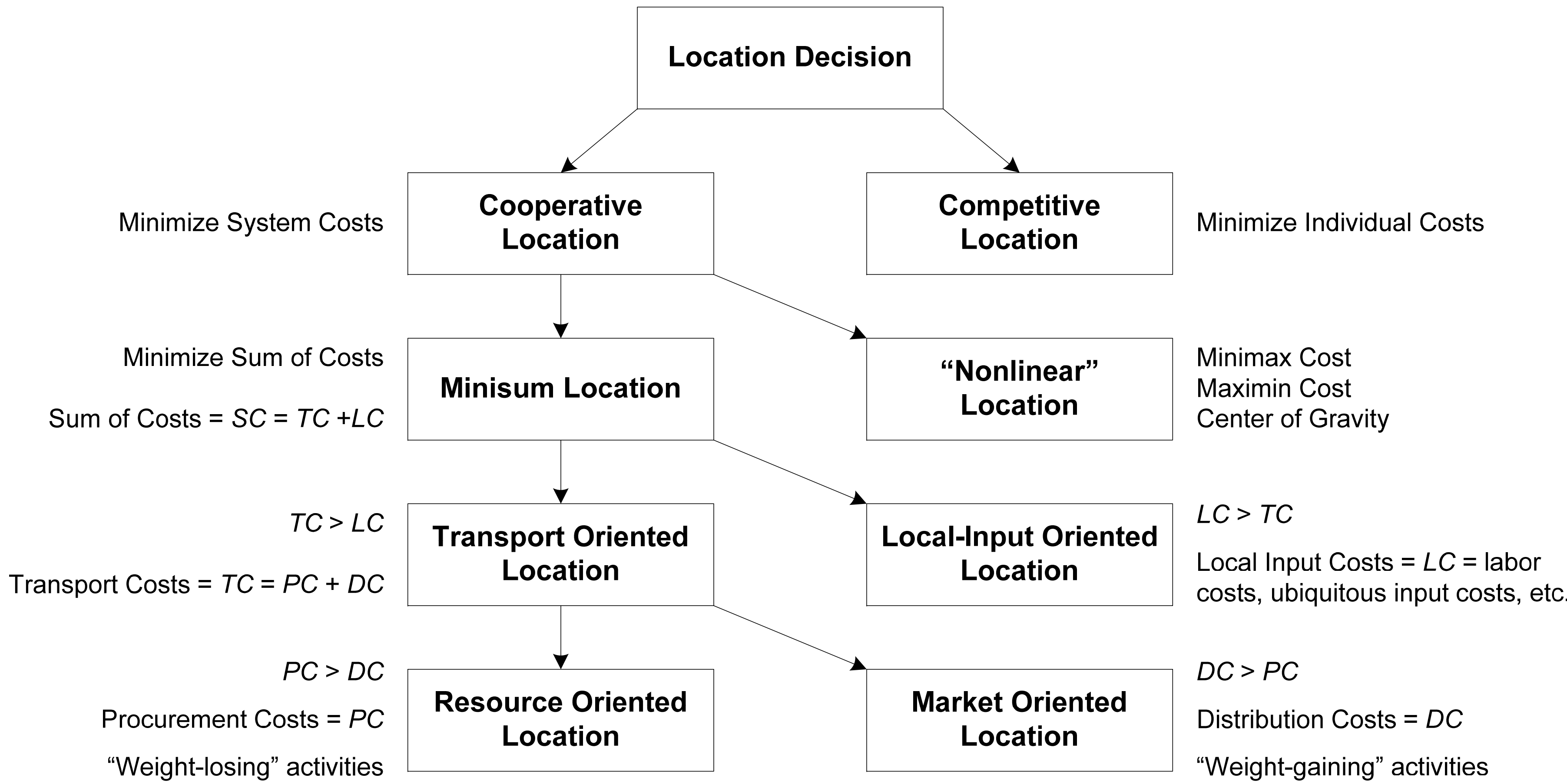
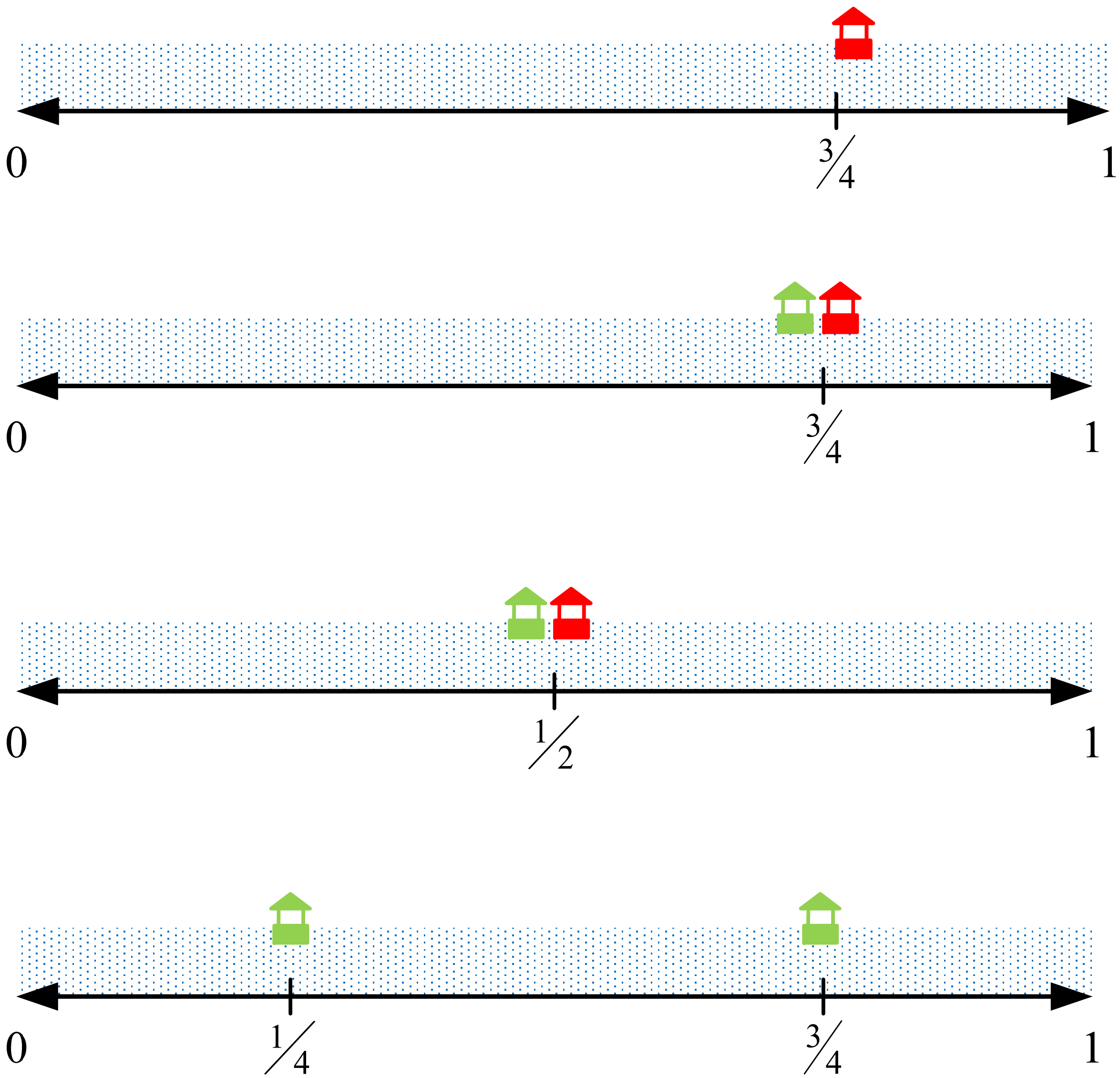
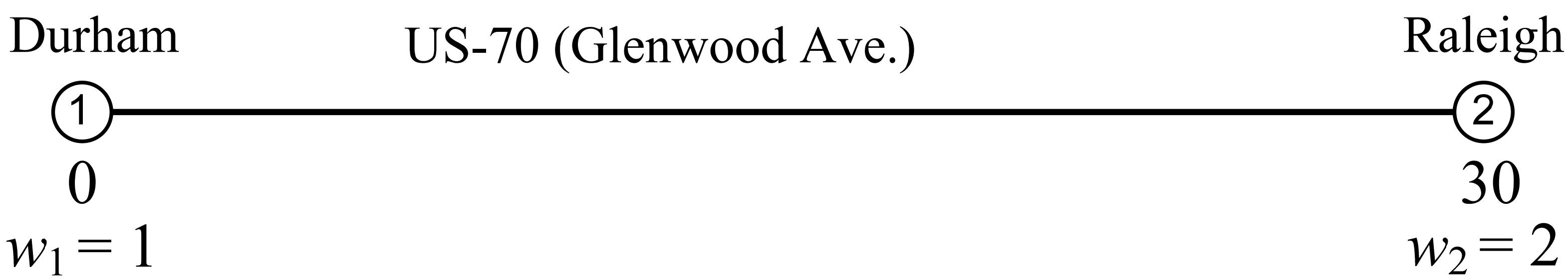
Taxonomy of Location Problems



Hotelling's Law



1-D Cooperative Location

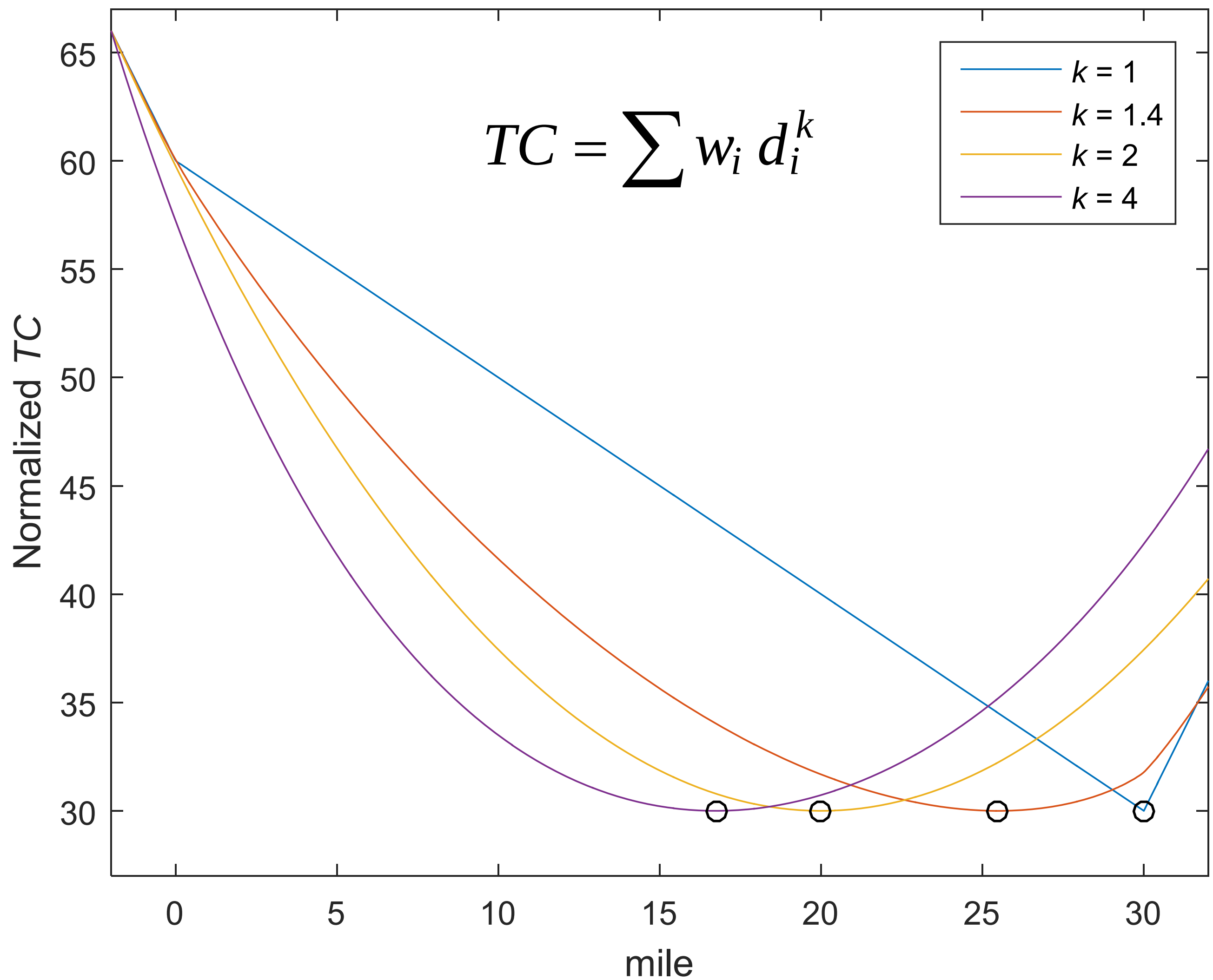


$$\text{Min } TC = \sum w_i \, d_i$$

$$\text{Min } TC = \sum w_i \, d_i^2$$

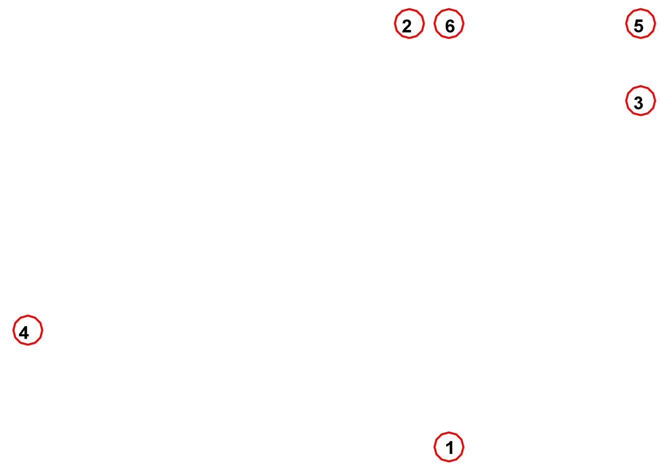
$$\text{Min } TC = \sum w_i \, d_i^k$$

"Nonlinear" Location



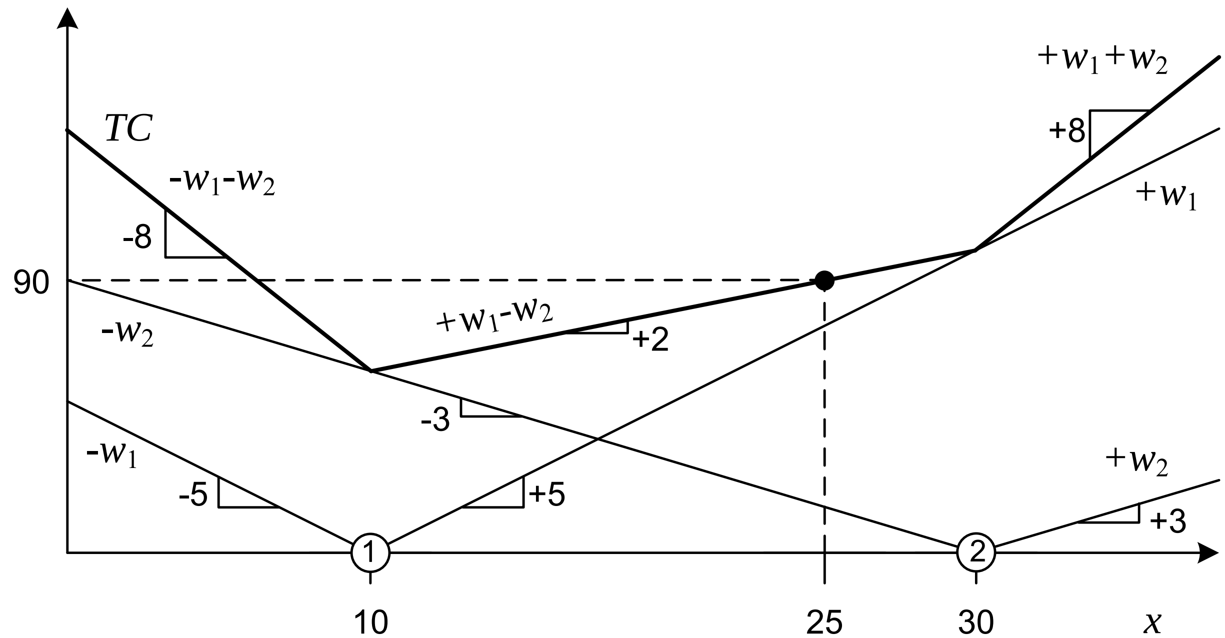
Minimax and Maximin Location

- Minimax
  - Min max distance
  - Set covering problem



- Maximin
  - Max min distance
  - AKA obnoxious facility location

2-EF Minisum Location



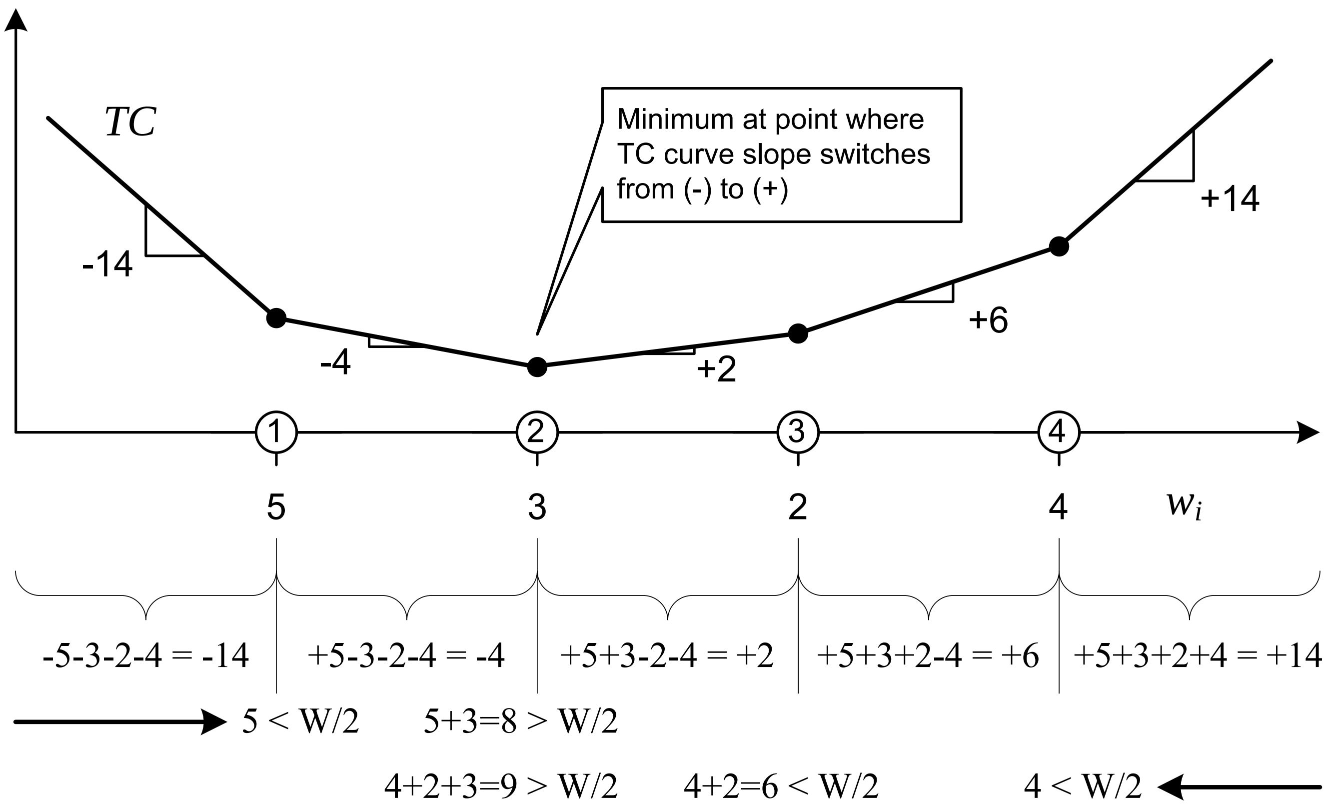
$$TC(x) = \sum w_i\, d_i = \beta_1(x - x_1) + \beta_2(x - x_2), \quad \text{where } \beta_i = \begin{cases} w_i, & \text{if } x \geq x_i \\ -w_i, & \text{if } x < x_i \end{cases}$$

$$\begin{aligned} TC(25) &= w_1(25 - 10) + (-w_2)(25 - 30) \\ &= 5(15) + (-3)(-5) = 90 \end{aligned}$$

Median Location: 1-D 4 EFs

Median location: For each dimension x of X:

1. Order EFs so that $|x_1| \le |x_2| \le \cdots \le |x_m|$
2. Locate x-dimension of NF at the first EFj where $\sum_{i=1}^{j} w_i \ge \frac{W}{2}$, where $W = \sum_{i=1}^{m} w_i$



Median Location: 1-D 7 EFs

Median location: For each dimension x of X:

1. Order EFs so that $|x_1| \le |x_2| \le \cdots \le |x_m|$

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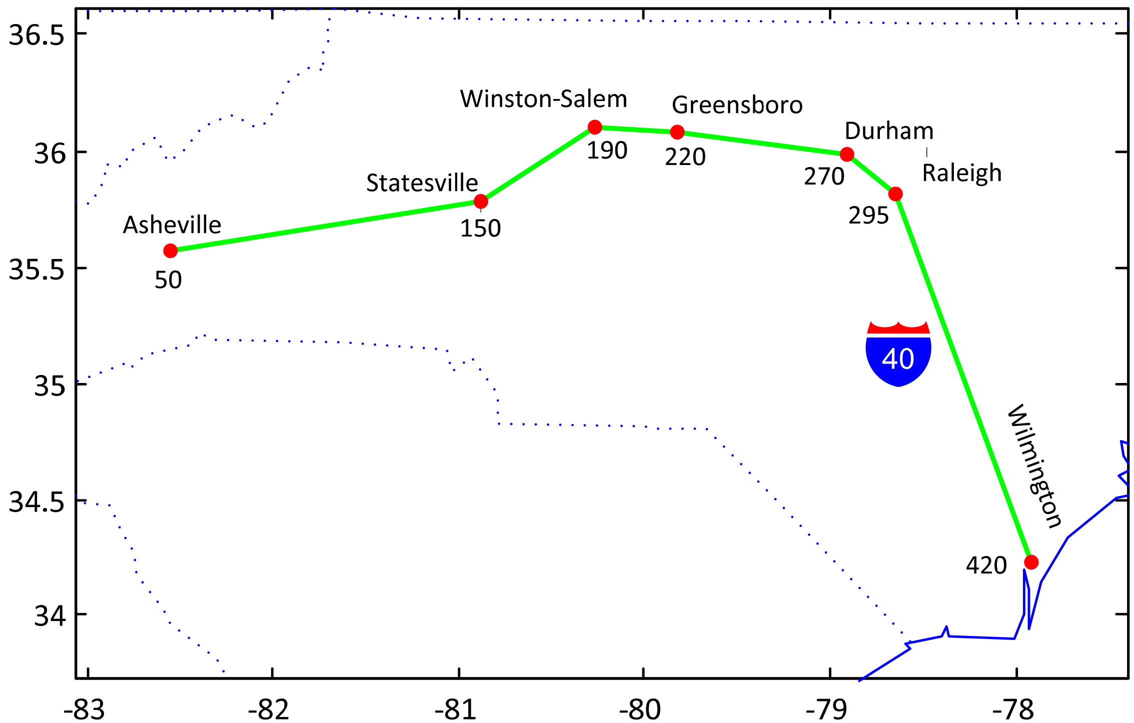
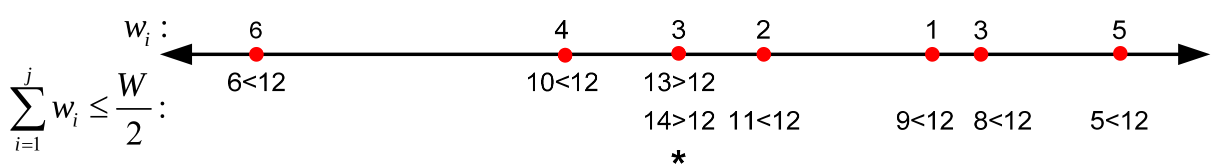
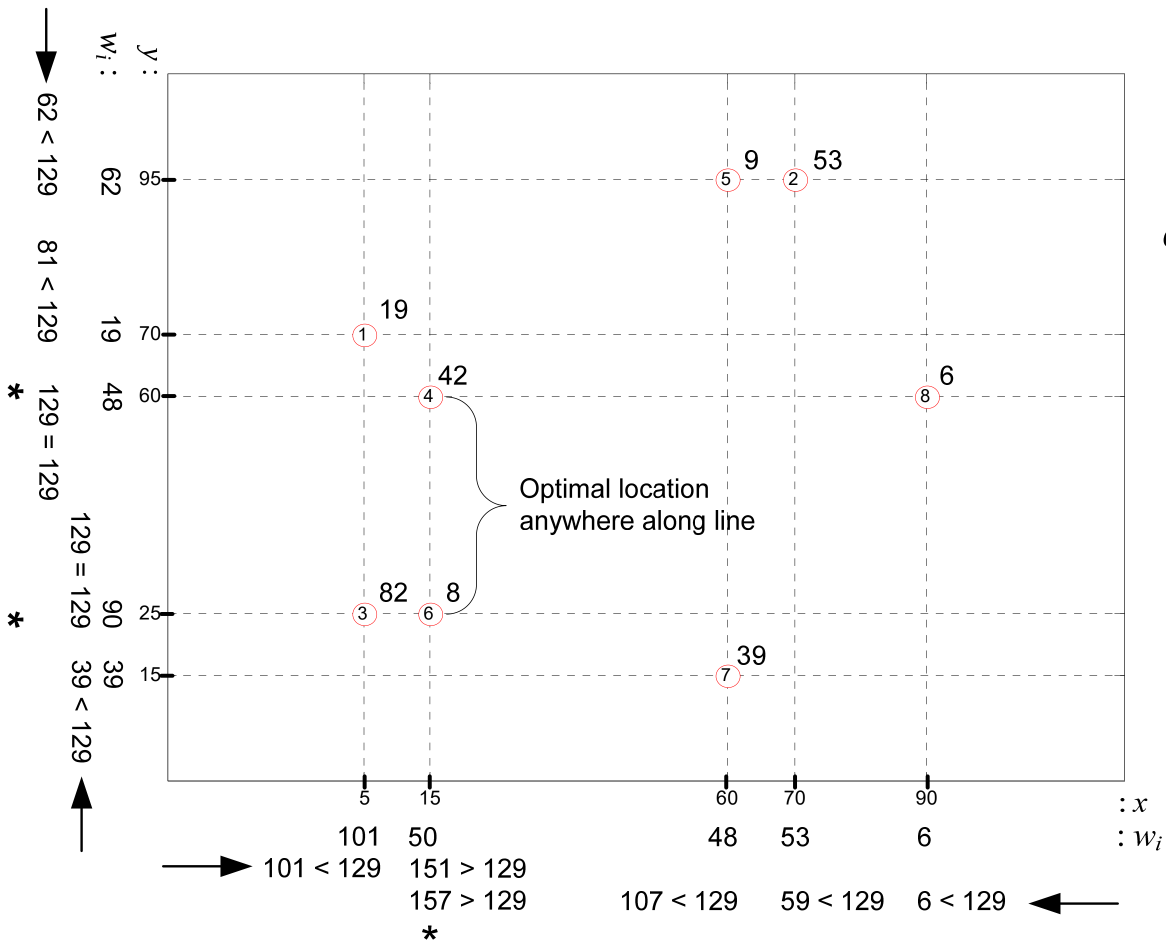




Median Location: 2-D Rectilinear Distance 8 EFs

Median location: For each dimension x of X:

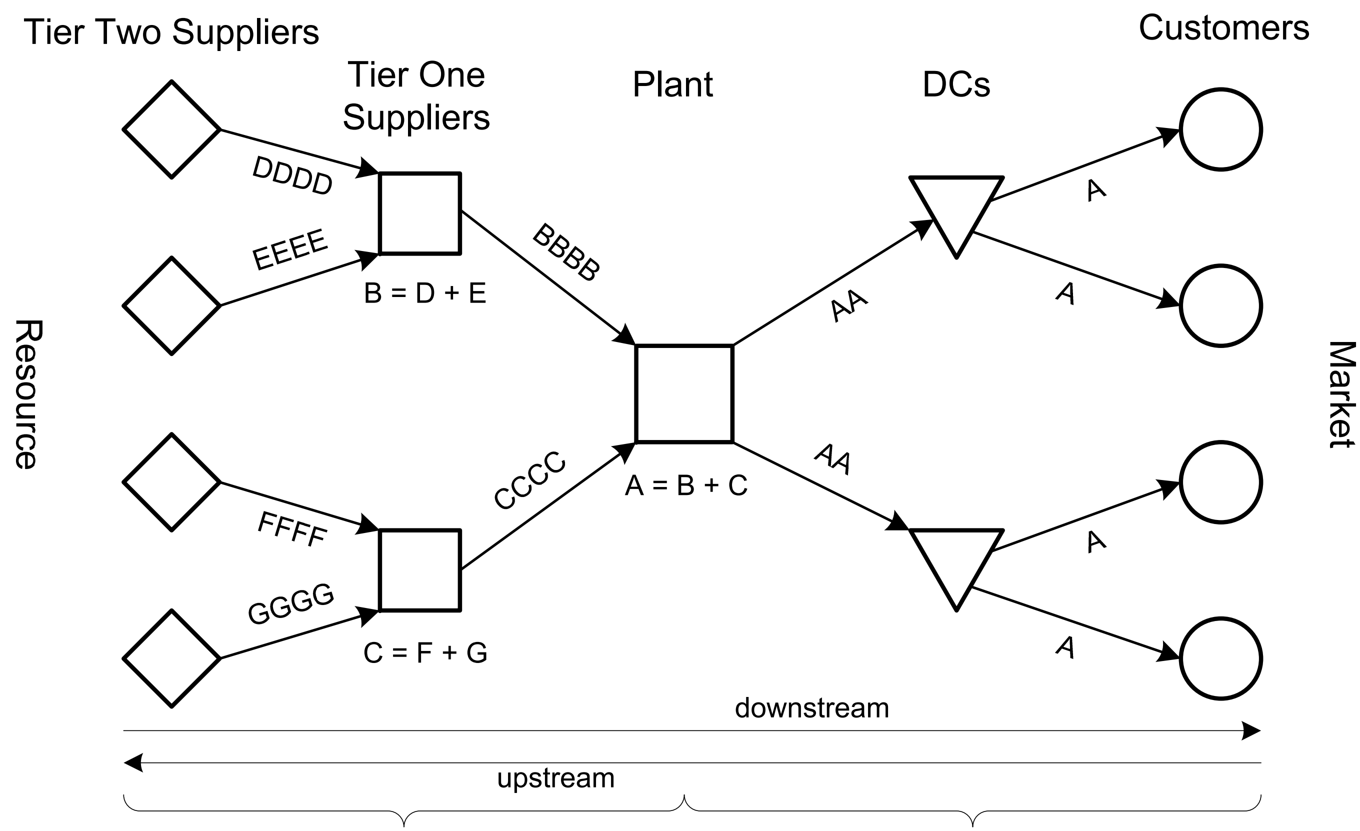
1. Order EFs so that $|x_1| \le |x_2| \le \cdots \le |x_m|$
2. Locate x-dimension of NF at the first EFj where $\sum_{i=1}^{j} w_i \ge \frac{W}{2}$, where $W = \sum_{i=1}^{m} w_i$



$$d_1(P_1, P_2) = |x_1 - x_2| + |y_1 - y_2|$$

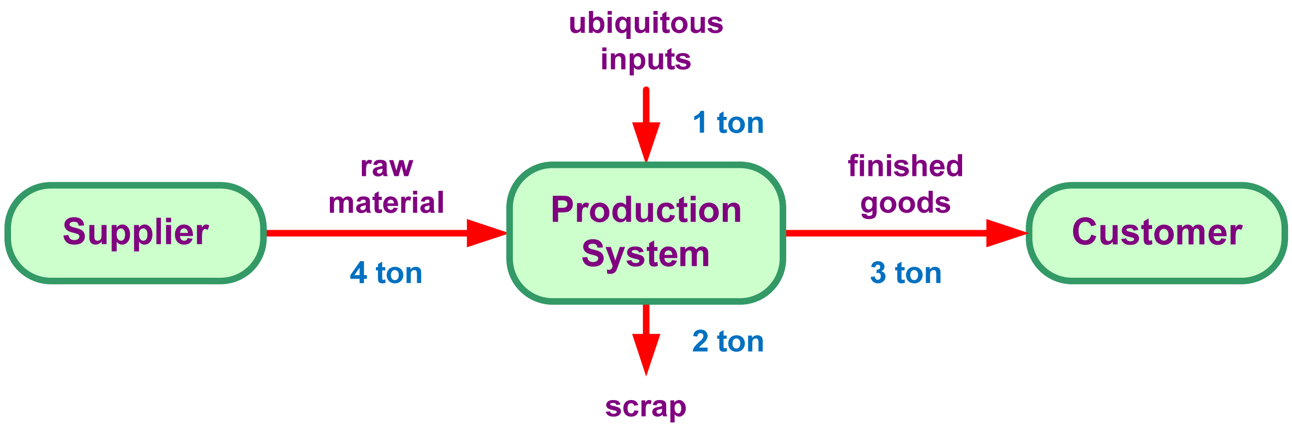
$$d_2(P_1, P_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

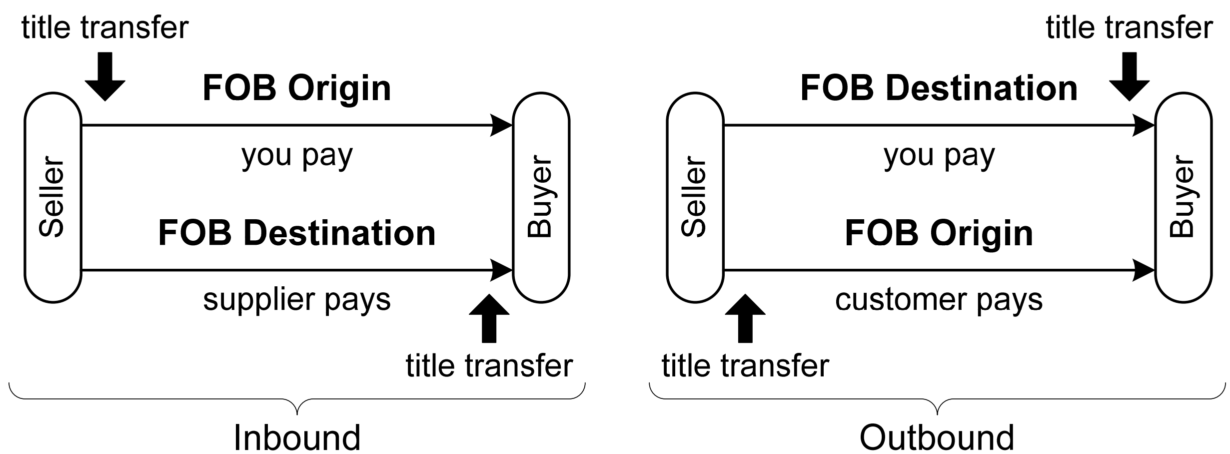
Logistics Network for a Plant



Assembly Network	vs.	Distribution Network
Procurement	vs.	Distribution
Inbound Logistics	vs.	Outbound Logistics
Raw Materials	vs.	Finished Goods

Basic Production System





FOB (free on board)

FOB and Location

- Choice of FOB terms (who directly pays for transport) usually does not impact location decisions:

Procurement cost	=	Landed cost at supplier	+	Inbound transport cost
Production cost	=	Procurement cost	+	Local resource cost (labor, etc.)
Total delivered cost	=	Production cost	+	Outbound transport cost
Transport cost (TC)	=	Inbound transport cost	+	Outbound transport cost

  – *Purchase price* from supplier and *sale price* to customer adjusted to reflect who is paying transport cost
  – Usually determined by who can provide the transport at the lowest cost
    - Savings in lower transport cost allocated (bargained) between parties

Monetary vs. Physical Weight

$$\min TC(X) = \sum_{i=1}^{m} w_i \, d(X, P_i) = \sum_{i=1}^{m} \underbrace{f_i \, r_i}_{w_i} d(X, P_i)$$

where TC = total transport cost (\$/yr)

w_i = monetary weight (\$/mi-yr)

f_i = physical weight rate (ton/yr)

r_i = transport rate (\$/ton-mi)

$d(X, P_i)$ = distance between NF at X and EF$_i$ at P_i (mi)

NF = new facility to be located

EF = existing facility

m = number of EFs

(Montetary) Weight Gaining: $\Sigma w_{\text{in}} < \Sigma w_{\text{out}}$

Physically Weight Losing: $\Sigma f_{\text{in}} > \Sigma f_{\text{out}}$

Minisum Location: TC vs. TD

- Assuming local input costs are
  - same at every location or
  - insignificant as compared to transport costs,

  the minisum transport-oriented single-facility location problem is to locate NF to minimize TC
- Can minimize total distance (TD) if transport rate is same:

$$\min TD(X) = \sum_{i=1}^{m} w_i \, d(X, P_i) = \sum_{i=1}^{m} \underbrace{f_i \, r_i}_{w_i} \, d(X, P_i)$$

where

$$TD = \text{total transport distance (mi/yr)}$$

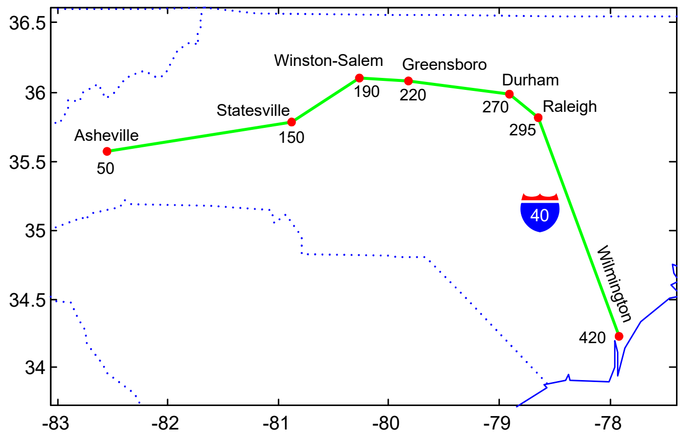
$$w_i = \text{monetary weight (trip/yr)}$$

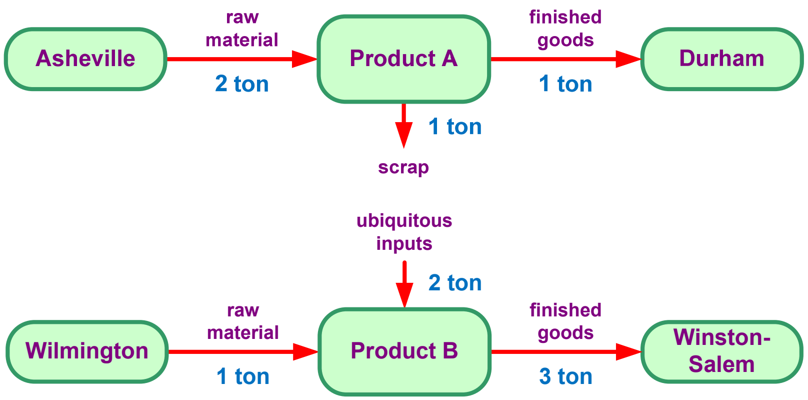
$$f_i = \text{trips per year (trip/yr)}$$

$$r_i = \text{transport rate} = 1$$

$$d(X, P_i) = \text{per-trip distance between NF and EF}_i \text{ (mi/trip)}$$

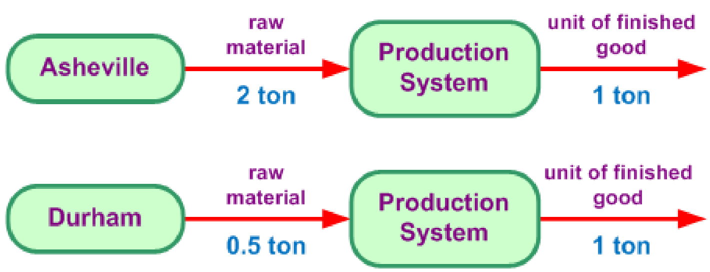
Example: Single Supplier/Customer



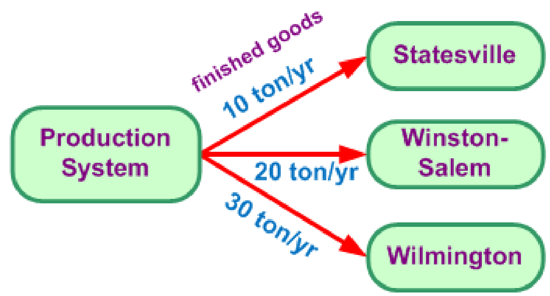


- The cost per ton-mile (i.e., the cost to ship one ton, one mile) for both raw materials and finished goods is $0.10.
  1. Where should the plant for each product be located?
  2. How would the location decision change if the customers paid for distribution costs (FOB Origin) instead of the producer (FOB Destination)?
  3. Which product is weight gaining and which is weight losing?
  4. If both products were produced in a single shared plant, why is it now necessary to know each product's annual demand (f_i)?

1-D Location with Procurement and Distribution Costs



Assume: all scrap is disposed of locally



$$\underset{(\$/\text{yr})}{TC} = \sum \underset{(\$/\text{mi-yr})}{w_i} \times \underset{(\text{mi})}{d_i}$$

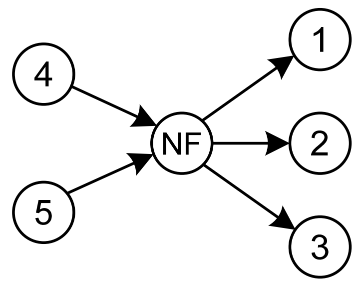
$$\underbrace{\underset{(\$/\text{mi-yr})}{w_i}}_{\text{monetary weight}} = \underbrace{\underset{(\text{ton/yr})}{f_i}}_{\text{physical weight}} \times \underset{(\$/\text{ton-mi})}{r_i}$$

$$r_{\text{in}} = \$0.33/\text{ton-mi}$$

$$r_{\text{out}} = \$1.00/\text{ton-mi}$$

$$f_4 = BOM_4 \sum_{i=1}^{3} f_i = 2(60) = 120, \quad w_4 = f_4 r_{\text{in}} = 40$$

$$f_5 = BOM_5 \sum_{i=1}^{3} f_i = 0.5(60) = 30, \quad w_5 = f_5 r_{\text{in}} = 10$$



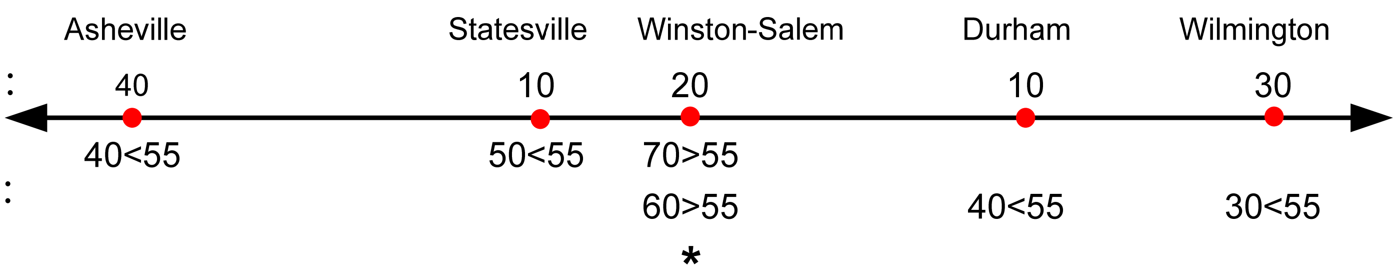
$$f_1 = 10, \quad w_1 = f_1 r_{\text{out}} = 10$$

$$f_2 = 20, \quad w_2 = f_2 r_{\text{out}} = 20$$

$$f_3 = 30, \quad w_3 = f_3 r_{\text{out}} = 30$$

w_i :

$\sum_{i=1}^{j} w_i \le \dfrac{W}{2}$:



$$\text{(Montetary) Weight Gaining:} \quad \Sigma w_{\text{in}} = 50 < \Sigma w_{\text{out}} = 60$$

$$\text{Physically Weight Losing:} \quad \Sigma f_{\text{in}} = 150 > \Sigma f_{\text{out}} = 60$$