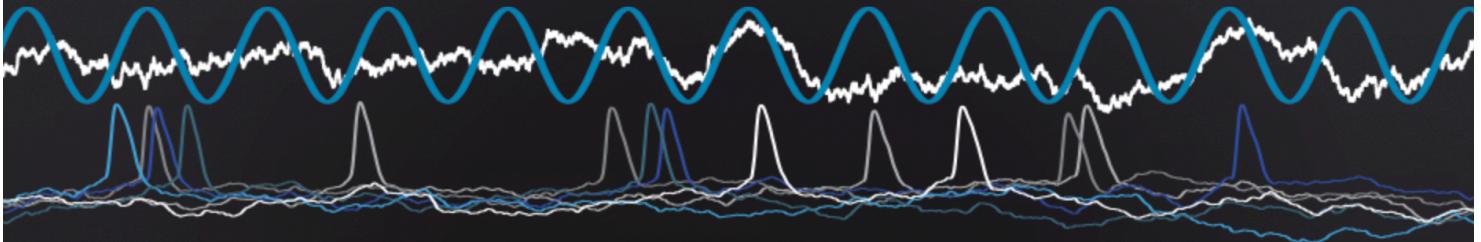


ELECTROPHYSIOLOGICAL SIGNALS



GENERATION AND CHARACTERISATION

Michele GIUGLIANO
Excitability Preliminaries

ATTENDANCE TRACKING - **code ???**
(for statistical purposes only)

<https://www.unimore.it/it/servizi/unimore-app>

Digression on (**mesoscopic**) (chemical) reactions (i.e., alias kinetic schemes): population descriptions!



Cato Maximilian **Guldberg**
Peter **Waage**, (Norway) 1865

The Mass Action Law

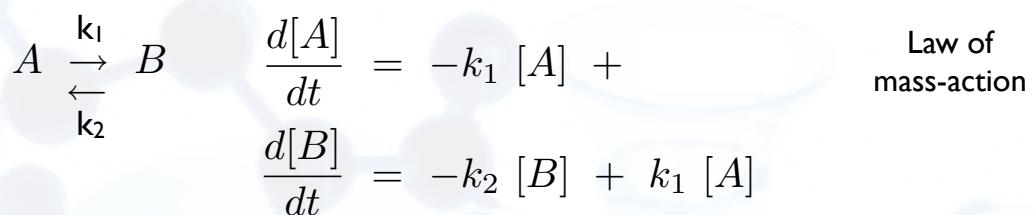
*if A reacts with B to form substance C,
then the rate of production of C is
proportional to the amount of A & B*



$$\begin{aligned} \text{Rate of Prod. of } C &= \\ &= k [A] [B] \end{aligned}$$

It follows from first principles of equilibrium thermodynamics.

Digression on (**mesoscopic**) (chemical) reactions (i.e., alias kinetic schemes): population descriptions!



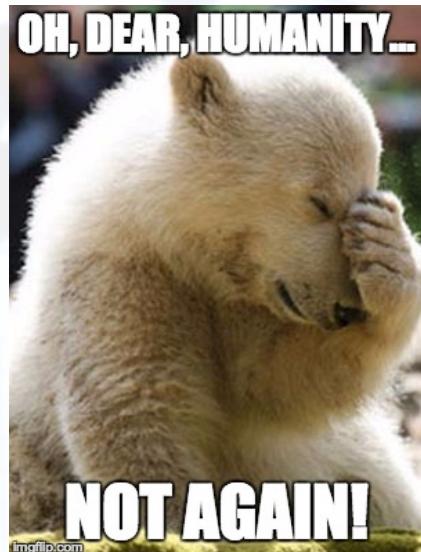
$$[A] + [B] = \text{constant} = W \quad \text{mass conservation!!} \quad a = \frac{[A]}{W} \quad \text{fraction}$$

$$\frac{d[A]}{dt} = -k_1 [A] + k_2 (W - [A])$$

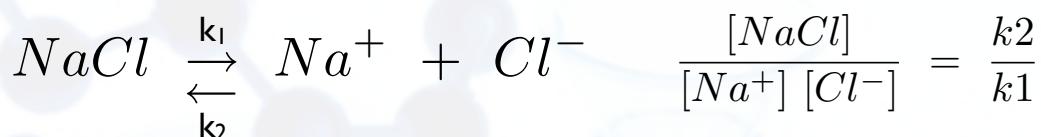
$$\frac{da}{dt} = -k_1 a + k_2 (1 - a) \quad \frac{da}{dt} = -(k_1 + k_2) a + k_2$$

The *reactions* are described at the ensemble level as first-order kinetics phenomena: they depend only on the concentration of the “reactants”.

$$\frac{da}{dt} = - (k_1 + k_2) a + k_2$$



Digression on (**mesoscopic**) (chemical) reactions (i.e., alias kinetic schemes): population descriptions!



$$[Na^+] = ??? \frac{\text{moles}}{\text{litre}} \quad \begin{aligned} \text{Avogadro's number} \\ 1 \text{ mole} &= 6.022 \cdot 10^{23} \text{ molecules} \\ 1 \text{ litre} &= 1 \text{ dm}^3 = 1000 \text{ cm}^3 \end{aligned}$$

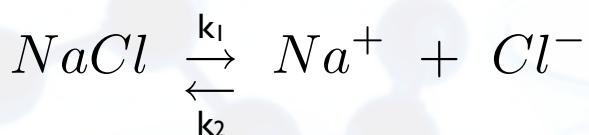


$$vol \ [Na^+] = ??? \text{ moles} = ??????? \text{ molecules}$$

A chemical reaction describes when (the density, or concentration of) “something” is *transformed* into “something else” (or gets another “form”).

$A \xrightarrow{k} C$ $\frac{d}{dt}[A] = -k [A]$ $\frac{d}{dt}[C] = k [A]$ $[A] + [C] = \text{fixed}$	$A \xrightleftharpoons[k_2]{k_1} C$ $\frac{d}{dt}[A] = -k_1 [A] + k_2 [C]$ $\frac{d}{dt}[C] = k_1 [A] - k_2 [C]$ $[A] + [C] = \text{fixed}$
$A + B \xrightarrow{k} C$ $\frac{d}{dt}[A] = -k [A][B]$ $\frac{d}{dt}[B] = -k [A][B]$ $\frac{d}{dt}[C] = k [A][B]$ $[A] + [B] + 2[C] = \text{fixed}$	$A + B \xrightleftharpoons[k_2]{k_1} C$ $\frac{d}{dt}[A] = -k_1 [A][B] + k_2 [C]$ $\frac{d}{dt}[B] = -k_1 [A][B] + k_2 [C]$ $\frac{d}{dt}[C] = k_1 [A][B] - k_2 [C]$ $[A] + [B] + 2[C] = \text{fixed}$

**Kinetic scheme is equivalent to a
(system of) o.d.e.!**



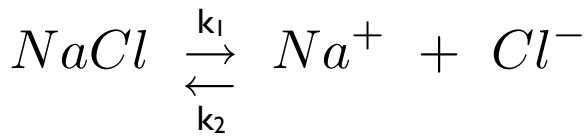
$$\frac{d[NaCl]}{dt} = -k_1[NaCl] + k_2[Na^+][Cl^-]$$

at steady-state ...

$$0 = -k_1[NaCl] + k_2[Na^+][Cl^-]$$

rearranging the terms proves the relationship:

$$\frac{[NaCl]}{[Na^+] [Cl^-]} = \frac{k_2}{k_1}$$



$$\frac{d[NaCl]}{dt} = -k_1[NaCl] + k_2[Na^+][Cl^-]$$

$$\frac{d[Na^+]}{dt} = k_1[NaCl] - k_2[Na^+][Cl^-]$$

heuristics: "sum twice the first equation and the remaining two equations"...

$$\frac{d[Cl^-]}{dt} = k_1[NaCl] - k_2[Na^+][Cl^-]$$

...subtracting the 2nd and 3rd equations:

$$[Na^+] = [Cl^-] + \text{constant}$$

...summing the first and the second:

$$[Na^+] + [NaCl] = \text{constant}$$

...summing the first and the third:

$$[Cl^-] + [NaCl] = \text{constant}$$

...summing the above two last expressions:

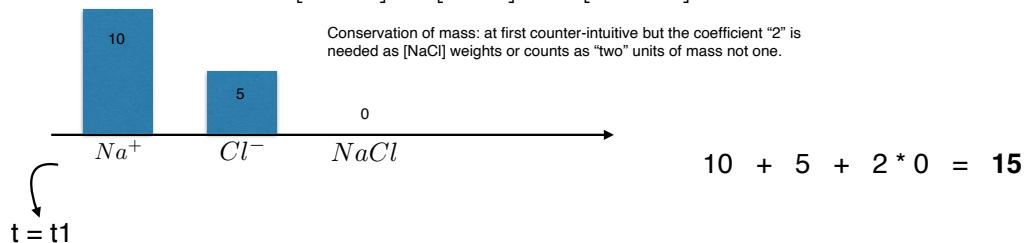
$$[Na^+] + [Cl^-] + 2[NaCl] = \text{constant}$$

Conservation of mass: in agreement with our intuition

$t = t_0$

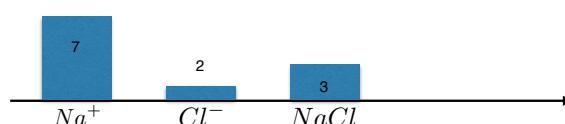
$$[Na^+] + [Cl^-] + 2[NaCl] = \text{constant}$$

Conservation of mass: at first counter-intuitive but the coefficient "2" is needed as $[NaCl]$ weights or counts as "two" units of mass not one.



$t = t_2$

$$7 + 2 + 2 * 3 = 15$$





$$\frac{1}{n} \frac{d}{dt}[A] = -k_1 [A]^n[B]$$

$$\frac{d}{dt}[B] = -k_1 [A]^n[B]$$

$$\frac{d}{dt}[C] = k_1 [A]^n[B]$$

$$[A] + [B] + (n+1)[C] = fixed$$



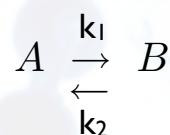
$$\frac{1}{n} \frac{d}{dt}[A] = -k_1 [A]^n[B] + k_2 [C]$$

$$\frac{d}{dt}[B] = -k_1 [A]^n[B] + k_2 [C]$$

$$\frac{d}{dt}[C] = k_1 [A]^n[B] + -k_2 [C]$$

$$[A] + [B] + (n+1)[C] = fixed$$

**What about a microscopic,
i.e., single-molecule description?**



$$Pr\{A \rightarrow B \text{ in } (t ; t + \Delta t) / \text{state } A \text{ at } t\} = k_1 \Delta t + O(\Delta t)$$

$$Pr\{B \rightarrow A \text{ in } (t ; t + \Delta t) / \text{state } B \text{ at } t\} = k_2 \Delta t + O(\Delta t)$$

$$Pr\{> \text{ one transition in } (t ; t + \Delta t)\} \approx 0$$

$$Pr = f(\Delta t) \approx = 0 + k \Delta t + \dots$$

Same notation, but different meaning:
NOT concentrations/densities, but occupancy probabilities.