ICA 4: MILP Formulation of UFL

ISE 453: Design of PLS Systems Fall 2018

Create a worksheet within a single spreadsheet that duplicates the following worksheet. This ICA has one question that can be answered using the worksheet you create.

UFL

Given m EFs and n sites at which NFs can be established, the uncapacitated facility location (UFL) problem can be formulated as the following mixed-integer linear programming (MILP) problem:

$$\min \quad \sum_{i \in N} k_i y_i + \sum_{i \in N} \sum_{j \in M} c_{ij} x_{ij}$$

$$\text{s.t.} \quad \sum_{i \in N} x_{ij} = 1, \qquad j \in M$$

$$y_i \geq x_{ij}, \qquad i \in N,\ j \in M$$

$$0 \leq x_{ij} \leq 1, \qquad i \in N,\ j \in M$$

$$y_i \in \{0,1\}, \qquad i \in N$$

where

k_i = fixed cost of NF at site $i \in N = \{1,...,n\}$

c_{ij} = variable cost from i to serve EF $j \in M = \{1,...,m\}$

$$y_i = \begin{cases} 1, & \text{if NF established at site } i \\ 0, & \text{otherwise} \end{cases}$$

x_{ij} = fraction of EF j demand served from NF at site i.

The UFL problem is a MILP because the y_i's are binary variables and the x_{ij}'s are real variables.

p-Median

When the number of NFs is specified and all of fixed costs are identical (or not stated), then the fixed costs will have no impact on the location decision and can be set to zero and the following constraint can be added to the UFL problem to formulate the *p-median problem*:

$$\sum_{i=1}^{n} y_i = p\,,$$

where p is the number of NFs to be located.

OpenSolver

OpenSolver (link on course webpage) is an open source alternative to Excel's Solver that can be used to solve mixed-integer linear programming (MILP) problems. Excel's Solver works for nonlinear optimization, but it is limited to solving MILPs with only a *very* few integer valuables. Unzip the OpenSolver download and place files in folder (Win): C:\Users\[username]\AppData\Roaming\Microsoft\Excel\XLSTART, which is a trusted location and will allow *OpenSolver* to load at startup just like Solver. Note:

- The folder "\AppData" is usually hidden, so you need to type the folder name in the address of the window.
- You only need the *OpenSolver.xlam* file and the *Solvers* folder in XLSTART.

- Press Control key to select multiple "Variable Cells" in Model.
- Clear **y** and **X** cells in worksheet before solving model.

UFL Example

Given $n = 6$ sites, $m = 6$ EFs, and fixed (k) and variable costs (c), the optimal solution is to establish two NFs at sites 3 and 6 serving EFs 2–4 and EFs 1, 5, and 6, respectively, for a $TC = 31$:

$$[k_i] = \begin{bmatrix} 8 \\ 8 \\ 10 \\ 8 \\ 9 \\ 8 \end{bmatrix}; \quad [c_{ij}] = \begin{bmatrix} 0 & 3 & 7 & 10 & 6 & 4 \\ 3 & 0 & 4 & 7 & 6 & 7 \\ 7 & 4 & 0 & 3 & 6 & 8 \\ 10 & 7 & 3 & 0 & 7 & 8 \\ 6 & 6 & 6 & 7 & 0 & 2 \\ 4 & 7 & 8 & 8 & 2 & 0 \end{bmatrix}; \quad [y_i] = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}; \quad [x_{ij}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Cells for **k** and **C** are the only inputs, and **y** and **X** cells are populated when the model is solved.

TC = =SUMPRODUCT(C4:C9,B4:B9) + SUMPRODUCT(F13:K18,F4:K9)

Uncapacitated Facility Location Example

Site	y	k
1	b 0	8
2	b 0	8
3	b 1	10
4	b 0	8
5	b 0	9
6	b 1	8
Sum	2	

X	1	2	3	4	5	6
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	1	1	1	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	1	0	0	0	1	1
Sum	1	1	1	1	1	1

1≥ (rows 1–6) 1= (Sum)

$x_{ij} - y_i$						
0≥	0	0	0	0	0	0
	0	0	0	0	0	0
	-1	0	0	0	-1	-1
	0	0	0	0	0	0
	0	0	0	0	0	0
	0	-1	-1	-1	0	0

TC = min 31

C	1	2	3	4	5	6
1	0	3	7	10	6	4
2	3	0	4	7	6	7
3	7	4	0	3	6	8
4	10	7	3	0	7	8
5	6	6	6	7	0	2
6	4	7	8	8	2	0

Objective Cell: $B$12

Variable Cells: $B$4:$B$9,$F$4:$K$9

Constraints:

<Add new constraint>
$M$4:$R$9 <= 0
$B$4:$B$9 bin
$F$4:$K$9 <= 1
$F$10:$K$10 = 1

Worksheet

Create a worksheet that duplicates the one above, which represents the optimal solution found by OpenSolver to the UFL example problem.

Question:

1. What is the *TC* if there must be exactly three NFs (the *p*-median problem, with $p = 3$)?