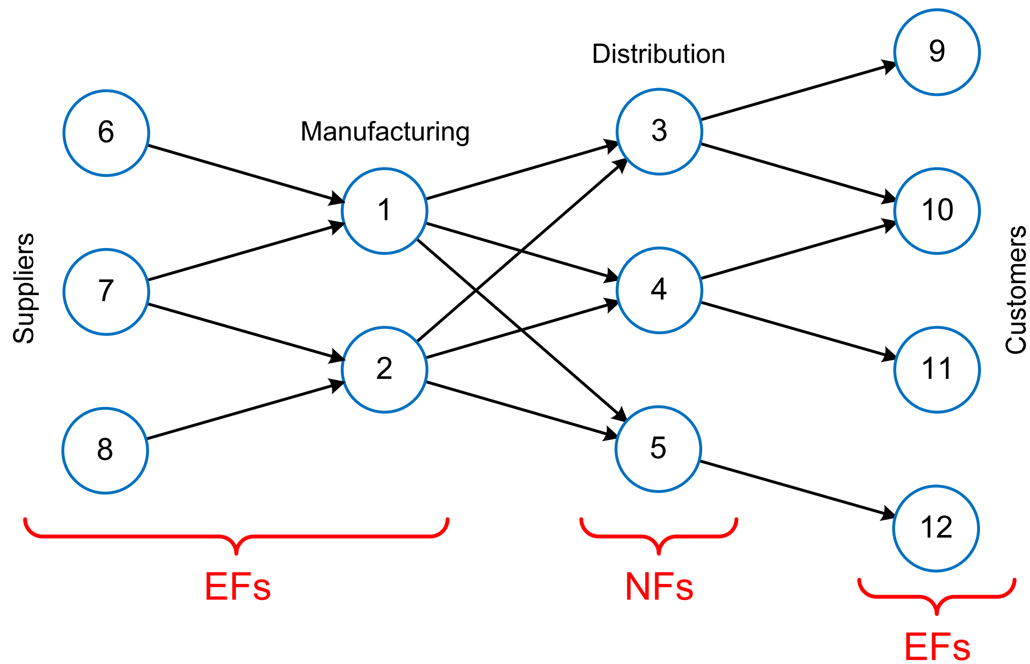
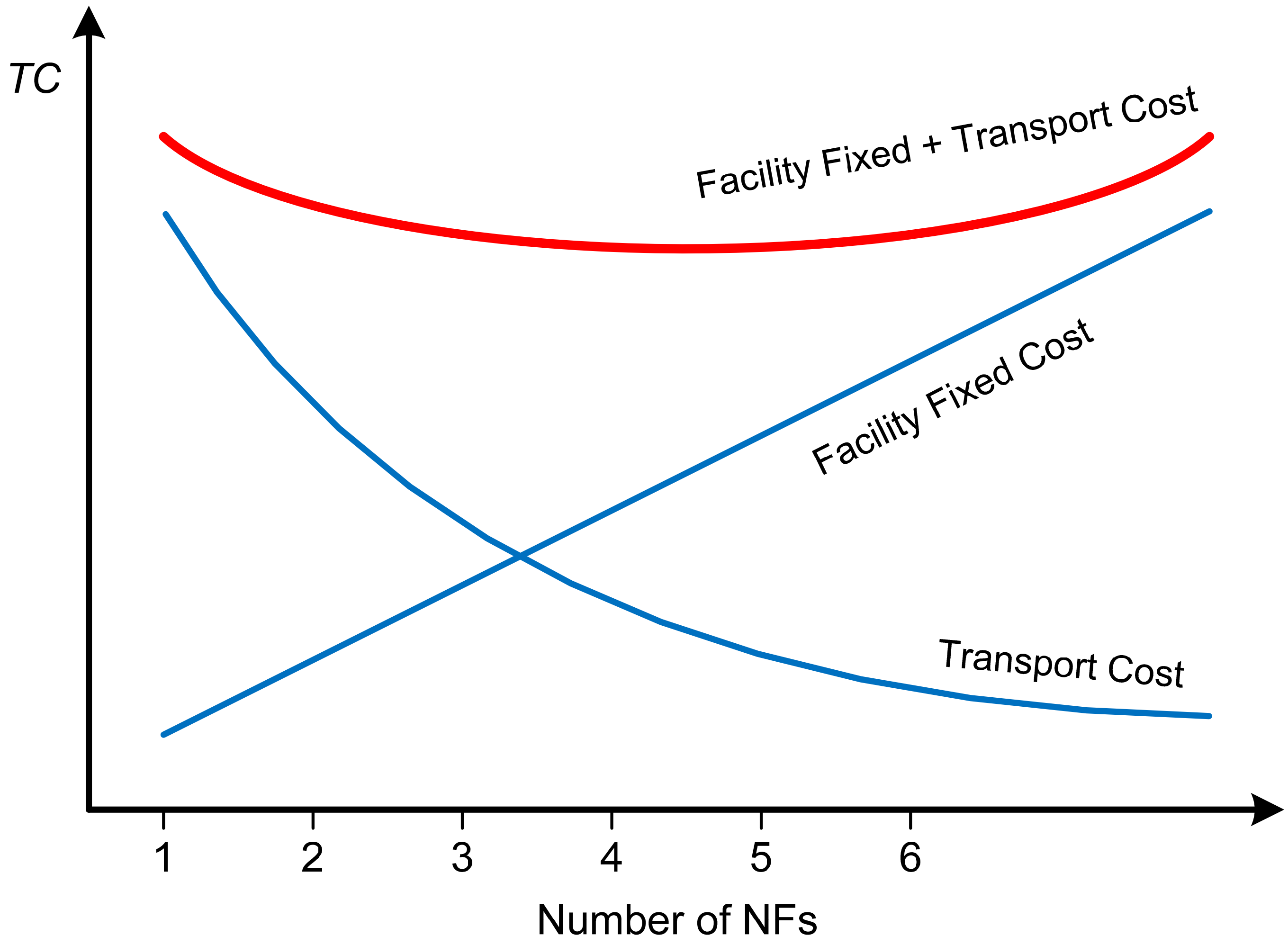
# Multiple Single-Facility Location



# Best Retail Warehouse Locations

| Number of Locations | Average Transit Time (days) | Warehouse Location | | |
|---|---|---|---|---|
| 1 | 2.20 | Bloomington, IN | | |
| 2 | 1.48 | Ashland, KY | Palmdale, CA | |
| 3 | 1.29 | Allentown, PA | Palmdale, CA | McKenzie, TN |
| 4 | 1.20 | Edison, NJ<br>Meridian, MS | Palmdale; CA | Chicago, IL |
| 5 | 1.13 | Madison, NJ<br>Dallas, TX | Palmdale, CA<br>Macon, GA | Chicago, IL |
| 6 | 1.08 | Madison, NJ<br>Dallas, TX | Pasadena, CA<br>Macon, GA | Chicago, IL<br>Tacoma, WA |
| 7 | 1.07 | Madison, NJ<br>Dallas, TX<br>Lakeland, FL | Pasadena, CA<br>Gainesville, GA | Chicago, IL<br>Tacoma, WA |
| 8 | 1.05 | Madison, NJ<br>Dallas, TX<br>Lakeland, FL | Pasadena, CA<br>Gainesville, GA<br>Denver, CO | Chicago, IL<br>Tacoma, WA |
| 9 | 1.04 | Madison, NJ<br>Dallas, TX<br>Lakeland. FL | Alhambra, CA<br>Gainesville, GA<br>Denver, CO | Chicago, IL<br>Tacoma, WA<br>Oakland, CA |
| 10 | 1.04 | Newark, NJ<br>Palistine, TX<br>Lakeland, FL<br>Mansfield, OH | Alhambra, CA<br>Gainesville, GA<br>Denver, CO | Rockford, IL<br>Tacoma, WA<br>Oakland. CA |

# Optimal Number of NFs



# MILP

LP: $\max \ \mathbf{c}'\mathbf{x}$

s.t. $\mathbf{A}\mathbf{x} \le \mathbf{b}$

$\mathbf{x} \ge 0$

MILP: some $x_i$ integer

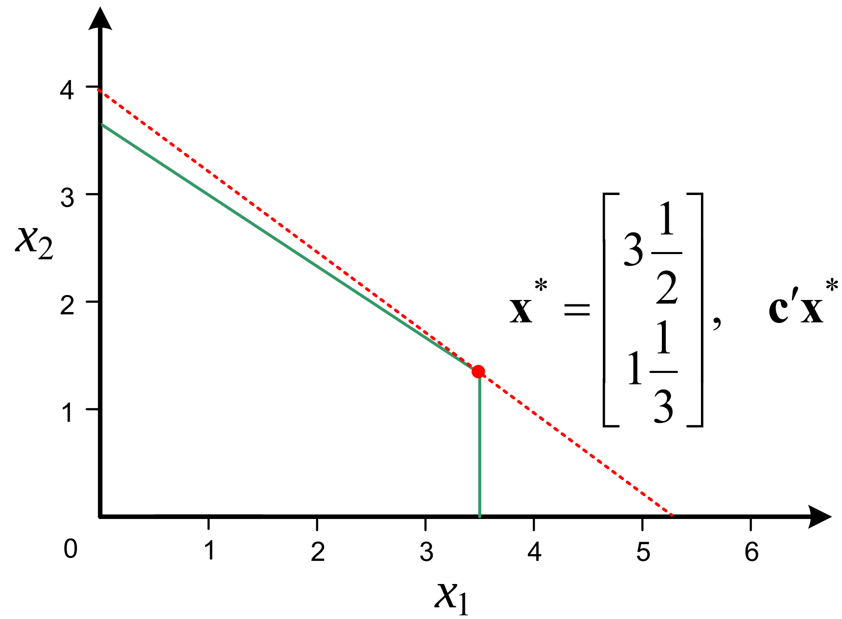
ILP: $\mathbf{x}$ integer

BLP: $\mathbf{x} \in \{0,1\}$

$$\begin{aligned} \max \quad & 6x_1 + 8x_2 \\ \text{s.t.} \quad & 2x_1 + 3x_2 \le 11 \\ & 2x_1 \qquad\ \ \le 7 \\ & x_1, x_2 \ge 0 \end{aligned}$$

$$\mathbf{c} = \begin{bmatrix} 6 & 8 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 2 & 0 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 11 \\ 7 \end{bmatrix}$$



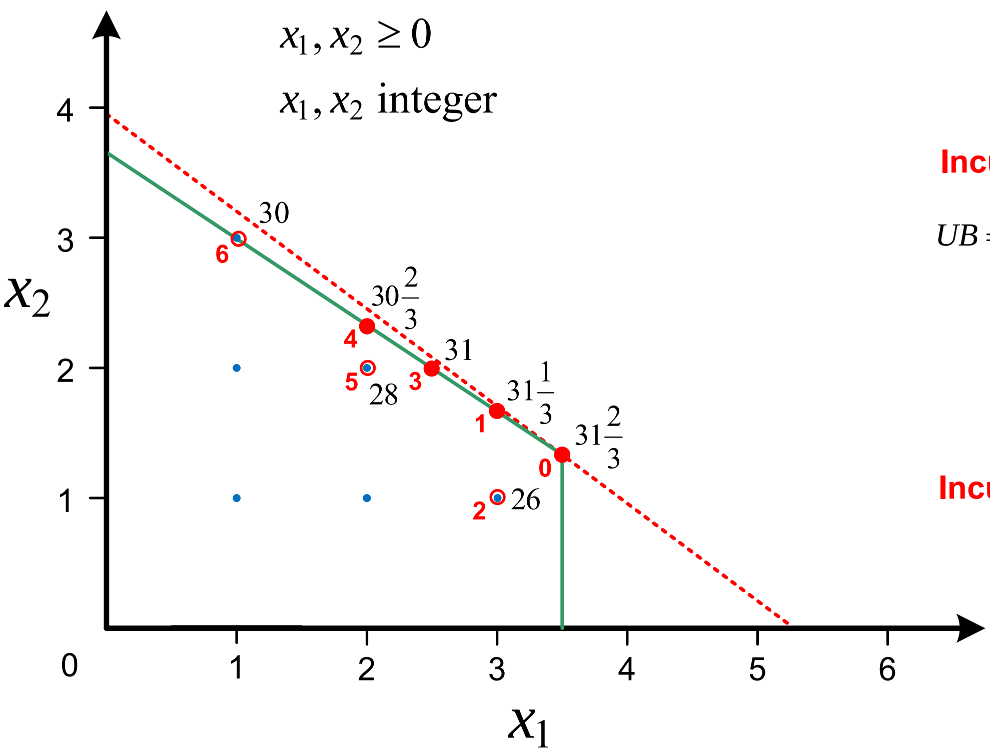

$$\mathbf{x}^* = \begin{bmatrix} 3\frac{1}{2} \\ 1\frac{1}{3} \end{bmatrix}, \quad \mathbf{c}'\mathbf{x}^* = 31\frac{2}{3}$$

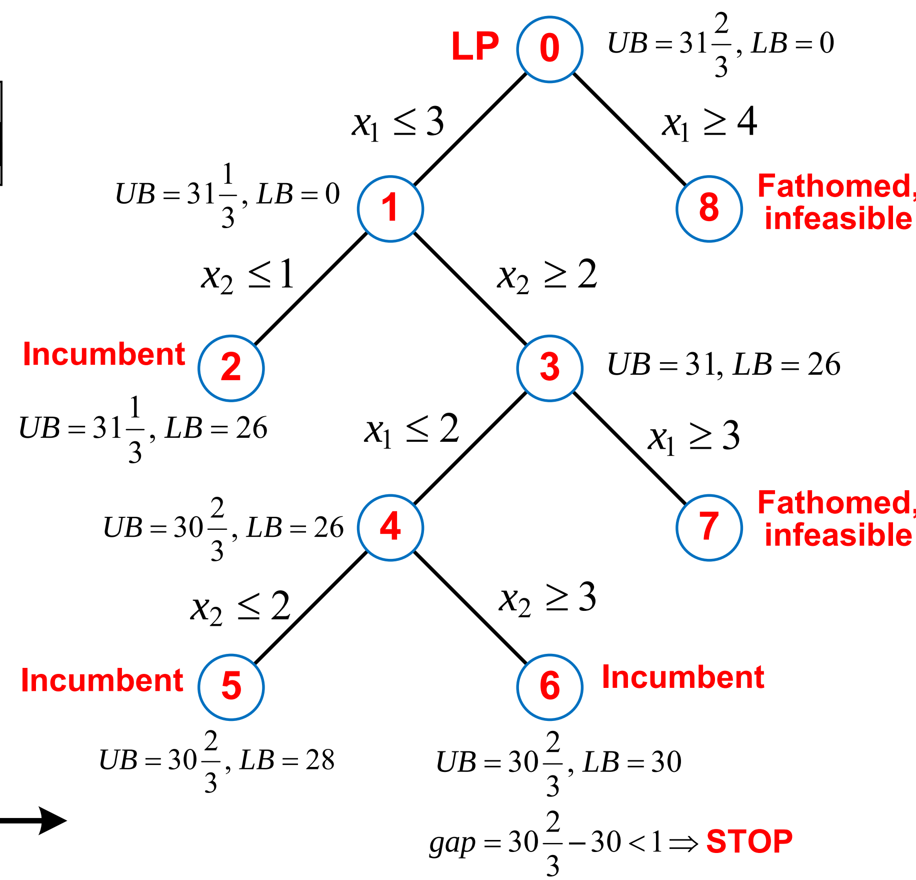
# Branch and Bound

$$\begin{aligned} \max \quad & 6x_1 + 8x_2 \\ \text{s.t.} \quad & 2x_1 + 3x_2 \le 11 \\ & 2x_1 \qquad\;\; \le 7 \\ & x_1, x_2 \ge 0 \\ & x_1, x_2 \text{ integer} \end{aligned}$$

$$\mathbf{c} = \begin{bmatrix} 6 & 8 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 2 & 0 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 11 \\ 7 \end{bmatrix}$$



# MILP Formulation of UFL

$$
\begin{aligned}
\min \quad & \sum_{i \in N} k_i y_i + \sum_{i \in N} \sum_{j \in M} c_{ij} x_{ij} \\
\text{s.t.} \quad & \sum_{i \in N} x_{ij} = 1, \qquad j \in M \\
& y_i \ge x_{ij}, \qquad i \in N,\ j \in M \\
& 0 \le x_{ij} \le 1, \qquad i \in N,\ j \in M \\
& y_i \in \{0,1\}, \qquad i \in N
\end{aligned}
$$

where

$k_i$ = fixed cost of NF at site $i \in N = \{1,\ldots,n\}$

$c_{ij}$ = variable cost from $i$ to serve EF $j \in M = \{1,\ldots,m\}$

$$y_i = \begin{cases} 1, & \text{if NF established at site } i \\ 0, & \text{otherwise} \end{cases}$$

$x_{ij}$ = fraction of EF $j$ demand served from NF at site $i$.

# MILP Formulation of *p*-Median

$$
\begin{aligned}
\min \quad & \sum_{i \in N} \sum_{j \in M} c_{ij} x_{ij} \\
\text{s.t.} \quad & \sum_{i \in N} y_i = p \\
& \sum_{i \in N} x_{ij} = 1, && j \in M \\
& y_i \ge x_{ij}, && i \in N, j \in M \\
& 0 \le x_{ij} \le 1, && i \in N, j \in M \\
& y_i \in \{0, 1\}, && i \in N
\end{aligned}
$$

where

$p$ = number of NF to establish

$c_{ij}$ = variable cost from $i$ to serve EF $j \in M = \{1, \ldots, m\}$

$$
y_i = \begin{cases} 1, & \text{if NF established at site } i \\ 0, & \text{otherwise} \end{cases}
$$

$x_{ij}$ = fraction of EF $j$ demand served from NF at site $i$.