Physics 360/Math 360   TF 1:00 - 2:20 PM, Room: CST 225

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Assignment 3, Due Thursday Feb 24, end of day.

Actually due early

The first part is actually due on Tuesday: we started working on a problem in class, where you were solving a differential equation via the table method. For Tuesday, get as far as you can on that problem. Please feel *encouraged* to post questions on Piazza! This is meant to be a group problem.

Due on the real due date

Table problem: Using a table like the one in Boas §12.2, solve problem 12.1.1

Legendre Problems:

Next set of problems: In this assignment, as in many of the future assignments, we will investigate a particular topic in depth, rather than solving several separate problems. This week, we focus on the Legendre polynomials, which have broad applicability in mathematical physics, especially in the modeling of spherically symmetric systems.

The text of the following problems is taken (with some small changes) from Boyce and DiPrima, Chapter 5, section 3.

The following problems deal with the Legendre equation:

$$(1-x^2)y'' - 2xy' + \alpha(\alpha+1)y = 0 \tag{1}$$

Following the convention of choosing a fundamental set of solutions such that

$$\begin{aligned} y_1(x) &= 1 + b_2(x-x_0)^2 + ... \\ y_2(x) &= (x-x_0) + c_2(x-x_0)^3 + ... \\ b_2 + c_2 &= a_2 \end{aligned}$$

(Note that these series have already included the fact that one will be even and one will be odd, a fact that you'll show below.)

Two solutions of the Legendre equation for $|x| < 1$ are

$$\begin{aligned} y_1(x) &= 1 - \frac{\alpha(\alpha+1)}{2!}x^2 + \frac{\alpha(\alpha-2)(\alpha+1)(\alpha+3)}{4!}x^4 \\ &\quad + \sum_{m=3}^{\infty}(-1)^m \frac{\alpha\cdots(\alpha-2m+2)(\alpha+1)\cdots(\alpha+2m-1)}{(2m)!}x^{2m}, \\ y_2(x) &= x - \frac{(\alpha-1)(\alpha+2)}{3!}x^3 + \frac{(\alpha-1)(\alpha-3)(\alpha+2)(\alpha+4)}{5!}x^5 \\ &\quad + \sum_{m=3}^{\infty} \frac{(\alpha-1)\cdots(\alpha-2m+1)(\alpha+2)\cdots(\alpha+2m)}{(2m+1)!}x^{2m+1} \end{aligned}$$

Legendre Problem 1 Write out the first 4 terms for y_1 and y_2.

Legendre Problem 2 Show that, if α is zero or a positive even integer $2n$, the series solution y_1 reduces to a polynomial of degree $2n$ containing only even powers of x. Find the polynomials corresponding to $\alpha = 0, 2, 4$. Similarly, show that if α is a positive odd integer $2n + 1$, the series solution y_2 reduces to a polynomial of degree $2n + 1$ containing only odd powers of x. Find the polynomials corresponding to $\alpha = 1, 3, 5$.

(There will be more Legendre problems on the next homework assignment)