1. Solve the differential equation $(y')^2 + 1 = ky^2$ for a constant k.

2. Solve the differential equation $x^2y'' + 5xy' + 4y = x \ln x$.

3. (a) Use the Laplace transform to evaluate the Dirichlet integral $\displaystyle\int_0^\infty \frac{\sin x}{x}\, dx$.
   (b) Solve the initial-value problem

$$y'' + 3y' + 2y = \frac{\sin x}{x}, \quad y(0) = -\frac{\pi}{6}, \quad y'(0) = 1.$$

4. Evaluate $\mathscr{L}\left\{\sqrt{t}\right\}$.

5. Let $y_1,\ y_2,\ \cdots,\ y_n$ be a fundamental set of solutions of the homogeneous linear nth-order differential equation

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y = 0$$

on an interval I. For a general linear nth-order differential equation

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y = f(x),$$

find a particular solution y_p on I using the method of variation of parameters.

6. Solve

$$\mathbf{X}' = \begin{pmatrix} 1 & -4 \\ 1 & 1 \end{pmatrix}\mathbf{X}.$$

Find and classify all critical points of the system. Draw a vector field of the system to describe the geometric behavior of the solution depending on initial values.

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