenomena in **large networks of neurons** where simpler neurons are *easier/faster* to simulate

where to stop???!

The data available.
it might (not) be possible to **constraint all parameters** of a complex model

The desired type of analysis.
one might want to investigate and analyze **collective properties** and not single-neuron phenomena

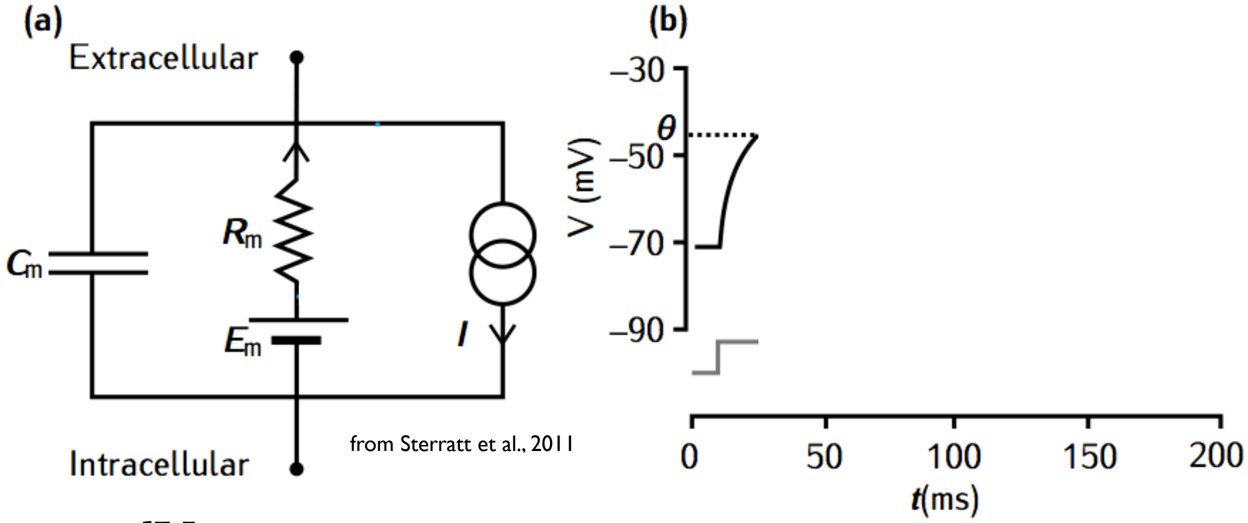
Computational resources.
Simpler models are **faster to simulate** than complex ones.

The level of explanation.
Correspondence between model parameters and physical elements (e.g. models of the appropriate types of ion channels are needed in order to predict what happens to a neuron when a particular neuromodulator is released or a particular type of channel is blocked).

from Sterratt et al., 2011

Integrate-and-Fire models

G = 1 / R



$$C\,\frac{dV}{dt} = G\,(E - V) + I_{ext} \qquad if\;\; V < \theta$$

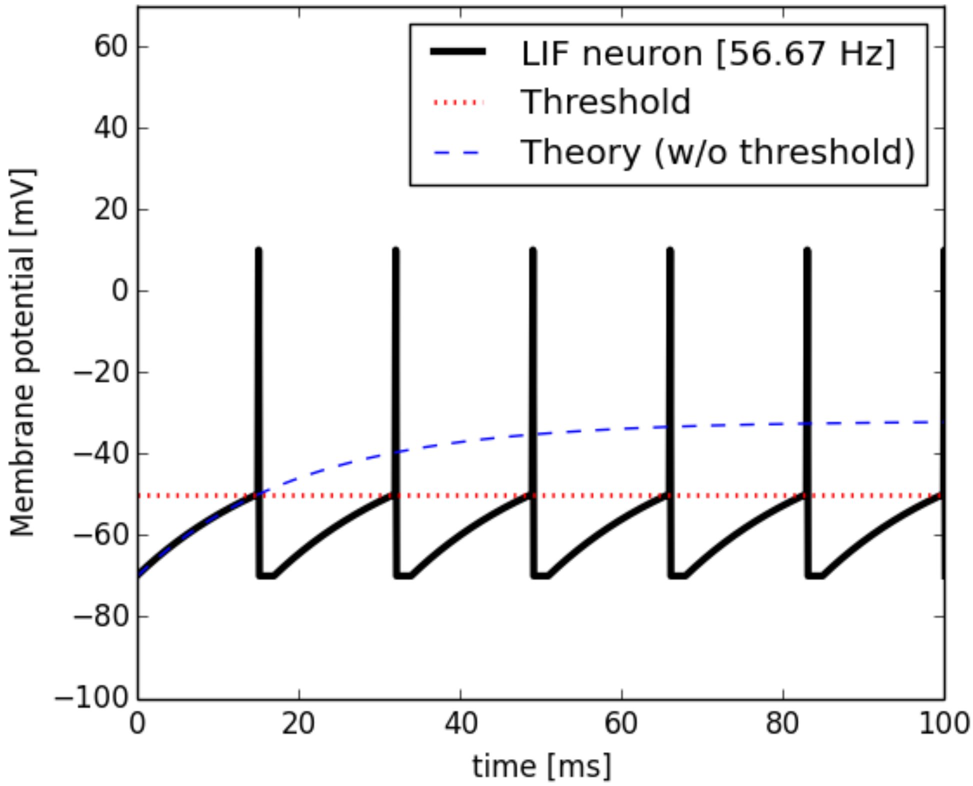
$$t^*\,spike,\quad V(t^*) \leftarrow H \qquad V(t)\;fixed\;at\;H,\;t \in [t^*; t^* + \tau_{arp})$$

Integrate-and-Fire models

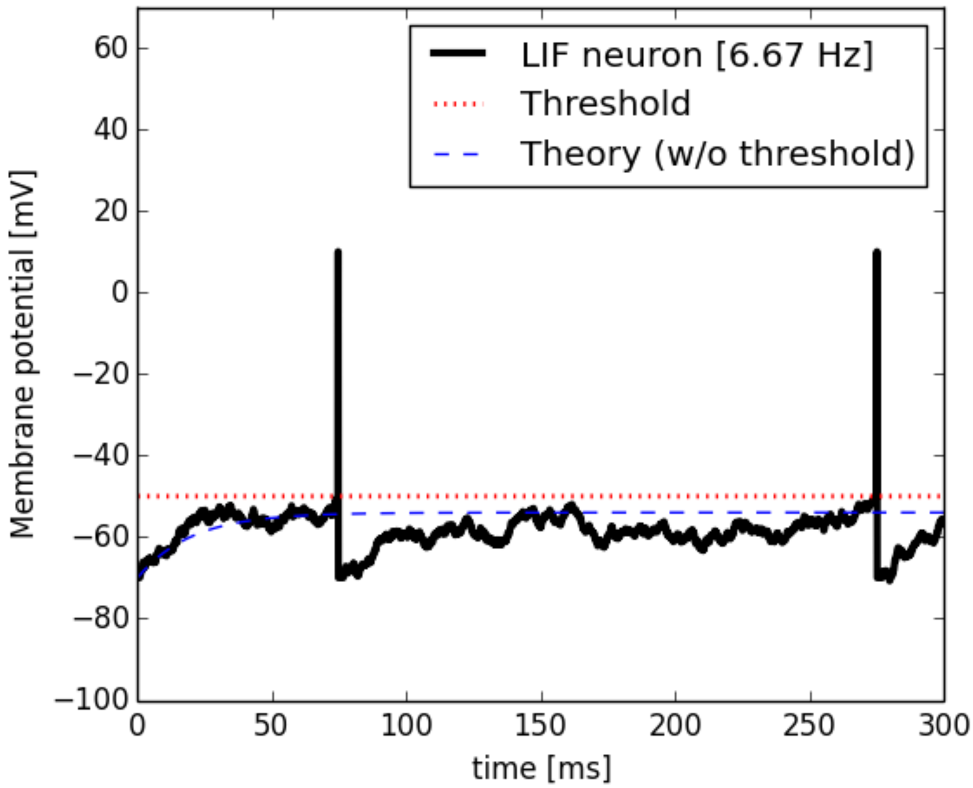
- there is a (fixed) explicit threshold (θ)
- a spike *is said* to occur when there is a threshold crossing
- after a spike, the membrane potential is clamped to H a (hyperpolarized) level, for a time interval τ_{arp}

```
if V >= theta
 V  = H
 to = t
elseif t < (to+Tarp)
 V = H
else
 V = V + dt*G/C * (E - V) + dt/C *I
end
```

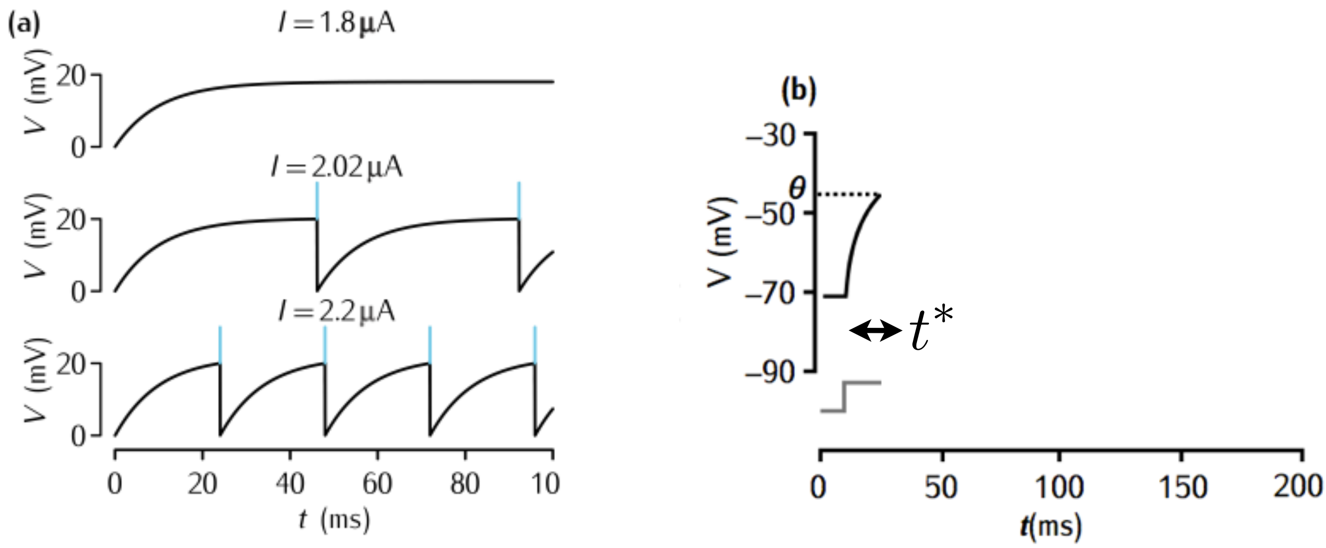
Leaky-Integrate-and-Fire model: V(t)



Leaky-Integrate-and-Fire model: V(t)



Frequency vs (DC) current curve for a I&F model



from Sterratt et al., 2011

Frequency vs (DC) current curve for a I&F model

G = 1 / R

$$C\,\frac{dV}{dt} = G\,(E - V) + I_{ext}$$

$$V(t) = E + I_0/G\left(1 - e^{-G/C\;t}\right)$$

$$V(t^*) = V_{th}$$

$$t^* = \frac{C}{G}\,log\left(\frac{I_0}{G(E - V_{th}) + I_0}\right)$$

$$f(I) = \frac{1}{\tau_{arp} + t^*}$$

Frequency vs (DC) current curve for a I&F model

- there is a minimal current (*rheobase*) for spikes to be fired (i.e. for the threshold to be crossed)

$$C \frac{dV}{dt} = G (E - V) + I_{ext}$$

...at the steady-state $$V = E + I_{ext}/G$$

$$I_{rhe} = G (V_{th} - E)$$

(or if you like math, then look at where ln(x) is defined...)

Frequency vs (DC) current curve for a I&F model

- if we neglect the refractoriness, for very large currents (i.e. far away the rheobase, where you have a strong non-linearity in the curve)...

$$f(I) \approx \frac{1}{t^*} \qquad t^* = -\frac{C}{G} \, log \left(1 + \frac{G(E - V_{th})}{I_0}\right)$$

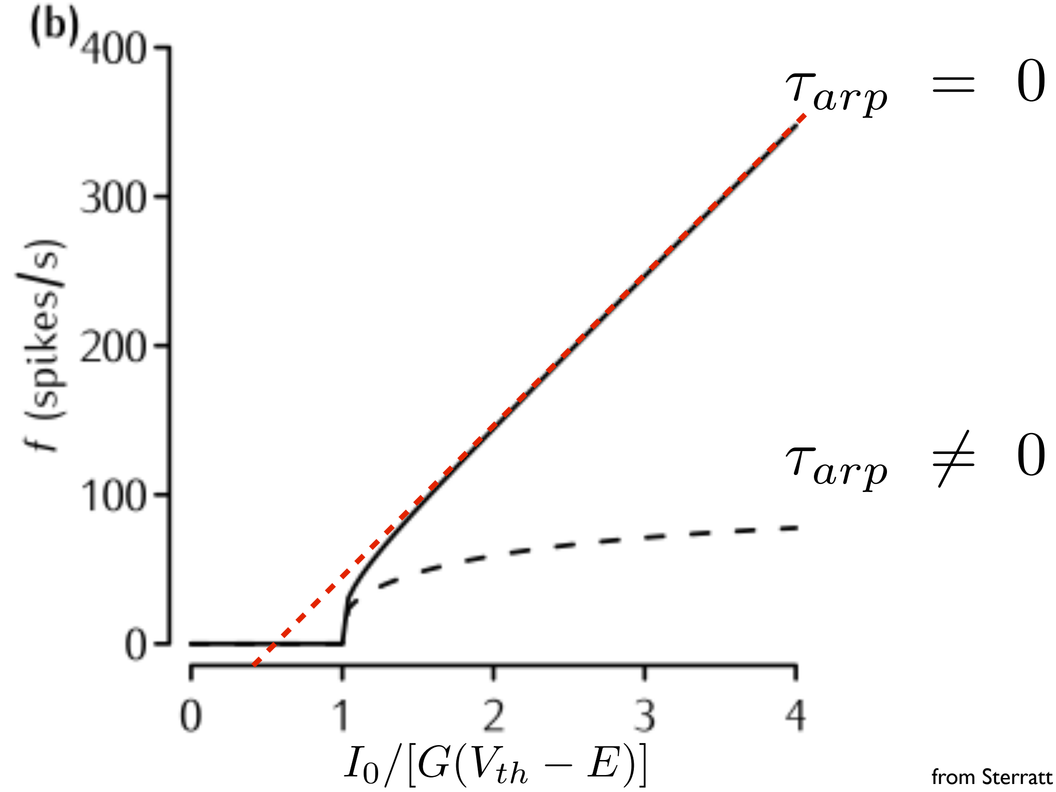
$$ln(1 + x) \approx x$$

$$f(I) \approx \frac{I_0}{C(V_{th} - E)}$$

$$t^* \approx -C \frac{(E - V_{th})}{I_0}$$

Then, the f-I curve is threshold-linear.

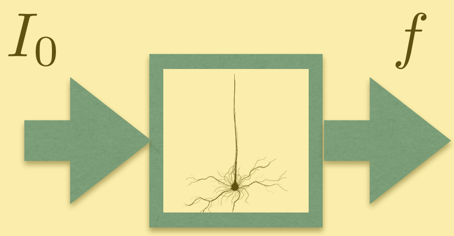
Frequency vs (DC) current curve for a I&F model



from Sterratt et al., 2011

Very rough (functional) approximation

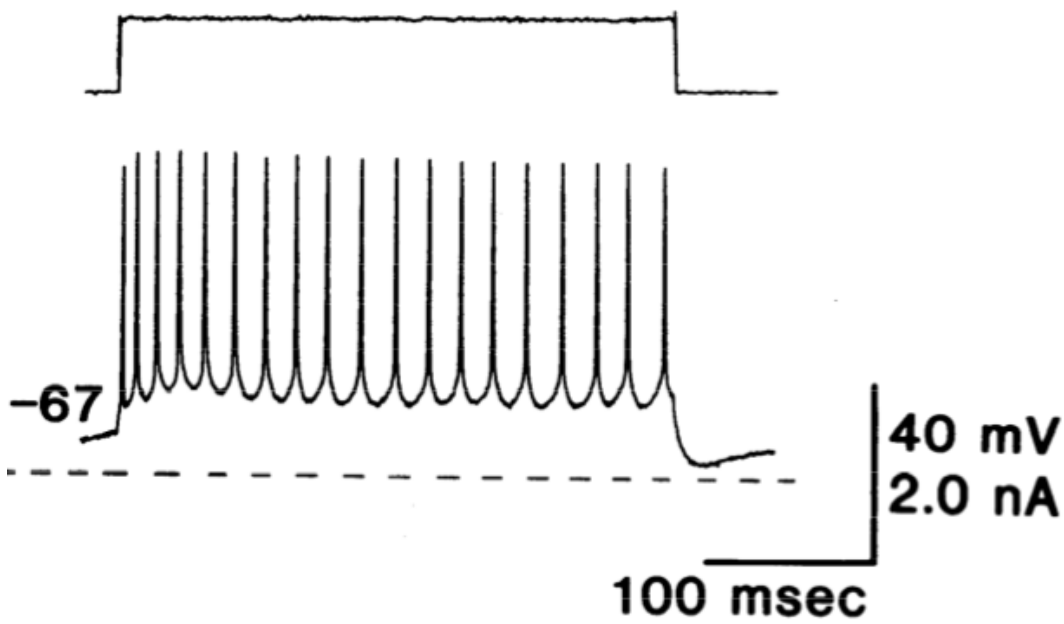
$$t^* = -\frac{C}{G} \, log \left(1 + \frac{G(E - V_{th})}{I_0} \right)$$



$$f(I) \approx \frac{I_0}{C(V_{th} - E)}$$

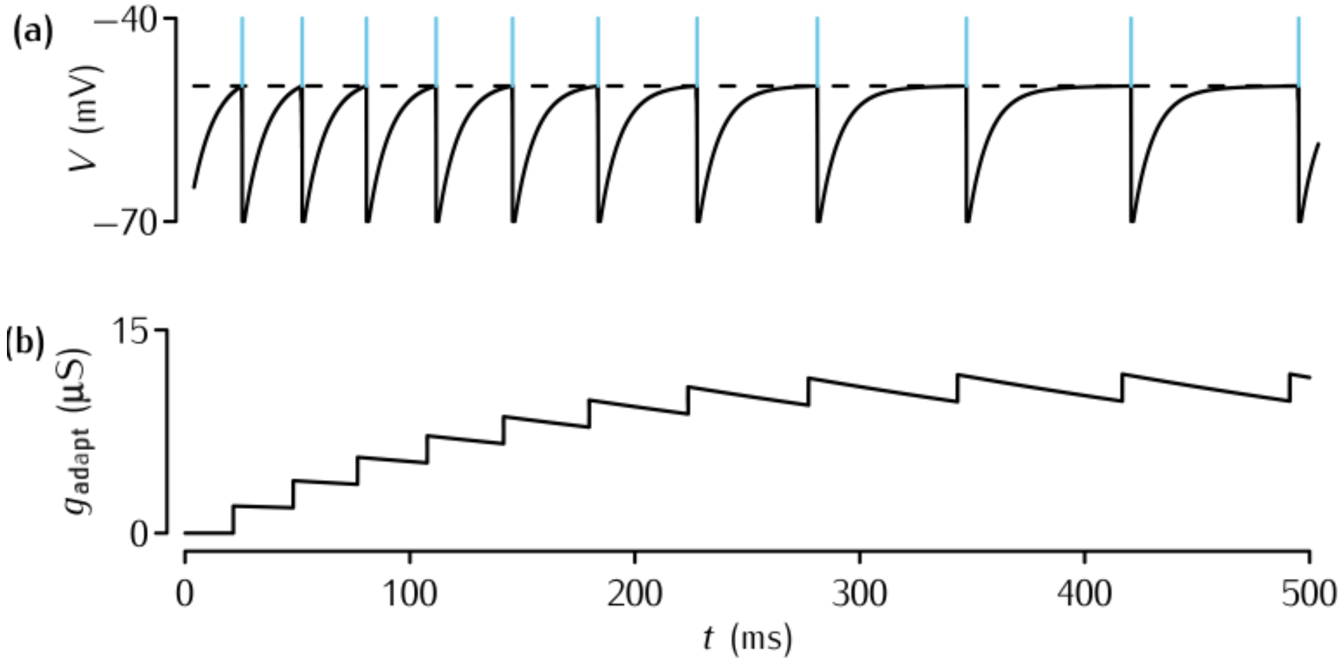
*A neuron is a device that converts input current (**amplitudes**) into a train of action potential with a certain **frequency**.*

But… cortical pyramidal neurons do display spike-frequency adaptation!



from McCormick et al., 1985

Integrate-&-Fire with extra "adaptation mechanism" (i.e. spike-frequency adaptation *current*)



from Sterratt et al., 2011

Integrate-&-Fire with extra "adaptation mechanism" (i.e. spike-frequency adaptation *current*)

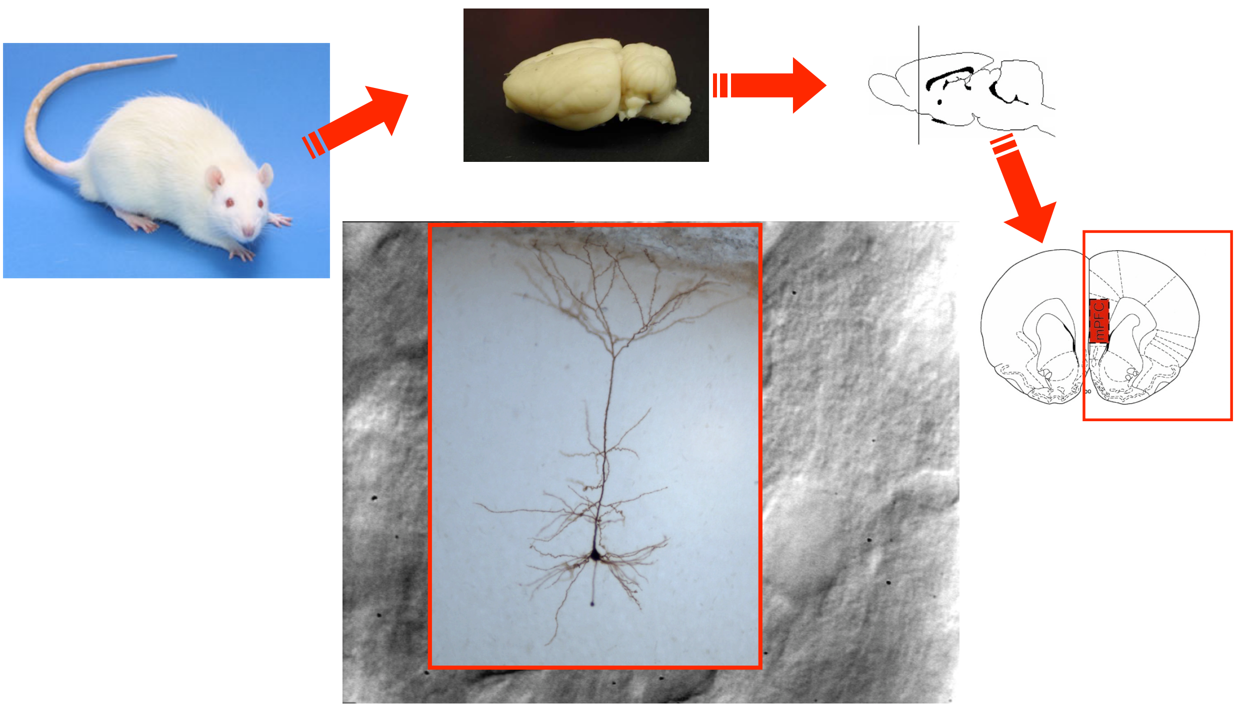
$$I_{adapt} \;=\; \bar{g}_{adapt}\; x\; (E - V)$$

$$\frac{dx}{dt} = -\frac{x}{\tau_{adapt}}$$ below threshold, $if \;\; V < \theta$

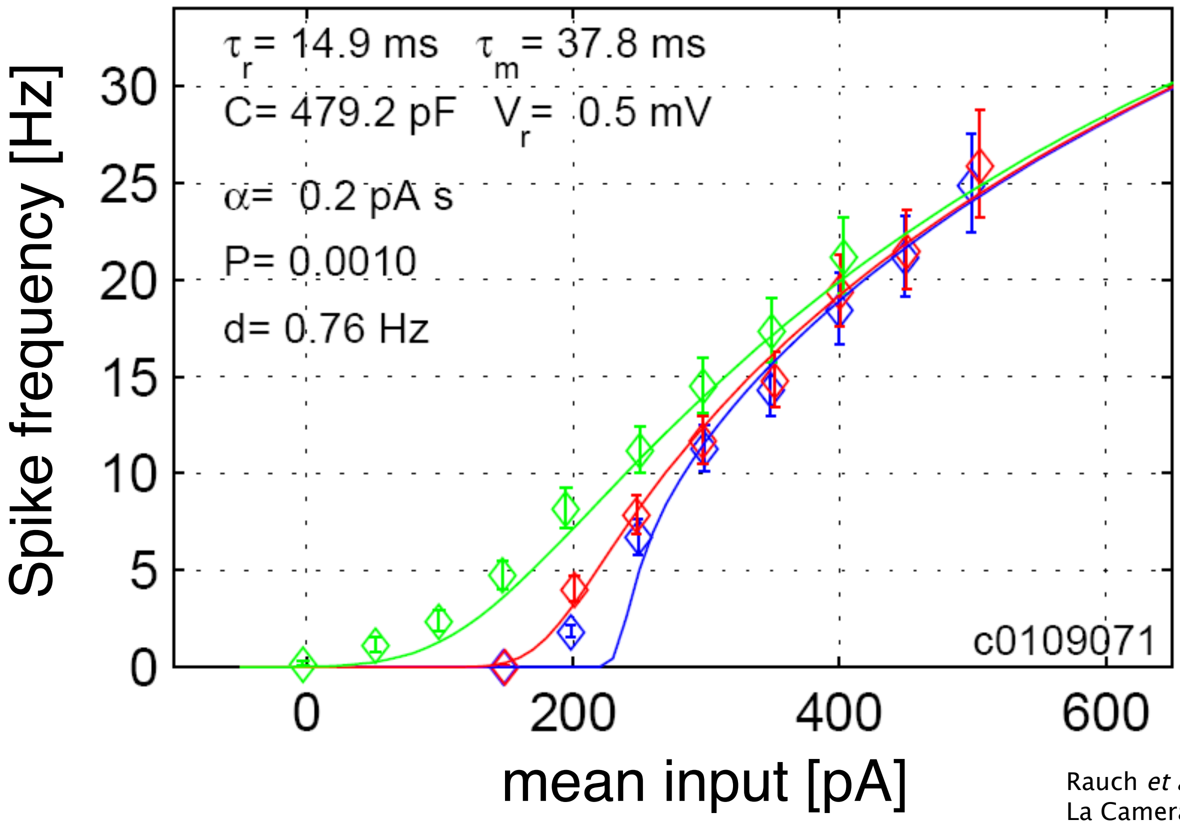
$$x \;\rightarrow x \;+\; \Delta_{adapt}$$ during a "spike"

$$\boxed{I_{adapt} \;\approx\; -\bar{g}_{adapt}\; x \quad \text{approx. equivalent}}$$

Frequency vs (DC) current curve for a real pyramidal neuron



Frequency vs current curve
for a pyramidal neuron: I&F are accurate enough!!



Rauch *et al.*, 2003
La Camera *et al.*, 2006