ICA 3: Great Circle Distances

ISE 453: Design of PLS Systems — Fall 2018

Create three separate worksheets within a single spreadsheet that duplicate the following worksheets (the final worksheet is a good template to keep and duplicate for later assignments). This ICA has three questions that can be answered using the third worksheet.

Worksheet 1: A product, that will use all ubiquitous raw materials, is to be produced in a plant that will be located along I-40. Finished product will be shipped from the plant to customers in Asheville, Winston-Salem, Durham, and Wilmington. The total demand of these customers is 150, 120, 75, and 40 tons, respectively, and it costs \$0.10 per ton-mile to ship finished goods from the plant. Determine where the plant should be located by creating a worksheet that duplicates the one below (where only shaded cells are inputs and all others are calculated), and then use Solver to determine the "optimal" location for the NF (the plant location) by changing the NF location cell (initial value 150) to minimize the total transport cost target cell (initial value \$3,960):

	x (mi mark)	**d** (mi)	**q** (tons)	**r** (\$/ton-mi)	**w** (\$/mi)	**TC** (\$)
NF	150					3,960.00
Asheville	50	100	150	0.10	15.00	1,500.00
Winston-Salem	190	40	120	0.10	12.00	480.00
Durham	270	120	75	0.10	7.50	900.00
Wilmington	420	270	40	0.10	4.00	1,080.00

Worksheet 2: This worksheet calculates the total great-circle distance from Raleigh, NC to Gainesville, FL, Baghdad, Iraq, and Rio de Janeiro, Brazil (your worksheet should *exactly* match the one below; if not, then there is an error somewhere).

	dd	**mm**	**ss**		**x** (deg)	**x** (rad)	**dd**	**mm**	**ss**		**y** (deg)	**y** (rad)	**d**(rad)	**d** (mi)	**q**	**r**	**w**	**TC**
Raleigh	78	39	32	W	-78.6589	-1.37286	35	49	19	N	35.82194	0.625211						11,561.33
Gainesville	82	20	11	W	-82.3364	-1.43704	29	40	27	N	29.67417	0.517912	0.120086	475.0745	1	1.00	1.00	475.07
Baghdad	44	22		E	44.36667	0.774344	33	14		N	33.23333	0.580031	1.6197	6407.166	1	1.00	1.00	6,407.17
Rio de Janeiro	43	12		W	-43.2	-0.75398	22	57		S	-22.95	-0.40055	1.181038	4679.089	1	1.00	1.00	4,679.09

$$(lon_1, lat_1) = (x_1, y_1), \quad (lon_2, lat_2) = (x_2, y_2)$$

$$x_{\text{deg}} = \begin{cases} DD + \dfrac{MM}{60} + \dfrac{SS}{3,600}, & \text{if } E \text{ or } N \\ -DD - \dfrac{MM}{60} - \dfrac{SS}{3,600}, & \text{if } W \text{ or } S \end{cases}$$

$$\begin{aligned} d_{rad} &= (\text{great circle distance in radians of a sphere}) \\ &= \cos^{-1}\left[\sin y_2 \sin y_1 + \cos y_2 \cos y_1 \cos(x_1 - x_2)\right] \end{aligned}$$

$$\begin{aligned} R &= (\text{radius of earth at equator}) - (\text{bulge from north pole to equator}) \\ &= 3,963.34 - 13.35 \sin\left(\frac{y_1 + y_2}{2}\right) \text{ mi}, \quad = 6,378.388 - 21.476 \sin\left(\frac{y_1 + y_2}{2}\right) \text{ km} \end{aligned}$$

$$d_{GC} = \text{distance } (x_1, y_1) \text{ to } (x_2, y_2) = \boxed{d_{rad} \cdot R}$$

$$x_{\text{rad}} = \frac{x_{\text{deg}}}{180}\pi \quad \text{and} \quad x_{\text{deg}} = \frac{x_{\text{rad}} \cdot 180}{\pi}$$

Worksheet 3: This worksheet is the same as the previous except that the first row, with a fixed location for Raleigh, is now replaced with a variable NF location, where the *x* and *y* location (in degrees) cells for the NF are now the cells that should be changed in Solver in order to determine the optimal location for a single NF. (Note: In Solver, the box "Make Unconstrained Variable Non-Negative" should be unchecked.)

	dd	mm	ss		x (deg)	x (rad)	dd	mm	ss		y (deg)	y (rad)	d(rad)	d (mi)	q	r	w	TC
NF					0	0					0	0						12,753.24
Gainesville	82	20	11	W	-82.3364	-1.43704	29	40	27	N	29.67417	0.517912	1.454668	5760.37	1	1.00	1.00	5,760.37
Baghdad	44	22		E	44.36667	0.774344	33	14		N	33.23333	0.580031	0.929845	3681.744	1	1.00	1.00	3,681.74
Rio de Janeiro	43	12		W	-43.2	-0.75398	22	57		S	-22.95	-0.40055	0.834879	3311.126	1	1.00	1.00	3,311.13

Questions:

1. Longitude of optimal NF location in decimal degrees
2. Latitude of optimal NF location in decimal degrees
3. Total cost at optimal NF location