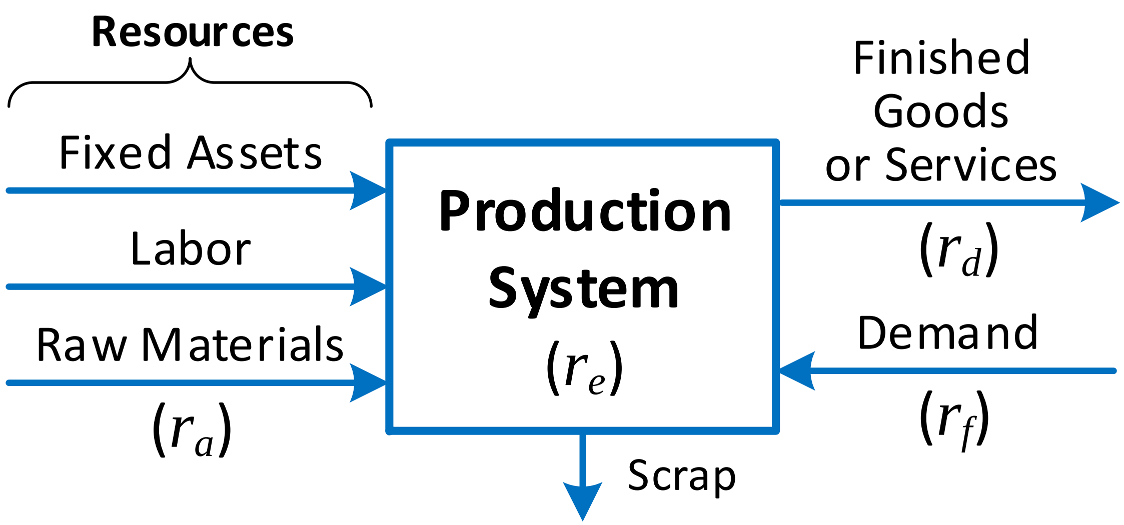
Production System



$$r_e = \text{effective production/service rate } (q/t)$$

$$= \textit{capacity} \text{ of production system}$$

$$r_f = \text{offered demand to production system}$$

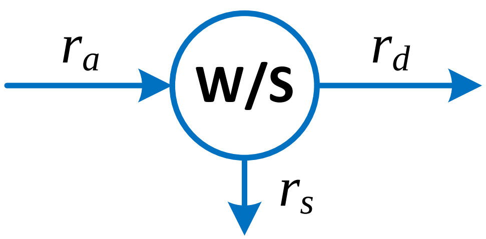
$$r_d = \text{departure rate of demand satisfied by production system}$$

$$r_a = \text{input (arrival) rate of raw material to production system}$$

$$r_a \ge r_d, \text{ where } r_a = r_d \text{ if no } \textit{yield loss} \text{ (true for most service systems)}$$

Yield Loss

- *Yield* is long-run average percentage of nondefective units produced
- Useful to represent production system as series of input-output nodes or *workstations* (W/S) that process "work"



$$r_a = \frac{r_d}{y} = \text{input (arrival) rate of work}$$

$$r_d = \text{output (departure) rate of nondefective work}$$

$$\leq r_f \ \text{(offered demand)}$$

$$y = \text{yield fraction}$$

$$r_s = r_a\left(1-y\right) = \text{scrap rate of defective work}$$

- Over long-run: Need RM at rate r_a (short-run $\Rightarrow$ 50% prob. out of RM)
  Can produce FG at rate r_d
- When scrap identified impacts capacity requirements
  - capacity still needs to be provided to produce scrapped work
- *Rework* of scrap sometimes possible (so it can be fed back in)

Examples: Yield Loss

1. Given a desired output rate of 200 nondefective parts (FG) per hour from a production system, what is required input of RM if the yield fraction is 0.8?

$$r_a = \frac{r_d}{y} = \frac{200}{0.8} = 250 \quad \text{parts per hour}$$

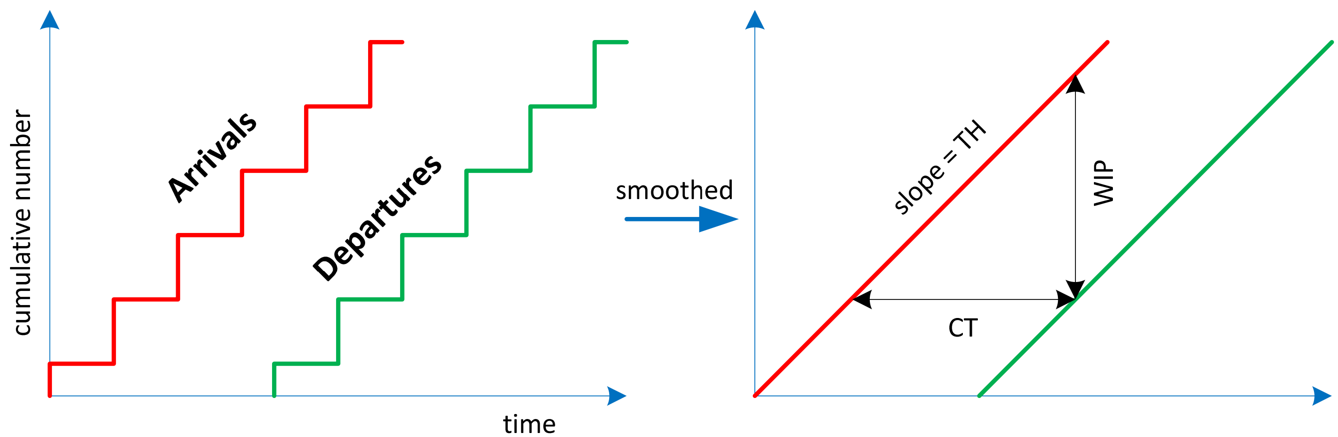
2. Currently, each unit of RM cost \$100 and is used to produce 18,000 units of FG per year with a yield fraction of 0.8. A device is available for \$500,000 to inspect RM that would increase the yield to 0.9. What is the payback of the device?

$$r_{a,\text{current}} = \frac{r_d}{y_{\text{current}}} = \frac{18,000}{0.8} = 22,500, \quad OC_{\text{current}} = \$100 \times r_{a,\text{current}} = \$2,250,000/\text{yr}$$

$$r_{a,\text{new}} = \frac{r_d}{y_{\text{new}}} = \frac{18,000}{0.9} = 20,000, \quad OC_{\text{new}} = \$100 \times r_{a,\text{new}} = \$2,000,000/\text{yr}$$

$$\text{payback} = \frac{IV_0}{OP} = \frac{IV_0}{OC_{\text{current}} - OC_{\text{new}}} = \frac{500,000}{250,000} = 2 \text{ years}$$

Little's Law



$$\textbf{Little's Law:} \quad \boxed{TH(r) = \frac{WIP(q)}{CT(t)}}, \qquad CT = \frac{WIP}{TH}, \quad WIP = TH \cdot CT$$

where $TH = r = throughput$

$= \text{average rate at which work is produced (units per hour)}$

$WIP = q = work\text{-}in\text{-}process$

$= \text{average number of units of work in a production system}$

$CT = t = cycle\ time$

$= \text{average time each unit of work is in a production system}$

Example: Little's Law

1. If the daily output of a production system is 25 units and the average number of units in process in the system is 50, what is the average amount of time each unit of product spends in production?

$$TH = 25 \text{ units/day}$$

$$WIP = 50 \text{ units}$$

$$CT = \frac{WIP}{TH} = \frac{50}{25} = 2 \text{ days}$$

- **Major assumption:** Little's Law only applies to steady state situations
  - doesn't work if TH, WIP, or CT are changing over time

Example: Little's Law

2. Inventory turnover is a measure of the number of times inventory is sold. If the value of inventory for a firm at the beginning of the year was \$10.2 million and \$12.4 million at the end of the year and its annual cost of goods sold was \$226 million, what was its inventory turnover during the year?

$$CT = \frac{WIP}{TH} = \frac{\left(\dfrac{10.2+12.4}{2}\right)}{226} = \frac{11.3}{226} = \frac{1}{20} \Rightarrow 20 \text{ turns per year}$$

Why is it necessary to assume that the difference in beginning and ending inventory is due to random variation and not due to a long-term increase in inventory?

Answer: Little's Law only works if WIP is not changing, and taking the average is a good estimate of the steady state inventory.

Yield-adjusted Throughput

- Since throughput includes work that is ultimately scrapped:

$$\text{Throughput: } r = \begin{cases} r_a\left(= r_d\right), & \text{if } y = 1 \text{ (no scrap)} \\ r_a\left[y + \gamma\left(1 - y\right)\right], & \text{otherwise} \end{cases}$$

$$\text{where } \ \gamma = \left[0,1\right] = \textit{yield occurance factor} = \begin{cases} 0, & \text{if scrap occurs at } \textit{start} \\ 1, & \text{if scrap occurs at } \textit{end} \end{cases}$$

$$r_a = \frac{r_d}{y}, \quad r_d = y r_a$$

$$r = r_d + \gamma\left(r_a - r_d\right) = y r_a + \gamma\left(r_a - y r_a\right) = r_a\left[y + \gamma\left(1 - y\right)\right]$$

- When scrap identified impacts capacity requirements
  - identified at *end* of processing $\Rightarrow \gamma = 1,\ r_e > r_a$
    (safe assumption, true if inspection done after processing)
  - at *any* time $\Rightarrow \gamma = 0.5$ (most likely value if no information)
  - at *start* of processing $\Rightarrow \gamma = 0,\ r_e > r_d$ (no capacity used for scrap)

Example: Little's Law

3. If the daily output of a production system is 18 units, the average number of units in process is 42, the yield fraction is 0.75, and scrap occurs any time during production, what is the average amount of time each unit of product spends in production?

$$TH = r = r_a\left[y + \gamma\left(1 - y\right)\right] = \frac{r_d}{y}\left[y + \gamma\left(1 - y\right)\right]$$

$$= \frac{18}{0.75}\left[0.75 + 0.5\left(1 - 0.75\right)\right] = 24(0.88) = 21 \text{ units/day}$$

$$CT = \frac{WIP}{TH} = \frac{42}{21} = 2 \text{ days}$$