Inventory 2: Periodic Safety Stock

- One-time safety stock: Uncertain demand and not able to carry inventory
 - Unmet demand is a lost sale
 - Excess product is disposed of
 - Optimal policy: Tradeoff between lost profit and disposal cost
- *Periodic* safety stock: Uncertain demand and/or replenishment lead time, and able to carry inventory
 - Unmet demand is either a lost sale or backordered
 - Excess product is carried over to the next period
 - Optimal policy: Tradeoff with lost-profit or backorder-cost, and inventory carrying cost

Squared Coefficient of Variation

 Provides a normalized measure used to estimate of variance of a process (demand, production, etc.)

$$c = \frac{\sigma}{t} = \text{coefficient of variation (CV)}$$

$$c^2 = \frac{\sigma^2}{t^2} = \text{squared coefficient of variation (SCV)}$$

$$\begin{cases} 0, & \text{deterministic/exactly (best case, } \textit{LB}) \\ < 0.75 & \text{low variability} \\ \geq 0.75, < 1.33, & \text{moderate variability} \\ 1, & \text{Poisson} \Leftrightarrow \text{totally random (practical worse case, } \textit{UB}) \\ \geq 1.33, & \text{high variability (bad control)} \end{cases}$$

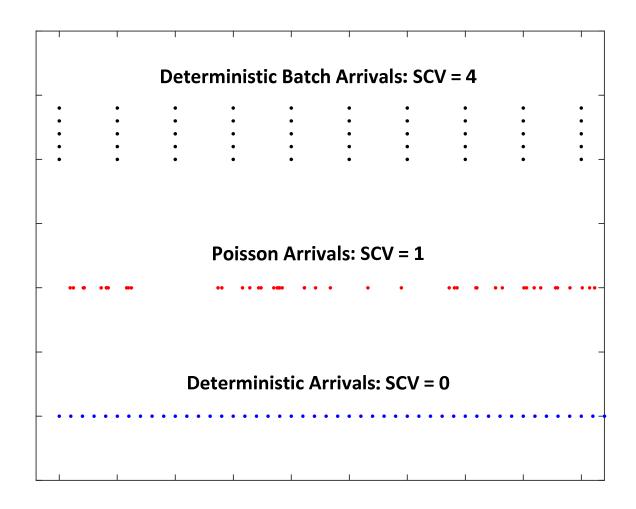
$$\sigma = \text{standard deviation of process}$$

$$t = \text{mean of process}$$

$$\sigma^2 = \text{variance of process}$$

Low, Moderate and High SCVs

All arrivals have same rate of 10 per hour



Base Stock with Lost Sales

- Pure safety stock
- Seller makes one decision:
 - 1. Maximum finished goods inventory level
- Control logic for seller:
 - Start with inventory at max level
 - Order replacement after each (unit) customer sale
- Customer fulfilment process:
 - If demand and inventory level > 0, make sale;
 otherwise, lost sale
- Performance measures:
 - 1. Out-of-stock percentage
 - 2. Average inventory level

Optimal Base-Stock Policy

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where  \begin{aligned} \max TP &= (p-c)(1-\pi_0) \, r_a - ch \overline{q} \\ q_{\text{max}} &= \text{maximum inventory level} \\ p &= \text{unit sales price} \\ c &= \text{unit operating cost} \\ \pi_0 &= \text{probability out of stock} \\ r_a &= \text{demand arrival rate} \\ h &= \text{inventory carrying rate} \\ \overline{q} &= \text{average inventory level} \end{aligned}
```

Order Point with Lost Sales

- Safety + cycle stock
- Seller makes two decisions:
 - 1. Maximum finished goods inventory level
 - 2. Order point (minimum inventory level)
- Control logic for seller:
 - Start with inventory at max level
 - Order up to max level when level falls below min level
- Customer fulfilment process:
 - If demand and inventory level > 0, make sale;
 otherwise, lost sale
- Performance measures:
 - 1. Out-of-stock percentage
 - 2. Average inventory level
 - 3. Average number of orders

Optimal Order-Point Policy

```
\max TP = (p-c)(1-\pi_0)r_a - ch\overline{q} - c_o\overline{n}_o
      q_{\max}, q_{\min}
             q_{\text{max}} = \text{maximum inventory level}
where
             q_{\min} = order point (minimum inventory level)
                 p = unit sales price
                 c = unit cost
               \pi_0 = probability out of stock
                r_a = demand arrival rate
                 h = inventory carrying rate
                 \overline{q} = average inventory level
                c_o = fixed cost per order
                \overline{n}_o = average number of orders
```