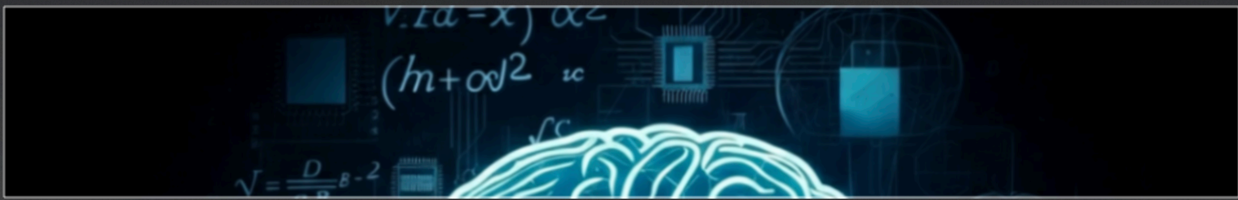
MODELLING NEURAL SYSTEMS



COMPUTATIONAL MODELLING OF NEURONS AND MICROCIRCUITS

Prof. Ing. Michele GIU**G**LIANO, PhD

Firing Rate Models

Plan for the day

- Simplified models of networks: mean-field hypothesis
- Recurrent excitation as a mechanism to amplify
- Recurrent excitation as a mechanism to change equilibria
- Feedback inhibition as a mechanism for rate oscillations
- Intrinsic cell properties: additional contributing mechanisms for (sparse) rate oscillations

Chemical synaptic transmission

[unirectional, fast, with "sign"]
model of a ionotropic receptor

$$C \underset{\beta}{\overset{\alpha[\mathrm{T}]}{\rightleftharpoons}} O \qquad g_{syn}(t) = \bar{g}_{syn} O(t)$$

$$\frac{dO(t)}{dt} = -\beta\, O + \alpha\, [T]\, C$$

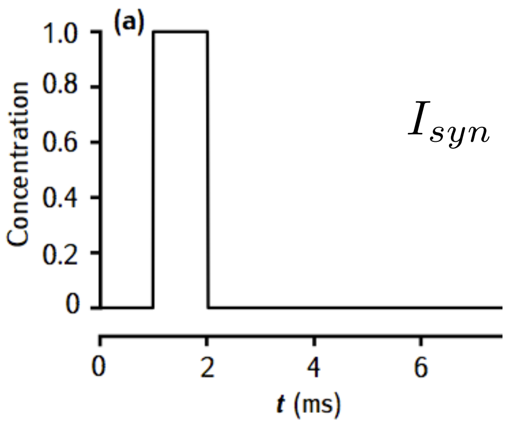
$$\frac{dC(t)}{dt} = +\beta\, O - \alpha\, [T]\, C$$

$$O(t) + C(t) = 1$$

Destexhe *et al.* [1994]

$$\frac{dO}{dt} = \frac{O_\infty - O}{\tau_O} \qquad O_\infty = \frac{\alpha\, T(t)}{\alpha\, T(t) + \beta}$$

$$\tau_O = \frac{1}{\alpha\, T(t) + \beta}$$



$$I_{syn} = \bar{g}_{syn}\, O(t)\, (E_{syn} - V_m)$$

from Sterratt et al., 2011

Chemical synaptic transmission

simplified description of [a population of] ionotropic receptors

$$C \underset{\beta}{\overset{\alpha[T]}{\rightleftharpoons}} O$$

[dynamical] steady-state??

$$\frac{dO(t)}{dt} = -\beta\, O + \alpha\, [T]\, C$$

$$\frac{dO}{dt} = -\beta\, O + \alpha\, T_{max}(1-O) \sum_k \delta(t-t_k)$$

$$\frac{dO}{dt} \approx -\beta\, O + \alpha\, T_{max} \sum_k \delta(t-t_k)$$

Even simpler models of synaptic transmission:

“mean-field” descriptions (*firing* rate models)

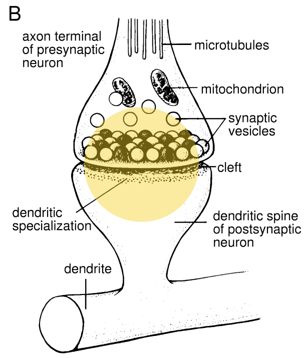
current-driven model synapse

$$I_{syn} \approx \bar{I}\; O(t)$$

$$\boxed{\bar{I} = \bar{g}_{syn}\;(E_{syn} - <V_m>)}$$

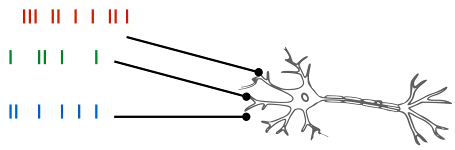
$$\frac{dI_{syn}}{dt} \approx -\beta I_{syn} + W \sum_k \delta\left(t - t_k\right)$$

$$W = \bar{g}\alpha T_{max}$$



TOTAL [current-driven] synaptic input

$$C\frac{dV}{dt} = ... + I_{syn_1} + I_{syn_2} + I_{syn_3}$$

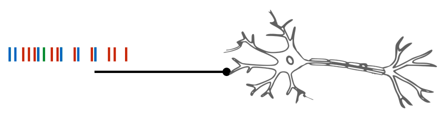


$$\frac{dI_{syn_1}}{dt} \approx -\beta I_{syn_1} + W_1 \sum_{k_1} \delta(t - t_{k_1})$$

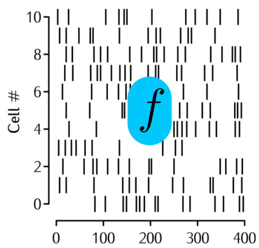
$$\frac{dI_{syn_2}}{dt} \approx -\beta I_{syn_2} + W_2 \sum_{k_2} \delta(t - t_{k_2})$$

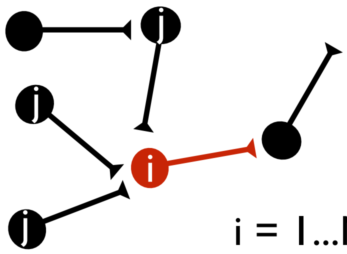
$$\frac{dI_{syn_3}}{dt} \approx -\beta I_{syn_3} + W_3 \sum_{k_3} \delta(t - t_{k_3})$$

$$\frac{d(\quad I_{syn} \quad)}{dt} \approx -\beta(\quad I_{syn} \quad) + \sum_{m=1}^{3} W_m \sum_{k_m} \delta(t - t_{k_m})$$



Large [feed-forward] network of neurons





i = 1...N

$$\frac{dI_{syn\ i}}{dt} \approx -\beta I_{syn\ i} + \sum_j C_{ij}\ W_{ij} \sum_{k_j} \delta\left(t - t_{k_j}\right)$$

- each neuron has identical [intrinsic] parameters...
- each neuron fires independently from each other...
- each one firing asynchronously, irregularly [Poisson], ~f

“Mean-field” approximation:

neurons are indistinguishable and
share the same ***average*** *synaptic input*

$$\frac{dI_{syn\ i}}{dt} \approx -\beta I_{syn\ i} + \sum_j C_{ij}\ W_{ij} \sum_{k_j} \delta\left(t - t_{k_j}\right)$$

$$\frac{d}{dt} < I_{syn} > \approx -\beta < I_{syn} > + \sum_j \sum_j \ldots >$$

$$< C_{ij} W_{ij} \sum_{k_j} \delta(t - t_{k_j}) > = ??$$

If feed-forward, easy!

If recurrent, … ??!?

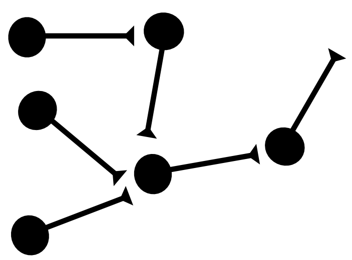
“Mean-field” approximation:

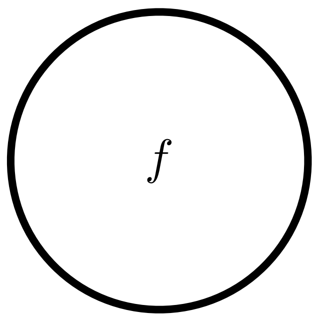
neurons are indistinguishable and
share the same average synaptic input

$$< C_{ij} >< W_{ij} >< \sum_{k_j} \delta(t - t_{k_j}) >= c\ w\ f$$

$$\frac{d}{dt} < I_{syn} > \approx -\beta < I_{syn} > +N\ c\ w\ f$$

- each neuron has identical [intrinsic] parameters...
- each neuron fires independently from each other...
- each one firing asynchronously, irregularly [Poisson], ~f



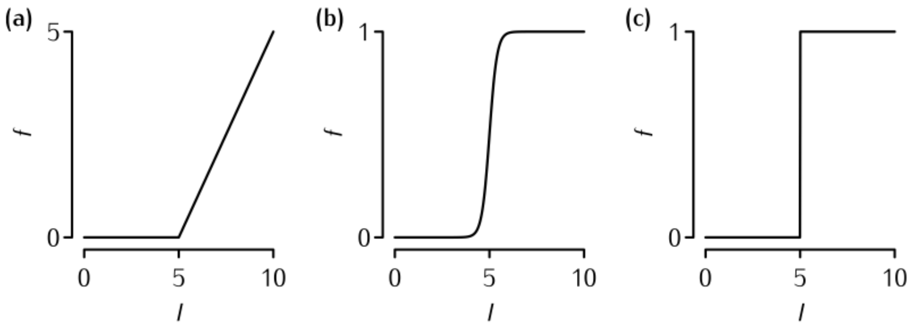


$$\frac{d}{dt} < I_{syn} > \approx -\beta < I_{syn} > +N\ c\ w\ f$$

$$f = F(I_{syn})$$

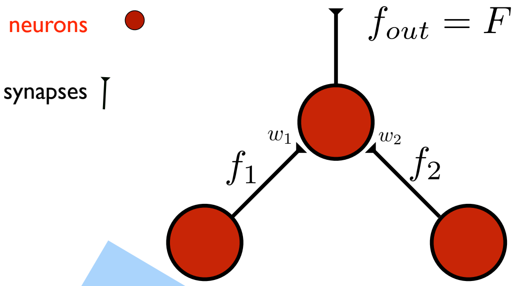
Rate models
[large populations]

$$f_i = F(I_{syn_i})$$

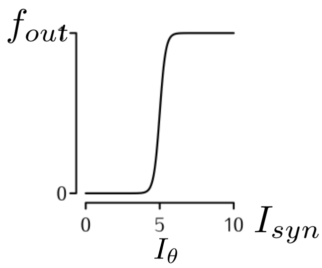


from Sterratt et al., 2011

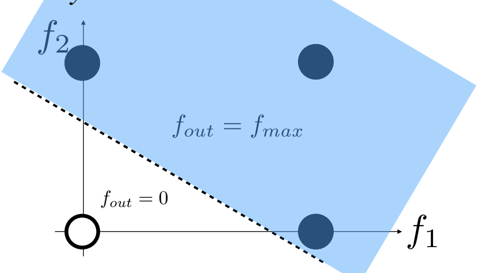
Rate models for the description of a **feedforward** cortical circuit



$$f_{out} = F(I_{syn})$$



$$I_{syn} = w_1 f_1 + w_2 f_2$$

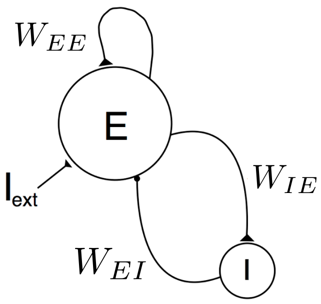


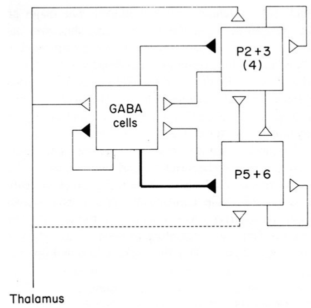
$$f_{out} = 0, \quad I_{syn} < I_\theta$$

$$f_{out} = f_{max}, \quad \boxed{I_{syn} > I_\theta}$$

Hertz, Krogh, Palmer, 1991, Addison-Wesley
Amit, 1989, Cambridge Univ. Press
Rolls & Treves, 1998, Oxford Univ. Press

Rate models for the description of a (cortical column) **recurrent** circuit?





$$f_E = F(< I_{syn\ E} >)$$

$$f_I = F(< I_{syn\ I} >)$$

$$\frac{d}{dt} < I_{syn\ E} > \approx -\beta_E < I_{syn\ E} > + N_E\ c_{EE}\ w_{EE} f_E - N_I\ c_{EI}\ w_{EI} f_I\ +\ I_{ext}$$

$$\frac{d}{dt} < I_{syn\ I} > \approx -\beta_I < I_{syn\ I} > + N_E\ c_{IE}\ w_{IE} f_E$$

Douglas & Martin, 1991

$$\frac{d}{dt} < I_{syn} > \approx -\beta < I_{syn} > + N\ c\ w\ f$$

$$f = F(I_{syn})$$

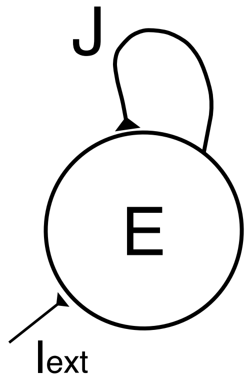
$$\tau \frac{dh}{dt} = -h\ +\ J\ E$$

$$E = F(h)$$

$$\boxed{\tau = \frac{1}{\beta}}$$

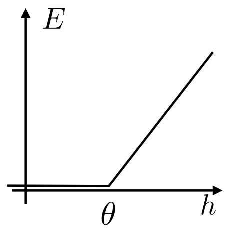
$$\boxed{J\ =\ N\ c\ w\ \frac{1}{\beta}}$$

Rate model describing a single large population of excitatory neurons **recurrently** connected



$$\tau \, \dot{h} \; = \; -h \; + \; J \; E \; + \; I_{ext}$$

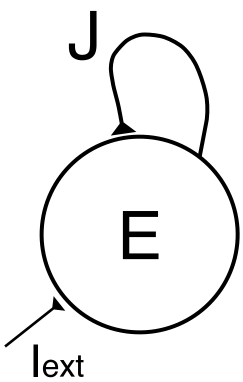
$$E \; = \; [\alpha \; (h \; - \; \theta)]_{+}$$



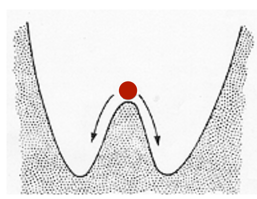
$$[x]_{+} = x \qquad for \;\; x > 0$$
$$[x]_{+} = 0 \qquad for \;\; x < 0$$

see e.g. Abbott & Dayan, 2000

What do we learn on recurrent networks using (mean-field) rate models?



- amplification of steady-state responses
- slowing down of *reaction times*
- altering of *equilibria*



"Amplification" by recurrent excitation (i.e., positive feedback)

$$h > \theta$$

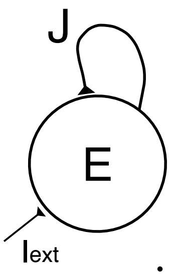
$$\tau \dot{h} = -h + J \alpha (h - \theta) + I_{ext}$$

$$\tau \dot{h} = -(1 - J \alpha) h + I_{ext} - J \alpha \theta$$

$$\dot{h} = \frac{h_\infty - h}{\tau_h} \qquad h_\infty = \frac{I_{ext} - J \alpha \theta}{1 - J\alpha}$$

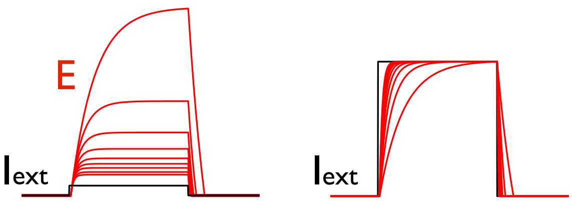
$$E = [\alpha (h - \theta)]_+ \qquad \tau_h = \frac{\tau}{1 - J\alpha}$$

Recurrent excitation (i.e., positive feedback)



- amplifies steady-state responses
- slows down the reaction times
- alters equilibrium points

$$\tau \dot{h} = -h + J E + I_{ext}$$



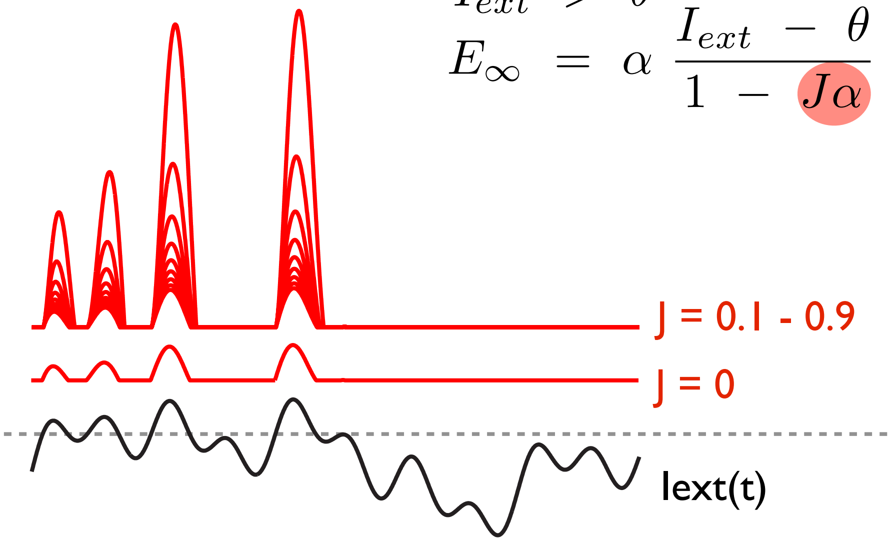
$$h_\infty = \frac{I_{ext} - J \alpha \theta}{1 - J\alpha}$$

$$\tau_h = \frac{\tau}{1 - J\alpha}$$

Amplification by recurrent excitation (i.e., positive feedback)

$$I_{ext} > \theta$$

$$E_\infty = \alpha \frac{I_{ext} - \theta}{1 - J\alpha}$$

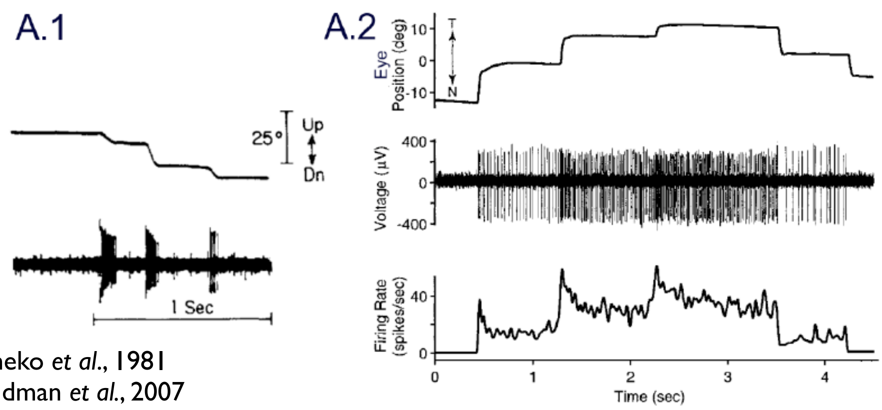


"Temporal integration"

Oculomotor system

Brainstem nuclei neurons encode (eye-velocity) motor commands for saccades as APs bursts. Neurons downstream fire tonically during fixations to maintain the eye muscles' tension and, thereby, the eye position stable.

A **neural "integrator" was hypothesised,** in order to explain how transient inputs as in A.1 result in sustained responses as in A.2. It is not yet known whether it is implemented with network mechanisms or single-cell properties (e.g. complicated extra ion-channels)...



Kaneko *et al.*, 1981
Goldman *et al.*, 2007

“Temporal integration” by recurrent excitation (i.e., finely-tuned positive feedback)

$$h \; > \; \theta$$

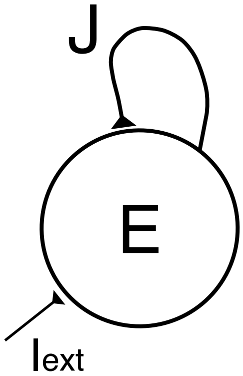
$$\tau \, \dot{h} \; = \; -h \; + \; J \, \alpha \; (h \; - \; \theta) \; + \; I_{ext}$$

$$\tau \, \dot{h} \; = \; -(1 \; - \; J \, \alpha) \; h \; + \; I_{ext} \; - \; J \, \alpha \, \theta$$

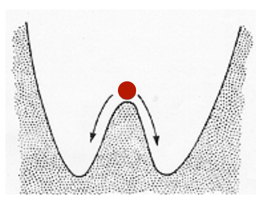
$$J \; = \; \alpha^{-1} \qquad \tau \, \dot{h} \; = \; I_{ext} \; - \; \theta$$

$$h(t) \; = \; h(0) \; + \; \int_0^t I_{ext}(t') \; - \; \theta \; dt'$$

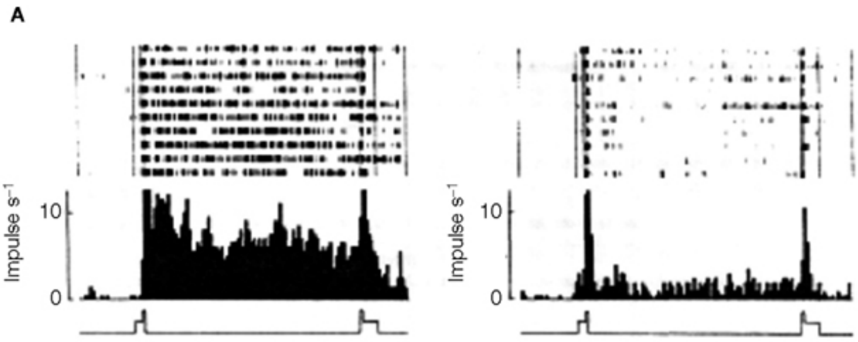
What do we learn on recurrent networks using (mean-field) rate models?



- amplification of steady-state responses
- slowing down of *reaction times*
- altering of *equilibria*



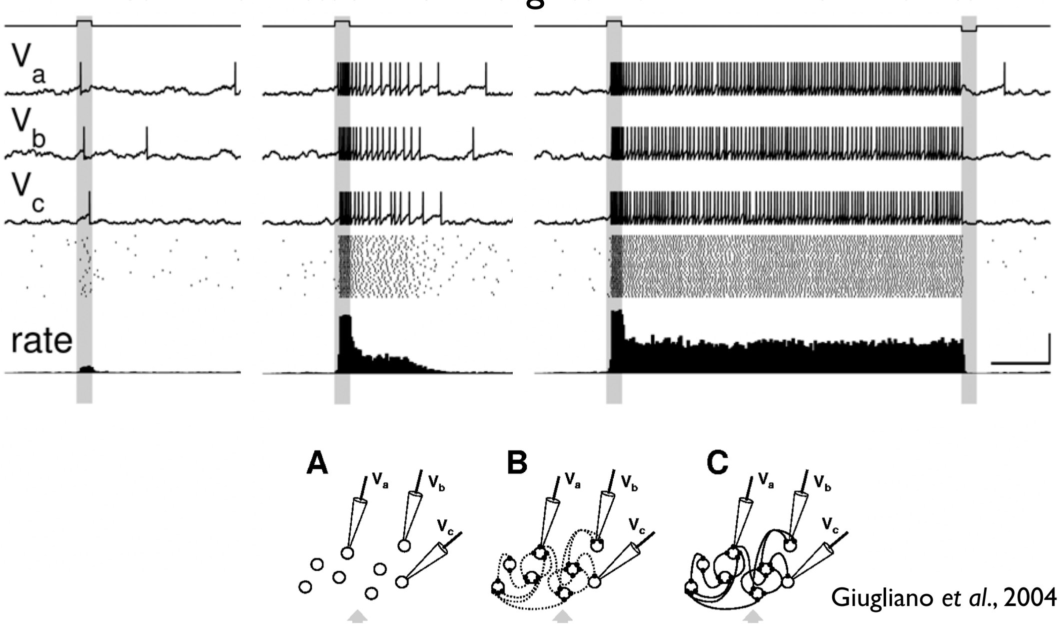
"Persistent activity" in the primate Infero-temporal cortex, during delay match-to-sample tasks



Miyashita, 1988

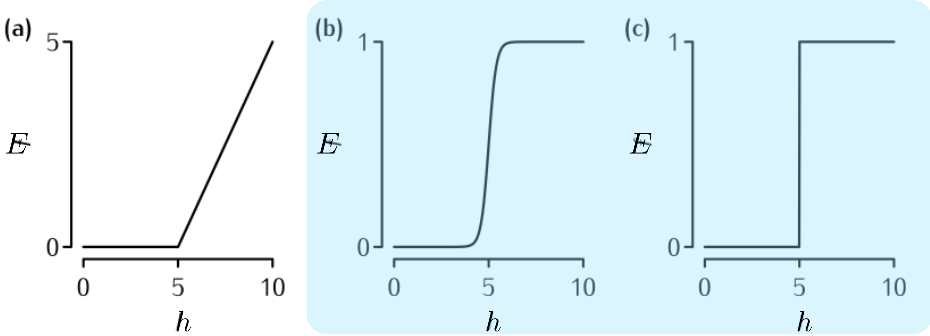
"Persistent activity" with a recurrent population of excitatory neurons

Recurrent network of Integrate-and-Fire model units



Giugliano *et al.*, 2004

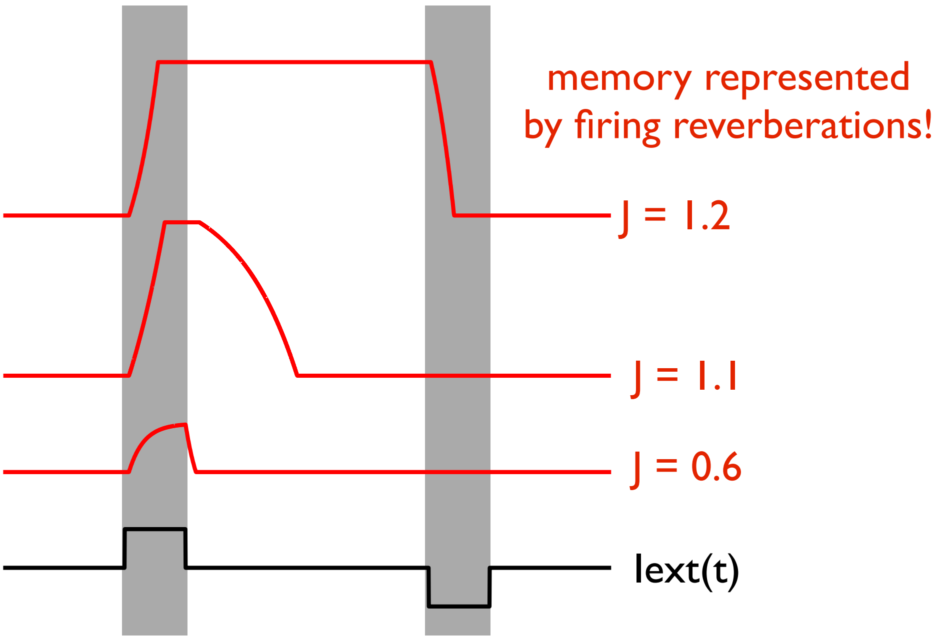
"Persistent activity" with a recurrent population of excitatory neurons



It is only possible with "saturating f-I" curve
(or with short-term depressing synapses...)

from Sterratt et al., 2011

"Persistent activity" with a recurrent population of excitatory neurons

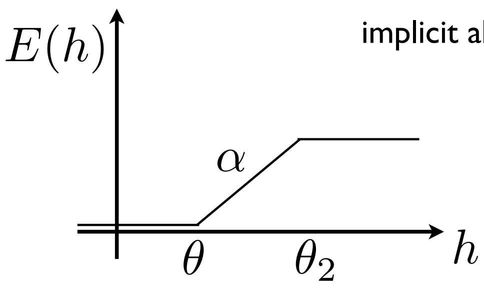


Analysis of the equilibrium points (i.e. steady-states or fixed-points)

$$\tau \, \dot{h} \; = \; -h \; + \; J \; E \; + \; I_{ext}$$

$$\dot{h}_{ss} \; = \; 0$$

$$h_{ss} \; = \; J \; E(h_{ss}) \; + \; I_{ext}$$



implicit algebraic equation: how to solve it??

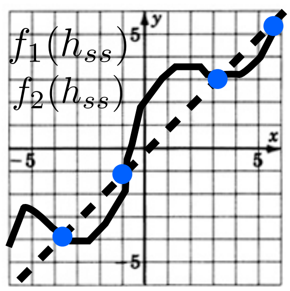
graphically

numerically with a PC

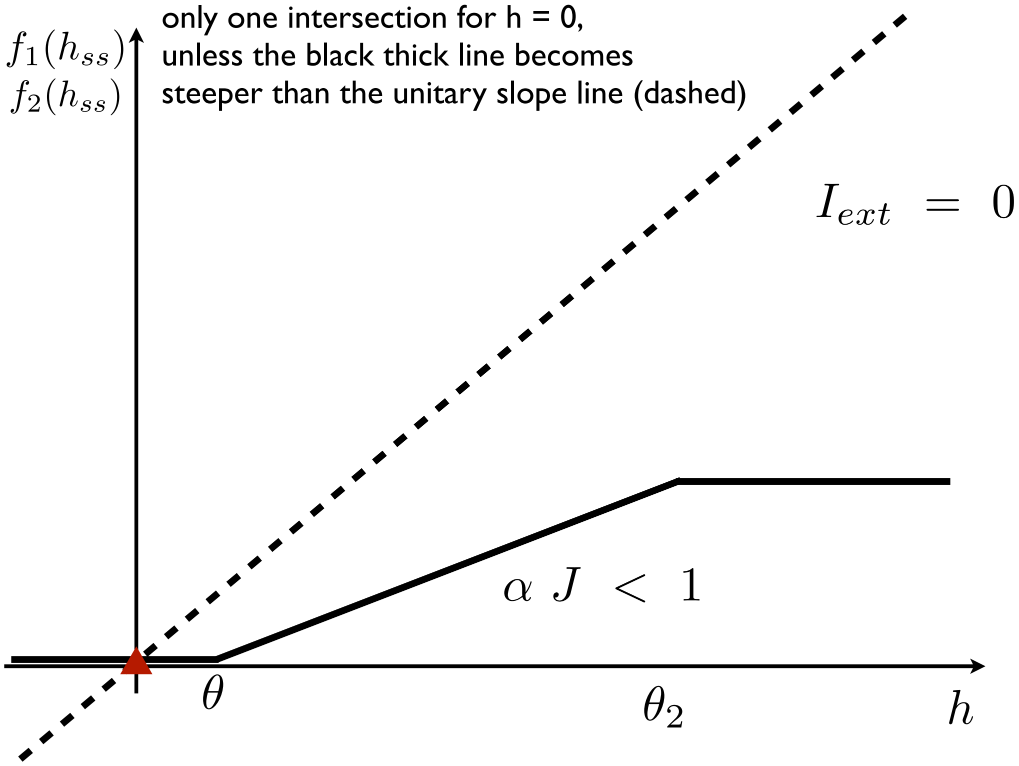
$$h_{ss} \; = \; J \; E(h_{ss}) \; + \; I_{ext}$$

this is the unitary slope line (easy to draw!!!)

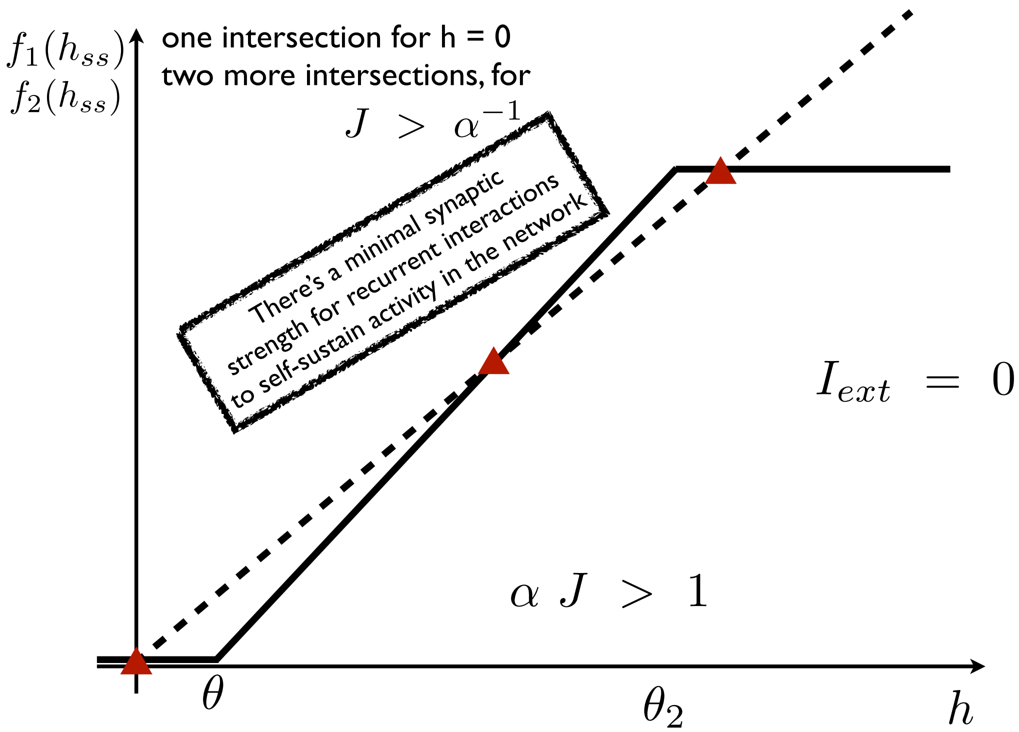
$$\begin{cases} f_1(h_{ss}) \; = \; h_{ss} \\ f_2(h_{ss}) \; = \; J \; E(h_{ss}) \; + \; I_{ext} \end{cases}$$



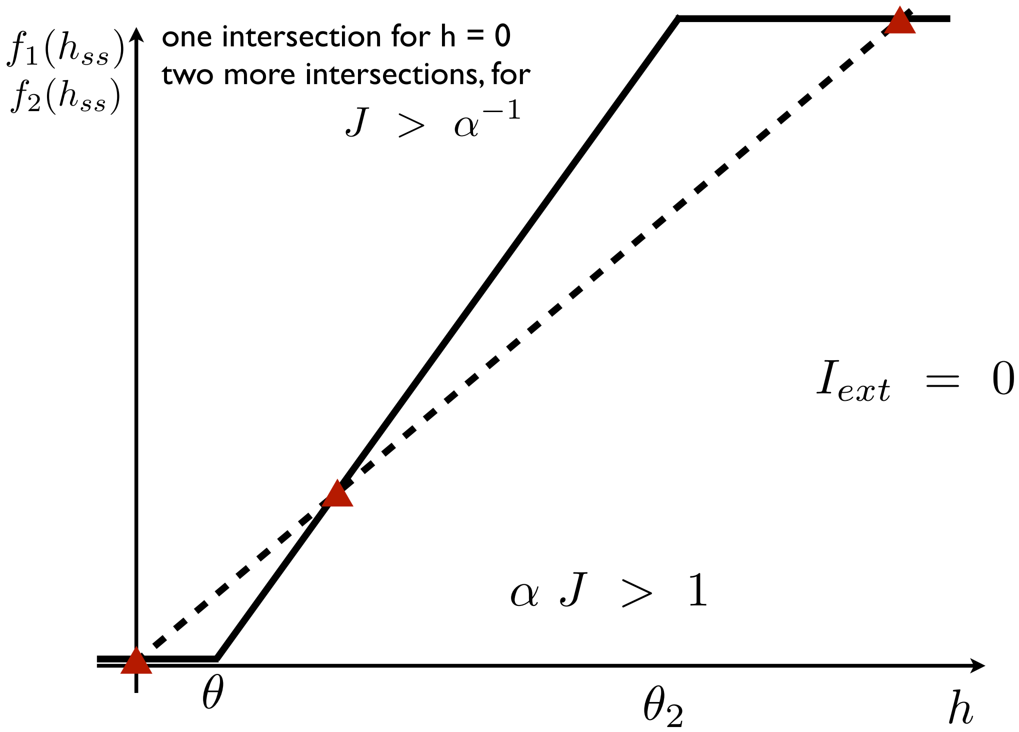
this is a piece-wise linear function (easy to draw)



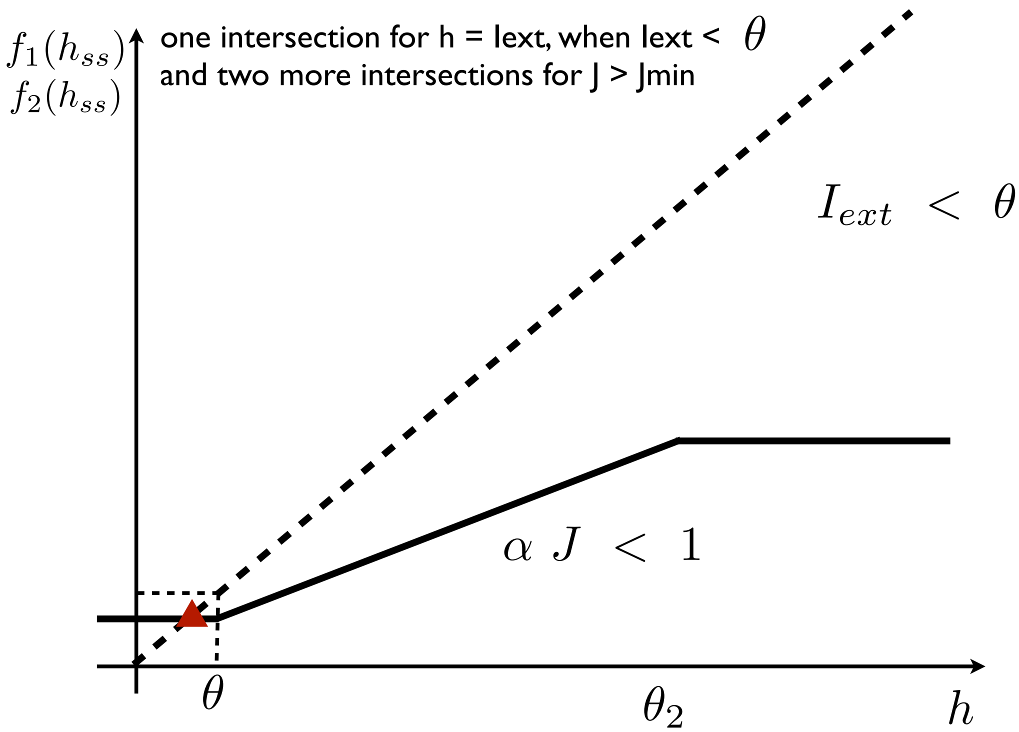
$f_1(h_{ss})$
$f_2(h_{ss})$
only one intersection for h = 0,
unless the black thick line becomes
steeper than the unitary slope line (dashed)
$I_{ext} = 0$
$\alpha J < 1$
θ
θ_2
h



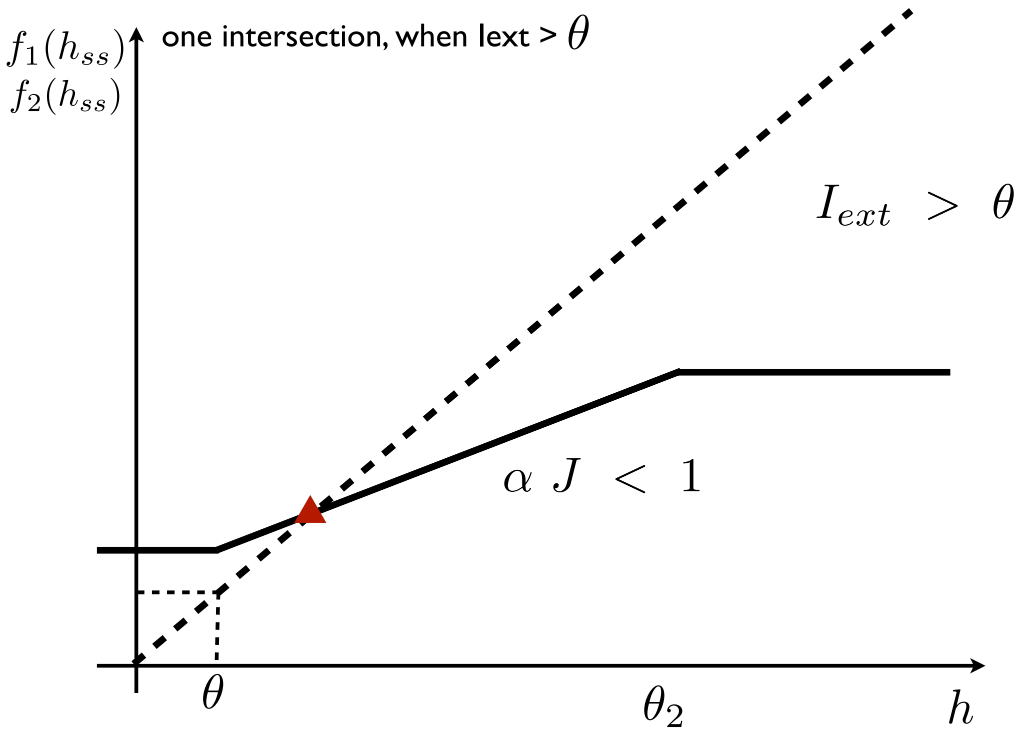
$f_1(h_{ss})$
$f_2(h_{ss})$
one intersection for h = 0
two more intersections, for
$J > \alpha^{-1}$
There's a minimal synaptic
strength for recurrent interactions
to self-sustain activity in the network
$I_{ext} = 0$
$\alpha J > 1$
θ
θ_2
h

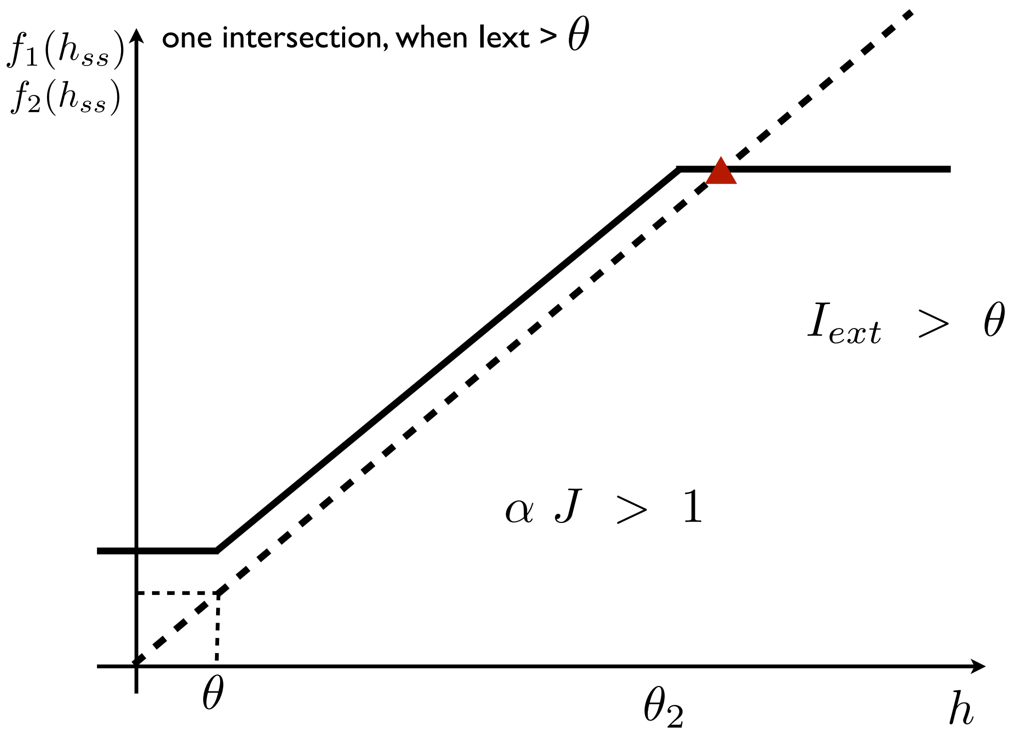


$f_1(h_{ss})$
$f_2(h_{ss})$
one intersection for h = 0
two more intersections, for
$J > \alpha^{-1}$
$I_{ext} = 0$
$\alpha J > 1$
θ
θ_2
h

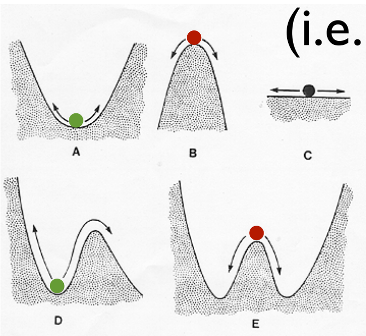


$f_1(h_{ss})$
$f_2(h_{ss})$
one intersection for h = Iext, when Iext < θ
and two more intersections for J > Jmin
$I_{ext} < \theta$
$\alpha J < 1$
θ
θ_2
h


$f_1(h_{ss})$
$f_2(h_{ss})$
one intersection, when Iext > θ
$I_{ext} > \theta$
$\alpha J < 1$
θ
θ_2
h


$f_1(h_{ss})$
$f_2(h_{ss})$
one intersection, when Iext > θ
$I_{ext} > \theta$
$\alpha J > 1$
θ
θ_2
h

Analysis of the stability of the equilibria
(i.e. stable or unstable)

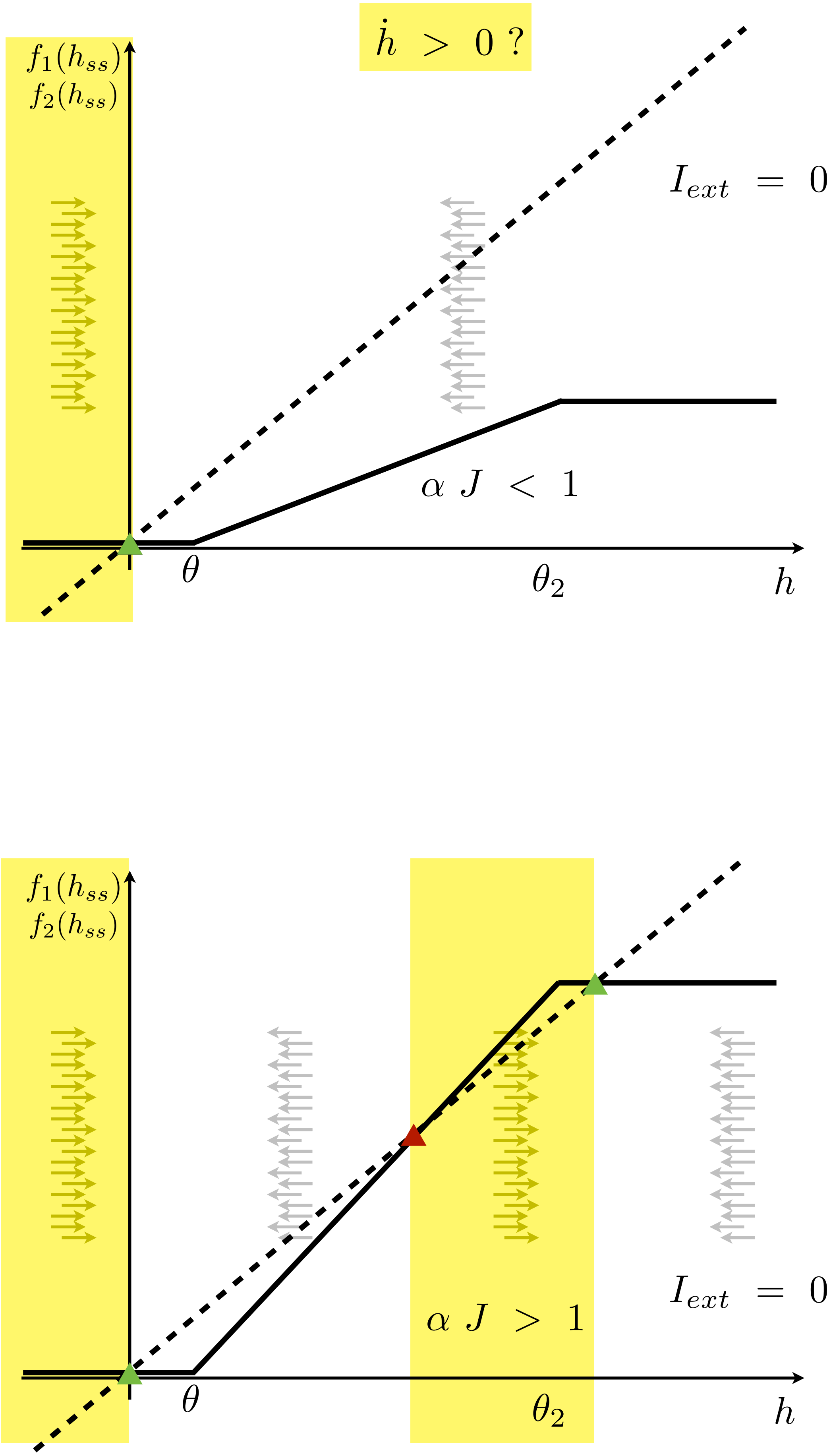


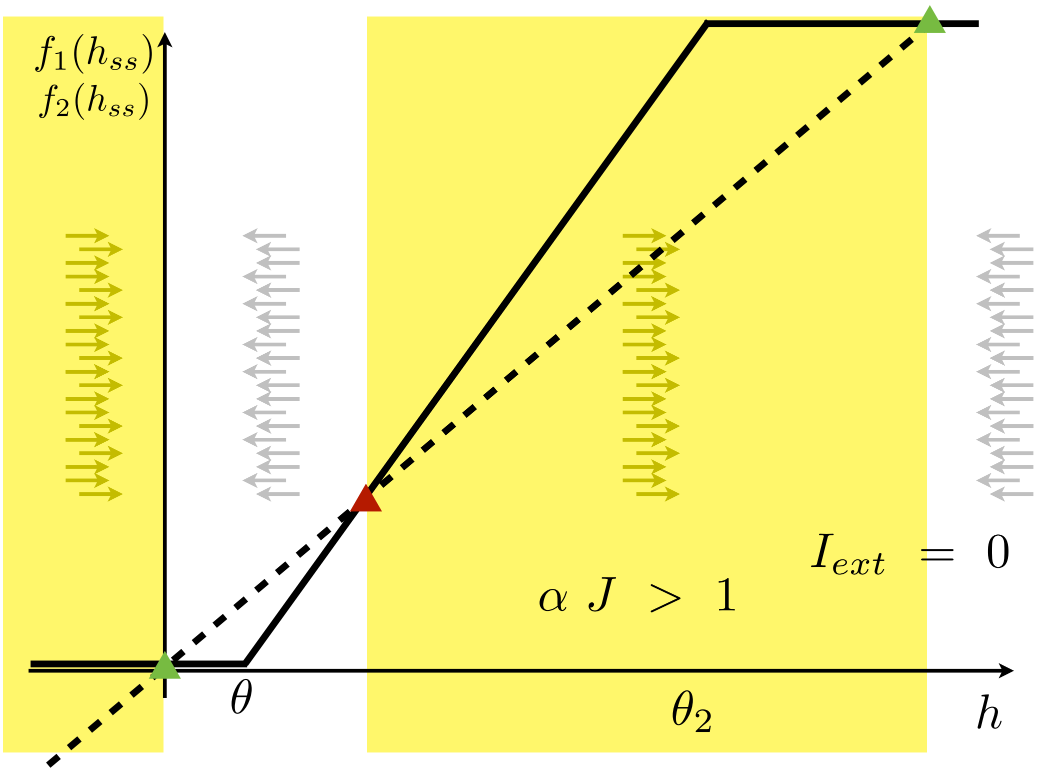
$$\tau \, \dot{h} \;=\; -h \;+\; J \; E \;+\; I_{ext}$$

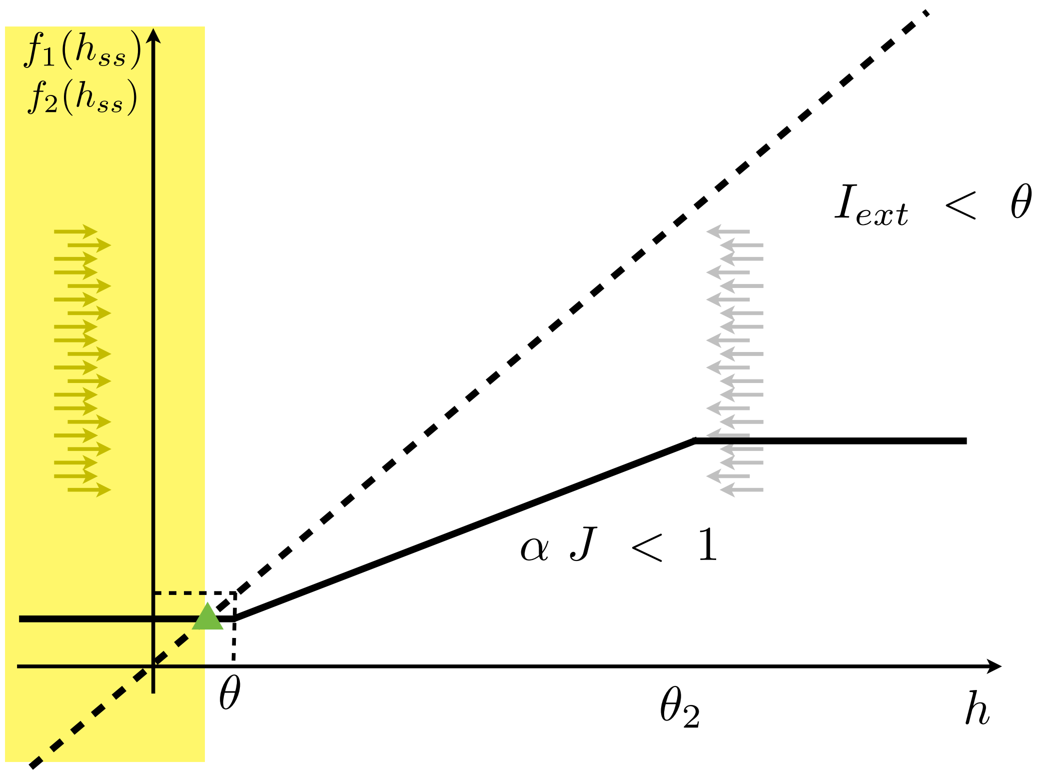
$\dot{h} \;>\; 0 \; ?$ $\qquad\qquad h \;<\; J \; E(h_{ss}) \;+\; I_{ext}$

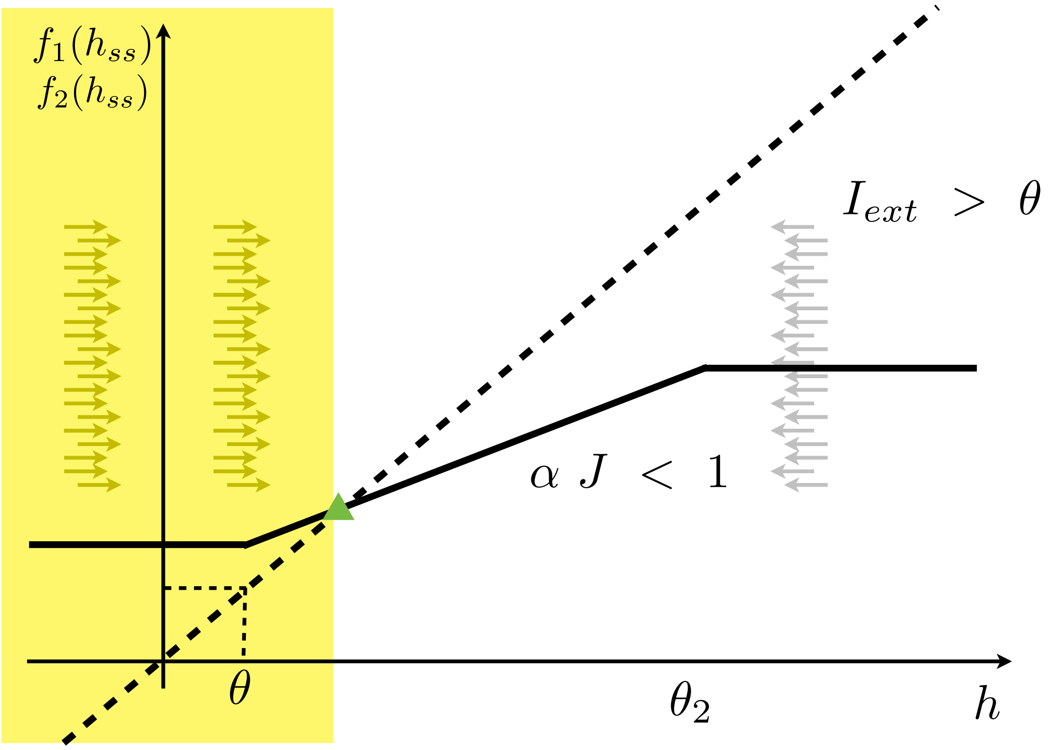
implicit algebraic inequality: how to solve it??

graphically

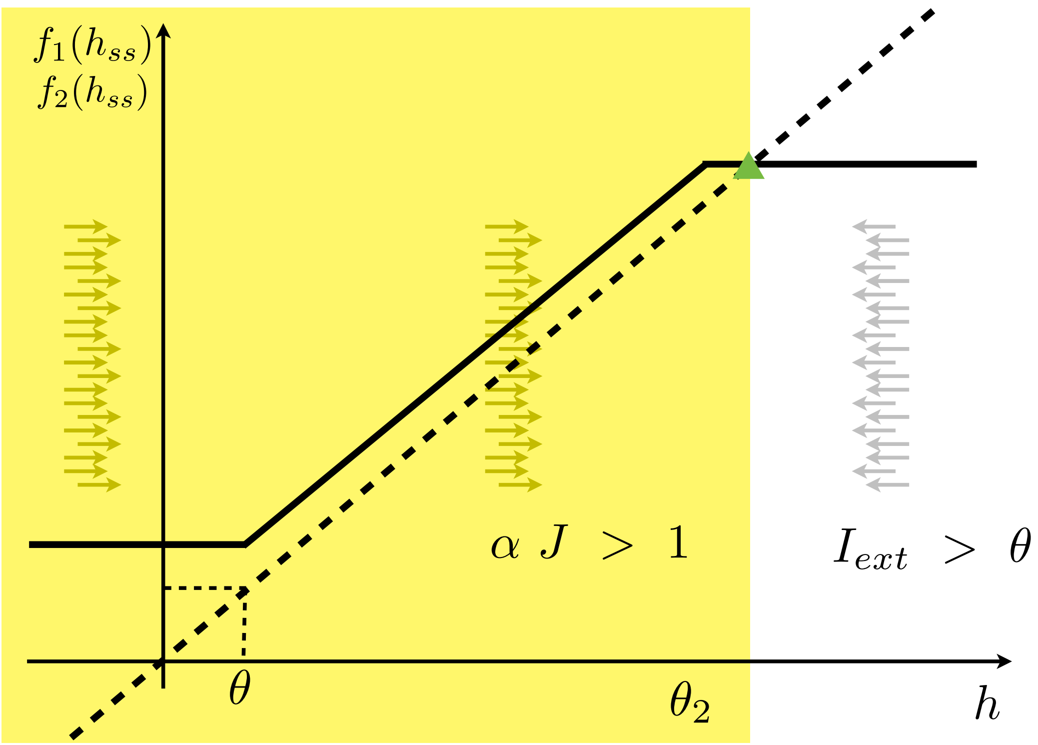

$\dot{h} > 0$?
$f_1(h_{ss})$
$f_2(h_{ss})$
$I_{ext} = 0$
$\alpha J < 1$
θ
θ_2
h
$f_1(h_{ss})$
$f_2(h_{ss})$
$I_{ext} = 0$
$\alpha J > 1$
θ
θ_2
h


$f_1(h_{ss})$
$f_2(h_{ss})$
$I_{ext} = 0$
$\alpha J > 1$
θ
θ_2
h


$f_1(h_{ss})$
$f_2(h_{ss})$
$I_{ext} < \theta$
$\alpha J < 1$
θ
θ_2
h



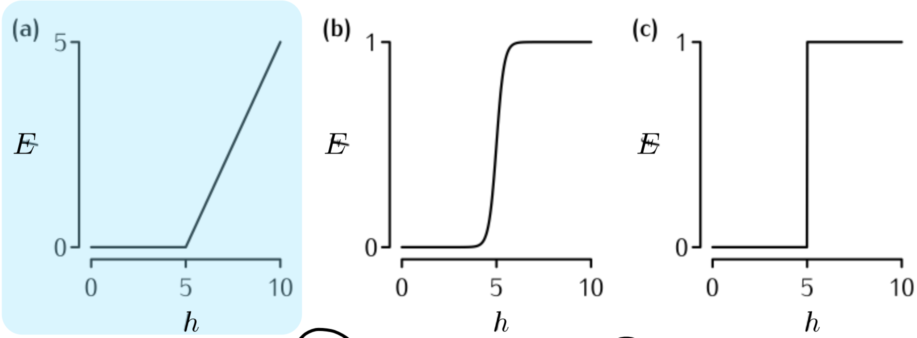
$f_1(h_{ss})$
$f_2(h_{ss})$
$I_{ext} > \theta$
$\alpha J < 1$
θ
θ_2
h

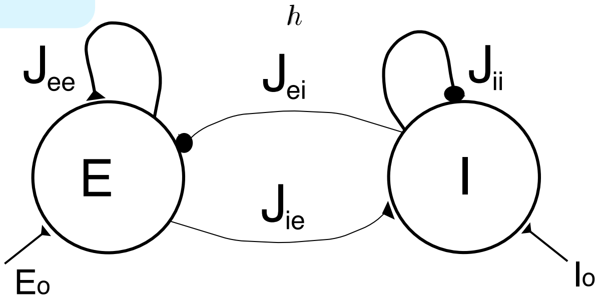


$f_1(h_{ss})$
$f_2(h_{ss})$
$\alpha J > 1$
$I_{ext} > \theta$
θ
θ_2
h

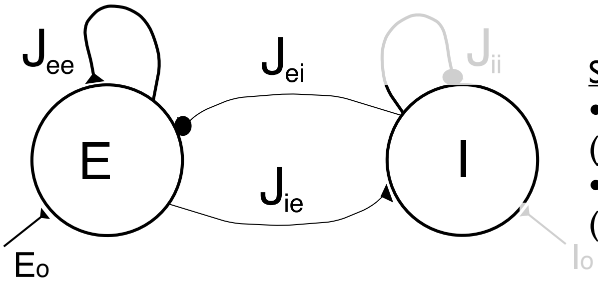
“Persistent activity” with two populations of excitatory and inhibitory neurons

No “saturating f-I” curve is necessary...





“Persistent activity” with two populations of excitatory and inhibitory neurons



<u>Simplifying hypotheses:</u>

- no recurrent inhibition
(J_{ii} = 0)
- no input to the inhibitory pop.
(I_o = 0)

$$\tau_e \, \dot{h}_e = -h_e + J_{ee} \, E - J_{ei} \, I + E_0$$

$$\tau_i \, \dot{h}_i = -h_i + J_{ie} \, E$$

$$E = [\alpha_e \, (h_e - \theta_e)]_+ \text{ and } I = [\alpha_i \, (h_i - \theta_i)]_+$$

Analysis of the equilibrium points (i.e. steady-states of fixed-points)

$$\tau_e \, \dot{h}_e = -h_e + J_{ee} \, E - J_{ei} \, I + E_0$$

$$\tau_i \, \dot{h}_i = -h_i + J_{ie} \, E$$

$$E = [\alpha_e \, (h_e - \theta_e)]_+ \text{ and } I = [\alpha_i \, (h_i - \theta_i)]_+$$

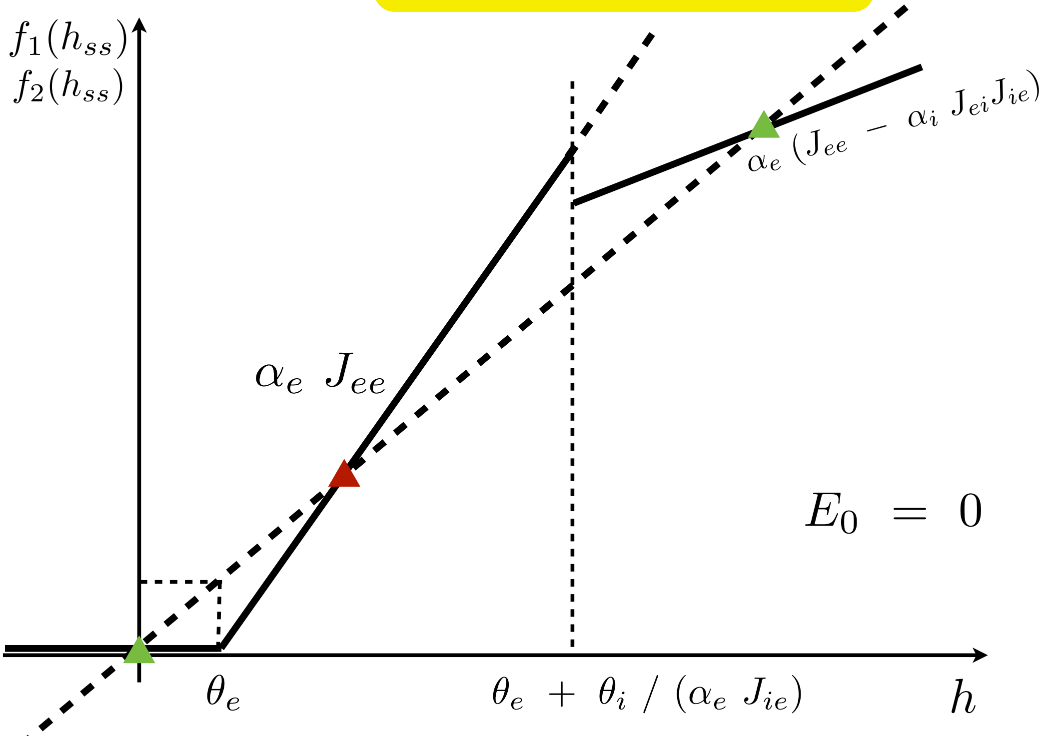
$$\dot{h}_e = \dot{h}_i = 0$$

$$h_e = J_{ee} \, E - J_{ei} \, I + E_0$$

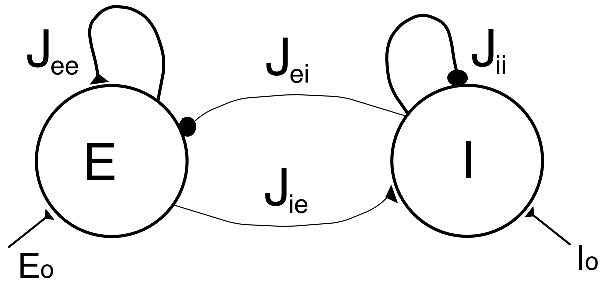
$$h_i = J_{ie} \, E$$

$$h_e = J_{ee} \, E - J_{ei} \, [\alpha_i \, (J_{ie} \, E - \theta_i)]_+ + E_0$$

$$h_e = J_{ee} \, E - J_{ei} \, [\alpha_i \, (J_{ie} \, E - \theta_i)]_+ + E_0$$



"Oscillations" with two populations of excitatory and inhibitory neurons

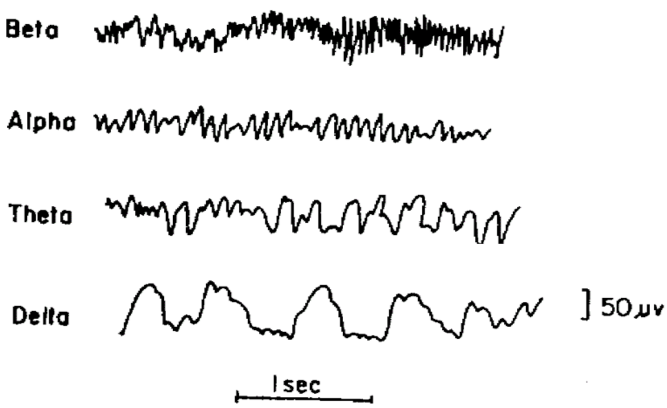


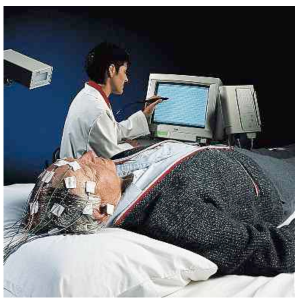
$$\tau_e \, \dot{h}_e = \; -h_e \; + \; J_{ee} \, E \; - \; J_{ei} \, I \; + \; E_0$$
$$\tau_i \, \dot{h}_i = \; -h_i \; + \; J_{ie} \, E \; - \; J_{ii} \, I \; + \; I_0$$

$$E \;=\; [\alpha_e \, (h_e - \theta_e)]_+ \text{ and } I \;=\; [\alpha_i \, (h_i - \theta_i)]_+$$

"Oscillations" in cortical networks revealed by EEG (electroencephalography)

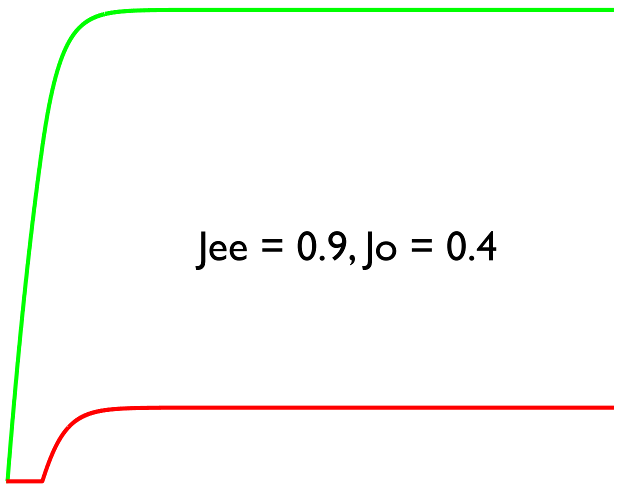
First EEG recorded by Hans Berger, circa 1928.





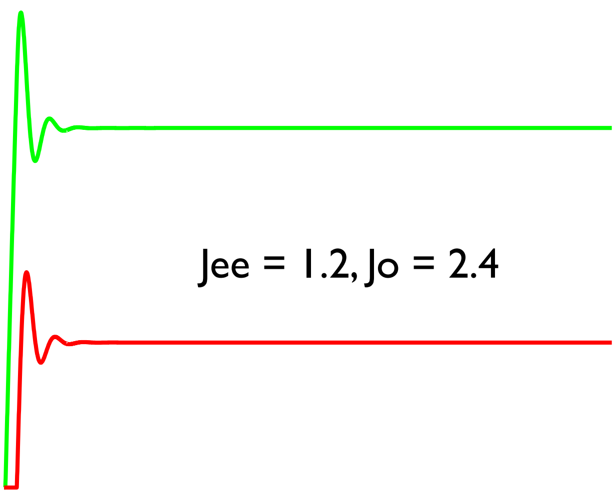
from Sabbatini, online

$$\begin{aligned} J_{ii} &= 0 \\ \tau_e &= \tau_i \\ J_{ie} &= J_{ei} = J_o \\ \alpha_e &= \alpha_i = 1 \\ \theta_e &= \theta_i \end{aligned}$$



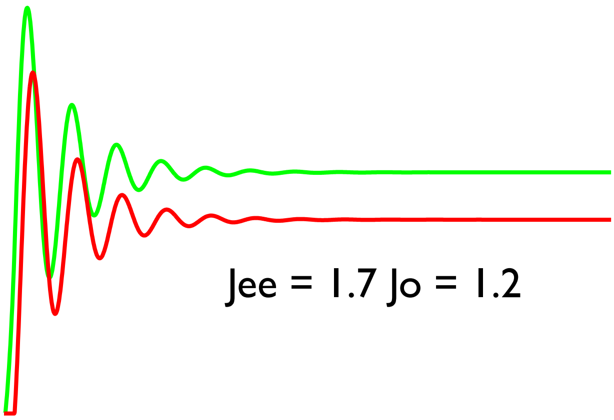
"Oscillations" with two populations of excitatory and inhibitory neurons

$$\begin{aligned} J_{ii} &= 0 \\ \tau_e &= \tau_i \\ J_{ie} &= J_{ei} = J_o \\ \alpha_e &= \alpha_i = 1 \\ \theta_e &= \theta_i \end{aligned}$$



“Oscillations” with two populations of excitatory and inhibitory neurons

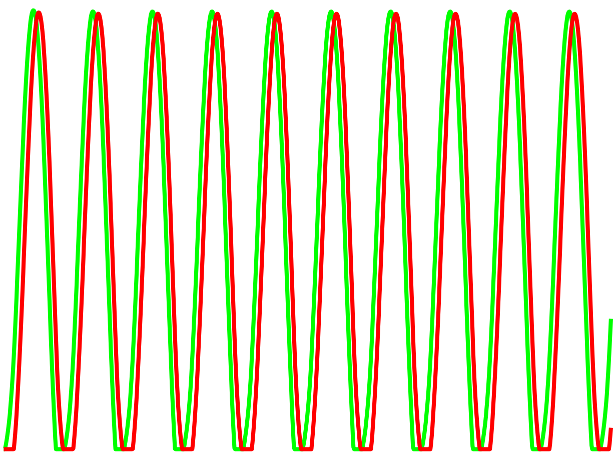
$$
\begin{aligned}
J_{ii} &= 0 \\
\tau_e &= \tau_i \\
J_{ie} &= J_{ei} = J_o \\
\alpha_e &= \alpha_i = 1 \\
\theta_e &= \theta_i
\end{aligned}
$$



Jee = 1.7 Jo = 1.2

“Oscillations” with two populations of excitatory and inhibitory neurons

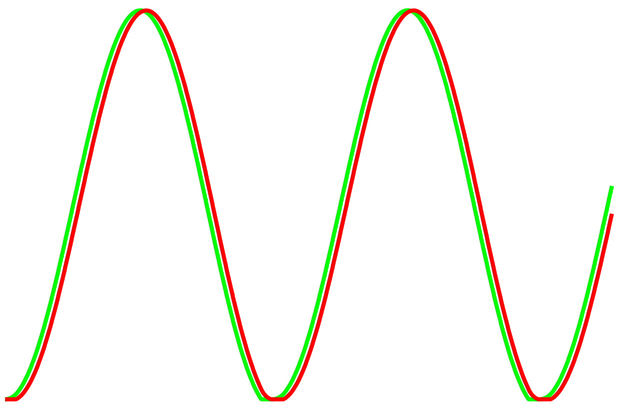
$$
\begin{aligned}
J_{ii} &= 0 \\
\tau_e &= \tau_i \\
J_{ie} &= J_{ei} = J_o \\
\alpha_e &= \alpha_i = 1 \\
\theta_e &= \theta_i
\end{aligned}
$$



Jee = 2 Jo = 1.2

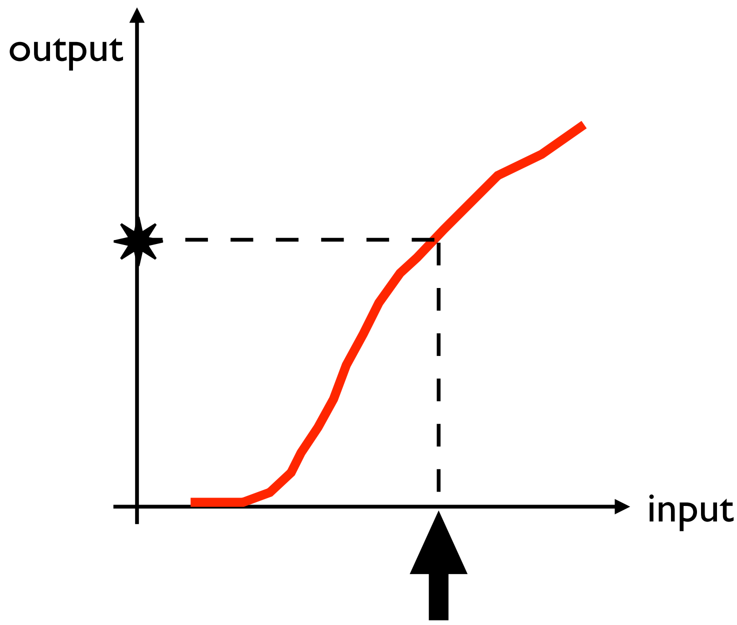
“Oscillations” with two populations of excitatory and inhibitory neurons

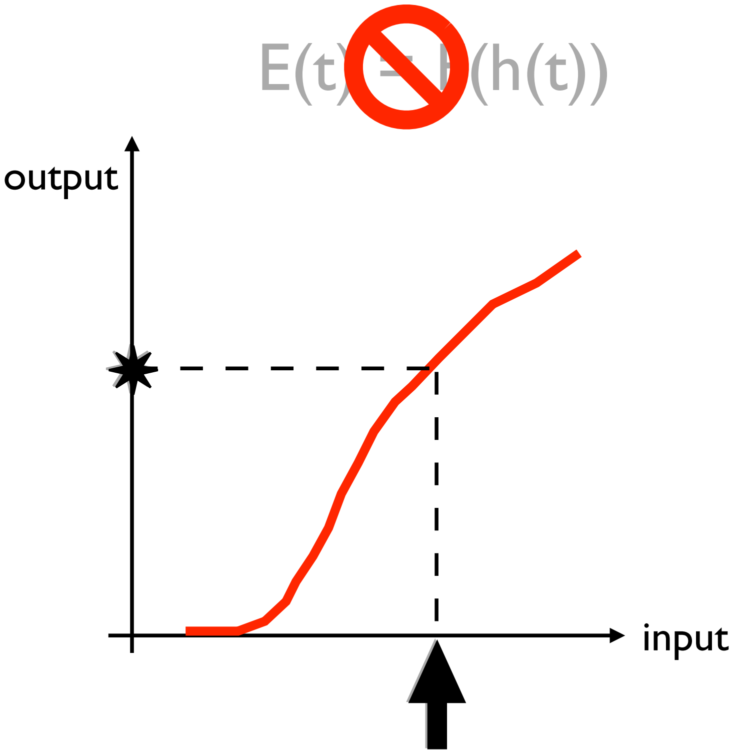
$$
\begin{aligned}
J_{ii} &= 0 \\
\tau_e &= \tau_i \\
J_{ie} &= J_{ei} = J_o \\
\alpha_e &= \alpha_i = 1 \\
\theta_e &= \theta_i
\end{aligned}
$$



Jee = 2 Jo = 1.01

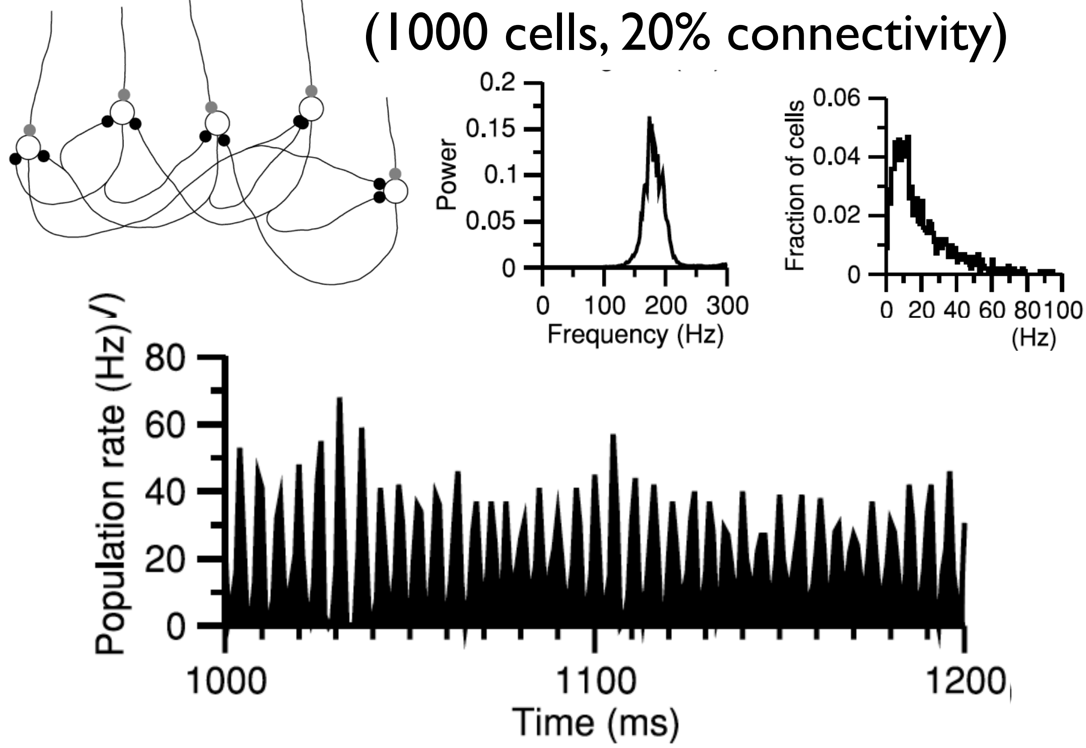
E = F(h)



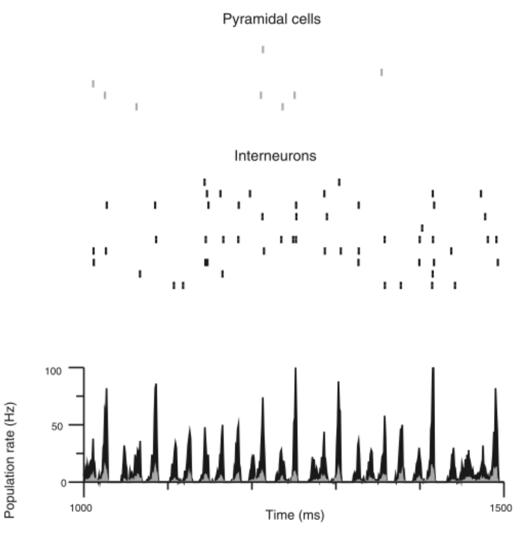


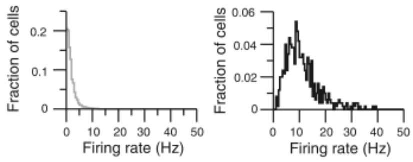
Mechanical, electrical, chemical, hydraulic physical systems have common properties of inertia and tend to attenuate and delay quick changes (e.g. Newton's laws)

recurrent network of inhibitory IF neurons (1000 cells, 20% connectivity)

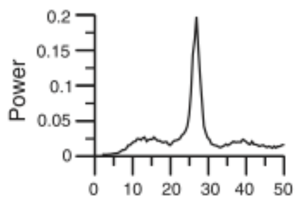


Coherent (fast) rhythms do emerge as a destabilisation of the asynchronous state: how?



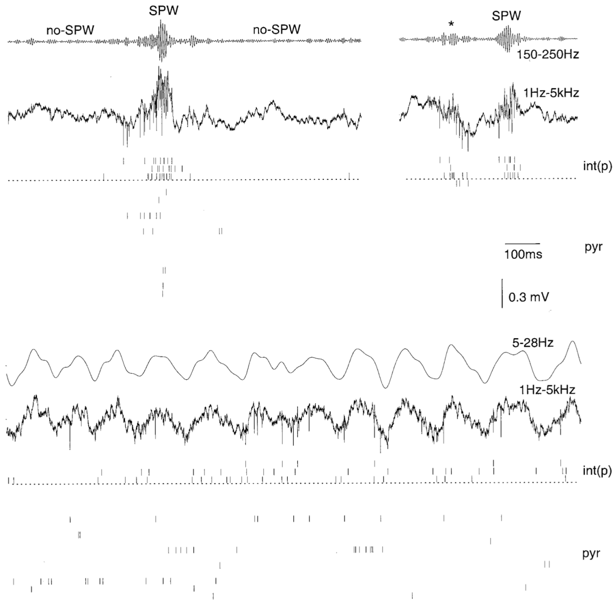


why 30 cycles/sec?

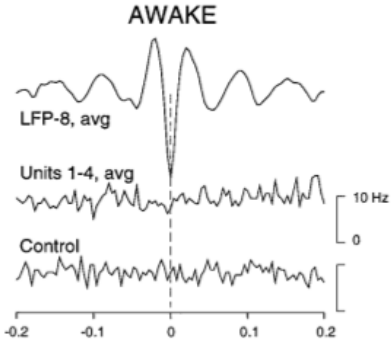


Wang, 2010
Geisler et al., 2005

Sparsely synchronised network oscillations:
coherent LFP fast oscillations, irregular & slow firing



Csicsvari et al., 1999



Destexhe et al., 1999

Global oscillations in neuronal networks, with irregular firing

LFP oscillations (from 2 to 200 Hz) *in vivo,* several areas

<u>cerebellum:</u> ~200Hz LFP oscillations in the Purkinje Cell layer
(Adrian, 1935; Isope et al., 2002; de Solages et al., 2008)

<u>rat hippocampus:</u> gamma 40–80Hz LFP rhythm (free movement, R.E.M.)
sharp-wave LFP ripples 200Hz (immobility, quiet sleep)
(Bragin et al. 1995; Buzsaki et al. 1992; Csicsvari et al. 1999b; Siapas and Wilson 1998)

<u>primates cortex:</u> often LFP has rhythmic component, but simultaneous spike trains appear irregular, with no clear-cut oscillation
(Fries et al. 2001; Logothetis et al. 2001).

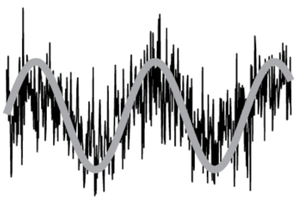
- Single-cell discharge rates < LFP oscillation frequency
- Rhythmicity not apparent in the raw spike trains of individual cells
- Single-cell behaves differently from the population activity during fast oscillations.

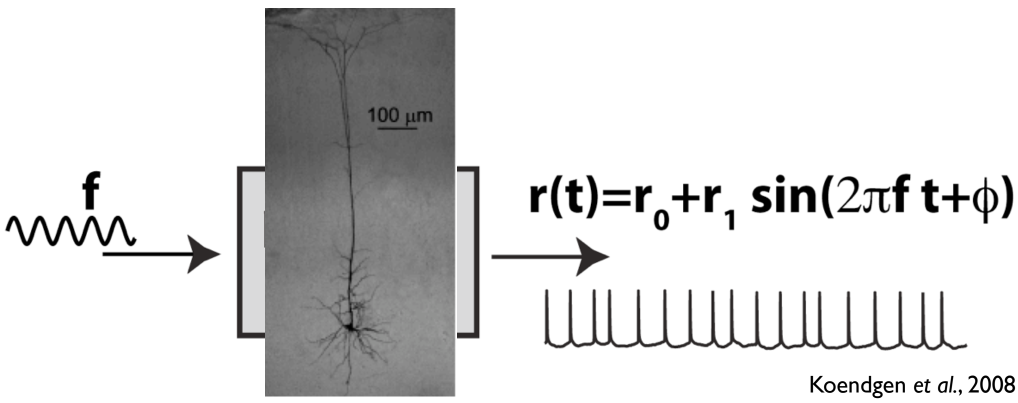
Brunel & Hakim (1999), Neural Computation 11, 1621–71
Geisler et al (2005), J Neurophysiol 94(6),4344-61
Brunel & Wang (2003), J Neurophysiol 90, 415–30

Sparsely synchronised network oscillations

Interplay among Synaptic Kinetics, Delays, Filtering, & Connectivity **and Intrinsic Neuronal Dynamics**

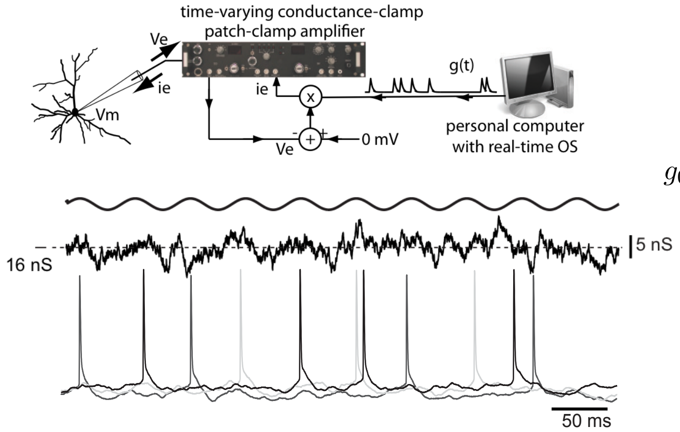
The dynamical response properties of cortical neurons: by Fourier Analysis!





Koendgen *et al.*, 2008

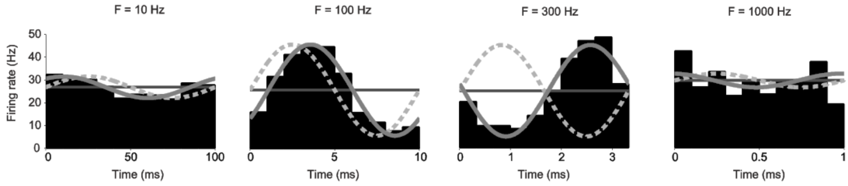
$$\Phi_{cell}(f) \;=?, \quad r_1(f) \;=?$$



$$g_0 \;+\; g_1 \; sin(2\pi f t \;+\; \Phi_{syn}(f)) \\ + \\ n(t)$$

$$\sum_k \delta(t - t_k)$$

$$r(t) \;=\; r_0 \;+\; r_1(f)\; sin(2\pi f t \;-\Phi_{syn}(f) \;-\Phi_{cell}(f))$$

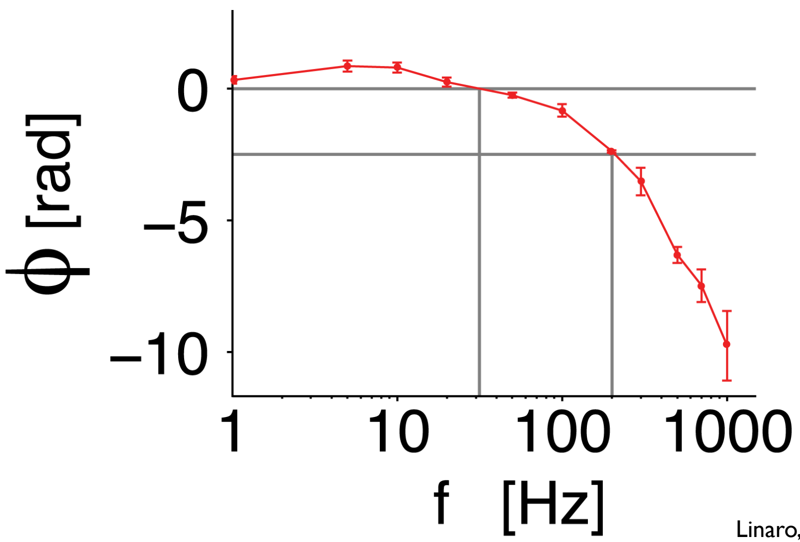


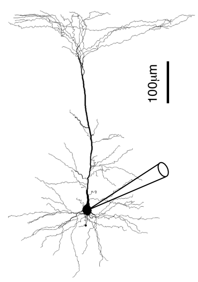
Linaro, Couto, Giugliano, 2014 J. Neurosci. Methods

Linaro, Biro, Giugliano 2017

$\Phi_{cell}(f)$

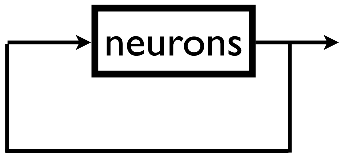
Phase of
the dynamical response of a pyramidal neuron
during ~20 spks/sec, low background noise





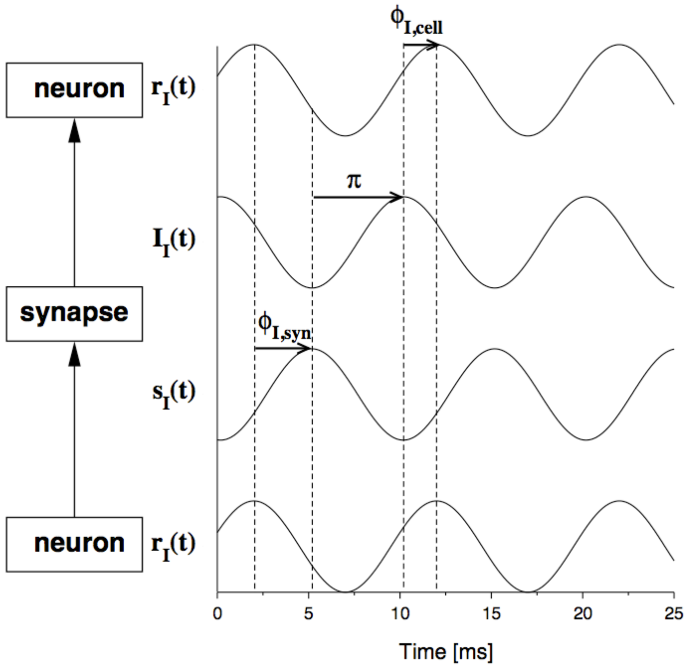
Linaro, Biro, Giugliano 2017

Oscillations in the global firing rate, if they occur, they occur at a self-consistent frequency



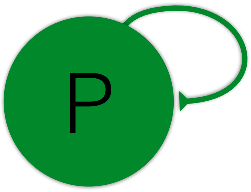
Brunel & Hakim (1999), Neural Computation 11, 1621–71
Geisler et al (2005), J Neurophysiol 94(6),4344-61
Brunel & Wang (2003), J Neurophysiol 90, 415–30

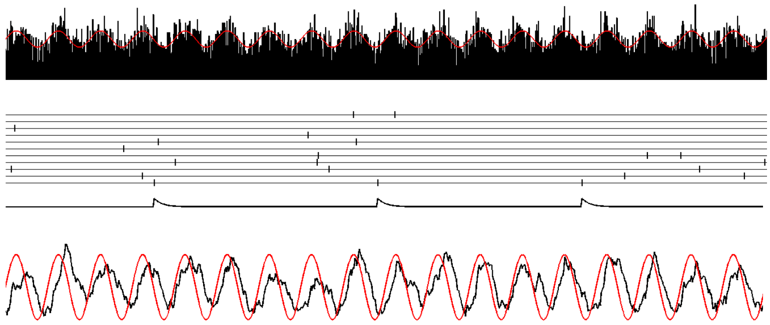
Oscillations in the global firing rate, if they occur, they occur at a self-consistent frequency



Geisler et al., 2005
Furhmann et al., 2003

$$r(t) \;=\; r_0 \;+\; r_1 \; sin(2\pi f \; t)$$





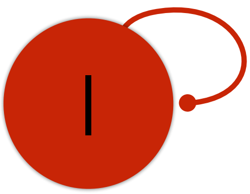
$$I_{syn}(t) \;=\; I_0 \;+\; I_1 sin(2\pi f t \;-\Phi_{syn}(f)) \;+\; n(t)$$

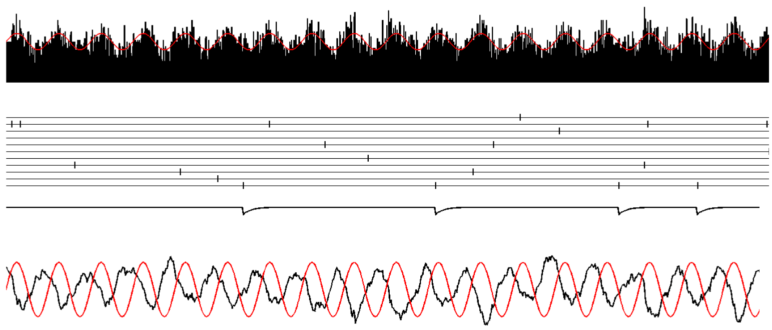
$$r(t) \;=\; r_0 \;+\; r_1(f) \, sin(2\pi f t \;-\Phi_{syn}(f) \;-\Phi_{cell}(f))$$

$$\Phi_{syn}(f) \;+\; \Phi_{cell}(f) \;=\; 0$$

Wang, 2010
Geisler et al., 2005

$$r(t) \;=\; r_0 \;+\; r_1 \; sin(2\pi f \; t)$$



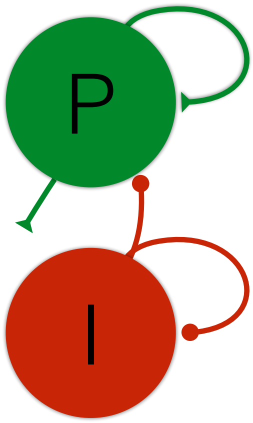


$$I_{syn}(t) \;=\; I_0 \;+\; I_1 sin(2\pi f t \;-\Phi_{syn}(f) \;-\pi) \;+\; n(t)$$

$$r(t) \;=\; r_0 \;+\; r_1(f) \; sin(2\pi f t \;-\Phi_{syn}(f) \;-\pi \;-\Phi_{cell}(f))$$

$$\Phi_{syn}(f) \;+\; \Phi_{cell}(f) \;=\; -\pi$$

Wang, 2010
Geisler et al., 2005



$$r_E(t) = r_{E0} + r_{E1}\ sin(2\pi f t)$$
$$r_I(t) = r_{I0} + r_{I1}\ sin(2\pi f t)$$

$$I_{syn}(t) = I_0 + I_{E1} sin(2\pi f t - \Phi_{Esyn}(f)) + \\ + I_{I1} sin(2\pi f t - \Phi_{Isyn}(f) - \pi) + n(t)$$

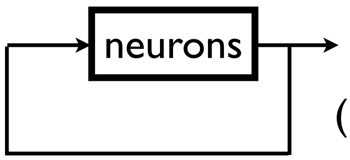
$$I_{syn}(t) = I_0 + I_1 sin(2\pi f t - \frac{\Phi_{Esyn}(f) + \Phi_{Isyn}(f) + \pi}{2}) + n(t)$$

$$\boxed{\frac{\Phi_{Esyn}(f) + \Phi_{Isyn}(f) + \pi}{2} - \Phi_{cellE}(f) = 0}$$

Wang, 2010
Geisler et al., 2005

recurrent network of *excitatory* neurons

$\Delta i(t) \approx cos(2\pi\ f_0\ t)$

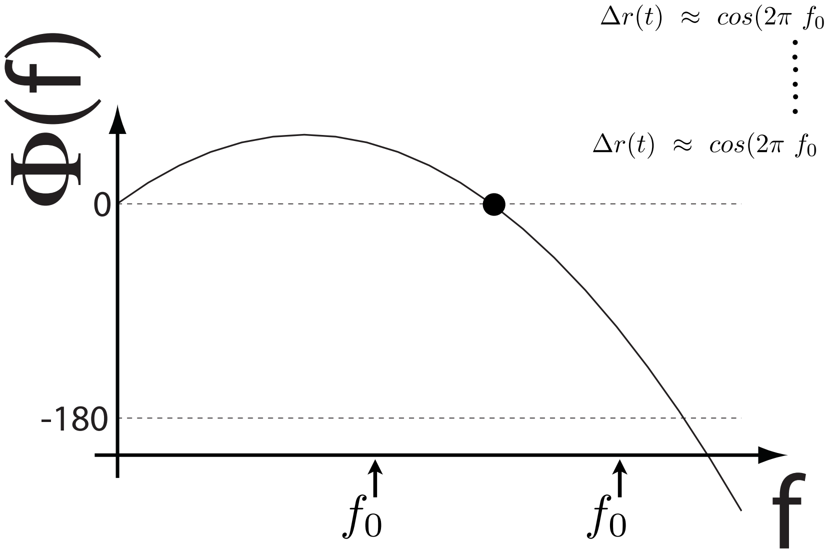


instantaneous
firing rate
(i.e. firing probability)

$\Delta r(t) \approx cos(2\pi\ f_0\ t + \Phi(f_0))$

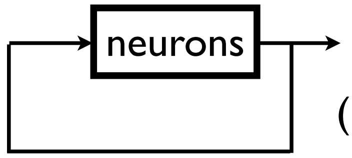
$\vdots$

$\Delta r(t) \approx cos(2\pi\ f_0\ t + \alpha\ t\ \Phi(f_0))$



recurrent network of *inhibitory* neurons

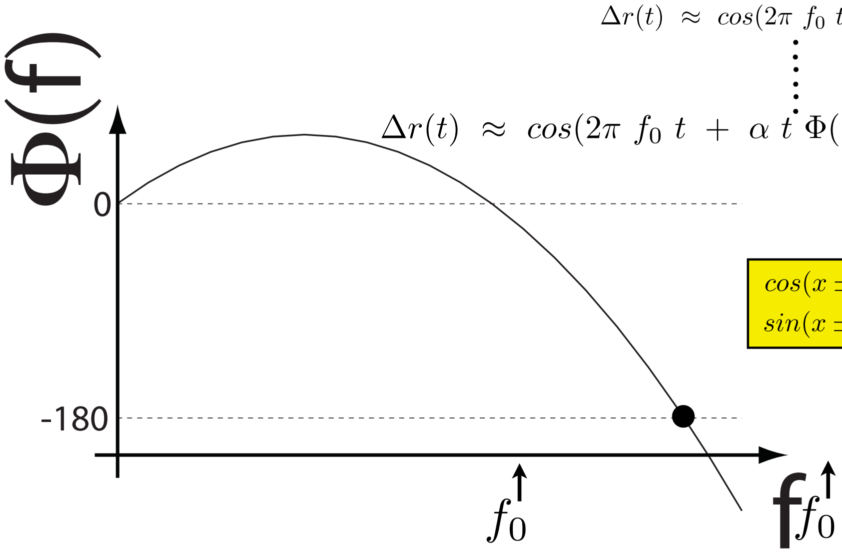
$\Delta i(t) \approx cos(2\pi\ f_0\ t)$



instantaneous
firing rate
(i.e. firing probability)

$$\Delta r(t)\ \approx\ cos(2\pi\ f_0\ t\ +\ \Phi(f_0))$$

$$\Delta r(t)\ \approx\ cos(2\pi\ f_0\ t\ +\ \alpha\ t\ \Phi(f_0)\ +\ \alpha\ t\ \pi)$$

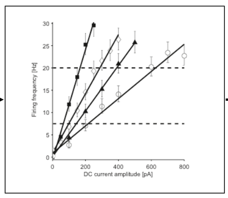


$$cos(x \pm \pi) = -cos(x)$$
$$sin(x \pm \pi) = -sin(x)$$

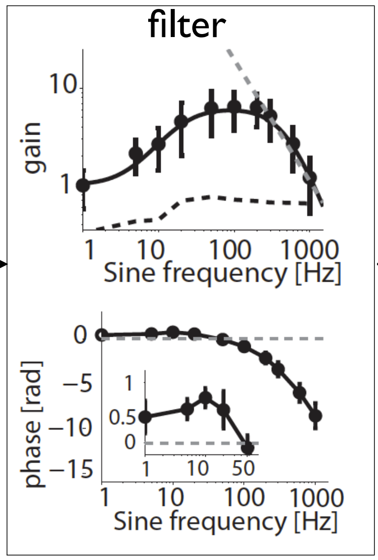
Significance of linear response properties

input

f-I curve



filter

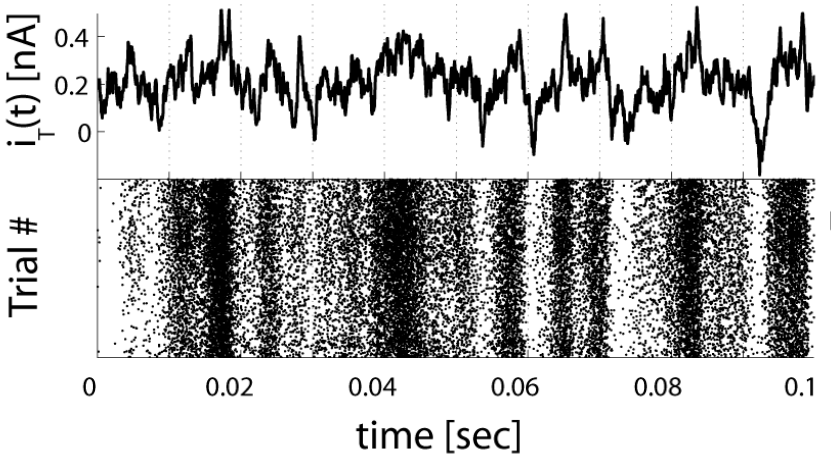


output

$$T(2\pi f)\ =\ K\ \frac{(2\pi f\ -\ z_1)}{(2\pi f\ -\ p_1)\,(2\pi f\ -\ p_2)}$$

Significance of linear response properties

H. Köndgen

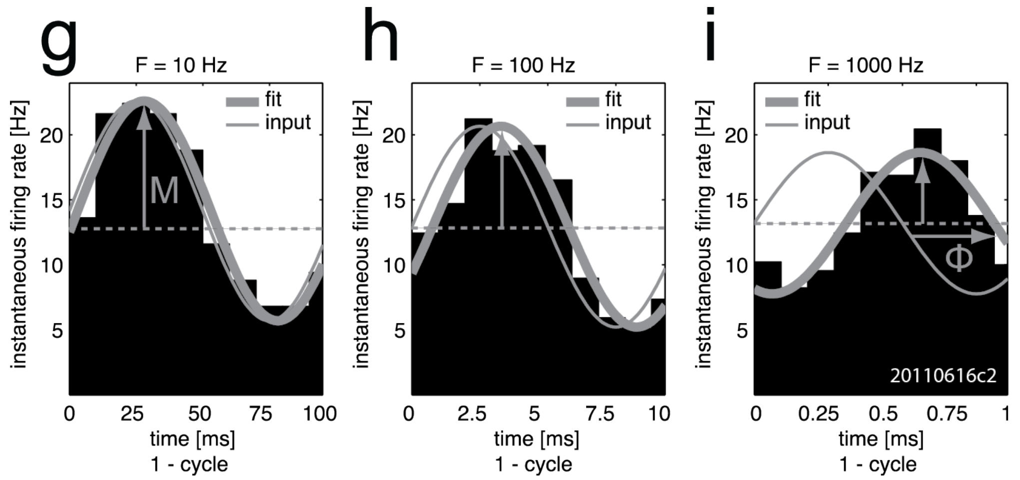


Köndgen et al., 2008, Cereb Cortex

- **A novel exp. description of excitability is imperative for predicting rhythms & reaction times;**
- the **phase** is roughly **independent** across conditions;
- the **magnitude** reveals to be **band-pass, with high cut-off frequency** (~200Hz), despite (capacitive) inertia or low firing rates;
- **background** uncorrelated firing does not speed up arbitrarily neuronal responses, but may boost existing (limited) bandwidth;
- **firing rate** may suppress, not extend, neuronal bandwidth;
- the dynamical properties account, to a large extent, for the neuronal response upon foreground broad-band inputs;

Human cortical neurons

even faster in relying downstream
ultra fast-varying temporal information

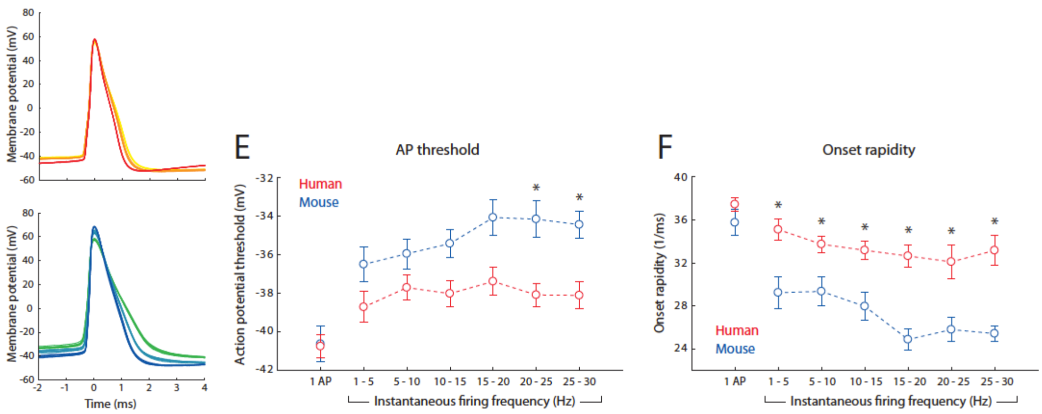


with G. Testa-Silva, H. Mansvelder, C. de Kock, J. Baayen

Human cortical neurons

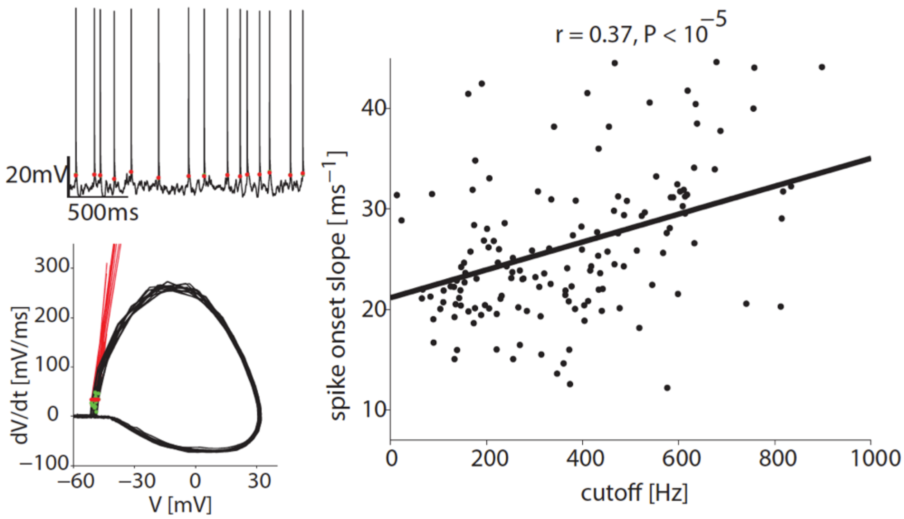
(from "healthy" tissue, Medial Temporal Lobe surgery)

G. Testa-Silva

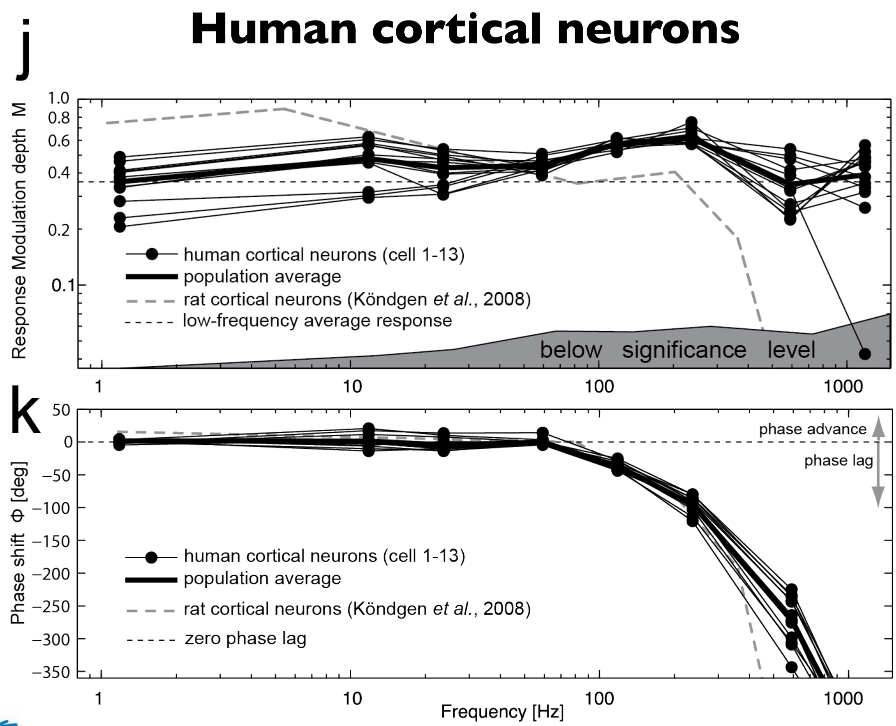


Testa-Silva et al., under review
with H. Mansvelder (Vrij Univ. Amsterdam, NL)

Does the cut-off frequency relates to AP speed-onset?
(and it is independent on the stimulation protocol)



Biro, Linaro, Giugliano et al., in preparation
Naundorf et al., 2005; McCormick et al., 2007; Yu et al., 2008



with G. Testa-Silva, H. Mansvelder, C. de Kock, J. Baayen