

# Truck Shipment Example: Periodic

11. Continuing with the example: assuming a constant annual demand for the product of 20 tons, what is the number of full truckloads per year?

$$f = 20 \text{ ton/yr}$$

$$q = q_{\max} = 6.1111 \text{ ton/ TL} \quad (\text{full truckload} \Rightarrow q \equiv q_{\max})$$

$$n = \frac{f}{q} = \frac{20}{6.1111} = 3.2727 \text{ TL/yr, average shipment frequency}$$

- Why should this number not be rounded to an integer value?

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12. What is the shipment interval?

$$t = \frac{1}{n} = \frac{q}{f} = \frac{6.1111}{20} = 0.3056 \text{ yr/TL, average shipment interval}$$

- How many days are there between shipments?

365.25 day/yr

$$t \times 365.25 = \frac{365.25}{n} = 111.6042 \text{ day/TL}$$

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13. What is the annual full-truckload transport cost?

$$d = 532 \text{ mi}, \quad r = \$2.5511 / \text{mi}$$

$$r_{FTL} = \frac{r}{q_{\max}} = \frac{2.5511}{6.1111} = \$0.4175 / \text{ton-mi}$$

$$\begin{aligned} TC_{FTL} &= f r_{FTL} d = n r d \quad (= w d, w = \text{monetary weight in } \$/\text{mi}) \\ &= 3.2727(2.5511)532 = \$4,441.73/\text{yr} \end{aligned}$$

- What would be the cost if the shipments were to be made at least every three months?

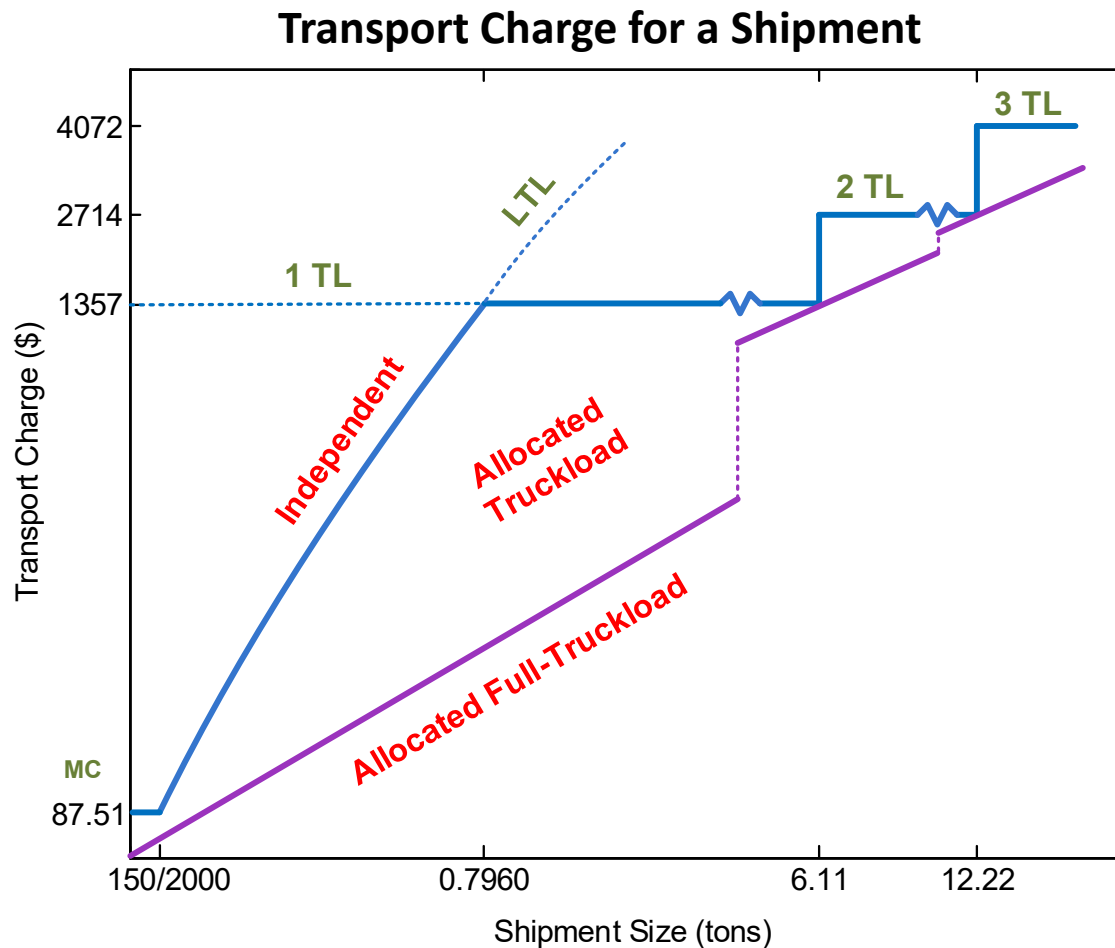
$$t_{\max} = \frac{3}{12} \text{ yr/TL} \Rightarrow n_{\min} = \frac{1}{t_{\max}} = 4 \text{ TL/yr} \Rightarrow q = \frac{f}{\max\{n, n_{\min}\}}$$

$$\begin{aligned} TC'_{FTL} &= \max\{n, n_{\min}\} r d \\ &= \max\{3.2727, 4\} 2.5511(532) = \$5,428.78/\text{yr} \end{aligned}$$

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- Independent and allocated full-truckload charges:

$$q \leq q_{\max} \Rightarrow [UB, LB] = [c_0(q), qr_{FTL}d]$$



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- *Total Logistics Cost* (TLC) includes all costs that could change as a result of a logistics-related decision

$$TLC = TC + IC + PC$$

$TC$  = transport cost

$IC$  = inventory cost

$$= IC_{\text{cycle}} + IC_{\text{pipeline}} + IC_{\text{safety}}$$

$PC$  = purchase cost

- *Cycle inventory*: held to allow cheaper large shipments
- *Pipeline inventory*: goods in transit or awaiting transshipment
- *Safety stock*: held due to transport uncertainty
- *Purchase cost*: can be different for different suppliers

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14. Since demand is constant throughout the year, one half of a shipment is stored at the destination, on average. Assuming that the production rate is also constant, one half of a shipment will also be stored at the origin, on average. Assuming each ton of the product is valued at \$25,000, what is a “reasonable estimate” for the total annual cost for this cycle inventory?

$$IC_{\text{cycle}} = (\text{annual cost of holding one ton})(\text{average annual inventory level}) \\ = (vh)(\alpha q)$$

$v$  = unit value of shipment (\$/ton)

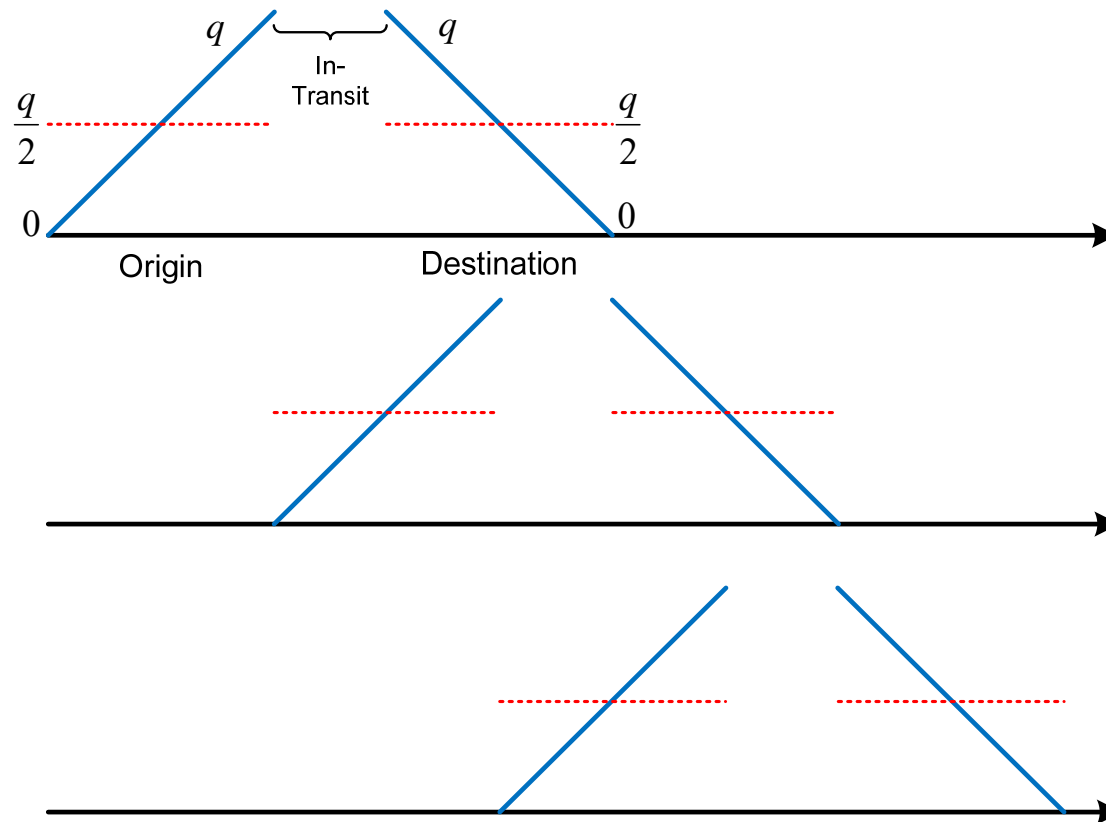
$h$  = inventory carrying rate, the cost per dollar of inventory per year (1/yr)

$\alpha$  = average inter-shipment inventory fraction at Origin and Destination

$q$  = shipment size (ton)

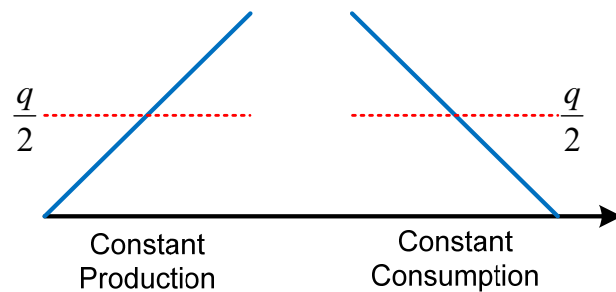
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- Average annual inventory level  $= \frac{q}{2} + \frac{q}{2} = \left(\frac{1}{2} + \frac{1}{2}\right)q = (1)q \Rightarrow \alpha = 1$

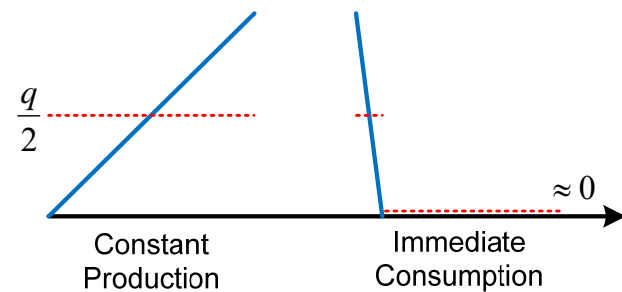


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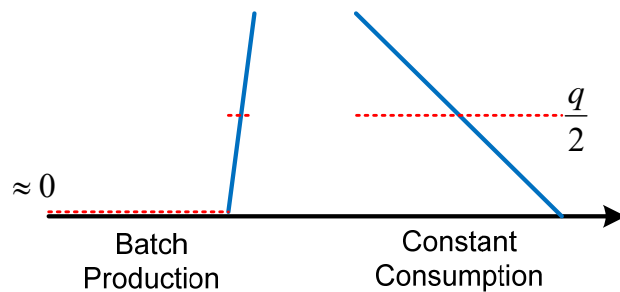
- Inter-shipment inventory fraction alternatives:  $\alpha = \alpha_O + \alpha_D$



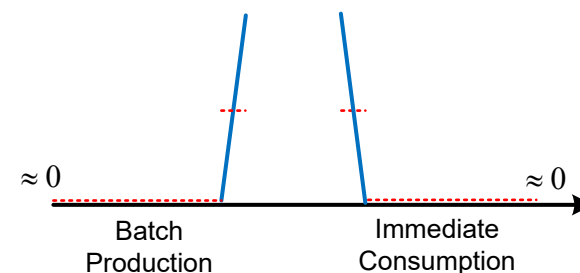
$$\alpha = \frac{1}{2} + \frac{1}{2} = 1$$



$$\alpha = \frac{1}{2} + 0 = \frac{1}{2}$$



$$\alpha = 0 + \frac{1}{2} = \frac{1}{2}$$



$$\alpha = 0 + 0 = 0$$



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- “Reasonable estimate” for the total annual cost for the cycle inventory:

$$\begin{aligned} IC_{\text{cycle}} &= \alpha v h q \\ &= (1)(25,000)(0.3)6.1111 \\ &= \$45,833.33 / \text{yr} \end{aligned}$$

where

$$\alpha = \frac{1}{2} \text{ at Origin} + \frac{1}{2} \text{ at Destination} = 1$$

$$v = \$25,000 = \text{unit value of shipment (\$/ton)}$$

$$h = 0.3 = \text{estimated carrying rate for manufactured products (1/yr)}$$

$$q = q_{\text{max}} = 6.111 = \text{FTL shipment size (ton)}$$

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15. What is the annual total logistics cost (TLC) for these full-truckload TL shipments?

$$\begin{aligned}TLC_{FTL} &= TC_{FTL} + IC_{\text{cycle}} \\&= nrd + \alpha v h q \\&= 3.2727(2.5511)532 + (1)(25,000)(0.3)6.1111 \\&= 4,441.73 + 45,833.33 \\&= \$50,275.06 / \text{yr}\end{aligned}$$

# Truck Shipment Example: Periodic

16. What is minimum possible annual total logistics cost for TL shipments, where the shipment size can now be less than a full truckload?

$$TLC_{TL}(q) = TC_{TL}(q) + IC(q) = \frac{f}{q} c_{TL}(q) + \alpha v h q = \frac{f}{q} r d + \alpha v h q$$

$$\frac{dTLC_{TL}(q)}{dq} = 0 \Rightarrow q_{TL}^* = \sqrt{\frac{f r d}{\alpha v h}} = \sqrt{\frac{20(2.5511)532}{(1)25000(0.3)}} = 1.9024 \text{ ton}$$

$$\begin{aligned} TLC_{TL}(q_{TL}^*) &= \frac{f}{q_{TL}^*} r d + \alpha v h q_{TL}^* \\ &= \frac{20}{1.8553} (2.5511)532 + (1)25000(0.3)1.8553 \\ &= 14,268.12 + 14,268.12 \\ &= \$28,536.25 / \text{yr} \end{aligned}$$

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- Including the minimum charge and maximum payload restrictions:

$$q_{TL}^* = \min \left\{ \sqrt{\frac{f \max \{rd, MC_{TL}\}}{\alpha vh}}, q_{\max} \right\} \approx \sqrt{\frac{frd}{\alpha vh}}$$

- What is the TLC if this size shipment could be made as an allocated full-truckload?

$$\begin{aligned} TLC_{AllocFTL}(q_{TL}^*) &= \frac{f}{q_{TL}^*} (q_{TL}^* r_{FTL} d) + \alpha vh q_{TL}^* = f \frac{r}{q_{\max}} d + \alpha vh q_{TL}^* \\ &= 20 \frac{2.5511}{6.1111} 532 + (1) 25000 (0.3) 1.9024 \\ &= 4,441.73 + 14,268.12 \\ &= \$18,709.85 / \text{yr} \quad (\text{vs. } \$28,536.25 \text{ as independent P2P TL}) \end{aligned}$$

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17. What is the optimal LTL shipment size?

$$TLC_{LTL}(q) = TC_{LTL}(q) + IC(q) = \frac{f}{q} c_{LTL}(q) + \alpha v h q$$

$$q_{LTL}^* = \arg \min_q TLC_{LTL}(q) = 0.7622 \text{ ton}$$

- Must be careful in picking starting point for optimization since LTL formula only valid for limited range of values:

$$r_{LTL} = PPI_{LTL} \left[ \frac{\frac{s^2}{8} + 14}{\left( q^{\frac{1}{7}} d^{\frac{15}{29}} - \frac{7}{2} \right) (s^2 + 2s + 14)} \right], \quad \begin{cases} 37 \leq d \leq 3354 \text{ (dist)} \\ \frac{150}{2,000} \leq q \leq \frac{10,000}{2,000} \text{ (wt)} \\ 2000 \frac{q}{s} \leq 650 \text{ ft}^3 \text{ (cube)} \end{cases}$$

$$\frac{150}{2000} \leq q \leq \min \left\{ \frac{10,000}{2,000}, \frac{650s}{2000} \right\} \Rightarrow 0.075 \leq q \leq 1.44$$

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18. Should the product be shipped TL or LTL?

$$TLC_{LTL}(q_{LTL}^*) = TC_{LTL}(q_{LTL}^*) + IC(q_{LTL}^*) = 34,349.19 + 5,716.40 = \$40,065.59 / \text{yr}$$

