Buffering Cost

Capacity	Time	Inventory	Production System
Low	Low	Low	Home production (a.k.a. putting-out system)
Low	High	Low	Dedicated make-to-stock (mass production)
Low	Low	High	Dedicated make-to-order, Home cooking
Low	High	High	Restaurant
High	Low	Low	Craft production, Process plant (continuous mfg)
High	High	Low	Shared make-to-stock (discrete part mfg)
High	Low	High	Shared make-to-order (job shop), Doctor's office
High	High	High	**Trauma unit at hospital, Additive manufacturing**

- Low capacity cost $\Rightarrow$ *dedicated* capacity for a single product
- High capacity cost $\Rightarrow$ capacity that is *shared* between multiple products
  - requiring set-ups/changeovers between production of batches of each product

Simple Service/Make-to-Order System

- Service and make-to-order systems do not carry FGI
- Producer makes one decision:
  1. Production rate (a.k.a. capacity/design of system)
- Control logic for producer:
  – **If** customer/order is waiting, produce;
    otherwise, shutdown production.
- Customer/order fulfilment process:
  – **Wait** for order to be produced
    (getting a discount in price based on wait time)

Service and Make-to-Order Systems

- Determine production rate that maximizes total profit:

$$\max_{r_e} TP = (p-c)\overbrace{(1-g t_{CT})}^{\text{time}} r_d - \overbrace{k r_e}^{\text{capacity}}$$

where

$$
\begin{aligned}
r_e &= \text{capacity of production system} \\
p &= \text{unit sales price} \\
c &= \text{unit operating cost} \\
g &= \text{delay discount factor} \\
t_{CT}(r_e) &= \text{cycle time of production system} \\
r_d &= \text{departure rate} \\
k &= \text{capital cost per unit of capacity}
\end{aligned}
$$

Delay Discount

$$\left(1 - g t_{CT}\right) = \text{discount applied to unit profit due to delay estimated by cycle time}$$

$$g = \text{delay discount factor } \left(0 \le g \le 1\right)$$

$$t_{CT}(r_e) = \text{cycle time (single machine), require } r_e > r_a \ge r_d \text{ so all demand satisfied}$$

$$= \underbrace{t_{CT_q}}_{\text{queuing time}} + \underbrace{t_e}_{\text{process time}} = \underbrace{\left(\frac{c_a^2 + c_e^2}{2}\right)}_{\text{variability}} \underbrace{\left(\frac{u}{1-u}\right)}_{\text{utilization}} \underbrace{t_e}_{\text{time}} + t_e \quad \text{(more later)}$$

$$= \text{cycle time (single machine + Poisson demand and processing)}$$

$$= \left(\frac{1+1}{2}\right)\left(\frac{\left(r_a/r_e\right)}{1-\left(r_a/r_e\right)}\right)\left(\frac{1}{r_e}\right)+\left(\frac{1}{r_e}\right) = \left(\frac{r_a}{r_e - r_a}\right)\left(\frac{1}{r_e}\right)+\left(\frac{1}{r_e}\right)$$

where $u = \text{utilization} = r_a / r_e$, for single machine (more later)

$t_e = \text{effective process time} = 1/r_e$, for single machine (more later)

$c_a^2, c_e^2 = $ squared coefficient of variation of demand and processing (more later)

$= 1$, for Poisson demand and processing

Delay Discount Factor

- In model, discount represents the reduction in unit operating *profit* for time demand waits to be filled/completed
  – Easier to estimate the deduction in unit *price*, then convert to profit
- Can estimate using "percent-reduction interval" method: given t_g when delay discount results in x_g-percent reduction in the original price p, find (price discount factor) g':

$$g' t_g \, p = x_g \, p \Rightarrow g' t_g = x_g \Rightarrow g' = \frac{x_g}{t_g}$$

- Convert price reduction to unit profit reduction to get g:

$$(p-c)g = pg'$$

$$(p-c)g = p\left(\frac{x_g}{t_g}\right) \Rightarrow \boxed{g = \frac{p x_g}{(p-c)t_g}}, \quad \text{and} \quad t_g = \frac{p x_g}{(p-c)g}$$

  – Important: t_g should be in same time units as t_{CT}

Example: Delay Discount Factor

- Assume p = \$100 and c = \$60
  - Low discount: If one year delay results in 80% price discount

$$t_g = 1 \text{ yr} \Rightarrow g = \frac{p x_g}{\left(p-c\right)t_g} = \frac{100(0.8)}{100-60} = 2$$

  - High discount: If 15 min delay results in 80% price discount

$$t_g = \frac{15}{60} = 0.25 \text{ hr} \Rightarrow g = \frac{100(0.8)}{\left(100-60\right)0.25} = 8$$

Service and Make-to-Order Systems

- Assume single-machine Poisson and $r_a = r_d$

$$t_{CT}(r_e) = \left(\frac{r_d}{r_e - r_d}\right)\left(\frac{1}{r_e}\right) + \left(\frac{1}{r_e}\right)$$

$$TP(r_e) = (p-c)\left[1 - g\,t_{CT}(r_e)\right] r_d - k r_e$$

Taking $\dfrac{dTP(r_e)}{dr_e} = 0$ and solving for r_e: $\boxed{r_e^* = r_d + \sqrt{\dfrac{(p-c)\,g r_d}{k}}}$

Example : $r_d = 10,\ p = \$70,\ c = \$50,\ g = 0.01,\ k = \$1$

$$r_e^* = 10 + \sqrt{\frac{(70-50)0.01(10)}{1}} = 11.41$$

Estimating Cost Data

- Cost inputs needed for model: p, c, and k
- Assume hour base time unit

  H = annual operating hours = 50 week/yr x 40 hr/week = 2000 hr/yr
- Unit sales price (p): assume given
- Unit operating cost (c):

  top-down: known annual OC and known demand F, then $c = OC/F$

  bottom-up: sum of raw material, labor, and energy cost per unit
- Unit capital cost (k):

  top-down: known K and known r_e then $k = (K/H)/r_e$

  if r_e not known, then can estimate from known r_d and estimated u, where

  r_d = (*Annual Demand*) $/H$

  $r_e = r_d/u$

  bottom-up: m identical machines, $k\,r_e = k_i\,m$, k_i = machine *i* cost

  (was assuming m = 1 in simple Poisson model, but still can est. CT for m > 1)

Example Cost Data

- Can use top-down approach since data from a similar production system known and can be used to estimate costs for new production system

	A	B	C	D	E
1	Cost of Capital	(i)	4%		0.04
2	Economic Life	(N, yr)	5		5
3	Annual Demand	(q /yr)	10,000		10000
4	Sale Price	(p , $/q)	70		70
5	Investment Cost	(IV , $)	59,000		59000
6	Salvage Percentage		25%		0.25
7	Salvage Value	(SV , $)	14,750		=C5*C6
8	Eff. Investment Cost	(IV^{eff}, $)	46,877		=C5-C7*(1+C1)^(-C2)
9	Cost Cap Recovery	(K , $/yr)	10,530		=C8*(C1/(1-(1+C1)^(-C2)))
10	Annual Operating Hours	(H , hr/yr)	2,000		2000
11	Known Departure Rate	(r_d , q/hr)	5.00		=C3/C10
12	Estimated Utilization	(u)	0.95		0.95
13	Estimated Capacity	(r_e , q/hr)	5.26		=C11/C12
14	Capital Cost per Unit	(k , $/q)	1.00		=(C9/C10)/C13
15	Operating Cost	(OC , $/yr)	500,000		500000
16	Oper Cost per Unit	(c , $/q)	50		=C15/C3