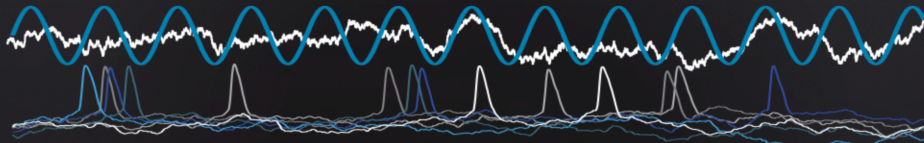


# PRINCIPLES OF COMPUTATIONAL NEUROSCIENCE



AN INTRODUCTORY COURSE OFFERED TO  
(UNDER)GRAD NEUROSCIENCES STUDENTS AT UNITS AND SISSA, TRIESTE (ITALY).



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**Hands-on session (M1)**

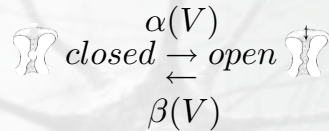


## Charge-balance equation (Neuroelectronics)

$$C \frac{d}{dt} V = [G_{Na}(E_{Na} - V) + G_K(E_K - V) + G_{Ca}(E_{Ca} - V) + \dots]$$

O.D.E., first-order, non-homogeneous,  
(non-linear, time-varying)  
Numerical methods and *in silico* studies

## Ion channels “gating” (Hodgkin-Huxley’s model)



$$\frac{dn}{dt} = -(\alpha + \beta)n + \alpha$$

O.D.E., first-order, non-homogeneous,  
(non-linear, time-varying)

Numerical methods and *in silico* studies

## Numerical solutions of an o.d.e.

### forward Euler’s method

• The independent variable is *discretised*  $x_0, x_1, x_2, \dots, x_{k-1}, x_k, x_{k+1}, \dots$

• e.g. uniformly  $x_k = k \Delta x$

• Derivatives are *approximated*  $\frac{df}{dx} \approx \frac{f(k\Delta x) - f((k-1)\Delta x)}{\Delta x}$

$$\frac{df}{dx} = -30 f(x) \quad \text{o.d.e.} \quad \longrightarrow \quad f_k \approx f_{k-1} - 30 \Delta x f_{k-1}$$

algebraic iterative equation

```
for k=2:N
    f[k] = f[k-1] - 30 * Δx * f[k-1]
end
```

### Joint exercise (formerly “Assignment 1”)

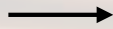
$$\frac{dV}{dt} = 0.15 (-70 - V(t)) + \sin(2 \pi F 0.001 t) + 1$$

$$V(0) = -70$$
$$F = 2 \quad \text{or} \quad F = 200$$

$$\frac{dV}{dt} \approx \frac{V(k\Delta t) - V((k-1)\Delta t)}{\Delta t}$$

$$t \rightarrow (k-1)\Delta t$$

...



...

o.d.e.

algebraic iterative equation

### Exercise (formerly “Assignment 1”)

- **write down on paper** the o.d.e. as a discrete-time (Euler's) approximation
- by a new *Julia-Jupyter Notebook* (inspired from the one provided)
  - **solve** the discrete-time numerical approximation of such an o.d.e.
  - **plot both the graphs of the function  $V(t)$  and of  $u(t) = \sin(2 \pi F 0.001 t) + 1$**
  - **plot also the graphs of  $(V(t)+70)$  and of  $(u(t)-1)$**
  - **describe** in your own words what the solution looks like...
- At home: **document your entire work in Markdown**

### Exercise (formerly “Assignment 1”)

- **beware** of properly “*defining*”  $u(t)$  (think about  $\Delta t$  and  $F$  - choose  $\Delta t$  wisely!!)
- **comment in your own words** every line of the code
- perform/explore **longer simulations**, until a state of “*steadyness*” is reached
- zoom on the last “cycles” of  $V(t)$  in your (long) simulation... and then
  - extract its **peak amplitude** (“peak” means relative to its offset)
- plot the **peak amplitude** for few values of  $F$  (e.g. 2, 5, 8, 10, 20, 50, 80, 100, 200, 500, 1000, 2000, 5000)
- do you “see” what happens to the **phase of  $V(t)$** , with respect to the *phase* of  $u(t)$  ?
- can you **interpret “functionally”** the input( $u$ )-output( $V$ ) transformation you see?