Squared Coefficient of Variation

- Provides a normalized measure used to estimate of variance of a process (demand, production, M/C repair, etc.)

$$c = \frac{\sigma}{t} = \text{coefficient of variation (CV)}$$

$$c^2 = \frac{\sigma^2}{t^2} = \text{squared coefficient of variation (SCV)}$$

$$= \begin{cases} 0, & \text{deterministic/exactly (best case, } LB\text{)} \\ < 0.75 & \text{low variability} \\ \geq 0.75, < 1.33, & \text{moderate variability} \\ 1, & \text{Poisson} \Leftrightarrow \text{totally random (practical worse case, } UB\text{)} \\ \geq 1.33, & \text{high variability (bad control)} \end{cases}$$

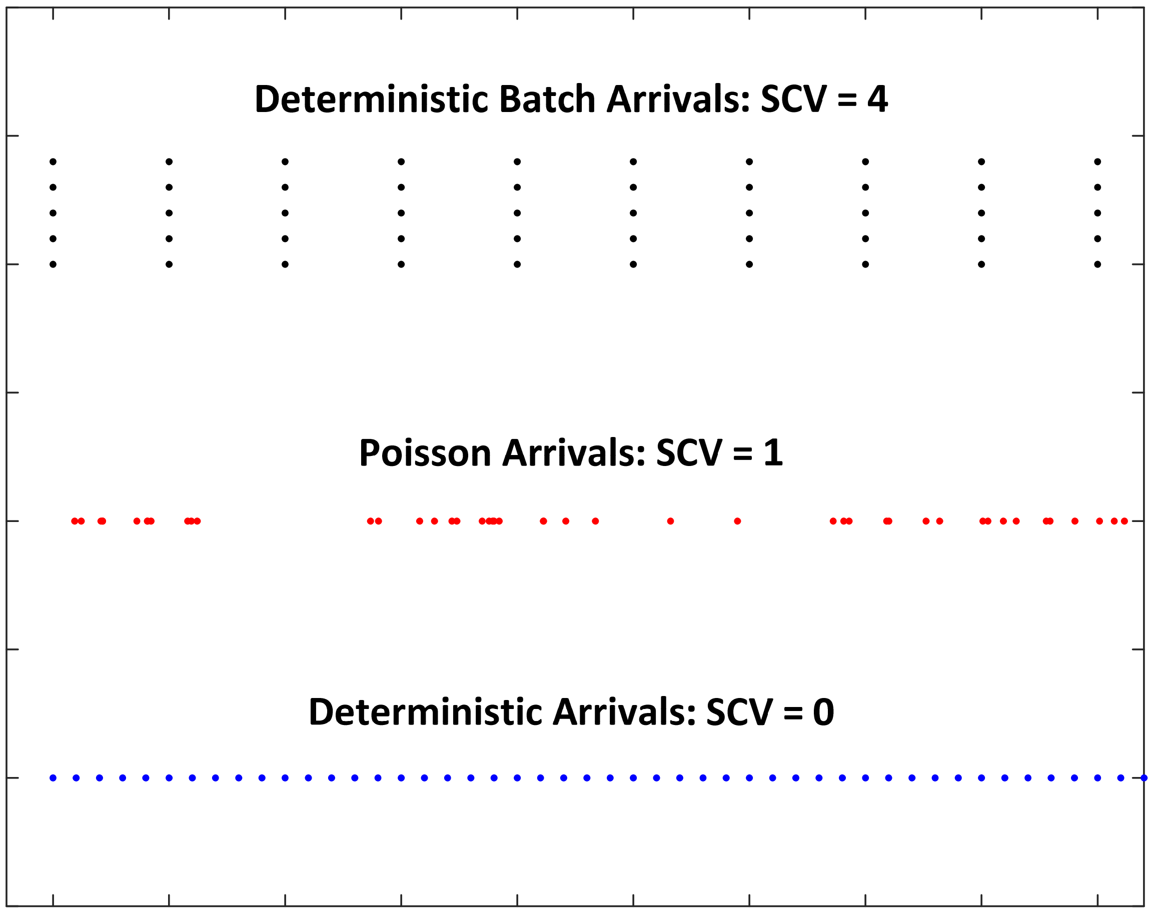
$$\sigma = \text{standard deviation of process}$$

$$t = \text{mean of process}$$

$$\sigma^2 = \text{variance of process}$$

Low, Moderate and High SCVs

- All arrivals have same rate of 10 per hour



Cycle-Time Estimation

$$t_{CT} = \underbrace{t_{CT_q}}_{\text{queuing time}} + \underbrace{t_e}_{\text{process time}} = \begin{cases} \underbrace{\left(\dfrac{c_a^2 + c_e^2}{2}\right)}_{\text{variability}} \underbrace{\left(\dfrac{u}{1-u}\right)}_{\text{utilization}} \underbrace{t_e}_{\text{time}} + t_e, & \text{if } m = 1 \\ \left(\dfrac{c_a^2 + c_e^2}{2}\right) \left[\dfrac{u^{\left(\sqrt{2(m+1)}-1\right)}}{m(1-u)}\right] t_e + t_e, & \text{if } m \geq 1 \end{cases}$$

$$c_a^2 = \frac{\sigma_a^2}{t_a^2} = \text{arrival SCV}$$

$$t_a = \frac{1}{r_a} = \text{mean time between arrivals}, \quad \sigma_a^2 = \text{variance of arrival time}$$

$$c_e^2 = c_0^2 + (1 + c_r^2) A (1 - A) \frac{MTTR}{t_0}$$

$$c_0^2 = \frac{\sigma_0^2}{t_0^2} = \text{natural process time SCV}, \quad \sigma_0^2 = \text{variance of natural process time}$$

$$c_r^2 = \frac{\sigma_r^2}{MTTR^2} = \text{repair time SCV}, \quad \sigma_r^2 = \text{variance of repair time}$$

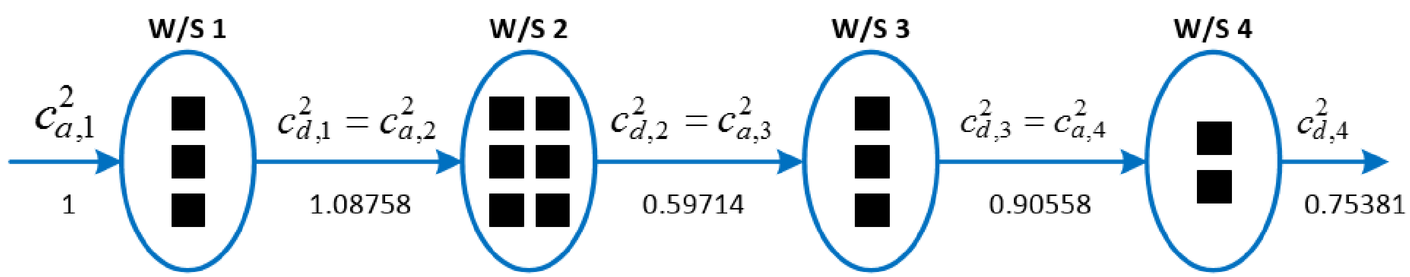
Min M/C W/S Cycle Time Example

	A	B	C	D	E
1	Arrival Rate	(r_a, q/hr)	18.75		=C15/C14
2	Arrival SCV	(c^2_a)	1		1
3	Natural Process Time	(t_0, hr)	0.2		=12/60
4	Var of Nat Proc Time	(hr^2)	0.04		=C3^2
5	Natural Process SCV	(c^2_0)	1		=C4/C3^2
6	MTTF	(hr)	20		20
7	MTTR	(hr)	2		2
8	Repair Time SCV	(c^2_r)	2.5		=10/C7^2
9	Availability	(A)	0.909091		=IF(ISBLANK(C6), 1, C6/(C6 + C7))
10	Effective Process Time	(t_e, hr)	0.22		=C3/C9
11	Eff Process Time SCV	(c^2_e)	3.892562		=C5+(1+C8)*C9*(1-C9)*C7/C3
12	Number of M/C	(m)	5		=FLOOR(C1*C10 + 1,1)
13	Utilization	(u)	0.825		=C1*C10/C12
14	Yield	(y)	0.8		0.8
15	Departure Rate (r_a*y)	(r_d, q/hr)	15		=120/8
16	Departure SCV	(c^2_d)	1.880452		=1 + (1 - C13^2)*(C2 - 1) + (C13^2/SQRT(C12))*(C11 - 1)
17	Cycle Time in Queue	(CT_q, hr)	0.382873		=((C2 + C11)/2)*((C13^(SQRT(2*(C12 + 1)) - 1))/(C12*(1 - C13)))*C10
18	Cycle Time at W/S	(CT, hr)	0.602873	(a)	=C17+C10

Departure SCV

- In a line of W/S, only first upstream sees demand variability
  - all other downstream W/S see mix of demand and upstream W/S processing variability

$$\textbf{Departure SCV:}\quad c_d^2 = \begin{cases} u^2 c_e^2 + \left(1-u^2\right) c_a^2, & \text{if } m = 1 \\ 1 + \left(1-u^2\right)\left(c_a^2 - 1\right) + \dfrac{u^2}{\sqrt{m}}\left(c_e^2 - 1\right), & \text{if } m \geq 1 \end{cases}$$



Cycle Time and Total Machine Cost Estimation

W/S		1	2	3	4	Total
Arrival Rate	(r_a, q/hr)	14.0407	11.9346	10.7411	10.2041	
Arrival SCV	($c^2{}_a$)	1	1.08758	0.59714	0.90558	
Natural Process Time	(t_0, hr)	0.2	0.5	0.25	0.15	1.1
Natural Process SCV	($c^2{}_0$)	0.25	0	0	0.5	
MTTF	(hr)	40		100		
MTTR	(hr)	2	0	5	0	
Repair Time SCV	($c^2{}_r$)	1	0	0	0	
Availability	(A)	0.95238	1	0.95238	1	
Effective Process Time	(t_e, hr)	0.21	0.5	0.2625	0.15	1.1225
Eff Process Time SCV	($c^2{}_e$)	1.15703	0	0.90703	0.5	
Number of M/C	(m)	3	6	3	2	
Utilization	(u)	0.98285	0.99455	0.93985	0.76531	
Yield	(y)	0.85	0.9	0.95	0.98	
Departure Rate (r_a*y)	(r_d, q/hr)	11.9346	10.7411	10.2041	10	
Departure SCV	($c^2{}_d$)	1.08758	0.59714	0.90558	0.75381	
Cycle Time in Queue	(CT_q, hr)	4.26486	8.19096	0.97673	0.15241	13.58496
Cycle Time at W/S	(CT, hr)	4.47486	8.69096	1.23923	0.30241	14.70746
WIP in Queue (r_a*CT_q)	(q)	59.8816	97.7558	10.4912	1.55518	169.6839
WIP at W/S	(q)	62.8302	103.723	13.3108	3.08579	182.9499
M/C Cost	($000)	12	18	2	6	
W/S Cost	($000)	36	108	6	12	162