Simple Production System

- Handles MTS and (with FGI = 0) service and MTO
- Producer makes two decisions:
  1. Production rate
  2. Maximum finished goods inventory (FGI) level
- Control logic for producer:
  – **If** customer order is waiting, produce;
  else, if FGI level < max level, produce;
  otherwise, shutdown production.
- Customer fulfilment process:
  – **If** FGI level > 0, fulfill from FGI;
  else, wait for order to be produced
  (getting a discount in price based on wait time)

Production System Design Model

$$\max_{r_e, q_{FG}^{\max}} TP = (p-c)\left[1-\overbrace{\pi_0 + \pi_0}^{\text{inventory}}\overbrace{(1-g t_{CT})}^{\text{time}}\right] r_d - \overbrace{(k+c) h q_{FG}}^{\text{inventory}} - \overbrace{k r_e}^{\text{capacity}}$$

where

r_e = capacity of production system

$q_{FG}^{\max}$ = maximum FGI held

p = unit sales price

c = unit operating cost

$\pi_0(r_e, q_{FG}^{\max})$ = probability out of (FGI) stock

g = delay discount factor

$t_{CT}(r_e)$ = cycle time of production system

r_d = departure rate

k = capital cost per unit of capacity

h = inventory carrying rate

$q_{FG}(r_e, q_{FG}^{\max})$ = average FGI level

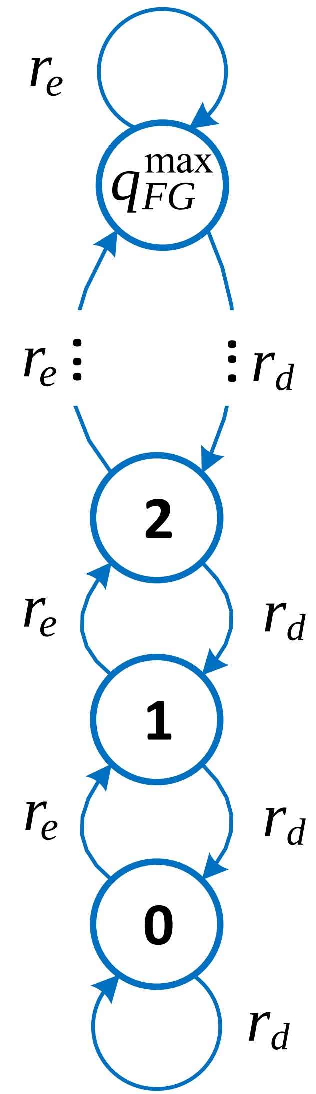
Poisson FG Inventory Model

- Finite birth-death process
  - production = birth
  - demand = death
  - birth (production) > death (demand)
- Poisson demand and production

$$\pi_0(r_e, q_{FG}^{\max}) = \frac{1 - \frac{r_e}{r_d}}{1 - \left(\frac{r_e}{r_d}\right)^{q_{FG}^{\max}+1}}$$

$$\pi_n = \pi_0 \left(\frac{r_e}{r_d}\right)^n, \quad \text{prob. } n \text{ units FGI}$$

$$q_{FG}(r_e, q_{FG}^{\max}) = \sum_{i=1}^{q_{FG}^{\max}} i\,\pi_i = \pi_0 \sum_{i=1}^{q_{FG}^{\max}} i \left(\frac{r_e}{r_d}\right)^i$$



Single-Machine Poisson Model

- Note: all costs p, c, and k are independent of r_d and r_e

$$\max_{r_e, q_{FG}^{\max}} TP = (p-c)\left[1-\pi_0+\pi_0\left(1-gt_{CT}\right)\right]r_d-(k+c)hq_{FG}-kr_e$$

$$\text{where} \quad \pi_0(r_e, q_{FG}^{\max}) = \frac{1-\dfrac{r_e}{r_d}}{1-\left(\dfrac{r_e}{r_d}\right)^{q_{FG}^{\max}+1}}, \quad [0,1]$$

$$t_{CT}(r_e) = \left(\frac{r_a}{r_e-r_a}\right)\left(\frac{1}{r_e}\right)+\left(\frac{1}{r_e}\right)$$

$$q_{FG}(r_e, q_{FG}^{\max}) = \pi_0 \sum_{i=1}^{q_{FG}^{\max}} i\left(\frac{r_e}{r_d}\right)^i$$

- Since $r_e > r_a$ and assuming $r_a = r_d$, $kr_e > kr_d$ in TP $\Rightarrow$ $\boxed{TP_{UB} = (p-c-k)r_d}$

Example of Model

- Both production rate and max FGI can be optimized

		Base	Opt FGI	Opt Cap	Opt
Unit Sales Price	(p, \$/q)	70	70	70	70
Unit Operating Cost	(c, \$/q)	50	50	50	50
Unit Capital Cost	(k, \$/q)	1	1	1	1
Discount Factor	(g)	0.2	0.2	0.2	0.2
Inventory Carrying Rate	(h)	0.01	0.01	0.01	0.01
Demand Rate	(r_f, q/hr)	10	10	10	10
Effective Production Rate	(r_e, q/hr)	15	15	10.7825	12.0739
Maximum FGI	(q^{max}_{FG})	20	3	20	6
Probability Out of FGI	(π_0)	0.0001	0.123077	0.020244	0.075675
Cycle Time	(t_{CT})	0.2	0.2	1.277955	0.482183
Average FGI Level	(q_{FG})	18.00421	1.984615	12.65341	3.732418
Total Profit	(TP, \$)	175.8171	183.0032	181.7294	184.563
Upper Bound on TP	(TP_{UB}, \$)	190	190	190	190
Utilization	(u)	0.666667	0.666667	0.927429	0.828233
Throughput	(r_d, q/hr)	10	10	10	10
WIP	(q_{WIP})	2	2	12.77955	4.821833

Example: Impact of Buffering Cost

		Buffering Cost: High/Low Capacity-Time-Inventory (k, g, h)							
		LLL	LLH	LHL	LHH	HLL	HLH	HHL	HHH
Unit Sales Price	(p , \$/q)	70	70	70	70	70	70	70	70
Unit Operating Cost	(c, \$/q)	50	50	50	50	50	50	50	50
Unit Capital Cost	(k , \$/q)	1	1	1	1	5	5	5	5
Discount Factor	(g)	0.01	0.01	0.7	0.7	0.01	0.01	0.7	0.7
Inventory Carrying Rate	(h)	0.00015	0.3	0.00015	0.3	0.00015	0.3	0.00015	0.3
Demand Rate	(r_a , q/hr)	10	10	10	10	10	10	10	10
Effective Production Rate	(r_e , q/hr)	10.1896	11.4127	10.3923	19.0637	10.062	10.629	10.0377	13.0747
Maximum FG Inventory	(q^{max}_{FG})	44	0	97	1	81	0	201	2
Probability Out of FGI	(π_0)	0.014272	1	0.000925	0.344072	0.009394	1	0.003311	0.248945
Cycle Time	(t_{CT})	5.274262	0.707864	2.54907	0.11033	16.12903	1.589825	26.5252	0.325235
Average FGI Level	(q_{FG})	25.13087	0	73.81924	0.655928	43.94809	0	113.1733	1.176621
Total Profit	(TP , \$)	189.4746	187.1695	188.8777	162.5376	149.0792	143.6847	148.3004	100.8451
Upper Bound on TP	(TP_{UB}, \$)	190	190	190	190	150	150	150	150
Utilization	(u)	0.981393	0.876217	0.962251	0.524557	0.993838	0.940822	0.996244	0.764836

- Both r_e and $q^{\max}_{FG}$ selected to maximize TP
- $q^{\max}_{FG}$ restricted to non-negative integers

Inventory Carrying Rate

- Rate (h) = sum of interest + warehousing + obsolescence rate
- Interest: 5% per Total U.S. Logistics Costs
- Warehousing: 6% per Total U.S. Logistics Costs
- Obsolescence: default rate $h_{\text{annual}} = 0.3 \Rightarrow h_{\text{obs}} \approx 0.2$
  - Low FGI cost (hr): $h = h_{\text{annual}}/H = 0.3/2000 = 0.00015$ (H = oper. hr/yr)
  - High FGI cost (hr): $h = h_{\text{obs}}$, can ignore interest & warehousing
  - Estimate h_{obs} using "percent-reduction interval" method: given time t_h when product loses x_h-percent of its original value v, find h ($h_{\text{obs}} \approx h$)

$$ht_h v = x_h v \Rightarrow ht_h = x_h \Rightarrow \boxed{h = \frac{x_h}{t_h}}, \quad \text{and} \quad t_h = \frac{x_h}{h}$$

  - Example: If a product loses 80% of its value after 2 hours 40 minutes:

$$t_h = 2 + \frac{40}{60} = 2.67 \text{ hr} \Rightarrow h = \frac{x_h}{t_h} = \frac{0.8}{2.67} = 0.3$$

  - Important: t_h should be in same time units as t_{CT}

Extensions

- Extensions to the basic model allow it to handle more realistic production scenarios:
  - Multiple identical machines
  - Non-Poisson demand and production
  - Serial production lines