Physics 360: Mathematical Methods of Physics

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Office Hours: TBD. I also have an open-door policy, and you're encouraged to stop in to ask questions whenever my door is open. That's most of the time.

When and where: Class meetings will be MWF 11:00-12:50 in CST 225. If Zoom sessions are needed, they will all use: https://us02web.zoom.us/j/88190538447

COVID: Campus-wide coronavirus policies and information: https://earlham.edu/coronavirus

Required Materials: Mathematical Methods in the Physical Sciences, Boas, 3E
Piazza account (piazza.com/earlham/spring2022/physmath360)

Pre- and Co-requisites: Math 320 (differential equations) and Math 250 (multivariate calculus).

Strongly-suggested: Physics 125, Matter in Motion **Suggested:** Math 310 (Linear Algebra)

Course Goals: One of the great strengths of a modern physics background is that it allows you to address problems in an enormous range of fields, from physics itself to economics, biology, ecology, computer science, complex systems, physical chemistry, geology, and engineering. The primary objective of this course is to provide a systematic introduction to mathematical techniques that will serve you both in future physics courses and in modeling interesting problems in your domain of choice. A significant secondary objective is to develop a level of mathematical sophistication that will allow you to confidently and competently explore further material on your own.

A significant fraction of the results of modern scientific research can only be accurately expressed in the language of advanced mathematics. A glance at any of the most prestigious scientific journals in your field of interest will quickly confirm this. Thus, even experimental scientists or those interested primarily in field work need to achieve a certain degree of mathematical sophistication if they are to understand the conceptual foundation and interpretation of research results in their own fields. This course is intended to provide you with a working knowledge of those techniques most commonly encountered in science and engineering. In the process of learning these techniques, you will also acquire experience in the physical interpretation of mathematical models of physical problems.

1. Students will learn to apply foundational topics of mathematical physics to a wide array of physical situations.
2. Students will chose a topic not covered in this class, model it with the techniques of mathematical physics, and present their work to the class.
3. Students will gain experience in the physical interpretation of mathematical models of physical systems.

This course contributes to several of Earlham's overall educational goals:

1. Critical reading and thoughtful reflection
2. Understanding the scientific process
3. Analytic reasoning
4. Mathematics proficiency
5. Group learning

Course Calendar*

Week	Date	Read	Topics	HW	Other
1		1.10-1.14	Power Series, Taylor Series		
1		1.15, 2.1-2.6	Accuracy of Approx, $\mathbb{C}$	1	
2		2.7-2.10, 2.11-2.13	Cmplx Power Series, $f(z)$,z^x		
2		2.14-2.15, 12.1-12.2	x^z, **Series Solns** of 2nd Order ODEs	2	
3		12.5, 12.7	Generating functions		
3		12.8, 12.11	Orthogonality, Frobenius	3	
4		12.12-12.13, 12.14	Bessel's Equation		Mini-Midterm
4				4	
5		7.1-7.4	**Fourier Series**		
5		7.5-7.7	More Fourier	5	
6		7.8-7.9,	Other Intervals, Even and Odd Fns		
6		7.12, [*]Extra Material	Fourier Transform, [*]Orthogonal Fns	6	
7		-7.11	Finish Fourier Series		
7		7.12, 8.8-8.9	Transforms: Fourier and Laplace	7	
8		**Spring Break**			
8		**Spring Break**			
9		8.10-8.11	Convolutions, Dirac Delta Functions		Midterm 1
9		8.11-8.12	Dirac Delta Functions, Green's functions	8	
10		6.1-6.6	**Vector Analysis** basic concepts, directional derivative		
10		(catch up)		9	
11		6.6-6.8	gradient, ∇, line integrals		Project Topics
11		6.9, 6.10	Green's Thm, Divergence Thm	10	Paragraph about project
12		6.11	Curl and Stokes Theorem		
12		**Project Workday**		11	
13		13.1-13.3	**PDEs** Laplace, heated plate		Project Draft 1
13		**Earlham Day**			Midterm 2
13		**Project Workday**		12	Midterm 2
14		13.4	Wave Equation, Vibrating String		Project Draft 2
14		**Project Workday**		13	
15		13.5, 13.6	Steady state temp in cylinder, circular membrane		
15		13.6, 13.7	circular membrane, sphere, diffusion	14	
		Final Exam	As per EC schedule. Take-home final, in-person presentations.		

Table 1: Course Schedule (HW, Reading, project work due on day listed; exams handed out on day listed)

*If we get substantially ahead of this syllabus, we can include extra topics like numerical integrators or Noether's theorem.

This class has, in some iterations, spent a week covering **infinite series** (Boas sections 1.1-1.8). This year, we will assume that students are sufficiently familiar with those topics as they were covered in Math 280. **If this is not the case, let me know as soon as possible**.

Major Topics:

Power Series
Computational Methods
Complex Numbers
Series Solution of Second Order Linear Equations
Vector Analysis
Fourier Series and Transforms
Laplace Transforms
Dirac Delta Functions
Partial Differential Equations
Bessel Functions
Numerical integration

Recommended Textbooks

Boyce and DiPrima, Elementary Differential Equations and BVPs This was the differential equations book last semester. It's an excellent reference for PDEs later in the term.

The Feynman Lectures on Physics, Volume II This has some very nice explanations of the vector calculus concepts that we'll be covering, taken from a physicist's perspective. This part of Feynman is also extremely conversational and readable. You can read the whole thing online with beautiful typesetting at http://www.feynmanlectures.info/

Gelfand and Fomin, Calculus of Variations The best introduction I know of to calculus of variations. It's an $11 Dover paperback.

Emmy Noether's Wonderful Theorem by Dwight E. Neuenschwander will be the basis for our discussion of Noether's Theorem, if we cover Noether's theorem. This book is really well written, and goes much farther than we'll have time for.

Kusse and Westwig, Mathematical Physics This text is extremely well written. It doesn't have quite the right focus for this particular class.

Arfken and Weber, Mathematical Methods for Physicists This encyclopedic volume makes for a good reference, but is a bit too dry to learn from in this course.

Schey, div grad curl and all that This is an extremely conversational introduction to/refresher on vector calculus

Communication: My preferred mode of communication is email. I will check my email multiple times per day during the work week. If you have sent me an email and have not heard back from me within 24 hours (excluding weekends), you should email me again or drop by my office to ask your question in person. I cannot guarantee that I will regularly check my email on evenings or on weekends.

I expect that you will check your Earlham email at least once per day. That is, 24 hours after sending an email to the class, it is my expectation that all of you will have read that email. Emailed assignments, due dates, policy changes, and other course information carry the same weight as information contained in the syllabus.

Information relevant to the course will also be posted to the Moodle page. You are responsible for checking Moodle at the same frequency and with the same conditions applied to email communication.

Workload Expectations: The expectation is that you will spend 2-3 hours outside class for every hour in class. So, for this three-credit class, you can expect to spend roughly 6-9 hours per week on reading, homework and exams. **If you find yourself spending significantly more (or less) than this, please let me know ASAP! That's an instructor-error that I can easily fix.**

Contacting me about sensitive issues: What if you have something you want to talk about, but don't feel comfortable doing so in a public (or non-anonymous) space? This often comes up with issues of racism, sexism, and other kinds of discrimination. It can come up with issues that relate to my behavior, the behavior of your classmates, or the world in general. Please feel *encouraged* to use either the anonymous forum on Moodle (I can't see who wrote something in the anonymous Moodle forum, though I *can* see who writes anonymous Piazza comments), or to leave a written note under my door. If you want to write a note, you may want to type it up and print it out so that your handwriting is further anonymized. Of course, if you feel comfortable and safe doing so, you are also encouraged to talk publicly or non-anonymously about these things!

If you are not comfortable with any of these options, you are encouraged to contact the Vice President for Student Life, Bonita Washington-Lacey (mailto:washibo@earlham.edu).

Academic Enrichment Center (tutoring, study help): The Academic Enrichment Center (AEC) provides assistance with study habits and skills as well as a peer tutoring service. The AEC is staffed by trained peer tutors for either pre-arranged group tutoring sessions (provided for many math, science and social science courses) or one-on-one tutoring sessions for other courses. Peer tutoring is a free service offered to all Earlham students. While we typically do not have tutors assigned to this class, we can definitely find one-on-one tutors for you at your request! Please visit https://earlham.edu/academics/academic-support-and-special-programs/academic-enrichment-center for more information.

Diversity Statement: I can hardly overstate how much inclusion, equity and diversity considerations have shaped my entire thinking around teaching and practicing physics. It affects everything from the way that physics is framed historically to the way that it is typically taught in your classrooms. My main goal is to create an environment that tells all of us that we can do physics; that invites people into physics who have never been invited, or who have been explicitly included. In the American physics context, these issues are most commonly framed in terms of gender, race, and the typical colonial framing of a physics curriculum. In most of our introductory classes, we have a "Non-Newtonian Physicist" assignment that focuses specifically on decolonising an introductory physics curriculum. In this class, you can choose whether to do this as a separate assignment, or whether to integrate into your independent project.

Physics cannot be divorced from the world in which we study it. I expect that we will recognize that in this class, and I expect that we will create an inclusive and welcoming environment. More specifically, I expect that we will strive *not* to create an environment that excludes people. This is hard work. We will get things wrong. You will get things wrong. *I* will get things wrong. Especially when I get things wrong, I encourage you to refer to the resources in the previous section of the syllabus (direct comments, anonymous Moodle comments, anonymous comments under my door, and comments to Bonita).

Grading Policy and Late Policy: Your grade in this class will be determined by a weighted combination of your scores from multiple categories. Within each category, each assignment is weighted equally. It is extremely hard to catch up on late work in normal times (usually things are late because students are busy, and people don't tend to get less busy!). This is especially true in a pandemic. So, I will drop a roughly a week worth of your lowest grades from most categories. If you have to turn things in late, my strong suggestion is that you prioritize getting your schedule back under control. Feel free to simply *not do* things that will not be graded. Especially if it helps you keep the rest of your life under control.

Because things can be even harder to schedule during a pandemic, I'm adding a second policy this year: a late pass policy that follows my policy from last year:

All students start the term with 5 free late passes. You can spend a late pass to extend the deadline of a single assignment to the following class period without any penalty to your score. Late passes cannot be applied to quizzes, or exams, but will work for every other type of assignment. Once you have spent your 5 free late passes, you can request additional extensions, but they come with a penalty to your score. For the 6th late pass, I will deduct 10% of your points on that assignment; for the 7th late pass, I will deduct 20% of your points on that assignment; and so on.

Late passes are not automatic. You must always email me to let me know that you plan to use a late pass on a given assignment. Once you declare to me that you are spending a late pass on an assignment, that late pass is spent whether or not you follow through and actually turn in the assignment by the modified due date.

Assignment	Number	Drop	Weight
Attendance/preparation/participation (including Piazza)	41	3	10
Homework	14	1	33
Non-Newtonian Physicist	1	0	3
Mini-midterm (open-book)	1	0	5
Two midterms, equally-weighted	2	0	20
Final exam	1	0	14
Independent Project	1	0	15

Final grades will be calculated according to the following scale, although individual quiz and exam scores may be curved.

Grade	Range
A+	99.1-100%
A	93.0-99.0%
A-	90.0-92.9%
B+	87.0-89.9%
B	83.0-86.9%
B-	80.0-82.3%
C+	77.0-79.9%
C	73.0-76.9%
C-	70.0-72.9%
D+	67.0-69.9%
D	63.0-66.9%
D-	60.0-62.9%
F	0.0-59.9%

Standard Grading Rubric: Several of the problems assigned in this class are quite challenging. Others are just rote computation. For the more challenging problems, my goal is to have you make the strongest possible effort towards **understanding** the solution. Thus, if you cannot fully solve the problem, say whatever you can about the way in which a solution would proceed from where you stop; say whatever you can about the qualitative behavior of a solution; say whatever you can about the physical meaning of the solution.

Homework problems will be graded on roughly the same scale as used in Physics 125 and 235:

Grade	Explanation
5	Solution is complete and well-written
4	Solution is missing minor parts or some important explanations
3	Solution is missing major parts and/or has few if any explanations
2	At least one major portion of the problem correct
1	Very little coherent initial effort was expended
0	No initial solution was submitted

Academic Accommodations: Students with a documented disability (e.g., physical, learning, psychiatric, visual, hearing, etc.) who need to arrange reasonable classroom accommodations must request accommodation memos from the Academic Enrichment Center(main floor of Lilly Library) and contact their instructors each semester. For greater success, students are strongly encouraged to visit the Academic Enrichment Center within the first two weeks of each semester to begin the process. https://www.earlham.edu/academic-enrichment-center/disability-services

We at Earlham appreciate the contribution that athletes make to the vitality of our community. If you must miss class due to a competition, notify me well in advance so we can make arrangements for you to make up the work. If you do not notify me in advance, there is every chance that I may not be able to make special arrangements.

Attendance/preparation/participation (including Piazza): Especially in a pandemic, attendance is (1) critical (2) sometimes hard to sustain. During class, we will cover material via lecture, and we will work on problems together and via groups in breakout rooms. This problem solving is a critical component of the class, and it is the main reason attendance is required. We all know that circumstances sometimes make it hard or impossible to attend, so I will drop your three lowest attendance and participation grades.

It is very important that students study the appropriate text assignments before coming to class and that they be prepared either to ask questions or to discuss questions related to the assigned material. The subject matter of this course is sufficiently complex that it is neither desirable nor possible to cover all relevant details in class. The primary value of lectures and class discussions in this course is to explain and illustrate points which students find difficult or obscure. It is essential, therefore, that students put in the advance work required to identify such points; the instructor cannot anticipate all of them. It is hoped that all of us will work together to produce a class atmosphere which is conducive to lively and interesting discussions of the material. Readings are assigned for every class period. By **midnight before each class period, you must comment on Piazza**. Your comment can be in one of the three following categories: (1) something interesting from the reading, (2) something confusing from the reading, (3) an answer to someone else's question. In order to make sure we have a good, constructive discussion atmosphere, please note that disrespectful comments will receive no credit and may be removed. You can post anonymously; instructors can see who made each post, but students cannot. **The piazza comments count for 5% of your grade.**

Exams: There will be three midterms and a final. The first midterm (the "mini-midterm") will be open-book and untimed. The other tests will be timed and closed-book. All will be self-scheduled take-home exams. The final will be cumulative, and we will have a clear discussion and review of topics before the exam. We will use the scheduled final exam time for project presentations. If you have a time conflict, you must reach out to me in advance of the date of the final.

Homework: This is basically a course in problem-solving techniques. The most common and the most damaging mistake a student can make in such a course is to yield to the temptation to try and master the material by reading the text and listening to the lectures while working a minimum of problems. While reading and listening can give you valuable new ideas, problem-solving skills are only efficiently developed by solving as many problems as possible. For most students the understanding and knowledge gained from this course will be in direct proportion to the number of problems which they successfully complete. The best approach is to maximize the percentage of your study time devoted to a disciplined effort to solve problems.

Homework will be assigned each approximately once per week. As with all Physics classes, you will find that your understanding is greatly improved if you start the assignments early in the week, rather than late in the week.

Independent project: The primary goals of this course are involve developing a level of mathematical sophistication that will allow you to confidently and competently explore further material outside of a lecture setting. So, in order to make sure we're walking the walk, you'll each pick either an interesting problem to model or an interesting technique to learn. You'll write a short paper and present the results to the class. You'll be expected to start on this halfway through the semester, and we'll discuss it in more detail at that point. Examples might include

- The Fast Fourier Transform (FFT)
- Physical representations of Fourier Transforms, including circuits and optics
- The discrete Fourier Transform, including application to time series data, consideration of how many useful samples you have in a dataset in Fourier space, etc.
- Struve functions
- Complex analysis including itegrating around poles, Cauchy Integral, Residue theorem (**this is a hard topic to nail down in terms of applications; you must carefully verify your proposal with me first!**)
- Tensors, covariant vs. contravariant, pseudo-vectors, index notation
- Black-Scholes Theorem (economics)
- Applications of Noether's Theorem to economic systems
- Modern techniques such as the immersed-boundary method
- Coding and examination of more complex numerical integrators
- Green-Kubo relations and the Fluctuation Theorem
- Wavelet Transforms
- Real world applications of any of the techniques studied in this course
- Numerical integration (e.g. write your own RK4 and Velocity Verlet, compare the two for some real application)
- Noether's Theorem
- The physics of a golf swing
- Building a better baseball lineup with Monte Carlo Methods
- Biomedical examples of Laplace transforms
- Interesting topic of your choice!

Note: If there is sufficient demand, it is likely possible to include at least one of the above topics in the main body of the course itself.

Academic Integrity: http://www.earlham.edu/curriculum-guide/academic-integrity/ As with all Earlham courses, this course is covered by the academic integrity policy. Much of the work in this class is group-based, but not all of it. Specific expectations are discussed in class and in this syllabus, but: daily problems should be attempted individually first. You may then ask for guidance in class, in the forums, and from your peers. Quizzes and exams must be done individually.

Sexual Harassment and Title IX Federal law, Title IX, and Earlham College policy prohibits discrimination, harassment, and violence based on sex and gender (including sexual harassment, sexual assault, domestic / dating violence, stalking, sexual exploitation, and retaliation). If you or someone you know has been harassed or assaulted, you can receive confidential support from a number of resources on and off campus, listed here: http://earlham.edu/counseling-services/confidential-resources-for-survivors-of-sexual-violence/.

Alleged violations can be reported non-confidentially to the Title IX Coordinator, Jenelle Job, at ext. 1346. Reports to law enforcement can be made to Earlham Public Safety at ext. 1400, or to the Richmond Police Department at 765-983-7247 (for non-emergency calls). I will seek to keep information you share with me private to the greatest extent possible, but as a professor I have mandatory reporting responsibilities regarding sexual misconduct and crimes I learn about to help make our campus a safer place for all.