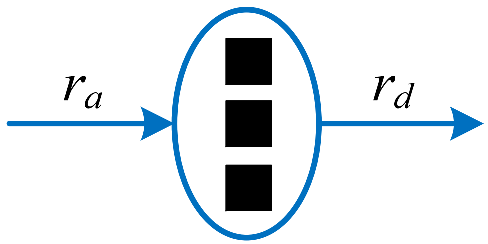
Extensions

- Extensions to the basic model allow it to handle more realistic production scenarios:
  - Multiple identical machines
  - Serial production lines
  - Non-Poisson demand and production

Multiple Identical Machines

- Most production capacity only available in discrete (lumpy) units of machines (M/C)



$m = 3$

$$r_a = r \Rightarrow \gamma = 1 \Rightarrow \text{scrap at end (default assumption)}$$

$$t_0 = \text{natural mean process time}$$

$$t_e = \frac{t_0}{A} = \text{effective mean process time with failures}$$

$$A = \frac{MTTF}{MTTF + MTTR} = \text{availability}$$

$$MTTF = \text{mean time to failure}$$

$$MTTR = \text{mean time to repair}$$

$$r_e = \frac{m}{t_e} = \text{effective capacity of } m\text{-machine workstation}$$

$$m_{\min} = \lfloor r_a t_e + 1 \rfloor = \text{minimum number of machines needed}$$

$$u = \frac{r_a}{r_e} = \frac{r_a t_e}{m} = \text{utilization of workstation}$$

- assume identical M/C at W/S
- different base and peak units possible
- failure (*preemptive outages*) $\Rightarrow$ increasing process time
- bottom-up: $k\, r_e = k_i\, m$, k_i = machine *i* cost

“Machine” Hours

- General method of determining resource requirements
  - resources can be machines, people, etc.
  - can be used to determine operating costs for economic justification

$$m = \text{number of machines}, \quad H = \text{ hours of operation}$$

$$mH = \text{available machine hours} \; = \; (\text{processing} + \text{repair} + \text{idle}) \text{ hours}$$

$$m_{\min} = \left\lfloor \frac{\text{machine hours needed to meet demand}}{\text{productive hours per machine}} + 1 \right\rfloor$$

$$= \left\lfloor \frac{r_a t_0 H}{AH} + 1 \right\rfloor = \left\lfloor r_a \frac{t_0}{A} + 1 \right\rfloor = \lfloor r_a t_e + 1 \rfloor$$

$$r_a t_0 H = \text{total processing hours}$$

$$r_a (t_e - t_0) H = \text{total repair hours}$$

$$(m - r_a t_e) H = \text{total idle hours}$$

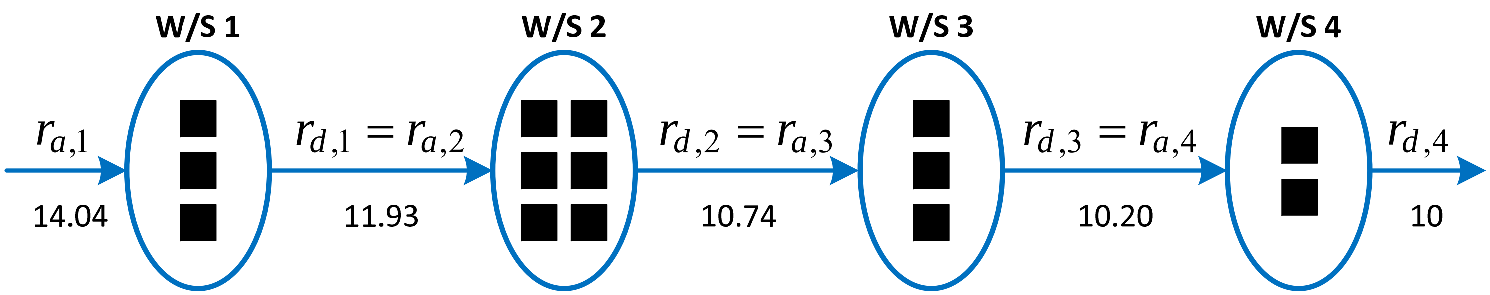
Line Yield

Given n operations at n workstations, each with yield fractions $y_1, \ldots, y_n$ and known $r_{d,n}$,

$$r_{a,1} = \frac{r_{d,n}}{Y_n}, \quad \text{where } Y_i = \prod_{j=1}^{i} y_j \text{ is the cumulative line yield from 1 to } i.$$

Example :

$$r_{a,1} = \frac{r_{d,4}}{Y_4} = \frac{10}{0.85 \cdot 0.9 \cdot 0.95 \cdot 0.98} = \frac{10}{0.71222} = 14.0407$$



W/S	1	2	3	4
Arrival Rate (r_a, q/hr)	14.0407	11.9346	10.7411	10.2041
Yield (y)	0.85	0.9	0.95	0.98
Departure Rate (r_d, q/hr)	11.9346	10.7411	10.2041	10

Throughput Feasible Capacity Plan

- Throughput feasible $\Rightarrow$ all $m_i = m_{\min,i}$

W/S	1	2	3	4
Arrival Rate (r_a, q/hr)	14.0407	11.9346	10.7411	10.2041
Natural Process Time (t_0, hr)	0.2	0.5	0.25	0.15
MTTF (hr)	40		100	
MTTR (hr)	2	0	5	0
Availability (A)	0.95238	1	0.95238	1
Effective Process Time (t_e, hr)	0.21	0.5	0.2625	0.15
Number of M/C (m)	3	6	3	2
Utilization (u)	0.98285	0.99455	0.93985	0.76531
Yield (y)	0.85	0.9	0.95	0.98
Departure Rate (r_d, q/hr)	11.9346	10.7411	10.2041	10

	A	B	C	D
1	W/S		1	2
2	Arrival Rate	(r_a, q/hr)	=C11/C10	=D11/D10
3	Natural Process Time	(t_0, hr)	0.2	0.5
4	MTTF	(hr)	40	
5	MTTR	(hr)	2	0
6	Availability	(A)	=IF(ISBLANK(C4), 1, C4/(C4 + C5))	=IF(ISBLANK(D4), 1, D4/(D4 + D5))
7	Effective Process Time	(t_e, hr)	=C3/C6	=D3/D6
8	Number of M/C	(m)	=FLOOR(C2*C7 + 1,1)	=FLOOR(D2*D7 + 1,1)
9	Utilization	(u)	=C2*C7/C8	=D2*D7/D8
10	Yield	(y)	0.85	0.9
11	Departure Rate	(r_d, q/hr)	=D2	=E2