A Diagnostic TANK Model for the Housing Market *

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Abstract

The U.S. housing market exhibits an unusually high degree of volatility, which challenges traditional models that rely on large preference shocks to explain such fluctuations. In this paper, I argue that the expectation channel plays a key role in driving this volatility. I incorporate Diagnostic Expectations (DE) within a Two-Agent New Keynesian (TANK) model featuring housing and banking sectors. Using Sequential Monte Carlo methods to estimate the model, I find that DE reduce the size of the housing preference shock by more than one-third relative to Rational Expectations, while reproducing the housing market fluctuations. This result holds whether agents' imperfect memory is based on recent or three-year past experiences. When the expectations channel is removed -i.e., when agents are rational- the model fails to generate the high volatility in house prices observed in the data. These findings highlight the importance of the expectation formation process in explaining a substantial part of unmodeled disturbances affecting the housing market.

Keywords: Diagnostic expectations, volatility, house prices.

JEL classification: D84, E32, E71, R31.

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"The mind of every man, in a longer or shorter time, returns to its natural and usual state of tranquillity. In prosperity, after a certain time, it falls back to that state; in adversity, after a certain time, it rises up to it" (Smith, 1759, p. 172)

1 Introduction

During the period spanning from the mid-eighties to the aftermath of the Great Financial Crisis, the U.S. housing market has been defined by its high volatility (Piazzesi & Schneider, 2016). Quantities and prices growth rates are three and six times more volatile than GDP, as seen in Figure 1, with standard deviations of 1.723% and 3.452%, respectively. Understanding what drives these dynamics is relevant, given the valuable information that the housing market provides about ongoing changes in economic activity (Chahrour & Gaballo, 2021) and the importance of housing in households' decisions and wealth (Davis & Heathcote, 2005).

Traditional models typically attribute pronounced house price movements to housing preference shocks, but this approach limits the insights offered for policy analysis by overlooking expectation-driven dynamics. Recent work, including Gelain, Lansing, and Mendicino (2012), suggests that agents' expectations can significantly influence monetary policy responses. Moreover, empirical studies challenge the rationality assumption in the housing market, indicating that expectations are the source of the pronounced fluctuations in the sector. The evidence reveals that housing market expectations strongly track recent observed house price changes (Kuchler, Piazzesi, & Stroebel, 2023; Adam, Pfäuti, & Reinelt, 2024), with price expectations showing short-run momentum (Gohl, Haan, Michelsen, & Weinhardt, 2024). Additionally, De Stefani (2021) finds that the risk of a downturn after a long period of growth in house prices is underestimated by consumers, generating predictable errors. Together, these findings position non-rational expectations as a promising explanation for the higher volatility in housing markets.

In this context, I develop a two-agent New Keynesian (TANK) model that incorporates a housing market inspired by Iacoviello and Neri (2010), a banking sector following the framework in Gertler and Karadi (2011), and diagnostic expectations (DE) as proposed by Bordalo, Gennaioli, and Shleifer (2018).[1] Diagnostic agents form beliefs influenced by recent (or not-so-recent) trends. For example, a history of rising (falling) house prices tend to make future prices following the same trend more prominent in diagnostic agents mind, but when these projections don't materialise, DE creates feedback loops that amplify optimism or pessimism. By introducing DE into this model, I aim to address the observed

[1] The authors build DE on Kahneman and Tversky (1972) concept of representativeness. This describes a judgemental process where the most distinguished characteristic of an event plays the main role in a human's mind when assigning probabilities.

volatility in the housing market without relying on large shocks. The main contribution of this paper, therefore, is to show that DE can account for approximately thirty to fifty percent of this volatility, offering a potentially more robust alternative to traditional explanations that attribute unexplained demand changes to housing preference shocks.

Figure 1: Real GDP, real house price and real residential investment in percentage change.

Building on this, I also explore an alternative structure for the reference group used by agents forming DE. Instead of relying solely on the immediate past, I extend the framework to examine how more distant memories might affect and shape agents' background context. Following Bordalo et al. (2018), I introduce a slow-moving reference by defining representativeness as a mixture of current and past likelihood ratios. This approach differs from Bianchi, Ilut, and Saijo (2024), as they consider a weighted average of lagged expectations as comparison group. To the best of my knowledge, this is the first attempt to incorporate and study DE with a slow-moving structure as reference in a model featuring heterogeneous agents, a housing sector, and a banking sector. Under this set up, I derive that diagnostic agents using a slow moving reference group misperceive the shock as an ARMA(1,S) process, where S represents the length of the periods used as the reference group.

I calibrate and estimate each model for the U.S. economy using recent advancements in macroeconomic model estimation by Herbst and Schorfheide (2014). The results supports the role of DE in driving housing market dynamics in the U.S. Compared to traditional RE models, DE reduces the standard deviation of the housing preference shock by at least one-third, suggesting DE could be a comprehensive alternative to the "catchall of all the unmodeled disturbances that can affect housing demand" (Iacoviello & Neri, 2010, p. 150). Though the evidence favours the DE model with a one-quarter lag reference, two key takeaways emerge from extending the comparison group: first, the prominence of recent events in shaping agents' expectations, and second, that most attention beyond this period centres between quarters three and ten.

In addition, a historical shock decomposition analysis indicates that the shock transmission mechanism in the economy remains stable regardless of agents' expectations formation process. The difference lies in the more volatile expectations intrinsic to DE, which amplify the impact of shocks without altering their transmission through the economy. I also examine the influence of DE on the economy using impulse responses. In general, both DE frameworks share similar characteristics: initial over-reactions, greater persistence and pronounced fluctuations. The extrapolation of shocks explains the initial over-reactions and subsequent reversals observed in both DE models, but other features are specific to each framework. In the DE model with a one-quarter reference, the economy's rigidities propagate the initial overreaction. On the other hand, the DE model with a twelve-quarters slow-moving reference exhibits more pronounced fluctuations, as agents may remain overly optimistic (pessimistic) due to the longer span memory in their expectation formation process.

A counterfactual analysis further disentangles the propagation and amplification mechanism of DE. When diagnostic agents who rely on the immediate past to form beliefs suddenly become rational, i.e. the diagnostic parameter equals zero, the model struggles to replicate house price volatility. This provides further evidence that it is the expectations mechanism, particularly DE, that drives cycles in the housing market. This paper thereby contributes to a growing body of research advocating for models that integrate expectation formation more closely aligned with observed economic behaviour, moving beyond preference shocks to examine the dynamics of household and market expectations.

Related literature

This paper is linked to recent articles that incorporate DE in macroeconomic models. One group of authors incorporates DE in macro-finance environments. Bordalo et al. (2018) find that such an extended macroeconomic model captures the empirical findings regarding credit cycles. Bordalo, Gennaioli, Shleifer, and Terry (2021) and Maxted (2024) combine DE in real business cycle models with financial frictions. Their main results are a greater variability in the macroeconomy and the ability to replicate financial crisis aspects, as well as the counter-cyclicality of credit spreads. More recently, L'Huillier, Singh, and Yoo (2024) derive a general framework to incorporate DE in linear models and show that DE are a viable behavioural alternative to generate fluctuations in business cycle models with shocks of more realistic size.

While the previous studies explored DE within a one-period reference framework, Bianchi et al. (2024) focus on distant memory and find that DE generate considerably rich dynamics, characterised by significant persistence and sudden changes in the way shocks propagate. In this line, and closely related to the current article, Qi (2021) and Bounader and Elekdag (2024) introduce DE and distant memory in New Keynesian models with heterogeneity in agents, a housing market and financial frictions. The first author finds a higher persistence and significant responses from house prices to a TFP shock in a TANK. The second authors contribute showing that DE and financial frictions reinforce shock amplification, especially after demand shocks.

The literature aiming to explain housing market dynamics using macro models is also related to this article. Within this literature, two groups stand out: (i) models explaining such dynamics using preference shocks and (ii) models relying on the inclusion of expectations-driven excess volatility by departures from rationality.

The first group of authors focuses on understanding the nature of shocks and movements in the housing market, as well as the effects that such variations have on the economy. The work of Iacoviello and Neri (2010) represents the cornerstone of this literature. They estimate a model akin to the one presented in this paper. They find that at least a quarter of the housing market's dynamics could be explained by a housing demand shock, which, in their words, is spontaneous, primitive and interpretable characteristics are questionable (p. 158). The authors estimate a shock size of approximately 4%. Other authors have estimated similar models for different countries. For example, Gerali, Neri, Sessa, and Signoretti (2010) use European data and obtain an estimate around 7%, while Funke and Paetz (2013) study the Hong Kong housing market and estimate the standard deviation of the preference shock to be around 10%. They acknowledge that this shock is the most important determinant of domestic property prices. Mendicino and Punzi (2014) reach a similar conclusion by analysing the relative importance of such shock in a theoretical model, where the preference housing shock accounts for 70% of the volatility in house prices. More recently, Ge, Li, Li, and Liu (2022) study the housing market dynamics in China, finding results consistent with previous works. They estimate the standard deviation of the housing preference shock to be around 7%, with this shock accounting for more than 80% of the volatility in the Chinese housing market.

In the second group of literature, deviations from RE are explored, such as adaptive expectations and learning. Some researchers, including Gelain et al. (2012), and Granziera and Kozicki (2015), argue that adaptive expectations can increase the volatility of the housing market due to overoptimism and overreaction to fundamentals. Moreover, this framework has been successful in generating momentum and volatility in models of the stock market, characteristics also presented in the housing sector. However, it is an ad hoc not micro founded approach, making DE a better choice. In addition, DE has already shown its ability to capture the run-ups and sharp decline behaviour present in the financial markets.

On the other hand, including learning suggests that individuals form mechanical backward-looking rules for belief updating. Chahrour and Gaballo (2021), Caines (2020) and Gandré (2022) provide evidence supporting the inclusion of learning about house prices as an amplification and propagation mechanism that helps to account for the dynamics of macro variables as well as credit and housing. However, this approach assumes that agents do not understand the true data generating process. In contrast, DE have three advantages over mechanical models of non-rational belief: it is forward-looking (no Lucas critique), it better accounts for measured expectations of financial analysts and macro forecasters, and its diagnostic parameters have been estimated in several data sets (Bordalo et al., 2021; L'Huillier et al., 2024; Bianchi et al., 2024). Additionally, DE have been successfully applied not only in macro and finance settings but also in, for example, modelling social stereotypes (Bordalo, Coffman, Gennaioli, & Shleifer, 2016).

Therefore, these reasons provide a solid basis for the inclusion of DE in a macro model to analyse the behaviour of the housing market. While Qi (2021) attempts to incorporate DE and distant memory in her model, and Bounader and Elekdag (2024) study the theoretical implications of including DE with a financial accelerator, my approach builds on these efforts and enhances them in several ways. First, I incorporate a banking sector, which introduces additional frictions and channels through which the expectations can lead to a more volatile economy. Second, I empirically estimate the diagnostic parameter value and weights assigned to past references. Finally, I extend my analysis beyond the housing market price and include quantities as well.

Structure of the paper

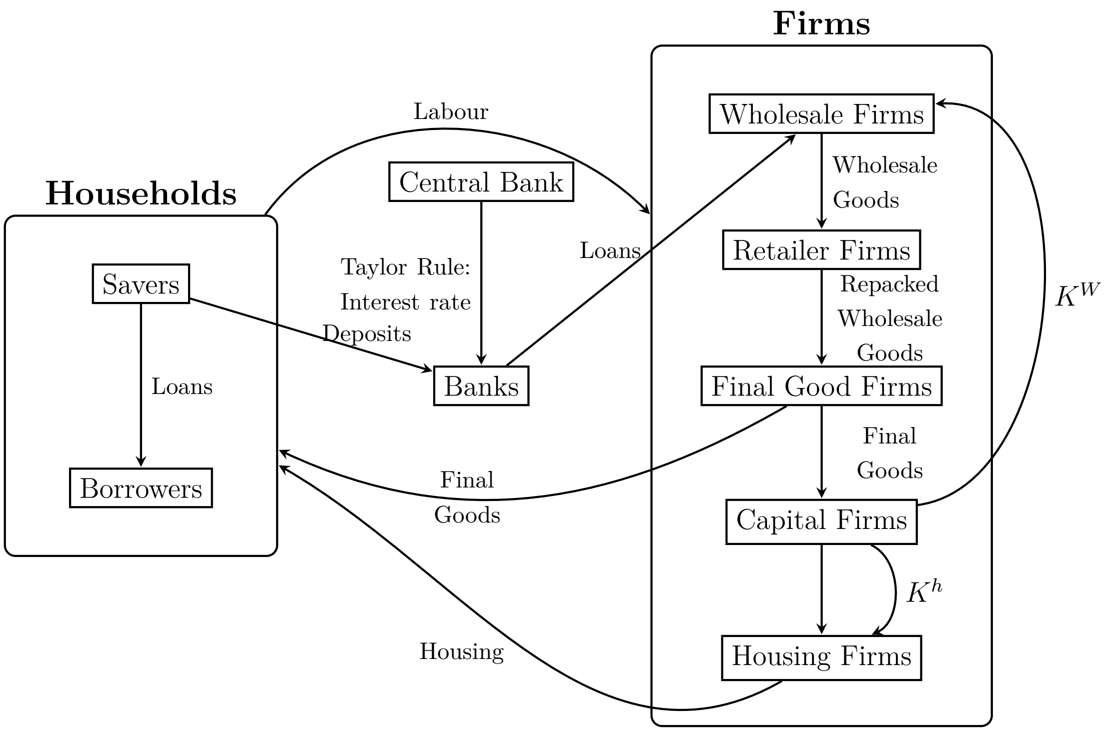
The rest of the paper is structured as follows. In section 2, I present the model. Section 3 explains how I include and solve the DE model. The calibration and estimation of the parameters are outlined in Section 4. Section 5 discusses the quantitative results. A counterfactual analysis is done in Section 6, and Section 7 concludes.

2 Model

The basic structure of the model is similar to Iacoviello (2005), Iacoviello and Neri (2010) and Gelain et al. (2012), although I extend it in some ways. First, I include capital producers which sell part of the total capital stock to wholesale firms and rent the rest to housing firms. This allows to derive an explicit expression for the real price of capital, as well as for the rental rate of capital in the housing sector (Gambacorta & Signoretti, 2014). Second, to model the housing market price and quantity dynamics, I introduce a housing production sector that produces houses using capital and labour services (Iacoviello &

Neri, 2010). Finally, I incorporate financial frictions using a banking sector as in Gertler and Karadi (2011). In this section, I present the derivations under RE, whereas in a later section I show how to modify the model to introduce DE.

Figure 2: Economy Model



The model summarised in Figure 2 consists of two types of households: patient and impatient, each of mass $1-n$ and n, respectively. The patient households are the savers in the economy. They provide liquidity to the impatient households, borrowers, in the form of loans. There are five types of firms: (i) wholesale firms producing wholesale goods, (ii) retailer firms re-packaging wholesale goods and introducing a price rigidity *à la Calvo*, (iii) a final good firm producing its output using goods from retailers as inputs, (iv) housing firms producing houses with labour and capital as inputs, and (v) capital good firms combining undepreciated capital and the final good to update and produce new capital. The model also features a banking sector as in Gertler and Karadi (2011). These banks act as financial intermediaries between patient households' deposits and wholesale firms' loans.[2] Finally, there is a central bank that sets the nominal interest rate following a simple Taylor-type rule. The model includes habit formation in consumption, investment adjustment costs, and nominal rigidities. Time is discrete, and one period in the model

[2]This version of the model does not allow for arbitrage between loan and deposit interest rates, this means the banks do not intermediate between households. The main reason behind this choice is to keep the banking problem easy to track. However, in a future version, banks will not only serve as intermediaries between patient households and firms, they will also mediate transactions with impatient households.

represents one quarter.

2.1 Households

The economy is populated by two type of households, patient and impatient denoted with subscripts "p" and "i", respectively. They consume non-durable goods, buy housing and supply labour. The patient households save in the form of deposits in banks and lend money to impatient households, who borrow using their housing as collateral.

2.1.1 Patient household

A representative patient household derives utility from consumption, $c_{p,t}$, and housing, $h_{p,t}$, and disutility from labour $n_{p,t}$. She discounts future utility flows by β_p and her lifetime utility is:

$$U_p = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_p^t \left[log(c_{p,t} - \gamma c_{p,t-1}) + \Gamma_t \nu_p^h log(h_{p,t}) - \nu_p^n \frac{n_{p,t}^{1+\varphi}}{1+\varphi} \right], \tag{1}$$

where Γ_t is a housing preference shock common to both agents that follows the AR(1) process $log(\Gamma_{t+1}) = \rho_\Gamma log(\Gamma_t) + \sigma_{\epsilon^\Gamma} \epsilon_{t+1}^\Gamma$, with $\rho_\Gamma \in (0,1)$ and $\epsilon_{t+1}^\Gamma \sim i.i.d.[0, \sigma_{\epsilon^\Gamma}^2]$. The habit formation parameter is $\gamma \in$ (0,1), and ν_p^h and ν_p^n govern the patient household's utility from housing and labour, respectively. The parameter φ is the inverse elasticity of labour supply.

The patient household maximises her utility subject to the following budget constraint:

$$c_{p,t} + q_t[h_{p,t} - (1-\delta_h)h_{p,t-1}] + d_t^B + d_t^l = \frac{d_{t-1}^B R_{t-1}^d}{\pi_t} + \frac{d_{t-1}^l R_{t-1}^l}{\pi_t} + w_t n_{p,t} + \Pi_{f,t} + \Pi_{B,t}. \tag{2}$$

q_t is real house prices, δ_h is the rate at which housing depreciates, and w_t is the real wage from supplying labour. The term d_{t-1}^B represents deposits held by the patient household in the bank at the end of time $t-1$, which yield a risk-less nominal return of R_{t-1}^d between period $t-1$ and t. d_{t-1}^l represents loans that the patient household lent to the impatient one, yielding a nominal return of R_{t-1}^l. π_t is the gross inflation rate and $\Pi_{f,t}$ and $\Pi_{B,t}$ are transfers of profits from the firms and the banks.

The resulting first order conditions of the patient household's maximisation problem with respect to $c_{p,t}$, $n_{p,t}$, $h_{p,t}$, d_t^B and d_t^l are:

$$\lambda_{p,t} = \frac{1}{(c_{p,t} - \gamma c_{p,t-1})} - \frac{\beta_p \gamma}{(c_{p,t+1} - \gamma c_{p,t})}. \tag{3}$$

$$\nu_p^n n_{p,t}^{\varphi} = w_t \lambda_{p,t}. \tag{4}$$

$$\lambda_{p,t} q_t = \frac{\Gamma_t \nu_p^h}{h_{p,t}} + \beta_p \mathbb{E}_t \Big[(1-\delta_h) q_{t+1} \lambda_{p,t+1}\Big]. \tag{5}$$

$$\lambda_{p,t} = \beta_p \mathbb{E}_t \Big[\lambda_{p,t+1} \frac{R_t^d}{\pi_{t+1}}\Big]. \tag{6}$$

$$\lambda_{p,t} = \beta_p \mathbb{E}_t \Big[\lambda_{p,t+1} \frac{R_t^l}{\pi_{t+1}}\Big]. \tag{7}$$

$\lambda_{p,t}$ denotes the marginal utility of consumption.

2.1.2 Impatient household

A representative impatient household also receives utility from consumption, $c_{i,t}$, and housing, $h_{i,t}$, and disutility from labour $n_{i,t}$. She discounts future utility flows by β_i, which is smaller than the patient household's discount factor, β_p, and her lifetime utility is:

$$U_i = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_i^t \Big[log(c_{i,t} - \gamma c_{i,t-1}) + \Gamma_t \nu_i^h log(h_{i,t}) - \nu_i^n \frac{n_{i,t}^{1+\varphi}}{1+\varphi}\Big], \tag{8}$$

where $\gamma \in (0{,}1)$ and φ are the same habit formation and inverse elasticity of labour supply parameters as for the patient household. ν_i^h and ν_i^n governs the utility from housing and labour for the impatient household. She faces the same housing preference shock Γ_t, and she maximises her utility subject to the following budget constraint:

$$c_{i,t} + q_t(h_{i,t} - (1-\delta_h) h_{i,t-1}) + \frac{l_{t-1} R_{t-1}^l}{\pi_t} = w_t n_{i,t} + l_t. \tag{9}$$

She also faces a limit to her liabilities during period t as a fraction χ of her expected housing value in period $t+1$:

$$l_t \leq \frac{\chi}{R_t^l} \mathbb{E}_t [q_{t+1} \pi_{t+1}] h_{i,t}. \tag{10}$$

Loans obtained by the impatient household from the patient household between period $t-1$ and t are l_{t-1}. The condition $(1-n) d_t^l = n l_t$ has to be satisfied. It implies that, in aggregation, money lent by patient households correspond to loans obtained by impatient households. The parameter χ denotes the loan-to-value ratio and measures the liquidity degree of housing.

The impatient household's optimisation problem leads to the following first order conditions with respect to $c_{i,t}$, $n_{i,t}$ $h_{i,t}$ and l_t:

$$\lambda_{i,t} = \frac{1}{(c_{i,t} - \gamma c_{i,t-1})} - \frac{\beta_b \gamma}{(c_{i,t+1} - \gamma c_{i,t})}. \tag{11}$$

$$\nu_i^n n_{i,t}^{\varphi} = w_t \lambda_{i,t}. \tag{12}$$

$$\lambda_{i,t} q_t = \frac{\Gamma_t \nu_i^h}{h_{i,t}} + \beta_i \mathbb{E}_t \Big[(1-\delta_h) q_{t+1} \lambda_{i,t+1} \Big] + \mu_{i,t} \frac{\chi}{R_t^l} \mathbb{E}_t [q_{t+1} \pi_{t+1}]. \tag{13}$$

$$\lambda_{i,t} - \mu_{i,t} = \beta_i \mathbb{E}_t \Big[\lambda_{i,t+1} \frac{R_t^l}{\pi_{t+1}} \Big], \tag{14}$$

where $\lambda_{i,t}$ is the marginal utility of consumption and $\mu_{i,t}$ is the Lagrange multiplier on the collateral constraint (10).

2.2 Firms

Firms in this economy are owned by the patient households. There are five types of firms: wholesale firms, retailer firms, final good producers, capital producers and housing producers.

2.2.1 Wholesale firms

These firms buy capital K_{t-1}^W, at the end of time $t-1$, from capital producers and they hire labour N_t^W from patient and impatient households. During time t, they produce wholesale goods Y_t^W using a Cobb-Douglas production function, and then they sell it to retailer firms:

$$Y_t^W = A_t {N_t^W}^{1-\alpha} {K_{t-1}^W}^{\alpha}, \tag{15}$$

where A_t is total factor productivity in the non-durable goods sector. It obeys an AR(1) process $log(A_{t+1}) = \rho_A log(A_t) + \sigma_{\epsilon^A} \epsilon_{t+1}^A$, where and $\rho_A \in (0,1)$ and $\epsilon_{t+1}^A \sim i.i.d.[0, \sigma_{\epsilon^A}^2]$.

At the end of period t, these firms obtain funds from the banking sector to finance the acquisition of capital K_t^W. In order to do so, they take S_t loans equal to the number of units of capital acquired (K_t^W), and price each at the unit price of capital q_t^K.

$$q_t^K K_t^W = q_t^K S_t. \tag{16}$$

After finishing production, these firms have the option of selling its undepreciated capital in the open market. Therefore, their earnings are conformed by the value of output and capital stock. Expenditure on labour and capital stock represent their total cost. The profit maximisation problem is:

$$\max_{N_t^W, K_t^W} \left[P_{m,t} Y_t^W + (1-\delta_k) q_{t-1}^K K_{t-1}^W - R_t^K q_{t-1}^K K_t^W - w_t N_t^W \right],$$

subject to the production function. $P_{m,t}$ is the relative intermediate output price, R_t^K is the state-contingent required return on capital during time t. The first order conditions for this firm, i.e. the demands for labour and capital, are:

$$w_t = P_{m,t}(1-\alpha) A_t \left(\frac{K_{t-1}^W}{N_t^W} \right)^{\alpha}, \tag{17}$$

$$q_{t-1}^K R_t^K = r_t^K + (1-\delta_k) q_t^K, \tag{18}$$

where $r_t^K = P_{m,t} \alpha A_t \left(\frac{N_t^W}{K_{t-1}^W} \right)^{1-\alpha}$ is the rental rate of capital. Solving for the labour to capital ratio, replacing it in equation (17) and equating the results, I obtain an expression for the marginal cost, which also satisfies $P_{m,t} = mc_t$:

$$mc_t = \frac{1}{A_t} \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{r_t^K}{\alpha} \right)^{\alpha}. \tag{19}$$

2.2.2 Retailers and final good firms

The final good firm aggregates the output of retailer firms $y_t(j)$ according to a Dixit-Stiglitz production technology and sells the final product in a perfectly competitive market:

$$Y_t = \left[\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}.$$

Y_t represents the final good, $y_t(j)$ denotes the $j'th$ retailer input used in the production of the final good, and ϵ denotes the elasticity of substitution between any two inputs, assumed to be bigger than 1. This firm's profit maximisation is a static problem, and from its first order condition I obtain the demand equation for each input as:

$$y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t.$$

Since this final good producing firm is competitive, it makes zero profit and its price is a function of the inputs' prices, i.e. an aggregate price index:

$$P_t = \left[\int_0^1 P_t(j)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}.$$

Retailers simply re-package intermediate output, i.e. wholesale production. It takes one intermediate output unit to make a unit of retail output. The marginal cost is thus the relative intermediate output price $P_{m,t}$. The retailer seeks to maximise its profit solving:

$$\max_{P_t(j)} P_t(j) \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t - mc_t \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t.$$

After optimising with respect to the choice variable $P_t(j)$, I obtain:

$$P_t(j) = \frac{\epsilon}{\epsilon - 1} mc_t.$$

This condition shows the market power that these firms have since they set their price, when there is no price rigidity, as a mark-up of the marginal cost. However, under the presence of some price rigidity, this result changes. Here I assume a price setting style *à la Calvo*. At each period, the firms receive a random draw from a Bernoulli distribution. This indicates that with a probability $1 - \theta$, $\theta \in [0, 1]$, the firm will be able to change its price. Conversely, with a probability θ, the firm will not be able to set a new price, keeping it unchanged.

$$P_t(j) = P_{t-1}(j), \forall j \in [0, \theta),$$
$$P_t(j) = P_t^*(j), \forall j \in [\theta, 1],$$

where $P_t^*(j)$ is determined by maximising the following problem:

$$\max_{P_t^*(j)} V_t(j) = \mathbb{E}_t \sum_{i=0}^{\infty} (\beta_p \theta)^i \left\{ \frac{\lambda_{p,t+i}}{\lambda_{p,t}} \left[\left(\frac{P_t^*(j)}{P_{t+i}} - mc_{t+i} \right) \left(\frac{P_t^*(j)}{P_{t+i}} \right)^{\epsilon} Y_{t+i} \right] \right\}.$$

The result determines that retailers who have obtained a successful draw will set their prices as a constant mark-up on an expression related to their expected discounted nominal total costs, relative to an expression related to their expected discounted real output.

$$P_t^*(j) = \frac{\epsilon}{\epsilon - 1}\left[\frac{\mathbb{E}_t \sum_{i=0}^{\infty}(\beta_p\theta)^i \lambda_{p,t+i} mc_{t+i} P_{t+i}^{\epsilon} Y_{t+i}}{\mathbb{E}_t \sum_{i=0}^{\infty}(\beta_p\theta)^i \lambda_{p,t+i} P_{t+i}^{\epsilon-1} Y_{t+i}}\right]. \tag{20}$$

The above equation does not depend on j, so every retailer firm that can set its price in period t will choose the same price. Moreover, in the limiting case of no price rigidity, the familiar expression of a firm's optimal price as a constant markup on real marginal costs is obtained. Given the previous result and the price rigidity mechanism, the Dixit-Stiglitz aggregate domestic price index evolves as:

$$P_t^{1-\epsilon} = (1-\theta)(P_t^*(j))^{1-\epsilon} + \theta P_{t-1}^{1-\epsilon}.$$

From the last equation, and defining gross inflation as $\left(\frac{P_t}{P_{t-1}}\right) = \pi_t$, I obtain:

$$\pi_t^{1-\epsilon} = (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon} + \theta\left(\frac{P_{t-1}}{P_{t-1}}\right)^{1-\epsilon}.$$

Solving for gross inflation reveals the relationship between inflation and the aggregate price level. Inflation turns out to be a function of the relative price (π_t^*) between the price optimally set by the firms (P_t^*) and the price of the final good.

$$\pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\pi_t^*\right)^{1-\epsilon}. \tag{21}$$

2.2.3 Capital good firms

Patient households own capital good firms. During period t, they transform output in the form of investment, I_t, and undepreciated capital, $(1-\delta_k)K_{t-1}$, to produce new capital K_t. Part of this new capital, K_t^W, is sold to wholesale firms, at the price q_t^K. The rest, K_t^h, is rented to housing firms at the rental rate r_t^h. The undepreciated capital, thus, is equal to undepreciated capital rented to housing firms and undepreciated capital bought from wholesale firms.

The representative capital producer maximises its expected discounted profits. At the end of period t, this firm receives income from selling capital to wholesale firms and renting capital to housing firms, while paying the costs of gross investment and undepreciated capital purchases from wholesale firms.

$$\mathbb{E}_0 \sum_{i=0}^{\infty} \beta_p^i \frac{\lambda_{p,t+i}}{\lambda_{p,t}} \Big[q_t^K K_t^W - q_t^K(1-\delta_k)K_{t-1}^W + r_t^{K,h} K_t^h - I_t\Big]. \tag{22}$$

The maximisation problem is subjected to total capital law of motion and the definition of aggregate capital stock.

$$K_t = (1-\delta_k)K_{t-1} + [1 - \frac{\psi}{2}(I_t/I_{t-1} - 1)^2]I_t, \tag{23}$$

$$K_t = K_t^W + K_t^h, \tag{24}$$

where δ_k is the capital depreciation rate and ψ is a parameter measuring the cost for adjusting investment. The law of motion implies that old capital can be converted one-to-one into new capital, while the transformation of general output is subject to a quadratic adjustment cost.

The optimality conditions with respect to K_t^W, K_t^h and I_t are:

$$q_t^K - \beta_p \frac{\lambda_{p,t+1}}{\lambda_{p,t}}(1-\delta_k)q_{t+1}^K = \lambda_{K,t} - \beta_p \frac{\lambda_{p,t+1}}{\lambda_{p,t}}(1-\delta_k)\lambda_{K,t+1}. \tag{25}$$

$$r_t^{K,h} = \lambda_{K,t} - \beta_p \frac{\lambda_{p,t+1}}{\lambda_{p,t}}(1-\delta_k)\lambda_{K,t+1}. \tag{26}$$

$$1 = \lambda_{K,t}\Big[1 - \frac{\psi}{2}\Big(\frac{I_t}{I_{t-1}} - 1\Big)^2 - \psi\Big(\frac{I_t}{I_{t-1}} - 1\Big)\Big(\frac{I_t}{I_{t-1}}\Big)\Big] + \beta_p \psi \mathbb{E}_t\Big[\frac{\lambda_{p,t+1}}{\lambda_{p,t}}\lambda_{K,t+1}\Big(\frac{I_{t+1}}{I_t}\Big)^2\Big(\frac{I_{t+1}}{I_t} - 1\Big)\Big], \tag{27}$$

where $\lambda_{K,t}$ denotes the Lagrange multiplier on the capital law of motion.

2.2.4 Housing firms

At time t, the housing firms produce new houses, I_t^h, using a Cobb-Douglas production technology. This process requires capital, K_{t-1}^h, rented from the capital producer and labour, N_t^h, hired from patient and impatient households at the real wage w_t.

$$I_t^h = Z_t {N_t^h}^{1-\mu_h} {K_{t-1}^h}^{\mu_h}, \tag{28}$$

where μ_h is the income share of capital used to produce new housing. Z_t is total factor productivity in the housing sector. It obeys an AR(1) process $log(Z_{t+1}) = \rho_Z log(Z_t) + \sigma_{\epsilon^Z}\epsilon_{t+1}^Z$, where and $\rho_Z \in (0,1)$ and $\epsilon_{t+1}^Z \sim i.i.d.[0, \sigma_{\epsilon^Z}^2]$.

These firms maximise the difference between their earnings from selling new houses and their costs in wages and rent. Denoting the price of new houses by q_t, the representative housing producer maximisation problem is:

$$\max_{N_t^h, K_{t-1}^h} \left[q_t I_t^h - r_t^{K,h} K_{t-1}^h - w_t N_t^h \right],$$

subject to the production technology I_t^h.

The first order conditions, with respect to N_t^h and K_{t-1}^h, yield the following demands for labour and capital:

$$w_t = (1 - \mu_h) q_t \frac{I_t^h}{N_t^h}. \tag{29}$$

$$r_t^{K,h} = \mu_h q_t \frac{I_t^h}{K_{t-1}^h}. \tag{30}$$

2.3 Banks

This sector closely follows the setting proposed by Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). In every period, each bank obtains funds in the form of deposits $D_{\tau,t}$ from patient households, which pays a nominal gross interest rate R_t^d in the next period. The banks transform these funds in loans for the wholesale firms. They take the form of equities $S_{\tau,t}$, which yield an ex-post return R_{t+1}^K.

Each bank τ has wealth -or net worth- $NW_{\tau,t}$ at the end of period t, and its balance sheet is given by:

$$q_t^K S_{\tau,t} = NW_{\tau,t} + D_{i,t}. \tag{31}$$

It states that a bank finances loans with newly issued deposits and net worth. Moreover, $D_{\tau,t}$ represents a bank's debt, while $S_{\tau,t}$ a bank's asset. Thus, $NW_{\tau,t}$ will be its equity capital, which evolves over time as the difference between expected earnings on loans to wholesale firms and interest payments on the borrowing from patient households[3]:

$$\begin{aligned} NW_{\tau,t+1} &= R_{t+1}^K q_t^K S_{\tau,t} - R_t^d D_{\tau,t}, \\ &= \left(R_{t+1}^K - R_t^d \right) q_t^K S_{\tau,t} + R_t^d NW_{\tau,t}. \end{aligned} \tag{32}$$

From expression (32), one can appreciate that net worth's growth, above the risk-less return R_t^d, depends on the risk premium $(R_{t+1}^K - R_t^d)$ and total loans. Defining $\beta_B^i \frac{\lambda_{p,t+i}}{\lambda_{p,t}}$

[3]Gertler and Karadi (2011) assume that banks can only accumulate net worth by retained earnings and do not issue new assets.

as the stochastic discount factor a banker τ at time t applies to earnings at time $t+i$, where $\beta_B = \beta_p \geq \beta_i$ because patient households own the banks, the bank will refuse to fund any loans with a discounted return smaller than the discounted cost of deposits. Therefore the following inequality must apply for the bank to operate:

$$\beta_B^i \mathbb{E}_t \left[\frac{\lambda_{p,t+i}}{\lambda_{p,t}} \left(R_{t+1}^K - R_t^d \right) \right] \geq 0, i \geq 0.$$

Gertler and Karadi (2011) summarise this stating: "as long as the bank earns a risk adjusted return greater than or equal to the return the household can earn on its deposits, it pays for the banker to keep building assets until exiting the industry"(p. 20).

Each bank has a probability σ to continue functioning until next period, and a probability to exit $1-\sigma$. This prevents the bank to overcome its financial constraint by saving indefinitely. In addition, it is assumed that the number of banks entering and exiting the sector are equal, keeping the total constant.

In each period, a banker's objective is to maximise her expected final wealth:

$$V_{\tau,t}^B = \max \mathbb{E}_t \sum_{i=0}^{\infty} (1-\sigma)\sigma^i \beta_B^{i+1} \frac{\lambda_{p,t+i}}{\lambda_{p,t}} NW_{\tau,t+i}, \tag{33}$$

subject to its balance sheet (31), equity capital law of motion (32) and an incentive constraint. This incentive constraint arises from introducing a moral hazard problem to limit the bank's ability to issue deposits. Following Gertler and Kiyotaki (2010), at the beginning of a period, and after the bank has accepted deposits, it has two options: (i) divert a fraction ζ of its assets to the patient households or (ii) hold its assets until the next period when payoffs are realised, and then pay its deposit obligations.[4] If the bank chooses the first option, it closes, following the default on its debt. The bank will need to afford the costs coming from creditors reclaiming their remaining fraction $(1-\zeta)$ of funds. Therefore, due to the risk that a bank may default on its debts, creditors will be reluctant to lend large amounts to the bank at the beginning of each period. This creates friction, acting as an incentive constraint for the bank when trying to obtain funds.

$$V_{\tau,t}^B \geq \zeta(q_t^K S_{t,\tau}). \tag{34}$$

Condition (34) suggests that the bank will refrain from diverting funds as long as its franchise value is greater than or equal to the portion it can divert. Thus, I re-write the bank's problem equation (33) in a Bellman equation form as:

[4]By assumption, patient households do not deposit funds in the banks they own.

$$V^B_{\tau,t} = \beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}} \{(1-\sigma) NW_{\tau,t} + \sigma \max V^B_{\tau,t+1}(NW_{\tau,t+1})\}, \tag{35}$$

which is subject to:

$$q^K_t S_{\tau,t} = NW_{\tau,t} + D_{\tau,t},$$
$$NW_{\tau,t+1} = \left(R^K_{t+1} - R^d_t\right) S_{\tau,t} + R^d_t NW_{\tau,t},$$
$$V^B_{\tau,t} \geq \zeta(q^k_{t,f} S_{\tau,t}).$$

Assuming that the value function $V^B_{\tau,t}$ is linear in $NW_{\tau,t}$, that is $V^B_{\tau,t} = \nu^B_t NW_{\tau,t}$, where ν^B_t depends only on aggregate quantities; and defining ξ_t as the Lagrange multiplier on the incentive constraint, the first order conditions for $S_{\tau,t}$ and $NW_{\tau,t}$ are:

$$\frac{\xi_t \zeta}{(1+\xi_t)} = \mathbb{E}_t \Big[(1 - \sigma + \sigma \nu^b_{t+1}) \Big(R^K_{t+1} - R^d_t \Big) \Big], \tag{36}$$

$$\frac{1}{(1+\xi_t)} = \mathbb{E}_t \Big[(1 - \sigma + \sigma \nu^b_{t+1}) R^d_t \Big], \tag{37}$$

where equation (36) makes the marginal benefit from increasing assets and the marginal cost of tightening the incentive constraint equal. Defining the bank's net worth adjusted marginal value as $\Omega_{\tau,t+1} = (1 - \sigma + \sigma \nu^b_{t+1})$, I re-express the value function:

$$V^B_{\tau,t} = \beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}} \Big\{ \Omega_{\tau,t+1} \Big[\Big(R^K_{t+1} - R^d_t \Big) q^K_t S_{\tau,t} + R^d_t NW_{\tau,t} \Big] \Big\}.$$

Multiplying and dividing this expression by $NW_{\tau,t}$, I obtain:

$$V^B_{\tau,t} = \beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}} \Big\{ \Omega_{\tau,t+1} \Big[\Big(R^K_{t+1} - R^d_t \Big) \phi_t + R^d_t \Big] \Big\} NW_{\tau,t}, \tag{38}$$

where $\phi_t = \frac{q^K_t S_{\tau,t}}{NW_{\tau,t}}$ and the term between curly brackets is ν^b_t. Therefore, if the incentive constraint is binding, $\nu^b_t = \zeta \phi_t$:

$$q^K_t S_{\tau,t} = \phi_t NW_{\tau,t}. \tag{39}$$

Using the result from the previous two equations, and after some rearranging, I obtain an expression for the leverage:

$$\phi_t = \frac{\beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}} \Omega_{\tau,t+1} R^d_t}{\zeta - \beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}} \Omega_{\tau,t+1} \Big(R^K_{t+1} - R^d_t \Big)}. \tag{40}$$

This expression does not depend on any firm-specific factor, making it possible to sum across wholesale firms, obtaining:

$$q^K_t S_t = \phi_t NW_t. \tag{41}$$

Finally, I derive a law of motion for NW_t as the sum of the old (existing) and young (new) banks net worth:

$$NW_t = NW_{o,t} + NW_{n,t}. \tag{42}$$

Given that a fraction σ of bankers at time $t-1$ survive until time t, $NW_{o,t}$ is:

$$NW_{o,t} = \sigma \Big(R^K_t q^K_{t-1} S_{t-1} - R^d_{t-1} D_{t-1} \Big). \tag{43}$$

As I described earlier, new banks receive funds from patient households, following Gertler and Karadi (2011), I assume this transfer equals to a small fraction of the assets intermediated by exiting banks in their final operating period. That is, banks exiting with an i.i.d probability have assets worth $(1-\sigma)(R^K_t q^K_{t-1} S_{t-1})$, from which a fraction $\omega/(1-\sigma)$ is transferred to the entering banks.

$$NW_{n,t} = \omega (R^K_t q^K_{t-1} S_{t-1}). \tag{44}$$

Combining these two conditions, I obtain the law of motion of NW_t:

$$NW_t = (\sigma + \omega)(R^K_t q^K_{t-1} S_{t-1}) - \sigma R^d_{t-1} D_{t-1}. \tag{45}$$

2.4 Central Bank

To close the model, the central bank sets the nominal interest rate R^d_t following a Taylor-type rule, which targets inflation and GDP growth stabilisation .

$$\frac{R^d_t}{\bar{R}^d} = \left(\frac{\pi_t}{\bar{\pi}} \right)^{\omega_\pi} \left(\frac{GDP_t}{GDP_{t-1}} \right)^{\omega_y} M_t, \tag{46}$$

where the steady state of the policy rate is $\bar{R}^d = (1/\beta_p)$. I follow Iacoviello and Neri (2010) and define GDP as the sum of consumption and investment, both non-residential and residential. That is, $GDP_t = C_t + I_t + \bar{q}I_t^h$, where $\bar{q}$ denotes the steady state value of real housing prices, so that short-run changes in real house prices do not affect GDP growth (Iacoviello & Neri, 2010, p. 132). M_t is a monetary policy shock, which follows an AR(1) process $log(M_{t+1}) = \rho_M log(M_t) + \sigma_{\epsilon^M}\epsilon_{t+1}^M$, where and $\rho_M \in (0,1)$ and $\epsilon_{t+1}^M \sim i.i.d.[0, \sigma_{\epsilon^M}^2]$.

2.5 Market clearing and aggregation

In equilibrium, each household's weighted contribution to consumption, labour and housing will determine the aggregates C_t, N_t and H_t, respectively.

$$C_t = (1-n)c_{p,t} + (n)c_{i,t}. \tag{47}$$

$$N_t = (1-n)n_{p,t} + (n)n_{i,t}. \tag{48}$$

$$H_t = (1-n)h_{p,t} + (n)h_{i,t}. \tag{49}$$

In addition, the amount of total labour demanded by wholesale firms and housing firms should equal the total amount of labour supplied by households.

$$N_t = N_t^W + N_t^h. \tag{50}$$

Total loans obtained by the impatient households must equal total loans provided by the patient household. Similarly, total deposits in the banking sector needs to equal aggregate deposits from the patient households.

$$(1-n)d_t^l = nl_t. \tag{51}$$

$$D_t = (1-n)d_t^B. \tag{52}$$

As previously specified, total loans issued by the wholesale firms to acquire funding for their capital acquisition must equal their demand of capital.

$$S_t = K_t^W. \tag{53}$$

From capital producers, total capital stock should equal the sum of capital supplied to wholesale firms and capital rented to housing firms.

$$K_t = K_t^W + K_t^h. \tag{54}$$

And its law of motion is:

$$K_t = (1 - \delta_k)K_{t-1} + [1 - \frac{\psi}{2}(I_t/I_{t-1} - 1)^2]I_t. \tag{55}$$

New housing or housing investment must also satisfy a law of motion. It establishes that new housing is equal to the difference between housing stock at time t net of undepreciated housing stock from time $t - 1$.

$$I_t^h = H_t - (1 - \delta_h)H_{t-1}. \tag{56}$$

The markets for non-durable goods must clear.

$$Y_t = C_t + I_t. \tag{57}$$

In addition, the link between final good and wholesale goods is given by[5]:

$$Y_t = \frac{Y_t^W}{\nu_t^j}, \tag{58}$$

where ν_t^j is a measure of price dispersion. Finally, I introduce the sum of durable and non-durable goods as GDP:

$$GDP_t = C_t + I_t + \bar{q}I_t^h. \tag{59}$$

3 Model solution

In this section I present the solution method I use to solve linear models with diagnostic agents. Using this strategy, I aim to obtain a RE representation of the DE model, in

[5]For the derivation of this condition, see Appendix 8.1

similar lines as L'Huillier et al. (2024).

3.1 Including diagnostic expectations

The main difference in this model is that agents are not rational; they are diagnostic. Consequently, when forming expectations, these agents' mind retrieves information influenced by past context. This directly impacts the way diagnostic agents assign probabilities to future scenarios, leading to mistakes, corrections, and exaggerated responses. Following Bordalo et al. (2021), I model this departure from rational expectations by assuming that agents misperceive the state of the economy. Following the shock processes included in the model, I assume that the state of the economy evolves as an AR(1) process, $x_{t+1} = \rho_x x_t + \epsilon_{t+1}$, where $\epsilon_{t+1} \sim \mathcal{N}(\mu, \sigma^2)$ and x_{t+1} has a probability density function (pdf):

$$f(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_{t+1}-\rho x_t)^2}{2\sigma^2}}. \tag{60}$$

At time t, the diagnostic agents form beliefs about the future state in $t+1$ by recalling past realisations of economic conditions that are at the forefront of their mind. That is, they compare information about the current economic conditions with what they already know or remember about past behaviour. During such process, they use a distorted density function instead of the rational pdf as defined by Bordalo et al. (2018):

$$f^{\phi}(x_{t+1}|x_t) = f(x_{t+1}|x_t = \bar{x}_t) \left[\frac{f(x_{t+1}|\bar{x}_t)}{f(x_{t+1}|\rho\bar{x}_{t-1})} \right]^{\phi} Z. \tag{61}$$

I denote the realisation of the variable by $\bar{x}_t$, thus the diagnostic distribution depends on realisations of x_t at the current time, $\bar{x}_t$, as well as in the past through the reference event, $\bar{x}_{t-1}$. Here, I assume that the agent only considers the most recent past when forming expectations. Z is a normalizing constant and $\phi \geq 0$ is the diagnostic parameter, which embeds the rational case when it is equal to 0. Replacing (60) in (61), I obtain:

$$f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_{t+1}-\rho\bar{x}_t)^2}{2\sigma^2}} \left[\frac{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_{t+1}-\rho\bar{x}_t)^2}{2\sigma^2}}}{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_{t+1}-\rho^2\bar{x}_{t-1})^2}{2\sigma^2}}} \right]^{\phi} Z, \tag{62}$$

After simplifying and grouping terms, I obtain:

$$f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} e^{\left\{-\frac{(x_{t+1}-\rho\bar{x}_t)^2}{2\sigma^2} - \frac{1}{2\sigma^2}\phi\left[(x_{t+1}-\rho\bar{x}_t)^2 - (x_{t+1}-\rho^2\bar{x}_{t-1})^2\right]\right\}} Z. \tag{63}$$

Expanding and re-writing the argument in the exponential:

$$
\begin{aligned}
f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} exp\Bigg(-\frac{1}{2\sigma^2}\Big\{ x_{t+1}^2 - 2x_{t+1}\left[\rho\bar{x}_t + \phi\left(\rho\bar{x}_t - \rho^2\bar{x}_{t-1}\right)\right] + (\rho\bar{x}_t)^2 \\
+ \phi\left[(\rho\bar{x}_t)^2 - (\rho^2\bar{x}_{t-1})^2\right]\Big\}\Bigg) Z.
\end{aligned} \tag{64}
$$

The constant Z is given by:

$$
\begin{aligned}
Z = exp\Bigg(-\frac{1}{2\sigma^2}\Big\{ -\phi\left[(\rho\bar{x}_t)^2 - (\rho^2\bar{x}_{t-1})^2\right] + 2\rho\bar{x}_t\phi\left[\rho\bar{x}_t - \rho^2\bar{x}_{t-1}\right] \\
+ \phi^2\left[\left(\rho\bar{x}_t - \rho^2\bar{x}_{t-1}\right)\right]^2\Big\}\Bigg).
\end{aligned} \tag{65}
$$

Therefore, after some algebra, the diagnostic pdf when the reference is the recent past is equal to:

$$
f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}\left\{\left[x_{t+1} - \left(\rho\bar{x}_t + \phi\left(\rho\bar{x}_t - \rho^2\bar{x}_{t-1}\right)\right)\right]^2\right\}}. \tag{66}
$$

The diagnostic distribution is characterised to be a normal distribution with variance σ^2 and a distorted mean.

Lemma 1: *Assume that the state of the economy x_{t+1} follows an AR(1) process but agents are diagnostic and just consider the most recent past when forming their expectations. Then, following Gennaioli and Shleifer (2018), equation (66) turns out to be a function that contains the kernel of a normal distribution with a distorted mean and the same variance:*

$$
\mathbb{E}_t^{\theta}(x_{t+1}) = \mathbb{E}_t(x_{t+1}) + \phi\left[\mathbb{E}_t(x_{t+1}) - \mathbb{E}_{t-1}(x_{t+1})\right], \tag{67}
$$

The results from Lemma 1 can be re-written in terms of the shock's realisation.

Lemma 2: *Using the assumption of x_{t+1} following an AR(1) process, the shock realisation can be obtained as $\epsilon_t = x_t - \rho x_{t-1}$, which after replacing it in equation (66) results:*

$$f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}\left\{[x_{t+1}-(\rho\bar{x}_t+\phi\rho\epsilon_t)]^2\right\}}. \tag{68}$$

Again, as in Gennaioli and Shleifer (2018) this is characterised by:

$$\mathbb{E}_t^{\phi}(x_{t+1}) = \rho x_t + \phi\rho\epsilon_t. \tag{69}$$

This is the key finding. It indicates that when agents are diagnostic ($\phi > 0$), there is extrapolation in the direction of the shock. This occurs because agents misperceive the shock to exhibit greater persistence than the true data generating process, mistakenly interpreting it as ARMA(1,1) process.

The results from Lemma 1 and Lemma 2 can be generalized to the case where remote memories influence the diagnostic agent's reference group.[6] The diagnostic pdf in this case is:

$$f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_{t+1}-\rho\bar{x}_t)^2}{2\sigma^2}} \left\{\left[\prod_{s=1}^{S} \frac{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_{t+1}-\rho^s\bar{x}_{t+1-s})^2}{2\sigma^2}}}{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_{t+1}-\rho^{s+1}\bar{x}_{t-s})^2}{2\sigma^2}}}\right]^{\alpha_s}\right\}^{\phi} Z, \tag{70}$$

where S represents the time span used by the diagnostic agent for reference group, while α_s denotes the weights the agent attaches to present and past representativeness.

Lemma 3: *Using the results from Lemma 1 and Lemma 2, and assuming that the agent has a slow moving reference group, the diagnostic pdf in (70) is characterised by:*

$$\mathbb{E}_t^{\phi}(x_{t+1}) = \mathbb{E}_t(x_{t+1}) + \phi\sum_{s=1}^{S}\alpha_s\left[\mathbb{E}_{t+1-s}(x_{t+1}) - \mathbb{E}_{t-s}(x_{t+1})\right], \tag{71}$$

This can also be re-written in terms of the realisation of the shocks as:

$$\mathbb{E}_t^{\phi}(x_{t+1}) = \rho x_t + \phi\sum_{s=1}^{S}\rho^s\alpha_s\epsilon_{t+s-1}. \tag{72}$$

Thus, when the agent is diagnostic and has distant memory through a slow moving reference group, the agent misperceives the shock as an ARMA(1,S) process, S being the length of the periods used as reference group.

[6]The full derivation for the slow moving reference group is presented in Appendix 8.4.

3.2 Solution procedure

I solve the model through first-order perturbation method following Klein (2000). First, I assume that diagnostic agents form beliefs based on a more distant past by using a moving average over the last twelve quarters as reference group.[7] Consequently, making use of expression (72), agents' concept of the state of the economy is as if it follows an ARMA(1,12) process instead of the true AR(1),

$$
\begin{aligned}
\mathbb{E}_t^{\phi}(x_{t+1}) = \rho x_t + \phi[(\rho\alpha_1\epsilon_t + \rho^2\alpha_2\epsilon_{t-1} + \rho^3\alpha_3\epsilon_{t-2} + \rho^4\alpha_4\epsilon_{t-3} + \rho^5\alpha_5\epsilon_{t-4} + \rho^6\alpha_6\epsilon_{t-5} \\
+ \rho^7\alpha_7\epsilon_{t-6} + \rho^8\alpha_8\epsilon_{t-7} + \rho^9\alpha_9\epsilon_{t-8} + \rho^{10}\alpha_{10}\epsilon_{t-9} + \rho^{11}\alpha_{11}\epsilon_{t-10} + \rho^{12}\alpha_{12}\epsilon_{t-11})]
\end{aligned} \tag{73}
$$

Second, I incorporate the MA components into the model as auxiliary variables and rewrite the exogenous shock processes as ARMA(1,12). Third, I compute the non-stochastic steady-state, point at which the model will be perturbed, by finding the fix-point of the system using the Newton method. Forth, I log-linearise the model variables around their steady state and solve the resulting system, which solution takes the following form:

$$
\begin{aligned}
\mathbf{x}_{t+1} &= \mathbf{h}\mathbf{x_t} + \mathbf{k}\epsilon_{\mathbf{t+1}} \\
\mathbf{y_t} &= \mathbf{g}\mathbf{x_t},
\end{aligned}
$$

where $\mathbf{y}_t$ denotes a (m x 1) vector of endogenous variables and $\mathbf{x}_t$ stacks a (n x 1) vector of state variables. The latter is comprised of three sub-vectors. The first one, of size (n_1 x 1), contains the auxiliary variables for the MA terms of the shock processes. The second, of size (n_2 x 1), includes the exogenous variables; and the third, of size (n_3 x 1), is composed of the predetermined variables. Therefore, matrix $\mathbf{g}$ linking the decision variables with the states can also be divided into three sub-matrices:

$$
\mathbf{g} = \left[\begin{array}{c|c|c} \mathbf{g_1} & \mathbf{g_2} & \mathbf{g_3} \end{array} \right].
$$

The sub-matrices $\mathbf{g_2}$ and $\mathbf{g_3}$ of size (m x n_2) and (m x n_3) connect decision variables to the exogenous states and the predetermined variables, respectively. A comparison between

[7] In the main body of the article, I also present results in which the reference group is the most recent past. In this case all the attention is on the previous quarter, i.e. α_1 is equal to 1, whereas the remaining weights are equal to zero. The choice of twelve quarters follows from the empirical evidence in the housing market found by Adam et al. (2024) and the estimation results from Bianchi et al. (2024).

the solutions of these sub-matrices under RE and DE reveals that they remain unchanged since they are independent of the diagnostic parameter. The distinction arises in sub-matrix $\mathbf{g_1}$, sized (m x n_1), which links the decision variables to the realized shocks. While the elements of this matrix are zero in the rational solution, in the diagnostic solution, they take on non-zero values. This reflects the source of the additional volatility that DE generate, and it aligns with the findings of L'Huillier et al. (2024).

As the DE solution is based on agents misperceving the state of the economy as an ARMA(1,12) rather than an AR(1) process, the first-order coefficients matrix in the state-transition equation, $\mathbf{h}$, still includes the parameters associated with the MA terms. The last step, therefore, is to mute these terms so that any further analysis is done under the true data-generating process, but with agents having DE regarding the state of the economy.

4 Model Estimation

I estimate the models using U.S. quarterly data for the period 1984:Q1 to 2019:Q4, which I describe in subsection 4.1. The estimation approach adopted is Sequential Monte Carlo, as outlined in subsection 4.2. Subsection 4.3 describes the parameters calibration, while subsection 4.4 shows the prior distributions of the estimated parameters. Subsection 4.5 exhibits the estimation results.

4.1 Data

I use eight macroeconomic time series for the calibration and estimation of the model. All variables are log-transformed, detrended using first-difference and demeaned, with the exception of the nominal interest rate which is transformed into quarterly rate and demeaned. Housing wealth is expressed in real per cápita terms as it is adjusted by the population level and the implicit price deflator, while the total amount of loans to households equals the sum of residential mortgages and consumer credit of households and non-profit organisations.[8] I obtained the data from the Board of Governors of the Federal Reserve System and the Bureau of Economic Analysis, using the National Accounts and Flow of Funds. I also use the Census Bureau House Price Index. The full set of variables is:

- Real Gross Domestic Product growth: $\Delta GDP_t = ln(GDP_t/GDP_{t-1})$
- GDP implicit price deflator: $\hat{\pi}_t = ln(P_t/P_{t-1})$
- Real Residential Investment growth: $\Delta I_t^h = ln(I_t^h/I_{t-1}^h)$

[8] A detailed explanation of the data series can be found in Appendix 8.6.

- Real House price growth: $\Delta q_t = ln(q_t/q_{t-1})$
- Nominal interest rate: $\hat{R}_t^d = ln(R_t/R_{t-1})$
- Real Loans growth: $\Delta l_t = ln(l_t/l_{t-1})$
- Real Non-residential investment growth: $\Delta I_t = ln(I_t/I_{t-1})$
- Real Housing wealth growth: $\Delta(qH_t) = ln(qH_t/qH_{t-1})$

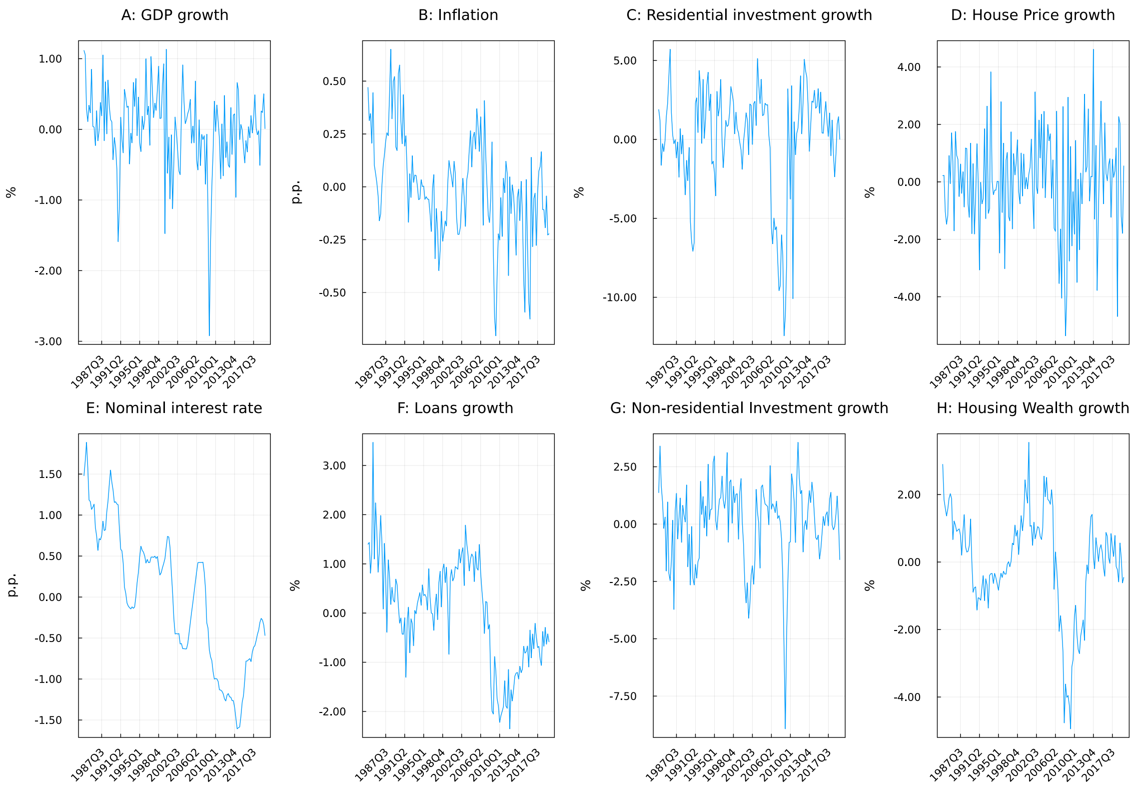


Figure 3: U.S. Macroeconomic variables.
Note: Real gross domestic product, real residential investment, real house price, real loans, real non-residential investment and real housing wealth growths are in percentages. Inflation and nominal interest rate are quarterly.

4.2 Methodology

I estimate the log-linearised models using a Bayesian strategy method, drawing on recent advancements in macroeconomic model estimation by Herbst and Schorfheide (2014). The authors introduce an alternative class of algorithms to the traditional random walk Metropolis-Hastings (RWMH) method, known as Sequential Monte Carlo (SMC). This estimation method for DSGE models combines features of classic importance sampling and MCMC techniques.

The aim is to infer the parameters' posterior distribution by combining the likelihood function, $p(\theta|Y)$, of a DSGE model with a prior distribution, $p(\theta)$, on its parameters:

$$p(\theta|Y) = \frac{p(Y|\theta)p(\theta)}{p(Y)},$$

where θ indicates a vector of parameters and Y represents the data set. The marginal data density is $p(Y) = \int p(Y|\theta)p(\theta)d\theta$.

In order to do so, the SMC relies on a candidate density, $g(\theta)$, from which it generates sample draws or particles, as it is an importance sampler at its core. Each of these particles has an associated importance weight, $w(\theta)$. This follows from the identity:

$$\mathbb{E}_t[h(\theta)] = \int h(\theta)\frac{p(Y|\theta)p(\theta)}{p(Y)}d\theta = \frac{1}{p(Y)}\int h(\theta)\frac{p(Y|\theta)p(\theta)}{g(\theta)}g(\theta)d\theta,$$

where the weights on the draws are $w(\theta) = \frac{p(Y|\theta)p(\theta)}{g(\theta)}$. This means that at each stage of the algorithm, a set of pairs $\{(\theta^i, w(\theta^i))\}_{i=1}^N$ will be an approximation of $p(\theta|Y)$.

The algorithm initiates by drawing initial particles from the prior distribution, as it represents a distribution that is easy to sample from. The algorithm ends with a sequence of pairs of particles and weights that embody the final importance sample approximation of the posterior. In between, the process recursively generates intermediate or "bridge" distributions. These bridge distributions serve as transitional steps in the iterative process that gradually shifts the initial prior distribution towards the final posterior distribution. Each iteration refines the approximation of the posterior by updating the particle weights and resampling. It can be thought as an iterative moulding process that refines and transition the distributions from their initial prior form to its posterior.[9]

I use SMC because it offers advantages over RWMH. For instance, it is suitable for parallel computing during the model's estimation step for many particles, thereby enhancing computational efficiency. Additionally, when new data become available, SMC facilitates the re-estimation of the model by picking up from where it was left off, saving computational resources and time. It also eliminates the need for additional computations since it approximates the marginal likelihood as a by product.

The estimation was performed using Julia 1.7.3 in Atom. I adapted the code for the SMC algorithm from Salazar-Perez and Seoane (2024). The current choice of hyperparameters for the SMC is constrained by available computational resource. The number of particles and stages are set to 500 and 200, respectively. The bending coefficient, is within the range used in the literature and it is borrowed from Salazar-Perez and Seoane (2024).[10]

[9]For a more detailed explanation of the algorithm see Herbst and Schorfheide (2014) and Cai et al. (2021).

[10]The bending coefficient controls the likelihood tempering in the algorithm. In this paper, I use the fixed tempering schedule from Herbst and Schorfheide (2014).

I adjust the scaling factor to target an acceptance rate around 25%, as it is done in Cai et al. (2021).

4.3 Calibration

In this section, I present the calibration of the model. Table 1 lists the structural parameters with their respective values and the corresponding source or target. Each period in the model represents one quarter. The discount factor for the patient households, β_p, is set to 0.9915. This implies a 3.42% annualized interest rate in steady state, which is close to the average 3.52% over the sample period.

Some parameters are calibrated to match first-order moments in the data. The housing depreciation rate, δ_h, is set to 0.0060 in order to generate an average total housing wealth to GDP of 145.53% as in the period analysed. This parameter value results slightly lower than the one from Iacoviello and Neri (2010) and Mendicino and Punzi (2014), and it implies an annual housing depreciation of around 2.5%. The loan-to-value ratio, χ, is calibrated at a value 0.8016. This choice aims to achieve a household credit to total housing wealth ratio of 35.24%, and it is consistent with the range found in the literature (Iacoviello & Neri, 2010; Gelain et al., 2012; Mendicino & Punzi, 2014, among others). The patient households' housing preference weight is set to 0.2361 so that the share of total housing wealth owned by patient households is 60%.[11] Moreover, the model generates a residential investment to GDP ratio equal to the sample period average of 3.44%, by setting the housing preference weight of the impatient households to 0.0906. Finally, I calibrate the elasticity of final good with respect to capital, α, to 0.3752, and I set the capital share in the housing production, μ_h, at 0.3 to obtain a non-residential investment to GDP ratio of 27%. The first parameter has a value that falls within the range commonly used in macroeconomic models, while the second results from the sum of the exponents for all the inputs that are not labour in Iacoviello and Neri (2010).

The remaining structural parameters are borrowed from the literature. For instance, the proportion of impatient households, n, follows from Gelain et al. (2012) and targets the top decile of households in the economy. The discount factor for the impatient household, β_i, is 0.9715. This value ensures that while linearising around the steady-state, these households' borrowing constraint is binding. The chosen value for the inverse labour supply elasticity φ is 0.1, which as in Gelain et al. (2012), implies a very flexible labour supply. This article is also the source of the labour disutility parameters for the patient and impatient household, ν_p^n and ν_i^n. The rate at which capital depreciates, δ, equals 0.025. This value is in line with standard values in the literature. The retailers target a 10% steady state mark-up, thus I set the elasticity of substitution, ϵ, to 11. Finally, the

[11]The total housing wealth share hold by the patient household is targeted following Wolff (2016).

Table 1: Calibration: Structural parameters

Description	Parameter	Value	Target/Source
Households			
Proportion of impatient households	n	0.9	Gelain et al. (2012)
Inverse elasticity of labour supply	φ	0.1	Gelain et al. (2012)
Patient Households			
Discount factor	β_p	0.9915	Annualized interest rate of 3.52%
Housing preference weight	ν_p^h	0.2361	Patient households share of housing wealth = 60%
Labour disutility	ν_p^n	1.19	Gelain et al. (2012)
Impatient Households			
Discount factor	β_i	0.9715	Borrowing constraint's binding
Housing preference weight	ν_i^h	0.0906	Residential investment/GDP = 3.44%
Labour disutility	ν_i^n	4.54	Gelain et al. (2012)
Loan-to-value ratio	χ	0.8016	Household credit to total housing wealth = 35.24%
Wholesale firms			
Elasticity of final good with respect to capital	α	0.3752	Investment/GDP = 27%
Final firms and Retailers firms			
Elasticity of substitution	ϵ	11	10 % markup
Capital good firms			
Capital depreciation rate	δ	0.025	Typical in macroeconomic model literature
Housing firms			
Housing depreciation rate	δ_h	0.0060	Housing wealth/GDP = 143.23%
Elasticity of housing with respect to capital	μ_h	0.3	Iacoviello and Neri (2010)
Banks			
Banks' surviving probability	σ	0.9725	Gertler and Karadi (2011)
Absconding rate of the bankers	ζ	0.383	Gertler and Karadi (2011)
Start up fund for the new bankers	ω	0.003	Gertler and Karadi (2011)

banking sector is characterised exactly as in Gertler and Karadi (2011). The parameters are consistent with the authors' goal to achieve an interest rate spread of around one hundred basis points, maintain a steady state leverage ratio at 4, and ensure an average banker lifespan of 10 years. Therefore, the banker's survival probability, σ, is 0.9725, the fraction of capital that the banker can steal, ζ, is equal to 0.383, and the start up fund for new bankers is 0.003. This implies a spread close to 1% and a leverage ratio slightly below 4.

4.4 Prior distributions

Table 2 summarises the prior distributions for the parameters to estimate. I set the shapes for each prior based on the feasible parameter support and in consistency with previous studies. Accordingly, for the standard errors of the shocks, I use an inverse gamma distribution as in L'Huillier et al. (2024) and Justiniano, Primiceri, and Tambalotti (2010).[12] For the persistence, since these parameters are bounded between 0 and

[12]The Inverse Gamma distribution is typically used as a prior for the variance estimation. However, as noted by Adjemian (2016), priors in practice are often defined over the standard deviation of a structural

1, I choose a loose beta prior with mean 0.5 and standard deviation 0.2. The values for the monetary policy rule feedback parameters were set equal to Taylor's original specifications. I choose a normal distribution with prior mean 1.5 for the response to inflation, and a 0.125 mean for the response to output growth; their standard deviations are 0.25 and 0.05, respectively. The prior on the investment adjustment costs follows Smets and Wouters (2007), it is a normal with mean 4.0 and standard deviation 1.5. Whereas for the habit formation and Calvo parameter, I chose the same beta priors as in Iacoviello and Neri (2010). The priors' means are 0.5 and 0.667, respectively, with standard deviations equal to 0.05 and 0.075.

Table 2: Prior distribution of the paramaters

Description	Parameter	Distribution	Mean	Std. dev
Structural Parameters				
Inv. adjustment cost	ψ	Normal	4.0	1.5
Habit formation	γ	Beta	0.667	0.05
Calvo parameter	θ	Beta	0.5	0.075
Taylor rule inflation	ω_π	Normal	1.50	0.25
Taylor rule output growth	$\omega_{\Delta y}$	Normal	0.125	0.05
Diagnostic parameters				
Diagnostic parameter	ϕ	Normal	1.0	0.3
1st quarter reference	α_1	Uniform	0.5	0.29
2nd quarter reference	α_2	Uniform	0.5	0.29
3rd quarter reference	α_3	Uniform	0.5	0.29
4th quarter reference	α_4	Uniform	0.5	0.29
5th quarter reference	α_5	Uniform	0.5	0.29
6th quarter reference	α_6	Uniform	0.5	0.29
7th quarter reference	α_7	Uniform	0.5	0.29
8th quarter reference	α_8	Uniform	0.5	0.29
9th quarter reference	α_9	Uniform	0.5	0.29
10th quarter reference	α_{10}	Uniform	0.5	0.29
11th quarter reference	α_{11}	Uniform	0.5	0.29
12th quarter reference	α_{12}	Uniform	0.5	0.29
Autoregressive coefficients				
Goods TFP	ρ_A	Beta	0.5	0.2
Housing TFP	ρ_Z	Beta	0.5	0.2
Monetary policy	ρ_M	Beta	0.5	0.2
Housing demand	ρ_Γ	Beta	0.5	0.2
Standard deviation of shocks				
Good TFP	$100*\sigma_{\epsilon^A}$	Inverse Gamma	0.5	2.0
Housing TFP	$100*\sigma_{\epsilon^Z}$	Inverse Gamma	0.5	2.0
Monetary policy	$100*\sigma_{\epsilon^M}$	Inverse Gamma	0.5	2.0
Housing demand	$100*\sigma_{\epsilon^\Gamma}$	Inverse Gamma	0.5	2.0

Note: The Inverse Gamma priors are of the form $p(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\frac{s}{2\sigma^2}}$. I borrow the function InverseGamma1.jl and *inverse_gamma_1_specification* from the Dynare package for Julia, developed by Adjemian et al. (2024), to obtain the parameters ν and s of and Inverse Gamma distribution characterised as in the table above.

For the diagnostic parameter, I employ a normal distribution with mean 1.0 and standard deviation 0.3, as in L'Huillier et al. (2024). Prior information on this parameter is limited given the scarcity of studies estimating it. A similar situation applies to the weights on backward references, with Bianchi et al. (2024) providing the only estimation example in the literature.[13] In their study, the approach involves modelling these weights

shock. Following this convention in the literature, I adopt the Type I Inverse Gamma distribution as defined by the author.

[13]It is worth noting that in this article, I follow Bordalo et al. (2018) when assuming that remote

using a beta distribution. They start by estimating the mean and standard deviation of this distribution, then they proceed with rescaling and discretizing it. Here, however, I estimate each weight employing a diffuse prior over the range (0,1), allowing the data to inform the analysis. The purpose behind estimating diagnostic and memory parameters is twofold: to support the presence of DE in the housing market, and to demonstrate that DSGE models incorporating DE can fit business cycle data better than those with RE.

4.5 Estimation results

I jointly estimate the remaining parameters to match second-order moments of four U.S. time series: real GDP growth, inflation, real residential investment growth and real house price growth, for the period 1984-2019. Table 3 gathers the results for the three estimated models: the diagnostic model with a twelve-quarters moving reference group, the diagnostic model with one-quarter reference group, and the rational model.[14] I report the mean posterior and the 90% high probability density credible interval (HPDI) for each parameter.[15] The last line exhibits the log of the marginal likelihood for each model.

The parameter values differ among the estimated models. Adjusting investment becomes more costly in the diagnostic model, more than twice as much when the reference period is the immediate past.[16] This tempers the diagnostic firms overreaction such that the fluctuations in investment are not as pronounced as they could be with a lower adjustment cost. The Calvo parameter estimates are very similar between the models. They indicate that firms' prices are sticky, as they can be reset once every seven quarters. These results are comparable with the values obtained by Iacoviello and Neri (2010). Households seem to have a somewhat high degree of habit formation with estimated values for this parameter slightly above 0.7. Nevertheless, it is a value close to the 0.6 proposed by Leith, Moldovan, and Rossi (2012) and supported by the meta-analysis in Havranek, Rusnak, and Sokolova (2017). Similarly, there is some variation in the central bank's Taylor rule parameters. In comparison to RE, the inflation feedback increases if agents form DE considering the

memories affect the way the diagnostic agent form expectations. Therefore, I define representativeness in terms of current and past likelihood ratios. On the other hand, Bianchi et al. (2024) stipulate that the comparison group the diagnostic agent uses as reference is an average of lagged rational expectations conditional on $t - J$ information, where J is the time span of the lag. In Appendix 8.4 I show how these two approaches are related.

[14] Note that the diagnostic framework with a twelve-quarters slow moving reference group encompasses both the diagnostic model using the last quarter as reference and the rational case. In the first, all attention is on the last quarter, meaning that $\alpha_1 = 1$ and the remaining αs are equal to zero. While, in the second, all weights and the diagnostic parameter are equal to zero.

[15] Figures 9, 10 and 11 in Appendix 8.5 show the posterior distributions for the variables from each model.

[16] Gabriel and Ghilardi (2012) estimate values within a similar range. They claim that such result arises from an interaction between investment costs and financial frictions.

most recent past, whereas it decreases when they use a more distant memory. On the other hand, the feedback on output growth shows the opposite behaviour. The central bank is less sensitive to output growth volatility when the diagnostic agent comparison group is the immediate past. However, when the reference group includes remoter memories, the central bank reacts as strong as in the rational case. These estimates suggest that agents' behaviour directly influences the central bank's trade-off between stabilising inflation and output growth volatility. This is in line with the conclusions of Bounader and Elekdag (2024).

Table 3: Estimation

Description	Parameter	DE Ref: Q12		DE Ref: Q1		RE	
		Mean	[0.05, 0.95]	Mean	[0.05, 0.95]	Mean	[0.05, 0.95]
Structural Parameters							
Inv. adjustment cost	ψ	0.8696	[0.5039,1.2422]	2.0600	[1.1548,3.3689]	0.8163	[0.4974,1.2026]
Habit formation	γ	0.7224	[0.6383,0.7896]	0.7415	[0.6558,0.7956]	0.7143	[0.6199,0.7763]
Calvo parameter	ϕ	0.8485	[0.8288,0.8637]	0.8732	[0.8604,0.8827]	0.8593	[0.8424,0.8718]
Taylor rule inflation	ω_π	1.6680	[1.4599,1.8769]	1.7381	[1.4643,2.0024]	1.7183	[1.4661,1.9764]
Taylor rule output growth	$\omega_{\Delta y}$	0.1972	[0.1249,0.2646]	0.1795	[0.0996,0.2455]	0.2029	[0.1102,0.2764]
Diagnostic parameters							
Diagnostic parameter	ϕ	0.1303	[0.0050,0.3265]	0.4555	[0.2819,0.6629]		
1st quarter reference	α_1	0.6714	[0.2689,0.9517]	1.0			
2nd quarter reference	α_2	0.2209	[0.0147,0.5706]				
3rd quarter reference	α_3	0.2054	[0.0055,0.6358]				
4th quarter reference	α_4	0.5226	[0.1065,0.9487]				
5th quarter reference	α_5	0.0990	[0.0057,0.3096]				
6th quarter reference	α_6	0.3797	[0.0380,0.8109]				
7th quarter reference	α_7	0.5930	[0.1513,0.9603]				
8th quarter reference	α_8	0.4963	[0.0910,0.8893]				
9th quarter reference	α_9	0.4775	[0.0829,0.9068]				
10th quarter reference	α_{10}	0.5157	[0.1629,0.8209]				
11th quarter reference	α_{11}	0.5219	[0.1789,0.8178]				
12th quarter reference	α_{12}	0.1340	[0.0087,0.4000]				
Autoregressive coefficients							
Goods TFP	ρ_A	0.8307	[0.7791,0.8906]	0.8691	[0.8153,0.9217]	0.8169	[0.7559,0.8750]
Housing TFP	ρ_Z	0.9413	[0.9212,0.9595]	0.9514	[0.9331,0.9660]	0.9546	[0.9379,0.9679]
Monetary policy	ρ_M	0.6561	[0.5444,0.7373]	0.7625	[0.6807,0.8115]	0.6896	[0.5965,0.7573]
Housing demand	ρ_Γ	0.9614	[0.9400,0.9800]	0.9445	[0.9080,0.9715]	0.9293	[0.8876,0.9633]
Standard deviation of shocks							
Good TFP	$100^*\sigma_{\epsilon^A}$	1.4435	[1.2829,1.6180]	1.3084	[1.1121,1.4724]	1.6550	[1.4820,1.8551]
Housing TFP	$100^*\sigma_{\epsilon^Z}$	3.9204	[3.3804,3.9830]	3.7382	[3.3965,4.1451]	3.7089	[3.3996,4.1035]
Monetary policy	$100^*\sigma_{\epsilon^M}$	0.3107	[0.2416,0.3896]	0.2222	[0.1719,0.2794]	0.2959	[0.2286,0.3771]
Housing demand	$100^*\sigma_{\epsilon^\Gamma}$	5.3588	[4.0070,6.6326]	7.2345	[4.6284,10.9440]	11.2891	[7.2150,16.4806]
Log marginal likelihood		568.67		598.91		591.63	

Note: The structural parameters include the investment adjustment cost (ψ), the habit formation (γ), the Calvo parameter (θ), the Central Bank Taylor rule inflation feedback (ω_π), and output growth feedback ($\omega_{\Delta y}$). The diagnostic parameters include the diagnosticity (ϕ) and the weights on past quarters as reference ($\alpha_{n=1}^{12}$). The autocorrelation coefficients measure the persistence of the goods TFP shock (ρ_A), housing TFP shock (ρ_Z), monetary shock (ρ_M), and housing demand (preference) shock (ρ_Γ), while $\sigma_{\epsilon_A}, \sigma_{\epsilon_Z}, \sigma_{\epsilon_M}, \sigma_{\epsilon_\Gamma}$ measure the standard deviations.

The key parameter in this analysis is the diagnostic parameter ϕ, which quantifies the size of the departure from rationality. For the DE model with one-quarter lag reference, the estimation places a substantial mass around a value of 0.4555, with a 90% HPDI away from zero, providing strong evidence in favour of DE. This value is consistent with the range found in the literature (L'Huillier et al., 2024; Bordalo et al., 2021). However, in the case with twelve-quarters lags reference, the posterior mean drops to 0.1303. This finding contrasts with Bianchi et al. (2024), who reported a diagnostic degree magnitude of around 2 when agents rely on distant memories. It is important to note that their estimate is relatively high compared to others in the literature. The authors obtained this under the assumption that the weights assigned to lagged expectations sum to one, which then requires a higher degree of diagnosticity to match initial overreactions. This is not the case in the current study, as I do not impose any constraints on the weights. Instead, I am interested in capturing whether there is a particular specification regarding their rate of decay.

Analysing the weights assigned to lagged representativeness as reference group, the estimates indicate two key findings. First, the diagnostic agents reliance on past information (as indicated by non-zeros $\alpha' s$ values) is inversely related to their degree of diagnosticity. This suggest that the slow moving reference group plays a crucial role in distributing the DE effects over time. Second, the most immediate quarter has the highest value, emphasizing the importance of recent events in shaping agents expectations. However, quarters three to ten account for approximately 70% of the overall weight. This observation is consistent with Bianchi et al. (2024), who found a similar concentration of attention within these quarters in their model.

Turning to the estimates of the shocks, I note that differences between most of the autoregressive coefficients do not exhibit a clear pattern. ρ_A is shown to be more persistent after the introduction of DE with the immediate past as reference, but when the comparison group for the diagnostic agent is expanded including distant lags, the value decreases towards the rational benchmark. ρ_M shows a similar outcome. In contrast, the autoregressive coefficients for the housing market behaves differently. ρ_Z remains relatively stable, while ρ_Γ turns out higher under both DE approaches. A different story holds for the standard deviations of the shocks. Overall, the estimated values are smaller in the DE models versus the rational model, apart from the housing TFP shock. This is consistent with evidence from previous articles pointing that DE is the channel through which shocks explain fluctuations (L'Huillier et al., 2024).

Here, I focus on the magnitude of the housing preference shock. This shock has been the major driver in rational models attempting to explain housing market dynamics, with estimates between 3% to 10% (Iacoviello & Neri, 2010; Iacoviello, 2015; Ge et al., 2022). Iacoviello and Neri (2010) describe this housing preference shock as either "genuine shifts in tastes for housing, or a catchall for all the unmodeled disturbances that can affect housing demand"(p. 150). The estimated standard deviation under rationality is 11.2891%. Instead, when agents are diagnostic, the values plummeted to 7.2345% and 5.3588%, contingent on whether their imperfect memory is driven by the immediate past or the last three years. This finding suggests that a significant part of that "catchall" seems to be related to the way agents form their expectations.[17] Specifically, DE help explain housing market dynamics while relying on a smaller preference shock. Gandré (2022) reaches a similar conclusion, highlighting the necessity for stronger shock variances under rationality compared to a model with behavioural agents.

5 Quantitative Results

This section evaluates the performance of the model in matching second order moments for selected variables. The bottom line in Table 3 summarises these findings. It shows that the log data density for the diagnostic model with 1 lag reference is 598.91, while for the rational model it is lower, 591.63. The difference between these measures, called Bayes factor (BF), is 7.28 in favour of the model with diagnostic agents, implying that its fit is better against the RE model.[18] This section proceeds showing how well the models do in fitting targeted moments of the data series. It also includes an analysis about the drivers of the business cycle.

5.1 Second order moments

I use the data in section 4.1 to calculate empirical moments. The time series variables are demeaned to make them comparable with their model counterpart, where there is no growth. I simulate series with the same length as the data, i.e. hundred and forty four observations, ten thousand times.

Table 4 compares the standard deviation (in %) of targeted variables in the data with that in the diagnostic models (DE Ref: Q12 and DE Ref: Q1), as well as in the rational model (RE). The three models perform reasonably well. Although real GDP growth appears more volatile in the models than in the data, the DE model with one lag as comparison group generates a value closer to the observed target. While the models tend to produce a more stable inflation, the results overall suggest that they successfully capture the excess volatility in the housing market.

Despite this, the RE model consistently underestimates the magnitude of the relative volatility observed in the data, whereas the evidence from the DE models is more accurate. Specifically, the DE Ref: Q1 model offers the closest fit. It achieves this by relying less on an ad hoc preference shock and more on the amplification mechanism inherent to the expectation formation process. This highlights that DE seem to better capture the dynamics of housing market fluctuations.

[17] By introducing DE with agents relying on the most recent past, the standard deviation estimate decreases by 35.91%, whereas if they use a longer time-span memory, it drops by 52.53%.

[18] Kass and Raftery (1995) classifies a statistic 2log(BF) = 14.56, as in this paper, to be a very strong evidence towards the diagnostic model over the rational model.

Table 4: Second-order moments in data and model

	Data	DE Ref:Q12	DE Ref:Q1	RE
Targeted moments				
Standard deviation				
Δ Real GDP	0.5764	0.8625	0.7271	0.8758
Relative standard deviation to GDP growth				
Inflation	0.4262	0.3025	0.3397	0.3057
Δ Real House prices	2.9896	2.4381	3.1882	2.3282
Δ Real Residential Investment	5.9893	5.0424	5.3184	4.7029

Note: Growth rates for real GDP, real house price, real residential investment. Model moments were obtained from averaging over ten thousand simulations of hundred and forty four observations each.

5.2 Historical shock decomposition

Figure 4 displays the historical shock decomposition for the model that better fits the data, the diagnostic model with the immediate past as comparison group.[19] This figure illustrates the nearly one-to-one relation between the four variables used in the estimation and the shocks. At each point in time, the bars indicate the proportion of the variable's deviation from its steady state that can be attributed to a particular shock, providing insight into the dynamic effects of these shocks on the variable over time. The orange bar represents the effect from the non-durable TFP shock, the green bar shows the impact of the housing TFP shock, the purple bar reflects the monetary policy shock, and the yellow bar represents the housing preference shock. The initial values, depicted in blue, show the impact of how far the variable is from its steady state at that moment. Since the data series do not begin at this point, the bars start out different from zero, but they gradually diminish over time.

Real GDP growth is in great proportion explained by the technology shock in the non-durable sector, as well as monetary policy shocks. Inflation, in contrast, is mainly influenced by monetary policy shocks. The latter is expected since most of the analysis covers the "Great Moderation" period. The housing shocks, both supply and demand, play a relevant part describing the swings in the housing market. Real residential investment growth is mainly driven by the technology shock in the durable sector. Whereas, the housing preference shock explains real house price growth by directly affecting the marginal utility of housing for both agents. Darracq Paries and Notarpietro (2008) report similar results in both the US and the euro area.

The results found here do not show significant differences to the rational case, nor to the diagnostic model with a twelve-quarters reference as shown in the Appendix. This suggests that shocks impact the economy in similar ways as in DE as in RE, however,

[19]Figures 12 and 13 in Appendix 8.5 show the same figure for the other two estimated models.

the amplification of these effects is driven by more volatile expectations inherent to the DE framework.

5.3 Impulse response function analysis

The following sections analyse the impact of the four shocks under the three estimated models using impulse responses. The responses are in log deviations from the steady state. I assume a 1% standard deviation shock in both durable and non-durable sector, as well as for the preference shock. The monetary shock, on the other hand, has a size of 25 basis points.

5.3.1 Effects of a non-durable goods productivity shock

Figure 5 displays the impulse responses to a positive productivity shock in the non-durable goods sector. The direction is as expected under RE. The shock increases labour and capital productivity, resulting in higher production (panel A) and consumption (panel B). Inflation decreases (panel C) as re-optimising firms adjust their prices in response to the fall in marginal cost. The central bank lowers the nominal interest rate (panel D), but the real interest rate increases (panel E). This has a positive effect on house prices (panel F) as they initially jump and gradually converge back to a steady state. Loans (panel H) exhibit a U-shaped response due to the behaviour of the interest rate, which influences two forces: patient households willing to lend and impatient households willing to borrow. Housing investment (panel G) reacts positively, driven by the increment in house prices.

In contrast, the responses under both DE frameworks are characterised by initial over-reactions, longer persistence and more pronounced fluctuations. The initial over-reactions are generated through the extrapolation of the shock and are common to both approaches. However, the persistence and pronounced fluctuations are more specific to each framework. In the DE model with a one-quarter reference, the model's rigidities propagate the initial overreaction throughout the economy. On the other hand, the DE model with a twelve-quarters slow-moving reference exhibits more pronounced ups and downs due to the longer span memory in their expectation formation process.

Agents misconceiving the shock to be an ARMA(1,1) or ARMA (1,12) generate optimism about their productivity in the future. As a result, households will assign higher probability to a scenario in which they are richer, leading to demand pressure (panel A and panel B). Firms experiencing higher labour costs will hire less workers, and thus decrease their marginal cost. This is reflected in the drop in domestic prices (panel C), firms cut prices more since they might not be able to re-optimise in the future. The central bank lowers the interest rate (panel D) by a larger amount when the diagnostic agent reference

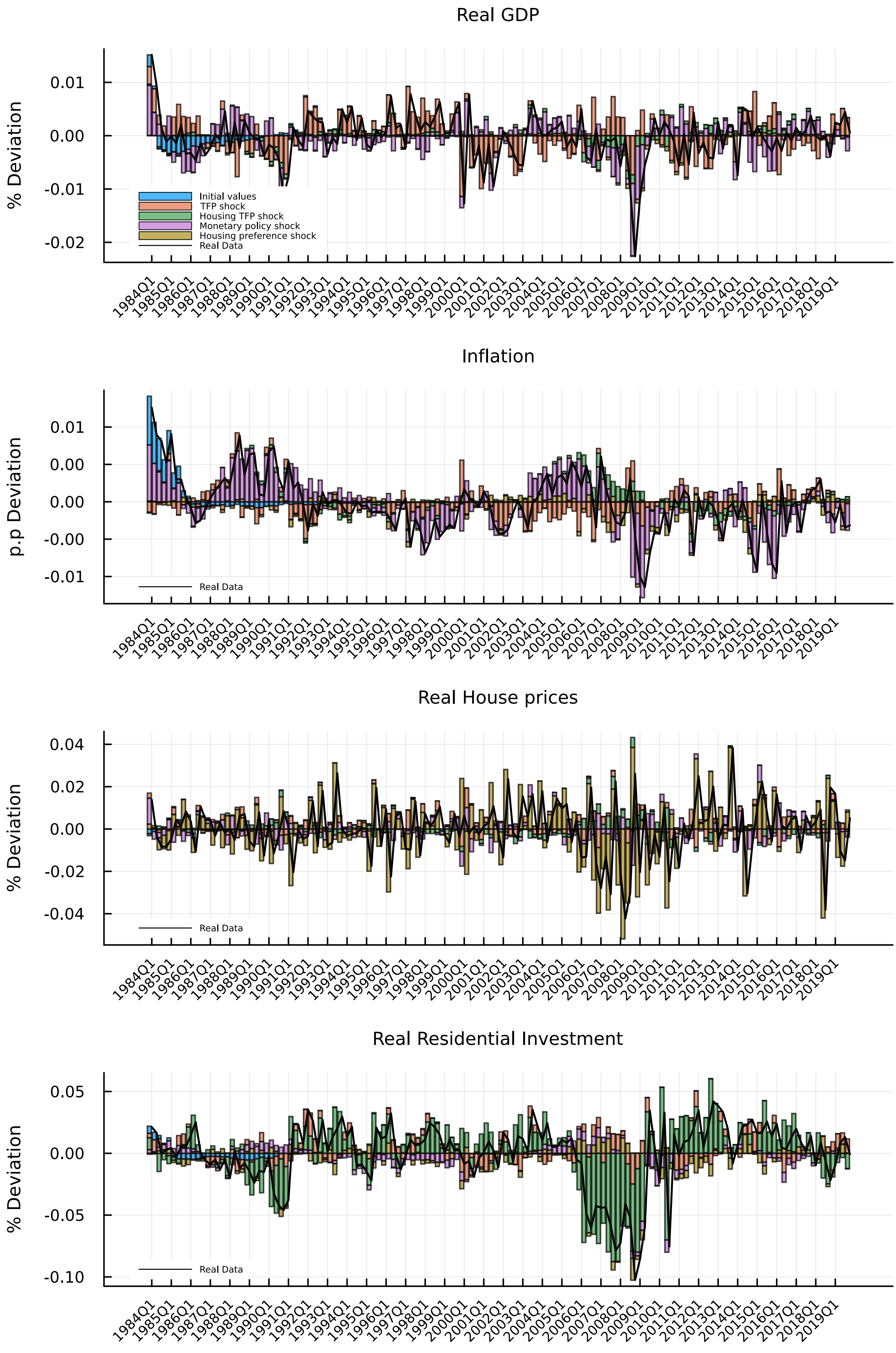


Figure 4: Historical shock decomposition under DE model with distant memory.

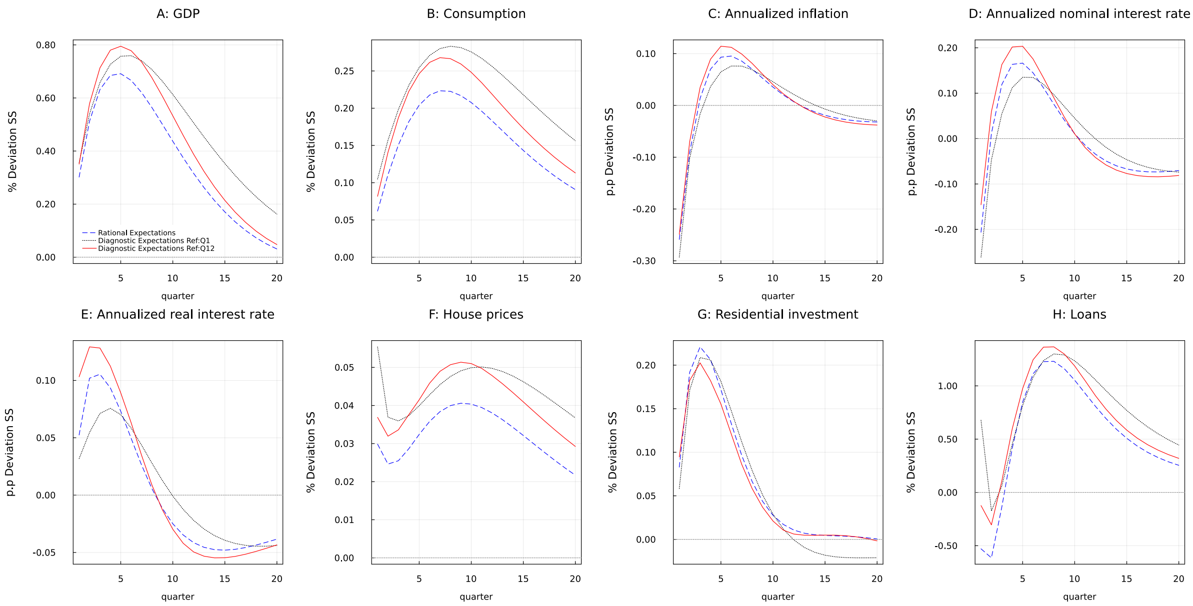


Figure 5: Impulse responses to non-durable goods productivity shock .
Note: The blue dashed line represents the responses when agents have rational expectations. The solid red line illustrates the impulse responses when agents have DE with a 3-year memory recall, while the black dotted line DE with the last quarter as reference.

is the most recent past as its Taylor rule reacts more strongly to deviations in inflation. These two forces are the core of the noticeable difference in the real interest rate response (panel E). Households extrapolate current surprise disinflation into the future, i.e. they expect inflation to further decrease. However, since the opposite happens, agents realise about their mistake, and so does the central bank, which hikes nominal interest rates, creating a boom-bust pattern in the real interest rate as L'Huillier et al. (2024). This alters the loans impulse response shape (panel H), as now impatient households will be more willing to borrow against their house value. This additional liquidity, puts pressure in house prices (panel F). They exhibit a boom-bust pattern which aligns with historical interpretations of bubbles, as suggested by Gelain et al. (2012), but this paper goes one step further by incorporating a micro and psychologically founded belief formation model.[20] Finally, this stimulates higher housing investment (panel G).

5.3.2 Effects of a housing sector productivity shock

Impulse responses to a positive productivity shock in the durable goods sector are shown in Figure 6. The impact primarily affects variables related to the housing market. Housing investment exhibits a positive response (panel G) due to the increased productivity of capital and labour in this sector. This, in turn, leads to a higher housing supply, resulting

[20] Greenspan (2002) defines: "Bubbles are often precipitated by perceptions of real improvements in the productivity and underlying profitability of the corporate economy. But as history attests, investors then too often exaggerate the extent of the improvement in economic fundamentals. Human psychology being what it is, bubbles tend to feed on themselves, and booms in their later stages are often supported by implausible projections of potential demand."

in a quite persistent decline in house prices (panel F). These movements affects consumption of non-durable goods (panel B) as there is a reallocation of resources, and a fall in annualized inflation (panel C) and nominal interest rate. GDP (panel A), nevertheless, is positively driven by the housing sector.

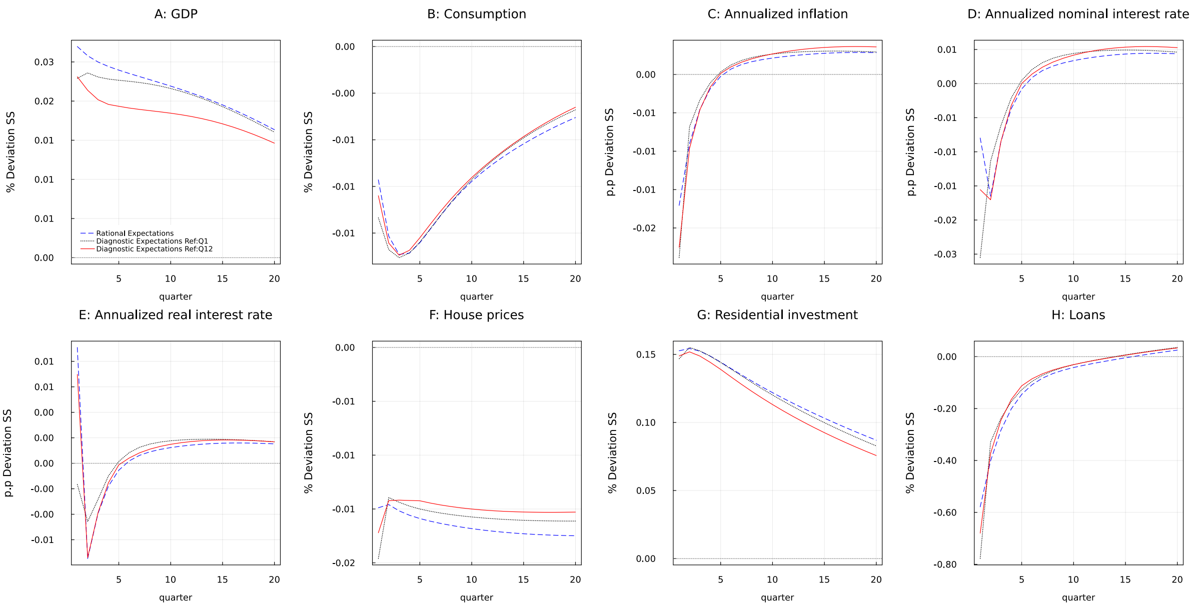


Figure 6: Impulse responses to a housing sector productivity shock .
Note: The blue dashed line represents the responses when agents have rational expectations. The solid red line illustrates the impulse responses when agents have DE with a 3-year memory recall, while the black dotted line DE with the last quarter as reference.

In comparison to the RE case, the DE models have a similar response, in magnitude, of housing investment (panel G). As the diagnostic agent believes that this TFP shock is more persistent than it actually is, they overestimate the future productivity of the housing sector and anticipate a higher housing supply. Consequently, they initially overreact, leading to a more pronounced decline in house prices compared to the rational case (panel F). This overreaction is stronger for the DE model with the recent quarter as reference. However, as agents realise that their beliefs are inconsistent with the true process of the shock, house prices correct and converge faster to steady state. In fact, this follows from households' disappointment, which result in less residential investment under DE compared to RE.

5.3.3 Effects of a tightening monetary policy

Turning to Figure 7, it illustrates the impulse responses of a tightening monetary policy shock. The direction under RE is as expected. The shock depresses the economy resulting in a negative deviation of GDP (panel A) and consumption (panel B) from their steady state. Inflation decreases (panel C) and the central bank react to these movements in output and inflation by lowering the nominal interest rate (panel D). However, the

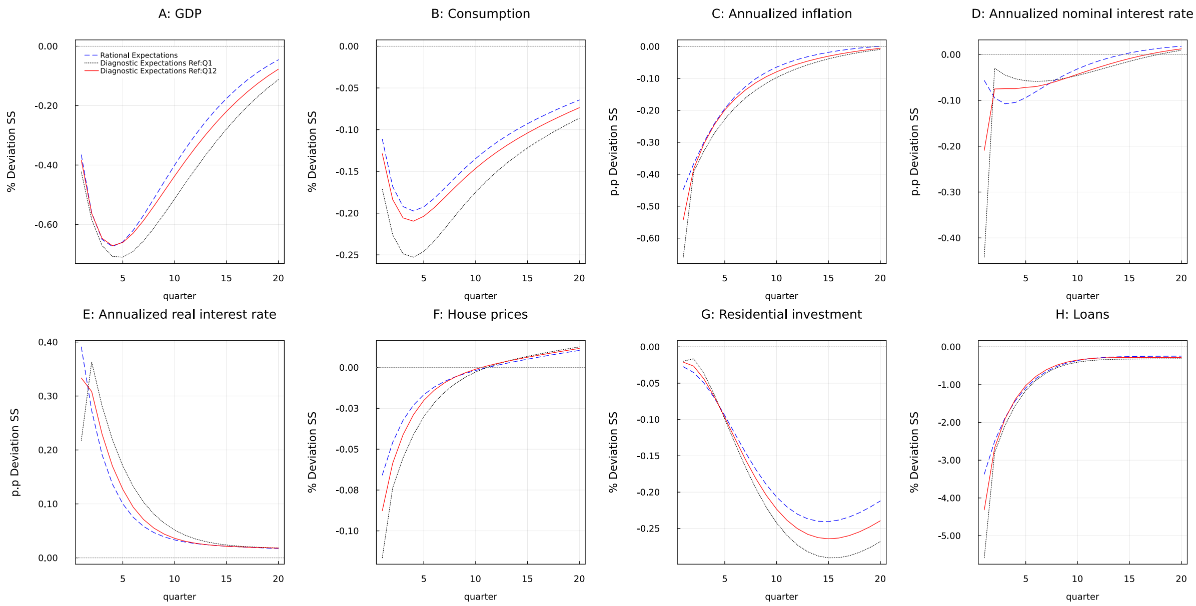


Figure 7: Impulse responses to monetary policy shock .
Note: The blue dashed line represents the responses when agents have rational expectations. The solid red line illustrates the impulse responses when agents have DE with a 3-year memory recall, while the black dotted line DE with the last quarter as reference.

decrease in inflation exceeds the adjustment in the nominal interest rate, resulting in a positive real interest rate (panel E). This has a negative effect in house prices (panel F) since mortgages for impatient households become more expensive (panel H). This, in turn, depresses housing demand and housing investment (panel G), as the increased cost of capital impact investment decisions.

When agents are diagnostic, the impulse responses exhibit some distinct features. The fall in GDP is relatively bigger and more persistent than in RE (panel A). This is because agents belief that the central bank will further tighten the monetary policy in the future. This explains the stronger initial fall in prices (panel C), which leads to stronger reactions in the nominal interest rate (panel D), and therefore slightly smaller real interest rate (panel E). Agents mistakenly expect the variables to follow this path, but as events unfold and there are no further surprises in monetary policy, they adjust their expectations. This explains the sudden rise in the nominal interest rate and the jump in inflation, which are more pronounced in the diagnostic model with agents who have short memory. Moreover, the behaviour of consumption (panel B) follows the Euler equation, and the u-shaped reaction is more persistent. A similar story holds for loans. The change in the real interest rate, as well as the decline in house prices (panel F), impacts the borrowing constraint of the impatient household. It decreases her collateral and so does her ability to obtain funds (panel H). This drags the housing demand and amplifies the fall in house prices. The posterior recovery follows from the relaxation of the impatient household's collateral constraint. De Stefani (2021) reports empirical results consistent with this.

5.3.4 Effects of a preference housing shock

Figure 8 provides details on the impact of the housing preference shock. Under RE, this shock shifts households' taste towards the housing sector as it directly hits the marginal utility of housing for both agents. This puts pressure on house prices (panel F). Although higher prices would make housing less desirable overall, the impatient households experience a loosing in their collateral constraint (panel H), reflecting their willingness to leverage their financial position, as they need to finance higher housing costs. However, this effect is insufficient to offset the decline in housing demand from patient agents. Additionally, the rise in interest rates (panel D and panel E) diverts funds away from the housing sector , causing a delayed increase in residential investment (panel G). This, once realised, stimulates GDP (panel A).

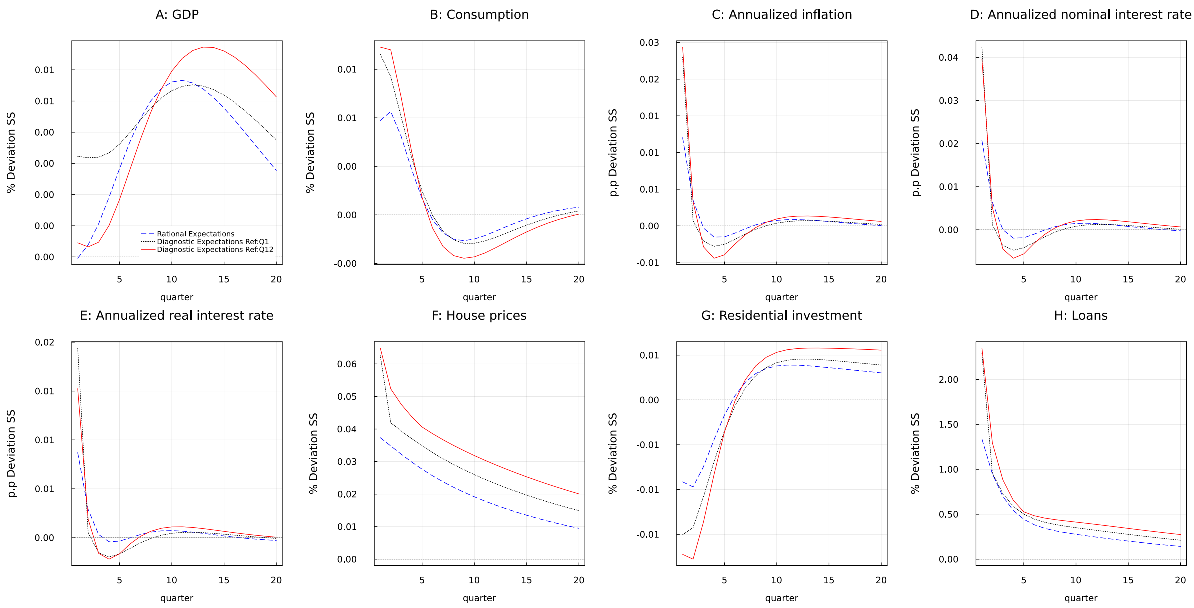


Figure 8: Impulse responses to a housing preference shock.
Note: The blue dashed line represents the responses when agents have rational expectations. The solid red line illustrates the impulse responses when agents have DE with a 3-year memory recall, while the black dotted line DE with the last quarter as reference.

The responses under both DE scenarios are in the same direction as in RE, although clearly amplified. When agents are diagnostic, after the shock hits, they expect further pressures in the housing market, anticipating house prices to rise even higher. This leads to an initial overreaction in house prices (panel F), under both short and long-term memory. Such significant shift impacts other variables in the economy in the same way as under rationality. Impatient households experience a greater loosening of their collateral constraint, but the stronger decline in patient households' housing demand, combined with the hike in interest rates, diverts more funds away from the housing sector (panel G). The main difference, however, occurs after agents realize that the true shock process is AR(1) rather than ARMA(1,1) or ARMA(1,12). In the first case, the rebound happens faster as agents rapidly revise their expectations, while in the second case, it takes longer. This difference is evident in the impulse responses for house prices, where the drop in period 2 is more pronounced when the reference group is the most recent past. Another key difference lies in the persistence and fluctuations of the responses. The initial overreaction, particularly when the comparison group spans twelve-quarters, takes longer to die out. Gandré (2022) suggests that these movements originate in households tastes swings, directly affecting intra-temporal and inter-temporal trade-offs.

6 Counterfactual analysis

In this section, I conclude the analysis presenting a counterfactual study where the expectations channel in the DE models is shut. I evaluate an alternative scenario in which agents from the estimated DE models become rational. I set the diagnostic parameter and the weights on past quarters to zero, while keeping all other parameters fixed.

Table 5: Real House price growth second-order moment

	Data	DE Ref:Q12	DE Ref:Q1	RE Ref:Q12 Counterfactual	RE Ref:Q1 Counterfactual
Volatility relative to GDP					
Real House price growth	2.9896	2.4381	3.1882	1.8877	2.4992

Note: House price growth rate is obtained from averaging over ten thousand simulations of hundred and forty four observations each.

The results in Table 5 suggest that the rational counterfactual models struggles to amplify house price volatility. Both counterfactual RE models produce a measure that is 22% lower than their diagnostic counterpart. Notably, the RE Ref: Q1 counterfactual generates a higher measure since the estimated size of the preference shock is higher than in the case of twelve-quarters as reference (7.23% vs 5.35%). This finding underscores the significant role that DE plays in driving housing market dynamics. In another words, around a third of the housing market volatility originates from the expectations channel, through DE.

7 Concluding remarks

This paper examines expectations as a central driver of housing market volatility by integrating Diagnostic Expectations (DE) with both short-term and long-term memory into a TANK model featuring housing and banking sectors. The results, based on diagnostic parameter and reference period weight estimates, validate the DE model empirically. Evidence favours the model in which diagnostic agents consider only the immediate past quarter when forming beliefs. It successfully accounts for economic fluctuations, particularly in the housing market, when conditioned on less volatile shocks. Specifically, the DE model explains housing price and quantity dynamics with a preference shock innovation size two-thirds of that under RE. This suggests DE as a more comprehensive alternative for the "catchall of all the unmodeled disturbances that can affect housing demand" (Iacoviello & Neri, 2010, p. 150).

Another noteworthy result is that, when the expectations channel in the DE models is shut down the models fail to generate the higher volatility in house prices relative to real GDP growth observed in the data. Together with the previous result, this suggests that DE drive cyclical dynamics in the housing market and, given the sector's significance in household decision-making, underline the need to consider DE in policy recommendations.

Future work would enhance the analysis. One direction I plan to explore is allowing the banking sector to intermediate between households, which would provide insights about the role of expectations in the housing credit market. Another possible extension would be to allow for heterogeneity in the degree of diagnosticity to capture diverse belief formation across households.

References

Adam, K., Pfäuti, O., & Reinelt, T. (2024). Subjective housing price expectations, falling natural rates and the optimal inflation target. *Journal of Monetary Economics*, 103647.

Adjemian, S. (2016). Prior distributions in dynare. *Manuscript: Université du Maine.*

Adjemian, S., Juillard, M., Karamé, F., Mutschler, W., Pfeifer, J., Ratto, M., … Villemot, S. (2024). *Dynare: Reference manual, version 6* (Dynare Working Papers No. 80). CEPREMAP.

Bianchi, F., Ilut, C., & Saijo, H. (2024). Diagnostic business cycles. *Review of Economic Studies*, *91*(1), 129–162.

Bordalo, P., Coffman, K., Gennaioli, N., & Shleifer, A. (2016). Stereotypes. *The Quarterly Journal of Economics*, *131*(4), 1753–1794.

Bordalo, P., Gennaioli, N., & Shleifer, A. (2018). Diagnostic expectations and credit cycles. *The Journal of Finance*, *73*(1), 199–227.

Bordalo, P., Gennaioli, N., Shleifer, A., & Terry, S. J. (2021). *Real credit cycles.* (Working Paper No. 28416). National Bureau of Economic Research.

Bounader, L., & Elekdag, S. A. (2024). *The diagnostic financial accelerator.* (Working Paper No. 2024/132). International Monetary Fund.

Cai, M., Del Negro, M., Herbst, E., Matlin, E., Sarfati, R., & Schorfheide, F. (2021). Online estimation of dsge models. *The Econometrics Journal*, *24*(1), C33–C58.

Caines, C. (2020). Can learning explain boom-bust cycles in asset prices? an application to the us housing boom. *Journal of Macroeconomics*, *66*, 103256.

Chahrour, R., & Gaballo, G. (2021). Learning from house prices: Amplification and business fluctuations. *The Review of Economic Studies*, *88*(4), 1720–1759.

Darracq Paries, M., & Notarpietro, A. (2008). *Monetary policy and housing prices in an estimated dsge model for the us and the euro area.* (Working Paper No. 972). European Central Bank Frankfurt.

Davis, M. A., & Heathcote, J. (2005). Housing and the business cycle. *International Economic Review*, *46*(3), 751–784.

De Stefani, A. (2021). House price history, biased expectations, and credit cycles: The role of housing investors. *Real Estate Economics*, *49*(4), 1238–1266.

Funke, M., & Paetz, M. (2013). Housing prices and the business cycle: An empirical application to hong kong. *Journal of Housing Economics*, *22*(1), 62–76.

Gabriel, V., & Ghilardi, M. F. (2012). *Financial frictions in an estimated dsge model* (Tech. Rep.). August 23, mimeo.

Gambacorta, L., & Signoretti, F. M. (2014). Should monetary policy lean against the wind?: An analysis based on a dsge model with banking. *Journal of Economic Dynamics and Control*, *43*, 146–174.

Gandré, P. (2022). A note on learning, house prices, and macro-financial linkages. *Macroeconomic Dynamics*, 1–16.

Ge, X., Li, X.-L., Li, Y., & Liu, Y. (2022). The driving forces of china's business cycles: Evidence from an estimated dsge model with housing and banking. *China Economic Review*, *72*, 101753.

Gelain, P., Lansing, K. J., & Mendicino, C. (2012). House prices, credit growth, and excess volatility: Implications for monetary and macroprudential policy. *International Journal of Central Banking*, *9*(2), 219-276.

Gennaioli, N., & Shleifer, A. (2018). *A crisis of beliefs: Investor psychology and financial fragility.* Princeton, NJ: Princeton University Press.

Gerali, A., Neri, S., Sessa, L., & Signoretti, F. M. (2010). Credit and banking in a dsge model of the euro area. *Journal of money, Credit and Banking*, *42*, 107–141.

Gertler, M., & Karadi, P. (2011). A model of unconventional monetary policy. *Journal of monetary Economics*, *58*(1), 17–34.

Gertler, M., & Kiyotaki, N. (2010). Financial intermediation and credit policy in business cycle analysis. In *Handbook of monetary economics* (Vol. 3, pp. 547–599). Elsevier.

Gohl, N., Haan, P., Michelsen, C., & Weinhardt, F. (2024). House price expectations. *Journal of Economic Behavior & Organization*, *218*, 379–398.

Granziera, E., & Kozicki, S. (2015). House price dynamics: Fundamentals and expectations. *Journal of Economic Dynamics and control*, *60*, 152–165.

Greenspan, A. (2002). Remarks by chairman alan greenspan. *Federal Reserve Board, Current Account, before the Economic Club of New York, New York*, *2*.

Havranek, T., Rusnak, M., & Sokolova, A. (2017). Habit formation in consumption: A meta-analysis. *European Economic Review*, *95*, 142–167.

Herbst, E., & Schorfheide, F. (2014). Sequential monte carlo sampling for dsge models. *Journal of Applied Econometrics*, *29*(7), 1073–1098.

Iacoviello, M. (2005). House prices, borrowing constraints, and monetary policy in the business cycle. *American economic review*, *95*(3), 739–764.

Iacoviello, M. (2015). Financial business cycles. *Review of Economic Dynamics*, *18*(1), 140–163.

Iacoviello, M., & Neri, S. (2010). Housing market spillovers: evidence from an estimated dsge model. *American Economic Journal: Macroeconomics*, *2*(2), 125–164.

Justiniano, A., Primiceri, G. E., & Tambalotti, A. (2010). Investment shocks and business cycles. *Journal of Monetary Economics*, *57*(2), 132–145.

Kahneman, D., & Tversky, A. (1972). Subjective probability: A judgment of representativeness. *Cognitive psychology*, *3*(3), 430–454.

Kass, R. E., & Raftery, A. E. (1995). Bayes factors. *Journal of the american statistical association*, *90*(430), 773–795.

Klein, P. (2000). Using the generalized schur form to solve a multivariate linear rational expectations model. *Journal of economic dynamics and control*, *24*(10), 1405–1423.

Kuchler, T., Piazzesi, M., & Stroebel, J. (2023). Housing market expectations. In *Handbook of economic expectations* (pp. 163–191). Elsevier.

Leith, C., Moldovan, I., & Rossi, R. (2012). Optimal monetary policy in a new keynesian model with habits in consumption. *Review of Economic Dynamics*, *15*(3), 416–435.

L'Huillier, J.-P., Singh, S. R., & Yoo, D. (2024). Incorporating diagnostic expectations into the new keynesian framework. *Review of Economic Studies*, *91*(5), 3013–3046.

Maxted, P. (2024). A macro-finance model with sentiment. *Review of Economic Studies*, *91*(1), 438–475.

Mendicino, C., & Punzi, M. T. (2014). House prices, capital inflows and macroprudential policy. *Journal of Banking & Finance*, *49*, 337–355.

Piazzesi, M., & Schneider, M. (2016). Housing and macroeconomics. *Handbook of macroeconomics*, *2*, 1547–1640.

Qi, Z. (2021). *Diagnostic expectations in housing price dynamics* (Unpublished doctoral dissertation). The Hong Kong University of Science and Technology.

Salazar-Perez, A., & Seoane, H. D. (2024). Perturbating and estimating dsge models in julia. *Computational Economics*, 1–18.

Smets, F., & Wouters, R. (2007). Shocks and frictions in us business cycles: A bayesian dsge approach. *American economic review*, *97*(3), 586–606.

Smith, A. (1759). *The theory of moral sentiments.* London: printed for A. Millar; and A. Kincaid and J. Bell, in Edinburgh.

Wolff, E. N. (2016). Household wealth trends in the united states, 1962 to 2013: What happened over the great recession? *RSF: The Russell Sage Foundation Journal of the Social Sciences*, *2*(6), 24–43.

Wu, J. C., & Xia, F. D. (2016). Measuring the macroeconomic impact of monetary policy at the zero lower bound. *Journal of Money, Credit and Banking*, *48*(2-3), 253–291.

8 Appendix

8.1 Model Derivations

8.1.1 Households

8.1.1.1 Patient

$$\begin{aligned}\mathcal{L}_p = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_p^t \Bigg\{ & \Bigg[log(c_{p,t} - \gamma c_{p,t-1}) + \Gamma_t \nu_p^h log(h_{p,t}) - \nu_p^n \frac{n_{p,t}^{1+\varphi}}{1+\varphi} \Bigg] - \\ & \lambda_{p,t} \Bigg[c_{p,t} + q_t[h_{p,t} - (1-\delta_h)h_{p,t-1}] + d_t^B + d_t^l - \frac{d_{t-1}^B R_{t-1}^d}{\pi_t} - \frac{d_{t-1}^l R_{t-1}^l}{\pi_t} - \\ & w_t n_{p,t} - \Pi_{f,t} - \Pi_{B,t} \Bigg] \Bigg\}\end{aligned} \tag{74}$$

The optimal conditions of this Lagrangian with respect to $c_{p,t}$, $n_{p,t}$, $h_{p,t}$, d_t^B and d_t^l are:

$$\frac{\partial \mathcal{L}_p}{\partial c_{p,t}} : \lambda_{p,t} = \frac{1}{(c_{p,t} - \gamma c_{p,t-1})} - \frac{\beta_p \gamma}{(c_{p,t+1} - \gamma c_{p,t})}. \tag{75}$$

$$\frac{\partial \mathcal{L}_p}{\partial n_{p,t}} : \nu_p^n n_{p,t}^{\varphi} = w_t \lambda_{p,t}. \tag{76}$$

$$\frac{\partial \mathcal{L}_p}{\partial h_{p,t}} : \lambda_{p,t} q_t = \frac{\Gamma_t \nu_p^h}{h_{p,t}} + \beta_p \mathbb{E}_t \Big[(1-\delta_h) q_{t+1} \lambda_{p,t+1} \Big]. \tag{77}$$

$$\frac{\partial \mathcal{L}_p}{\partial d_t^B} : \lambda_{p,t} = \beta_p \mathbb{E}_t \Big[\lambda_{p,t+1} \frac{R_t^d}{\pi_{t+1}} \Big]. \tag{78}$$

$$\frac{\partial \mathcal{L}_p}{\partial d_t^l} : \lambda_{p,t} = \beta_p \mathbb{E}_t \Big[\lambda_{p,t+1} \frac{R_t^l}{\pi_{t+1}} \Big]. \tag{79}$$

8.1.1.2 Impatient

$$
\begin{aligned}
\mathcal{L}_i =&\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_i^t \Bigg\{ \Bigg[log(c_{i,t} - \gamma c_{i,t-1}) + \Gamma_t \nu_i^h log(h_{i,t}) - \nu_i^n \frac{n_{i,t}^{1+\varphi}}{1+\varphi} \Bigg] - \\
&\lambda_{i,t} \Bigg[c_{i,t} + q_t(h_{i,t} - (1-\delta_h) h_{i,t-1}) + \frac{l_{t-1} R_{t-1}^l}{\pi_t} - l_t - w_t n_{i,t} \Bigg] - \\
&\mu_{i,t} \Bigg[l_t - \frac{\chi}{R_t^l} (q_{t+1}\pi_{t+1}) h_{i,t} \Bigg] \Bigg\}
\end{aligned} \tag{80}
$$

The optimal conditions of this Lagrangian with respect to $c_{i,t}$, $n_{i,t}$ $h_{i,t}$ and l_t are:

$$
\frac{\partial \mathcal{L}_i}{\partial c_{i,t}} : \lambda_{i,t} = \frac{1}{(c_{i,t} - \gamma c_{i,t-1})} - \frac{\beta_i \gamma}{(c_{i,t+1} - \gamma c_{i,t})}. \tag{81}
$$

$$
\frac{\partial \mathcal{L}_i}{\partial n_{i,t}} : \nu_i^n n_{i,t}^{\varphi} = w_t \lambda_{i,t}. \tag{82}
$$

$$
\frac{\partial \mathcal{L}_i}{\partial h_{i,t}} : \lambda_{i,t} q_t = \frac{\Gamma_t \nu_i^h}{h_{i,t}} + \beta_i \mathbb{E}_t \Big[(1-\delta_h) q_{t+1} \lambda_{i,t+1} \Big] + \mu_{i,t} \frac{\chi}{R_t^l} \mathbb{E}_t [q_{t+1}\pi_{t+1}], \tag{83}
$$

$$
\frac{\partial \mathcal{L}_i}{\partial l_t} : \lambda_{i,t} - \mu_{i,t} = \beta_i \mathbb{E}_t \Big[\lambda_{i,t+1} \frac{R_t^l}{\pi_{t+1}} \Big] \tag{84}
$$

8.1.2 Firms

8.1.2.1 Wholesale firms

$$
\max_{N_t^W, K_t^W} \Pi_{t+1}^{w,f} = \Big[P_{m,t} Y_t^W + (1-\delta_k) q_{t-1}^K K_{t-1}^W - R_t^K q_{t-1}^K K_t^W - w_t N_t^W \Big] \tag{85}
$$

subject to

$$
Y_t^W = A_t {N_t^W}^{1-\alpha} {K_{t-1}^W}^{\alpha}, \tag{86}
$$

The first order conditions with respect to N_t^W and K_t^W are:

$$
w_t = P_{m,t} (1-\alpha) A_t \left(\frac{K_{t-1}^W}{N_t^W} \right)^{\alpha}, \tag{87}
$$

$$
q_{t-1}^K R_t^K = r_t^K + (1-\delta_k) q_t^K, \tag{88}
$$

where $r_t^K = P_{m,t}\alpha A_t\left(\frac{N_t^W}{K_{t-1}^W}\right)^{1-\alpha}$ is the rental rate of capital. Obtaining the ratio $\frac{N_t^W}{K_{t-1}^W}$ from the wage and rental rate expressions and equating them, I obtain an equation for $P_{m,t}$, which turns out to be the marginal cost, therefore:

$$mc_t = \frac{1}{A_t}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t^K}{\alpha}\right)^{\alpha}. \tag{89}$$

8.1.2.2 Final good firm

The final good producer purchases goods re-packaged by the retailers and aggregates them according to a Dixit-Stiglitz production technology. After it, they sell the final product in a perfect competitive market at the price P_t.

$$Y_t = \left[\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}} \tag{90}$$

Y_t represents the final good, $y_t(j)$ denotes the $j'th$ retailer input. This firm's profit maximisation is a static problem and can be stated as:

$$\max_{y_t(j)} P_t\left[\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}} - \int_0^1 p_t(j)y_t(j)dj, \tag{91}$$

where Y_t was replaced using its definition. The first order condition of this decision problem by choosing $\{y_t(j)\}_{j=0}^1$ is given by:

$$P_t\frac{\epsilon}{\epsilon-1}\left[\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}-1}\frac{\epsilon-1}{\epsilon}y_t(j)^{\frac{\epsilon-1}{\epsilon}-1} = P_t(j), \forall j$$

$$\Rightarrow P_t\left[\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{1}{\epsilon-1}} y_t(j)^{-\frac{1}{\epsilon}} = P_t(j)$$

$$\Rightarrow y_t(j)^{-\frac{1}{\epsilon}} = \left(\frac{P_t(j)}{P_t}\right)\left[\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{-\frac{1}{\epsilon-1}}$$

$$\Rightarrow y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon}\left[\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{-\frac{\epsilon}{\epsilon-1}}. \tag{92}$$

Which after using the definition of Y_t, the demand equation for each input turns out to be:

$$y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t. \tag{93}$$

Since this final good producing firm acts in a competitive market, it makes zero profit. Replacing the demand equation in the maximisation problem, I obtain:

$$P_t Y_t = \int_0^1 P_t(j) Y_t(j) dj$$

$$\Rightarrow P_t Y_t = \int_0^1 P_t(j) \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t dj$$

$$\Rightarrow P_t Y_t = P_t^{\epsilon} Y_t \int_0^1 P_t(j)^{1-\epsilon} dj$$

$$P_t^{1-\epsilon} = \int_0^1 P_t(j)^{1-\epsilon} dj. \tag{94}$$

Rearranging this equation yields an expression of the price of the final good as a function of the intermediate inputs' prices, i.e. an aggregate price index:

$$P_t = \left[\int_0^1 P_t(j)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}} \tag{95}$$

8.1.2.3 Retailers firms

In the presence of price rigidity *à la Calvo*, retailers will be able to change their price with a probability $(1-\theta)$, while with a probability θ they will not. To determine the new price $P_t^*(j)$, retailers' maximise:

$$V_t(j) = \mathbb{E}_t \sum_{i=0}^{\infty} (\beta_p \theta)^i \Big\{ \frac{\lambda_{p,t+i}}{\lambda_{p,t}} \left[\left(\frac{P_t^*(j)}{P_{t+i}} - mc_{t+i} \right) \left(\frac{P_t^*(j)}{P_{t+i}} \right)^{\epsilon} Y_{t+i} \right] \Big\}.$$

The first order condition of this problem is:

$$\frac{\partial V_t(j)}{\partial P_t^*(j)} : \mathbb{E}_t \sum_{i=0}^{\infty} (\beta\theta)^i \left\{ \frac{\lambda_{p,t+i}}{\lambda_{p,t}} \left[\left(\frac{P_t^*(j)}{P_{t+i}} \right)^{-\epsilon} - \epsilon \left(\frac{P_t^*(j)}{P_{t+i}} - mc_{t+i} \right) \left(\frac{P_t^*(j)}{P_{t+i}} \right)^{-(\epsilon+1)} \right] \frac{Y_{t+i}}{P_{t+i}} \right\} = 0 \tag{96}$$

$$\mathbb{E}_t \sum_{i=0}^{\infty} (\beta\theta)^i \frac{\lambda_{p,t+i}}{\lambda_{p,t}} \left[(1-\epsilon)(P_t^*(j))^{-\epsilon} P_{t+i}^{\epsilon-1} + \epsilon m c_{t+i} (P_t^*(j))^{-(\epsilon-1)} P_{t+i}^{\epsilon} \right] Y_{t+i} = 0$$

$$\mathbb{E}_t \sum_{i=0}^{\infty} (\beta\theta)^t \frac{\lambda_{p,t+i}}{\lambda_{p,t}} \left[(1-\epsilon)(P_t^*(j))^{-\epsilon} P_{t+i}^{\epsilon-1} Y_{t+1} + \epsilon m c_{t+i} P_{t+i}^{\epsilon} Y_{t+i} \right] = 0$$

After rearranging, the result of this maximisation problem determines that retailer firms, which have obtained a successful draw, will set their price as a constant mark-up on an expression related to their expected discounted nominal total costs relative to an expression related to their expected discounted real output.

$$P_t^*(j) = \frac{\epsilon}{\epsilon - 1} \left[\frac{\mathbb{E}_t \sum_{i=0}^{\infty} (\beta_p \theta)^i \lambda_{p,t+i} m c_{t+i} P_{t+i}^{\epsilon} Y_{t+i}}{\mathbb{E}_t \sum_{i=0}^{\infty} (\beta_p \theta)^i \lambda_{p,t+i} P_{t+i}^{\epsilon-1} Y_{t+i}} \right]. \tag{97}$$

The above equation does not depend on j, this implies that every retailer firm that is able to set its price in period t will choose the same price. Moreover, in the limiting case, with no price rigidity, the firm's optimal price is a constant markup on real marginal costs. This expression can be writen in terms of two auxiliary variables, $x_{1,t}$ and $x_{2,t}$:

$$\pi_t^* = \frac{\epsilon}{\epsilon - 1} \frac{x_{1,t}}{x_{2,t}}, \tag{98}$$

where the auxiliary variables take the following recursive forms:

$$x_{1,t} = \lambda_{p,t} m c_t Y_t + \theta \beta_p \mathbb{E}_t (\pi_{t+1})^{\epsilon} x_{1,t+1}. \tag{99}$$

$$x_{2,t} = \lambda_{p,t} Y_t + \theta \beta_p \mathbb{E}_t (\pi_{t+1})^{\epsilon-1} x_{2,t+1}. \tag{100}$$

Next, I define an auxiliary variable ν_t^j for the measure of price dispersion:

$$\nu_t^j = \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} dj. \tag{101}$$

Making use of the fact that a proportion of firms are able to reset their price, while others are not, the price dispersion can be re-written as:

$$\nu_t^j = \int_0^{1-\theta} \left(\frac{P_t^*(j)}{P_t} \right)^{-\epsilon} dj + \int_{1-\theta}^1 \left(\frac{P_{t-1}(j)}{P_t} \right)^{-\epsilon} dj \tag{102}$$

To obtain an expression of the price dispersion in terms of inflation rate, I multiply and divide by powers of P_{t-1} where necessary, given:

$$\nu_t^j = \int_0^{1-\theta} \left(\frac{P_t^*(j)}{P_t}\right)^{-\epsilon} dj + \int_{1-\theta}^1 \left(\frac{P_{t-1}(j)}{P_{t-1}}\right)^{-\epsilon} \left(\frac{P_{t-1}}{P_t}\right)^{-\epsilon} dj.$$

Using the definition of π_t^*, and of gross inflation π_t, the previous expression becomes:

$$\nu_t^j = (1-\theta)(\pi_t^*)^{-\epsilon} + (\pi_t)^{\epsilon} \int_{1-\theta}^1 \left(\frac{P_{t-1}(j)}{P_{t-1}}\right)^{-\epsilon} dj$$

where the last term, using the definition of the auxiliary variable, is equal to $\theta\nu_{t-1}^j$. Replacing it yields:

$$\nu_t^j = (1-\theta)(\pi_t^*)^{-\epsilon} + (\pi_t)^{\epsilon}\theta\nu_{t-1}^j. \tag{103}$$

8.1.2.4 Capital good firms

The capital good producers maximise:

$$\mathbb{E}_0 \sum_{i=0}^{\infty} \beta_p^t \frac{\lambda_{p,t+i}}{\lambda_{p,t}} \Big[q_t^K K_t^W - q_t^K (1-\delta_k) K_{t-1}^W + r_t^{K,h} K_t^h - I_t\Big], \tag{104}$$

subject to the law of motion of total capital and the definition of aggregate capital.

$$K_t = (1-\delta_k)K_{t-1} + [1 - \frac{\psi}{2}(I_t/I_{t-1} - 1)^2]I_t, \tag{105}$$

$$K_t = K_t^W + K_t^h. \tag{106}$$

I write the problem in Lagrangian form as:

$$\begin{aligned}\mathcal{L}_K = \mathbb{E}_0 \sum_{i=0}^{\infty} \beta_p^i \frac{\lambda_{p,t+i}}{\lambda_{p,t}} \Bigg\{ \Big[q_t^K K_t^W - q_t^K (1-\delta_k) K_{t-1}^W + r_t^{K,h} K_t^h - I_t\Big] - \\ \lambda_{K,t}\Big[K_t^W + K_t^h - (1-\delta_K)(K_{t-1}^W + K_{t-1}^h) - \Big(1 - \frac{\psi}{2}\Big(\frac{I_t}{I_{t-1}} - 1\Big)^2\Big) I_t\Big] \Bigg\}\end{aligned} \tag{107}$$

The optimality conditions with respect to K_t^W, K_t^h and I_t are:

$$\frac{\partial \mathcal{L}_K}{\partial K_t^W} : q_t^K - \beta_p \frac{\lambda_{p,t+1}}{\lambda_{p,t}}(1-\delta_k) q_{t+1}^K = \lambda_{K,t} - \beta_p \frac{\lambda_{p,t+1}}{\lambda_{p,t}}(1-\delta_k)\lambda_{K,t+1}. \tag{108}$$

$$\frac{\partial \mathcal{L}_K}{\partial K_t^h} : r_t^{K,h} = \lambda_{K,t} - \beta_p \frac{\lambda_{p,t+1}}{\lambda_{p,t}}(1-\delta_k)\lambda_{K,t+1}. \tag{109}$$

$$\begin{aligned}\frac{\partial \mathcal{L}_K}{\partial I_t} : 1 = \lambda_{K,t}\Big[1 - \frac{\psi}{2}\Big(\frac{I_t}{I_{t-1}} - 1\Big)^2 - \psi\Big(\frac{I_t}{I_{t-1}} - 1\Big)\Big(\frac{I_t}{I_{t-1}}\Big)\Big] + \\ \beta_p \psi \mathbb{E}_t\Big[\frac{\lambda_{p,t+1}}{\lambda_{p,t}}\lambda_{K,t+1}\Big(\frac{I_{t+1}}{I_t}\Big)^2\Big(\frac{I_{t+1}}{I_t} - 1\Big)\Big],\end{aligned} \tag{110}$$

8.1.2.5 Housing firms

This firm's profit maximisation is a static problem and can be stated as:

$$\max_{N_t^h, K_{t-1}^h} \Pi_t^h = \big[q_t I_t^h - r_t^{K,h} K_{t-1}^h - w_t N_t^h\big], \tag{111}$$

subject to

$$I_t^h = Z_t {N_t^h}^{1-\mu_h} {K_{t-1}^h}^{\mu_h}, \tag{112}$$

After replacing the production function in the profit expression, I re-write the problem as following:

$$\max_{N_t^h, K_{t-1}^h} \Pi_t^h = \big[q_t (Z_t {N_t^h}^{1-\mu_h} {K_{t-1}^h}^{\mu_h}) - r_t^{K,h} K_{t-1}^h - w_t N_t^h\big], \tag{113}$$

The first order conditions of this maximisation problem with respect to N_t^h and K_t^h are:

$$w_t = (1-\mu_h) q_t \frac{I_t^h}{N_t^h}. \tag{114}$$

$$r_t^{K,h} = \mu_h q_t \frac{I_t^h}{K_t^h}. \tag{115}$$

8.1.3 Banks

To solve the optimisation problem of bank's τ, I write it in a Bellman equation form as:

$$V^B_{\tau,t}(NW_{i,t}) = \beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}} \{(1-\sigma)NW_{\tau,t} + \sigma \max V^B_{\tau,t+1}(NW_{\tau,t+1})\}, \tag{116}$$

which is subject to:

$$q^K_t S_{\tau,t} = NW_{\tau,t} + D_{\tau,t},$$
$$NW_{\tau,t+1} = \Big(R^K_{t+1} - R^d_t\Big) S_{\tau,t} + R^d_t NW_{\tau,t},$$
$$V^B_{\tau,t} \geq \zeta(q^k_{t,f} S_{\tau,t}).$$

I start guessing that the value function $V^B_{\tau,t}$ is linear in $NW_{\tau,t}$, $V^B_{\tau,t} = \nu^B_t NW_{\tau,t}$, where ν^B_t depends only on aggregate quantities. Then, I replace the balance sheet in the evolution of the net-worth equation, which then I plug in the Bellman equation. The problem now is to maximise the new Bellman equation subject to the incentive constraint. I re-express bank's i problem using the Lagrangian as:

$$\mathcal{L}_B = \Big[(1-\sigma+\sigma\nu^B_{t+1})\Big(\Big(R^K_{t+1} - R^d_t\Big) q^K_t S_{\tau,t} + R^d_t NW_{\tau,t}\Big)\Big](1+\xi_t) - \xi_t(\zeta(q^K_t S_{\tau,t})), \tag{117}$$

where ξ_t is the Lagrange multiplier with respect to the incentive constraint, and the first order condition with respect to $S_{\tau,t}$ and $NW_{\tau,t}$ are:

$$\frac{\partial \mathcal{L}_B}{\partial S_{\tau,t}} : \frac{\xi_t \zeta}{(1+\xi_t)} = \mathbb{E}_t\Big[(1-\sigma+\sigma\nu^b_{t+1})\Big(R^K_{t+1} - R^d_t\Big)\Big]. \tag{118}$$

$$\frac{\partial \mathcal{L}_B}{\partial NW_{\tau,t}} : \frac{1}{(1+\xi_t)} = \mathbb{E}_t\Big[(1-\sigma+\sigma\nu^b_{t+1})\Big(\frac{R^d_t}{\pi_{t+1}}\Big)\Big]. \tag{119}$$

Defining the adjusted marginal value of the net worth as $\Omega_{\tau,t+1} = (1-\sigma+\sigma\nu^b_{t+1})$, the value function can be re-expressed as:

$$V^B_{\tau,t} = \beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}} \Big\{\Omega_{\tau,t+1}\Big[\Big(R^K_{t+1} - R^d_t\Big) q^K_t S_{\tau,t} + R^d_t NW_{\tau,t}\Big]\Big\}$$

Multiplying and dividing this expression by $NW_{\tau,t}$ I obtain:

$$V^B_{\tau,t} = \beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}} \Big\{\Omega_{\tau,t+1}\Big[\Big(R^K_{t+1} - R^d_t\Big)\phi_t + R^d_t\Big]\Big\} NW_{\tau,t}, \tag{120}$$

where $\phi_t = \frac{q^K_t S_{\tau,t}}{NW_{\tau,t}}$ and the term between curly brackets is ν^b_t. Therefore, if the incentive constraint is binding, i.e. $\nu^b_t = \zeta\phi_t$, replacing the previous result:

$$\beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}} \Big\{ \Omega_{\tau,t+1} \Big[\Big(R^K_{t+1} - R^d_t \Big) \phi_t + R^d_t \Big] \Big\} = \zeta \phi_t.$$

Which, after rearranging, implies that the leverage is equal to:

$$\phi_t = \frac{\beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}} \Omega_{\tau,t+1} R^d_t}{\zeta - \beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}} \Omega_{\tau,t+1} \Big(R^K_{t+1} - R^d_t \Big)}. \tag{121}$$

8.2 Equilibrium conditions

The model is characterised by 47 equations, with 43 endogenous variables $\{\lambda_{p,t}, c_{p,t}, n_{p,t}, h_{p,t}, d^B_{p,t}, d^l_{p,t}, \lambda_{i,t}, c_{i,t}, n_{i,t}, h_{i,t}, l_{i,t}, \mu_{i,t}, I_t, K_t, K^W_t, K^h_t, \lambda_{K,t}, q^K_t, I^h_t, H_t, q_t, r^K_t, r^{K,h}_t, w_t, R^d_t, R^l_t, R^K_t, mc_t, N_t, N^W_t, N^h_t, C_t, Y_t, x_{1,t}, x_{2,t}, \pi_t, \pi^*_t, \nu^j_t, \phi_t, \Omega_t, NW_t, D_t, S_t\}$ and 4 exogenous shocks $\{A_t, Z_t, M_t, \Gamma_t\}$.

8.2.1 Patient Households

$$\lambda_{p,t} = \frac{1}{(c_{p,t} - \gamma c_{p,t-1})} - \frac{\beta_p \gamma}{(c_{p,t+1} - \gamma c_{p,t})}. \tag{122}$$

$$\nu^n_p n^\varphi_{p,t} = w_t \lambda_{p,t}. \tag{123}$$

$$\lambda_{p,t} q_t = \frac{\Gamma_t \nu^h_p}{h_{p,t}} + \beta_p \mathbb{E}_t \Big[(1-\delta_h) q_{t+1} \lambda_{p,t+1} \Big]. \tag{124}$$

$$\lambda_{p,t} = \beta_p \mathbb{E}_t \Big[\lambda_{p,t+1} \frac{R^d_t}{\pi_{t+1}} \Big]. \tag{125}$$

$$\lambda_{p,t} = \beta_p \mathbb{E}_t \Big[\lambda_{p,t+1} \frac{R^l_t}{\pi_{t+1}} \Big]. \tag{126}$$

$$c_{p,t} + q_t[h_{p,t} - (1-\delta_h) h_{p,t-1}] + d^B_t + d^l_t = \frac{d^B_{t-1} R^d_{t-1}}{\pi_t} + \frac{d^l_{t-1} R^l_{t-1}}{\pi_t} + w_t n_{p,t} + \Pi_{f,t} + \Pi_{B,t}. \tag{127}$$

8.2.2 Impatient Households

$$\lambda_{i,t} = \frac{1}{(c_{i,t} - \gamma c_{i,t-1})} - \frac{\beta_i \gamma}{(c_{i,t+1} - \gamma c_{i,t})}. \tag{128}$$

$$\nu^n_i n^\varphi_{i,t} = w_t \lambda_{i,t}. \tag{129}$$

$$\lambda_{i,t} q_t = \frac{\Gamma_t \nu^h_i}{h_{i,t}} + \beta_i \mathbb{E}_t \Big[(1-\delta_h) q_{t+1} \lambda_{i,t+1} \Big] + \mu_{i,t} \frac{\chi}{R^l_t} \mathbb{E}_t[q_{t+1} \pi_{t+1}]. \tag{130}$$

$$\lambda_{i,t} - \mu_{i,t} = \beta_i \mathbb{E}_t \Big[\lambda_{i,t+1} \frac{R^l_t}{\pi_{t+1}} \Big], \tag{131}$$

$$c_{i,t} + q_t(h_{i,t} - (1-\delta_h)h_{i,t-1}) + \frac{l_{t-1}R^l_{t-1}}{\pi_t} = w_t n_{i,t} + l_t. \tag{132}$$

$$l_t \leq \frac{\chi}{R^l_t}\mathbb{E}_t[q_{t+1}\pi_{t+1}]h_{i,t}. \tag{133}$$

8.2.3 Goods firms

$$Y^W_t = A_t {N^W_t}^{1-\alpha} {K^W_{t-1}}^{\alpha}. \tag{134}$$

$$\nu^j_t = (1-\theta)(\pi^*_t)^{-\epsilon} + (\pi_t)^{\epsilon}\theta\nu^j_{t-1}. \tag{135}$$

$$mc_t = \frac{1}{A_t}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r^K_t}{\alpha}\right)^{\alpha}. \tag{136}$$

$$\frac{r^K_t}{w_t} = \frac{\alpha N^W_t}{(1-\alpha)K^W_{t-1}} \tag{137}$$

$$q^K_{t-1}R^K_t = r^K_t + (1-\delta_k)q^K_t. \tag{138}$$

I define two auxiliary variables to re-express pricing as:

$$\pi^*_t = \frac{\epsilon}{\epsilon-1}\frac{x_{1,t}}{x_{2,t}}. \tag{139}$$

These variables have a recursive representation given by:

$$x_{1,t} = \lambda_{p,t} mc_t Y_t + \theta\beta_p\mathbb{E}_t(\pi_{t+1})^{\epsilon}x_{1,t+1}. \tag{140}$$

$$x_{2,t} = \lambda_{p,t} Y_t + \theta\beta_p\mathbb{E}_t(\pi_{t+1})^{\epsilon-1}x_{2,t+1}. \tag{141}$$

$$\pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\pi^*_t\right)^{1-\epsilon}. \tag{142}$$

8.2.4 Housing firms

$$I^h_t = Z_t {N^h_t}^{1-\mu_h} {K^h_{t-1}}^{\mu_h}, \tag{143}$$

$$\frac{r^{K,h}_t}{w_t} = \frac{\mu_h N^h_t}{(1-\mu_h)K^h_{t-1}} \tag{144}$$

8.2.5 Capital firms

$$q^K_t - \beta_p\frac{\lambda_{p,t+1}}{\lambda_{p,t}}(1-\delta_k)q^K_{t+1} = \lambda_{K,t} - \beta_p\frac{\lambda_{p,t+1}}{\lambda_{p,t}}(1-\delta_k)\lambda_{K,t+1}. \tag{145}$$

$$r_t^{K,h} = \lambda_{K,t} - \beta_p \frac{\lambda_{p,t+1}}{\lambda_{p,t}} (1-\delta_k)\lambda_{K,t+1}. \tag{146}$$

$$\frac{1}{\lambda_{K,t}} = 1 - \frac{\psi}{2}\Big(\frac{I_t}{I_{t-1}} - 1\Big)^2 - \psi\Big(\frac{I_t}{I_{t-1}} - 1\Big)\Big(\frac{I_t}{I_{t-1}}\Big) + \beta_p \psi \mathbb{E}_t\Big[\frac{\lambda_{p,t+1}}{\lambda_{p,t}}\lambda_{K,t+1}\Big(\frac{I_{t+1}}{I_t}\Big)^2\Big(\frac{I_{t+1}}{I_t} - 1\Big)\Big]. \tag{147}$$

8.2.6 Banks

$$\Omega_{\tau,t+1} = (1 - \sigma + \sigma\zeta\phi_t). \tag{148}$$

$$q_t^K S_t = \phi_t N W_t. \tag{149}$$

$$NW_t = (\sigma + \omega)(R_t^K q_{t-1}^K S_{t-1}) - \sigma R_{t-1}^d D_{t-1}. \tag{150}$$

$$\phi_t = \frac{\beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}} \Omega_{\tau,t+1} R_t^d}{\zeta - \beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}} \Omega_{\tau,t+1}\Big(R_{t+1}^K - R_t^d\Big)}. \tag{151}$$

$$q_t^K S_t = NW_t + D_t. \tag{152}$$

8.2.7 Central Bank

$$R_t^d = (1/\beta_p)\left(\frac{\pi_t}{\bar{\pi}}\right)^{\omega_\pi}\left(\frac{GDP_t}{GDP_{t-1}}\right)^{\omega_y} M_t, \tag{153}$$

8.2.8 Aggregation

$$C_t = (1-n)c_{p,t} + (n)c_{i,t} \tag{154}$$

$$N_t = (1-n)n_{p,t} + (n)n_{i,t}. \tag{155}$$

$$H_t = (1-n)h_{p,t} + (n)h_{i,t} \tag{156}$$

$$N_t = N_t^W + N_t^h. \tag{157}$$

$$Y_t = \frac{Y_t^W}{\nu_t^j} \tag{158}$$

$$GDP_t = C_t + I_t + \bar{q} I_t^h. \tag{159}$$

$$D_t = (1-n)d_t^B \tag{160}$$

$$(1-n)d_t^l = n l_t \tag{161}$$

$$S_t = K_t^W \tag{162}$$

$$K_t = (1-\delta_k)K_{t-1} + [1 - \frac{\psi}{2}(I_t/I_{t-1} - 1)^2]I_t \tag{163}$$

$$K_t = K_t^W + K_t^h \tag{164}$$

$$I_t^h = H_t - (1-\delta_h)H_{t-1} \tag{165}$$

8.2.9 Shocks

$$log(A_{t+1}) = \rho_A log(A_t) + \sigma_{\epsilon^A}\epsilon_{t+1}^A. \tag{166}$$

$$log(Z_{t+1}) = \rho_Z log(Z_t) + \sigma_{\epsilon^Z}\epsilon_{t+1}^Z. \tag{167}$$

$$log(M_{t+1}) = \rho_M log(M_t) + \sigma_{\epsilon^M}\epsilon_{t+1}^M. \tag{168}$$

$$log(\Gamma_{t+1}) = \rho_\Gamma log(\Gamma_t) + \sigma_{\epsilon^\Gamma}\epsilon_{t+1}^\Gamma. \tag{169}$$

8.3 Steady State

8.3.1 Patient

$$R^d = \frac{1}{\beta_p}. \tag{170}$$

$$\lambda_p = \frac{(1-\beta_p\gamma)}{(1-\gamma)c_p}. \tag{171}$$

$$\nu_p^n n_p^\varphi = w\lambda_p. \tag{172}$$

$$\frac{1}{h_p\lambda_p q} = \frac{1-\beta_p(1-\delta_h)}{\nu_p^h}. \tag{173}$$

8.3.2 Impatient

$$\lambda_i = \frac{(1-\beta_i\gamma)}{(1-\gamma)c_i}. \tag{174}$$

$$\nu_i^n n_i^\varphi = w\lambda_i. \tag{175}$$

$$\mu_i = (1-\beta_i R^l)\lambda_i. \tag{176}$$

$$\frac{1}{h_i\lambda_i q} = \frac{1-\beta_i(1-\delta_h)-(1-\beta_i R^l)\frac{\chi}{R^l}}{\nu_i^h}. \tag{177}$$

$$c_i = wn_i + (1-R^l)l - qh_i\delta_h. \tag{178}$$

$$\frac{lR^l}{\chi} = qh_i. \tag{179}$$

8.3.3 Goods firms

$$\frac{r^K}{w} = \frac{\alpha N^W}{(1-\alpha)K^W}. \tag{180}$$

$$R^K = r^K + (1-\delta_k). \tag{181}$$

$$mc = \left(\frac{w}{1-\alpha}\right)^{1-\alpha}\left(\frac{r^K}{\alpha}\right)^{\alpha}. \tag{182}$$

$$\frac{Y}{K^W} = \left(\frac{N^W}{K^W}\right)^{1-\alpha}. \tag{183}$$

$$x_1 = \frac{\lambda_p mcY}{1-\theta\beta}. \tag{184}$$

$$x_2 = \frac{\lambda_p Y}{1-\theta\beta}. \tag{185}$$

$$\pi^* = \frac{\epsilon}{\epsilon-1}\frac{x_1}{x_2}. \tag{186}$$

$$\pi^{1-\epsilon} = \theta + (1-\theta)\left(\pi^*\right)^{1-\epsilon}. \tag{187}$$

$$\nu^j = \frac{(1-\theta)(\pi^*)^{-\epsilon}}{1-\theta\pi^{\epsilon}}. \tag{188}$$

8.3.4 Housing firms

$$I^h = N^{h^{1-\mu_h}} K^{h^{\mu_h}}. \tag{189}$$

$$\frac{r^{K,h}}{w} = \frac{\mu_h N^h}{(1-\mu_h)K^h}. \tag{190}$$

8.3.5 Capital firms

$$q^K = 1. \tag{191}$$

$$r^{K,h} = 1 - \beta_p(1-\delta_k). \tag{192}$$

$$\frac{1}{\lambda_K} = 1. \tag{193}$$

8.3.6 Banks

$$\Omega = (1-\sigma+\sigma\zeta\phi). \tag{194}$$

$$\frac{NW}{K^W} = \frac{1}{\phi}. \tag{195}$$

$$\frac{NW}{K^W} = 1 - \frac{D}{K^W}. \tag{196}$$

$$\frac{NW}{K^W} = \frac{(\sigma+\omega)R^K - \frac{\sigma}{\beta_p}}{1-\frac{\sigma}{\beta_p}} = \frac{1}{\phi}. \tag{197}$$

$$\phi = \frac{\beta_p \Omega R^d}{\zeta - \beta_p\Omega(R^K - R^d)}. \tag{198}$$

8.3.7 Central Bank

$$R^d = \frac{1}{\beta_p}. \tag{199}$$

8.3.8 Aggregation

$$C = (1-n)c_p + (n)c_i. \tag{200}$$

$$N = (1-n)n_p + (n)n_i. \tag{201}$$

$$H = (1-n)h_p + (n)h_i. \tag{202}$$

$$N = N^f + N^h. \tag{203}$$

$$Y = Y^W. \tag{204}$$

$$GDP = C + I + qI^h. \tag{205}$$

$$D = (1-n)d^B. \tag{206}$$

$$(1-n)d^l = nl. \tag{207}$$

$$S = K^W. \tag{208}$$

$$\delta_K = \frac{I}{K}. \tag{209}$$

$$K = K^W + K^h. \tag{210}$$

$$\delta_h = \frac{I^h}{H}. \tag{211}$$

8.4 Diagnostic probability density function

To obtain the diagnostic probability density function of the economy's state, I use the assumption that it follows an AR(1) process, and that a standard probability density function of a normally distributed variable x_{t+1} is:

$$f(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_{t+1}-\rho x_t)^2}{2\sigma^2}}.$$

Recalling the definition of the diagnostic probability density function under a slow moving reference group $f^{\phi}(x_{t+1}) =$ $f(x_{t+1}|x_t = \rho\bar{x}_t)\left\{\left[\prod_{s=1}^{S} \frac{f(x_{t+1}|\rho^s \bar{x}_{t+1-s})}{f(x_{t+1}|\rho^{s+1}\bar{x}_{t-s})}\right]^{\alpha_s}\right\}^{\phi} Z,$, and using the previous expression:

$$f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_{t+1}-\rho\bar{x}_t)^2}{2\sigma^2}} \left\{\left[\prod_{s=1}^{S} \frac{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_{t+1}-\rho^s \bar{x}_{t+1-s})^2}{2\sigma^2}}}{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_{t+1}-\rho^{s+1}\bar{x}_{t-s})^2}{2\sigma^2}}}\right]^{\alpha_s}\right\}^{\phi} Z, \tag{212}$$

Simplifying and rewriting, I obtain:

$$f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} e^{\left\{-\frac{(x_{t+1}-\rho\bar{x}_t)^2}{2\sigma^2} - \frac{1}{2\sigma^2}\phi\left[\sum_{s=1}^{S}\alpha_s\left((x_{t+1}-\rho^s\bar{x}_{t+1-s})^2-(x_{t+1}-\rho^{s+1}\bar{x}_{t-s})^2\right)\right]\right\}} Z. \quad (213)$$

Let's expand and re-write the argument of the exponential as:

$$\begin{aligned} f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} exp\Bigg(- \frac{1}{2\sigma^2}\Big\{&(x_{t+1}^2 - 2x_{t+1}\rho\bar{x}_t + (\rho\bar{x}_t)^2) + \\ &\phi\Big[\sum_{s=1}^{S}\alpha_s\Big((x_{t+1}^2 - 2x_{t+1}\rho^s\bar{x}_{t+1-s} + (\rho^s\bar{x}_{t+1-s})^2) - \\ &(x_{t+1}^2 - 2x_{t+1}\rho^{s+1}\bar{x}_{t-s} + (\rho^{s+1}\bar{x}_{t-s})^2)\Big)\Big]\Big\}\Bigg) Z. \end{aligned} \quad (214)$$

This can be further expanded and rearranged, after taking $2x$ as common factor:

$$\begin{aligned} f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} exp\Bigg(- \frac{1}{2\sigma^2}\Big\{ x_{t+1}^2 - 2x_{t+1}\left[\rho\bar{x}_t + \phi\left[\sum_{s=1}^{S}\alpha_s\left(\rho^s\bar{x}_{t+1-s} - \rho^{s+1}\bar{x}_{t-s}\right)\right]\right] + \\ (\rho\bar{x}_t)^2 + \phi\left[\sum_{s=1}^{S}\alpha_s\left((\rho^s\bar{x}_{t+1-s})^2 - (\rho^{s+1}\bar{x}_{t-s})^2\right)\right]\Big\}\Bigg) Z, \end{aligned} \quad (215)$$

where the constant Z is given by:

$$\begin{aligned} Z = exp\Bigg(- \frac{1}{2\sigma^2}\Big\{ - \phi\left[\sum_{s=1}^{S}\alpha_s\left((\rho^s\bar{x}_{t+1-s})^2 - (\rho^{s+1}\bar{x}_{t-s})^2\right)\right] + 2\rho\bar{x}_t\phi\left[\sum_{s=1}^{S}\alpha_s\left(\rho^s\bar{x}_{t+1-s} - \rho^{s+1}\bar{x}_{t-s}\right)\right] \\ + \phi^2\left[\sum_{s=1}^{S}\alpha_s\left(\rho^s\bar{x}_{t+1-s} - \rho^{s+1}\bar{x}_{t-s}\right)\right]^2\Big\}\Bigg). \end{aligned} \quad (216)$$

After some algebra, the diagnostic pdf is equal to:

$$f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}\left\{\left[x_{t+1}-\left(\rho\bar{x}_t+\phi\sum_{s=1}^{S}\alpha_s\left(\rho^s\bar{x}_{t+1-s}-\rho^{s+1}\bar{x}_{t-s}\right)\right)\right]^2\right\}}. \quad (217)$$

Which, as Gennaioli and Shleifer (2018) states, contains the kernel of a normal distribution with a distorted mean and the same variance. Therefore:

$$\mathbb{E}_t^{\phi}(x_{t+1}) = \mathbb{E}_t(x_{t+1}) + \phi \sum_{s=1}^{S} \alpha_s \left[\mathbb{E}_{t+1-s}(x_{t+1}) - \mathbb{E}_{t-s}(x_{t+1})\right]. \tag{218}$$

Expression (217) can be extended in order to re-write it in terms of the realisations of the shocks:

$$f^{\phi}(x_{t+1}|x_t) =$$
$$\frac{1}{\sigma\sqrt{2\pi}} exp\Bigg\{ - \frac{1}{2\sigma^2}\Big\{x_{t+1}^2 - 2x_{t+1}\left[\rho\bar{x}_t + \phi \textstyle\sum_{s=1}^{S} \alpha_s \left(\rho^s \bar{x}_{t+1-s} - \rho^{s+1}\bar{x}_{t-s}\right)\right] + (\rho\bar{x}_t)^2 +$$
$$2\rho\bar{x}_t\phi \left[\textstyle\sum_{s=1}^{S} \alpha_s \left(\rho^s \bar{x}_{t+1-s} - \rho^{s+1}\bar{x}_{t-s}\right)\right] + \phi^2 \left[\textstyle\sum_{s=1}^{S} \alpha_s \left(\rho^s \bar{x}_{t+1-s} - \rho^{s+1}\bar{x}_{t-s}\right)\right]^2 \Big\}\Bigg\},$$

which can be rewritten using the AR(1) process definition as:

$$f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} exp\Bigg\{ - \frac{1}{2\sigma^2}\Big\{x_{t+1}^2 - 2x_{t+1}\left[\rho\bar{x}_t + \phi \textstyle\sum_{s=1}^{S} \rho^s \alpha_s \epsilon_{t+s-1}\right] + (\rho\bar{x}_t)^2 +$$
$$2\rho\bar{x}_t\phi \left[\textstyle\sum_{s=1}^{S} \rho^s \alpha_s \epsilon_{t+s-1}\right] + \phi^2 \left[\textstyle\sum_{s=1}^{S} \rho^s \alpha_s \epsilon_{t+s-1}\right]^2 \Bigg\},$$

This can be rearrange as:

$$f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}\left\{\left[x_{t+1} - \left(\rho\bar{x}_t + \phi \sum_{s=1}^{S} \rho^s \alpha_s \epsilon_{t+s-1}\right)\right]^2\right\}}. \tag{219}$$

Again, this function contains the kernel of a normal distribution with a distorted mean:

$$\mathbb{E}_t^{\phi}(x_{t+1}) = \rho x_t + \phi \sum_{s=1}^{S} \rho^s \alpha_s \epsilon_{t+s-1}. \tag{220}$$

This way of modelling DE with slow moving reference embeds the one from Bianchi et al. (2024) as a special case. This occurs when $\alpha_1 = 1$ and the rest are such that $\sum_{s=1}^{S} \alpha_s^{'} = 1$, where $\alpha_s^{'} = (\alpha_s - \alpha_{s+1})$. In that case, the diagnostic expectation will be defined as:

$$\mathbb{E}_t^{\phi}(X_{t+1}) = \mathbb{E}_t(X_{t+1}) + \phi \left[\mathbb{E}_t(X_{t+1}) - \mathbb{E}_t^{r}(X_{t+1})\right], \tag{221}$$

where $\mathbb{E}_t^{r}(X_{t+1}) = \sum_{s=1}^{S} \alpha_s^{'} \mathbb{E}_{t-s}(X_{t+1})$

8.4.1 Diagnostic distribution using last twelve-quarters as reference

Using the previous result, if the Diagnostic agent form expectations taking into account the last twelve-quarters, I obtain the following probability density function:

$$f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} exp\Bigg\{ - \frac{1}{2\sigma^2}\Big\{x_{t+1}^2 - 2x[\rho\bar{x}_t + \phi[\rho\alpha_1\left(\bar{x}_t - \rho\bar{x}_{t-1}\right) + \rho^2\alpha_2\left(\bar{x}_{t-1} - \rho\bar{x}_{t-2}\right) +$$

$$\rho^3\alpha_3\left(\bar{x}_{t-2} - \rho\bar{x}_{t-3}\right) + \rho^4\alpha_4\left(\bar{x}_{t-3} - \rho\bar{x}_{t-4}\right) + \rho^5\alpha_5\left(\bar{x}_{t-4} - \rho\bar{x}_{t-5}\right) + \rho^6\alpha_6\left(\bar{x}_{t-5} - \rho\bar{x}_{t-6}\right) +$$

$$\rho^7\alpha_7\left(\bar{x}_{t-6} - \rho\bar{x}_{t-7}\right) + \rho^8\alpha_8\left(\bar{x}_{t-7} - \rho\bar{x}_{t-8}\right) + \rho^9\alpha_9\left(\bar{x}_{t-8} - \rho\bar{x}_{t-9}\right) + \rho^{10}\alpha_{10}\left(\bar{x}_{t-9} - \rho\bar{x}_{t-10}\right) +$$
$$\rho^{11}\alpha_{11}\left(\bar{x}_{t-10} - \rho\bar{x}_{t-11}\right) + \rho^{12}\alpha_{12}\left(\bar{x}_{t-11} - \rho\bar{x}_{t-12}\right)] + (\rho\bar{x}_t)^2 + 2\rho x_t\phi[\rho\alpha_1\left(\bar{x}_t - \rho\bar{x}_{t-1}\right) +$$

$$\rho^2\alpha_2\left(\bar{x}_{t-1} - \rho\bar{x}_{t-2}\right) + \rho^3\alpha_3\left(\bar{x}_{t-2} - \rho\bar{x}_{t-3}\right) + \rho^4\alpha_4\left(\bar{x}_{t-3} - \rho\bar{x}_{t-4}\right) + \rho^5\alpha_5\left(\bar{x}_{t-4} - \rho\bar{x}_{t-5}\right) +$$

$$\rho^6\alpha_6\left(\bar{x}_{t-5} - \rho\bar{x}_{t-6}\right) + \rho^7\alpha_7\left(\bar{x}_{t-6} - \rho\bar{x}_{t-7}\right) + \rho^8\alpha_8\left(\bar{x}_{t-7} - \rho\bar{x}_{t-8}\right) + \rho^9\alpha_9\left(\bar{x}_{t-8} - \rho\bar{x}_{t-9}\right) +$$
$$\rho^{10}\alpha_{10}\left(\bar{x}_{t-9} - \rho\bar{x}_{t-10}\right) + \rho^{11}\alpha_{11}\left(\bar{x}_{t-10} - \rho\bar{x}_{t-11}\right) + \rho^{12}\alpha_{12}\left(\bar{x}_{t-11} - \rho\bar{x}_{t-12}\right)] +$$

$$\phi^2[\rho\alpha_1\left(\bar{x}_t - \rho\bar{x}_{t-1}\right) + \rho^2\alpha_2\left(\bar{x}_{t-1} - \rho\bar{x}_{t-2}\right) + \rho^3\alpha_3\left(\bar{x}_{t-2} - \rho\bar{x}_{t-3}\right) + \rho^4\alpha_4\left(\bar{x}_{t-3} - \rho\bar{x}_{t-4}\right) +$$
$$\rho^5\alpha_5\left(\bar{x}_{t-4} - \rho\bar{x}_{t-5}\right) + \rho^6\alpha_6\left(\bar{x}_{t-5} - \rho\bar{x}_{t-6}\right) + \rho^7\alpha_7\left(\bar{x}_{t-6} - \rho\bar{x}_{t-7}\right) + \rho^8\alpha_8\left(\bar{x}_{t-7} - \rho\bar{x}_{t-8}\right) +$$
$$\rho^9\alpha_9\left(\bar{x}_{t-8} - \rho\bar{x}_{t-9}\right) + \rho^{10}\alpha_{10}\left(\bar{x}_{t-9} - \rho\bar{x}_{t-10}\right) + \rho^{11}\alpha_{11}\left(\bar{x}_{t-10} - \rho\bar{x}_{t-11}\right) +$$
$$\rho^{12}\alpha_{12}\left(\bar{x}_{t-11} - \rho\bar{x}_{t-12}\right)]^2\Big\}\Bigg\},$$

which implies:

$$\begin{aligned}\mathbb{E}_t^{\phi}(\bar{x}_{t+1}) = \rho\bar{x}_t + \phi[&\rho\alpha_1(\bar{x}_t - \rho\bar{x}_{t-1}) + \rho^2\alpha_2(\bar{x}_{t-1} - \rho\bar{x}_{t-2}) + \rho^3\alpha_3(\bar{x}_{t-2} - \rho\bar{x}_{t-3}) + \rho^4\alpha_4(\bar{x}_{t-3} - \rho\bar{x}_{t-4}) \\ &+ \rho^5\alpha_5(\bar{x}_{t-4} - \rho\bar{x}_{t-5}) + \rho^6\alpha_6(\bar{x}_{t-5} - \rho\bar{x}_{t-6}) + \rho^7\alpha_7(\bar{x}_{t-6} - \rho\bar{x}_{t-7}) + \rho^8\alpha_8(\bar{x}_{t-7} - \rho\bar{x}_{t-8}) \\ &+ \rho^9\alpha_9(\bar{x}_{t-8} - \rho\bar{x}_{t-9}) + \rho^{10}\alpha_{10}(\bar{x}_{t-9} - \rho\bar{x}_{t-10}) + \rho^{11}\alpha_{11}(\bar{x}_{t-10} - \rho\bar{x}_{t-11}) \\ &+ \rho^{12}\alpha_{12}(\bar{x}_{t-11} - \rho\bar{x}_{t-12})].\end{aligned} \tag{222}$$

After using the definition of the AR(1) process:

$$\begin{aligned}\mathbb{E}_t^{\phi}(x_{t+1}) = \rho x_t + \phi[(&\rho\alpha_1\epsilon_t + \rho^2\alpha_2\epsilon_{t-1} + \rho^3\alpha_3\epsilon_{t-2} + \rho^4\alpha_4\epsilon_{t-3} + \rho^5\alpha_5\epsilon_{t-4} + \rho^6\alpha_6\epsilon_{t-5} \\ &+ \rho^7\alpha_7\epsilon_{t-6} + \rho^8\alpha_8\epsilon_{t-7} + \rho^9\alpha_9\epsilon_{t-8} + \rho^{10}\alpha_{10}\epsilon_{t-9} + \rho^{11}\alpha_{11}\epsilon_{t-10} + \rho^{12}\alpha_{12}\epsilon_{t-11})]\end{aligned} \tag{223}$$

Thus, agents wrongly perceive the AR(1) shock as an ARMA(1,12).

8.5 Additional results

8.5.1 Posterior distributions and historical decomposition

This sub-section presents figures showing the posterior distributions from the SMC of the DE model with twelve-quarters reference, DE model with one-quarter reference, and the RE model. It also exhibits the historical decomposition for the RE model and the DE model with twelve-quarters reference.

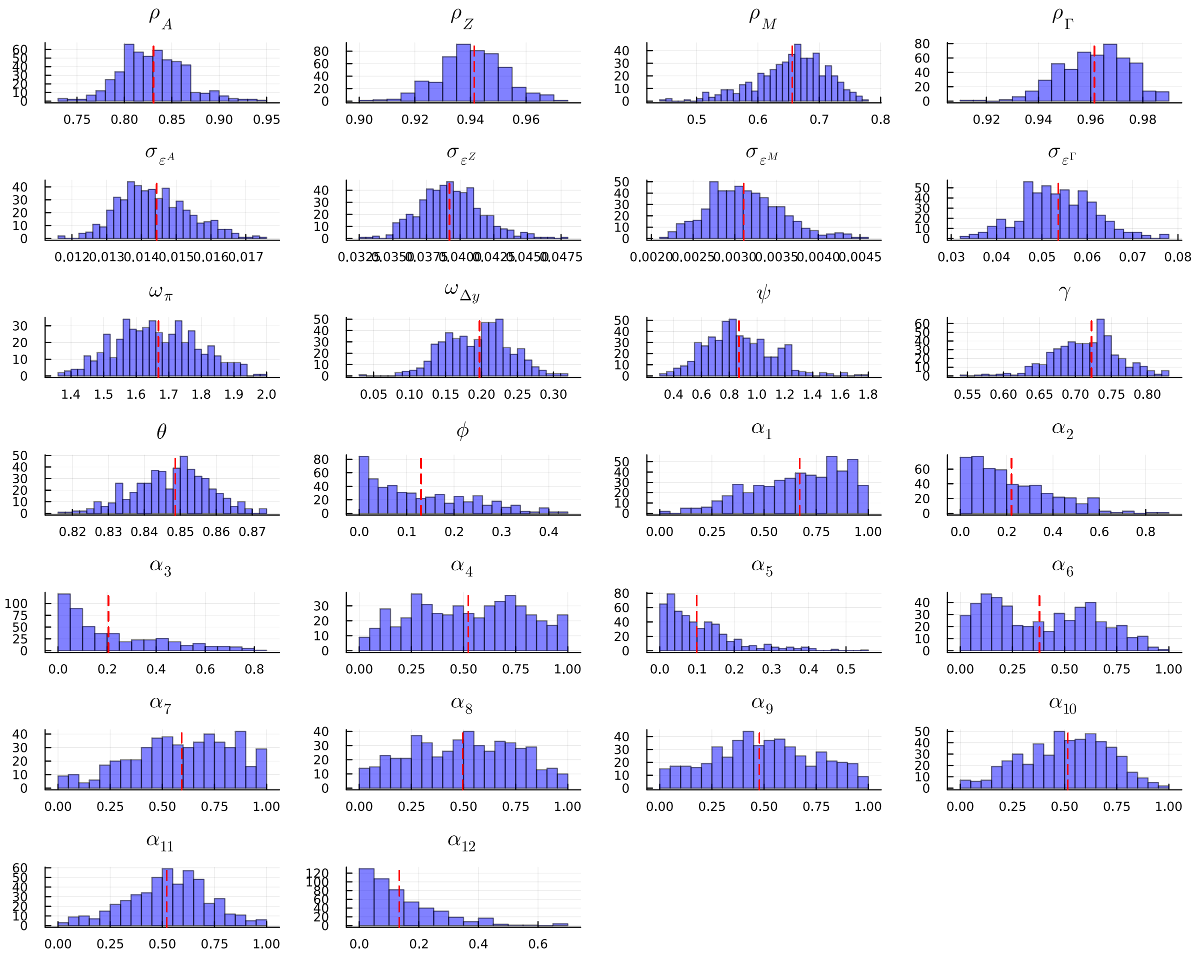


Figure 9: Posterior distributions diagnostic expectations ref: Q12.
Note: The red dashed line represents the mean of each posterior distribution.

8.6 Definition of data variables

I calibrate the model using quarterly data for the United States. I obtained the data from the Board of Governors of the Federal Reserve System and the Bureau of Economic Analysis, using the National Accounts and Flow of Funds. I also use the Census Bureau House Price Index. The sample period begins in 1984:Q1 and ends in 2019:Q4, i.e. pre-pandemic. The variables I use are:

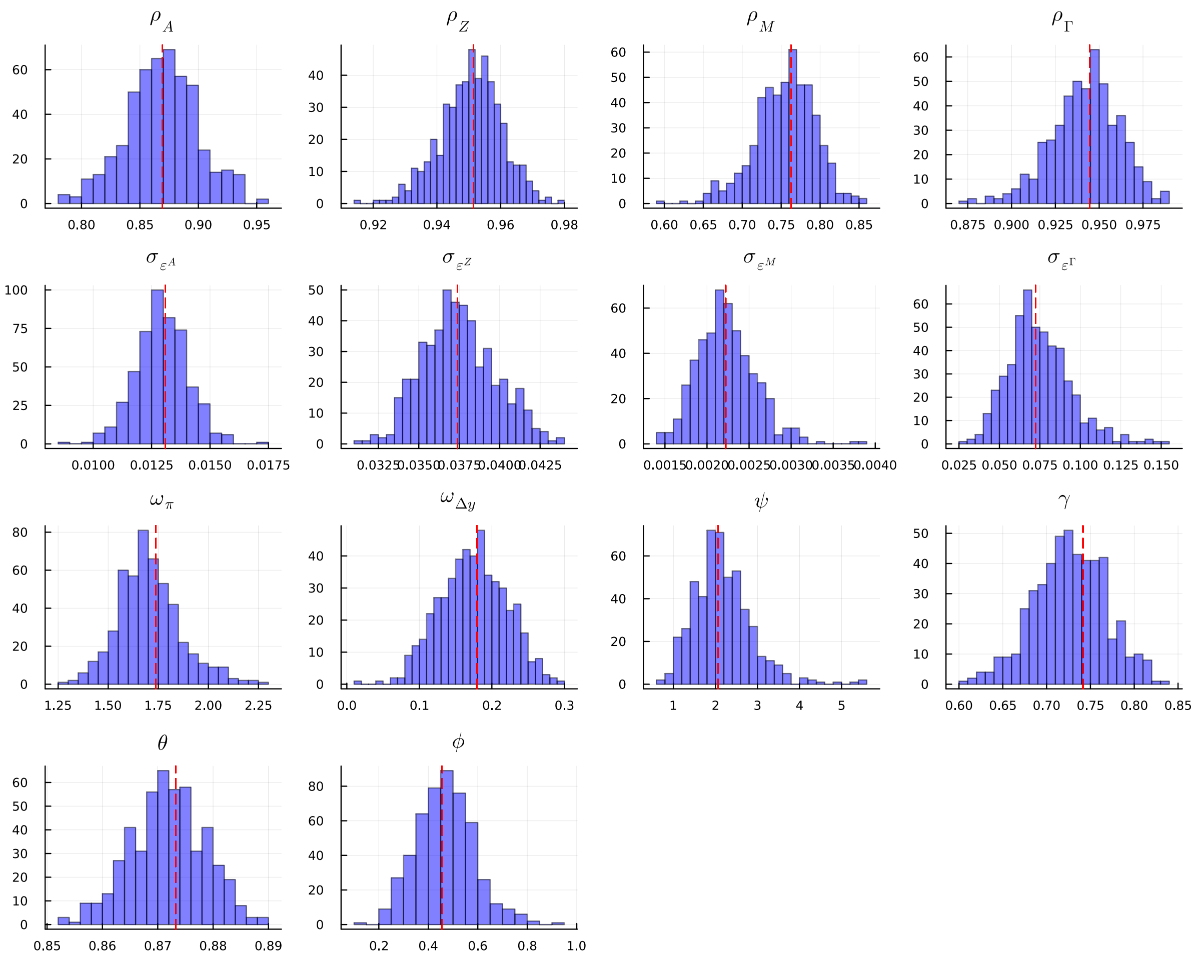


Figure 10: Posterior distributions diagnostic expectations ref: Q1.
Note: The red dashed line represents the mean of each posterior distribution.

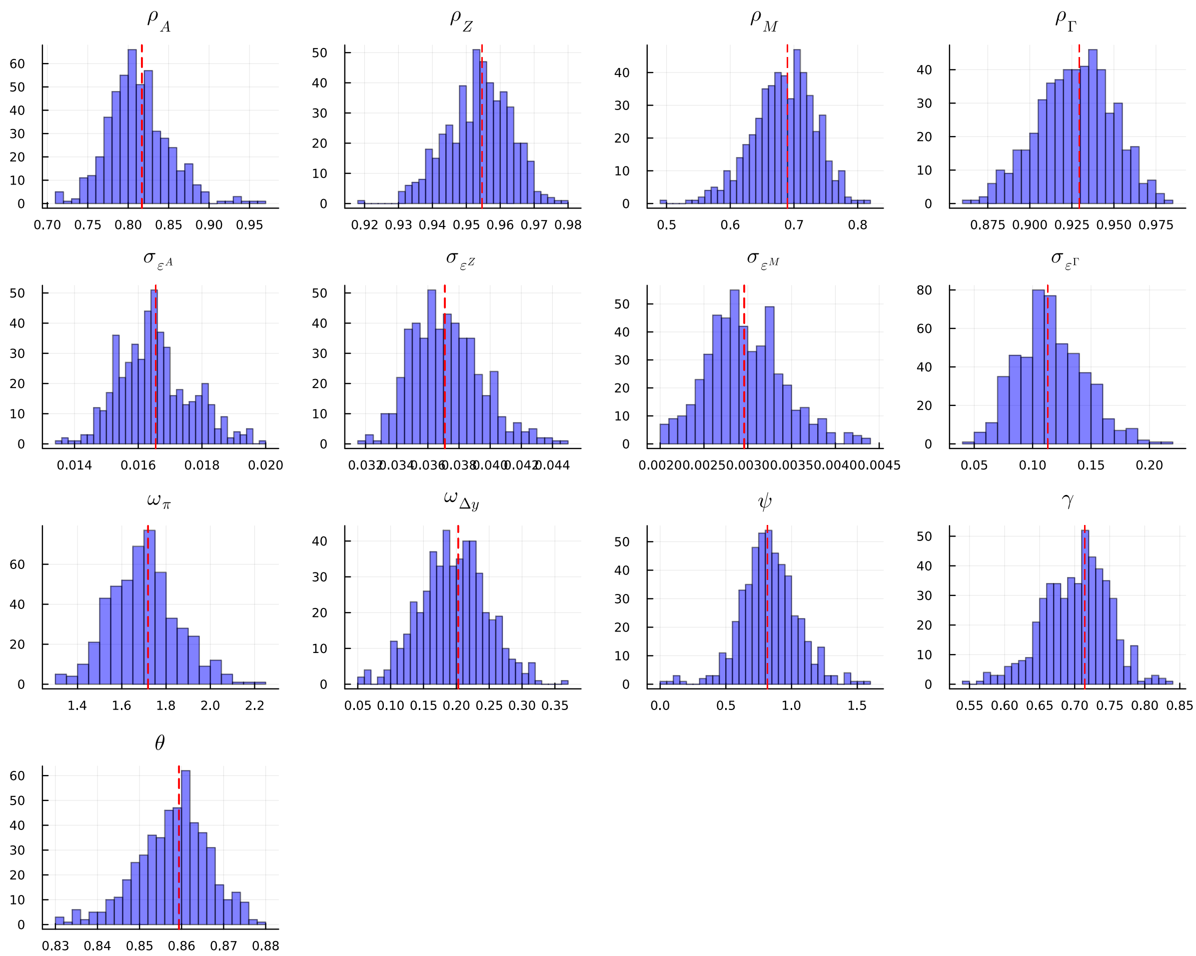


Figure 11: Posterior distributions rational expectations.
Note: The red dashed line represents the mean of each posterior distribution.

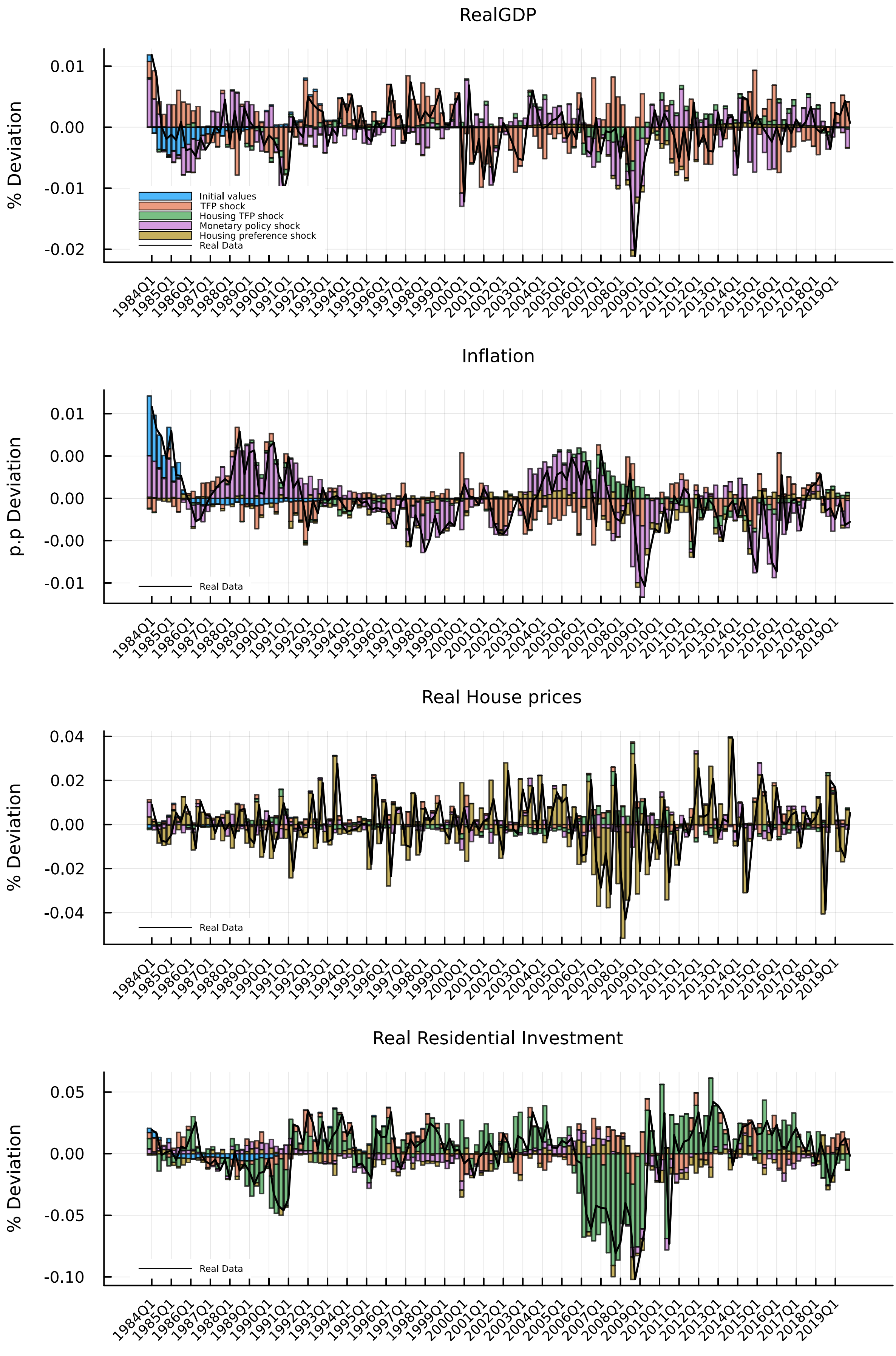


Figure 12: Historical shock decomposition under RE model.

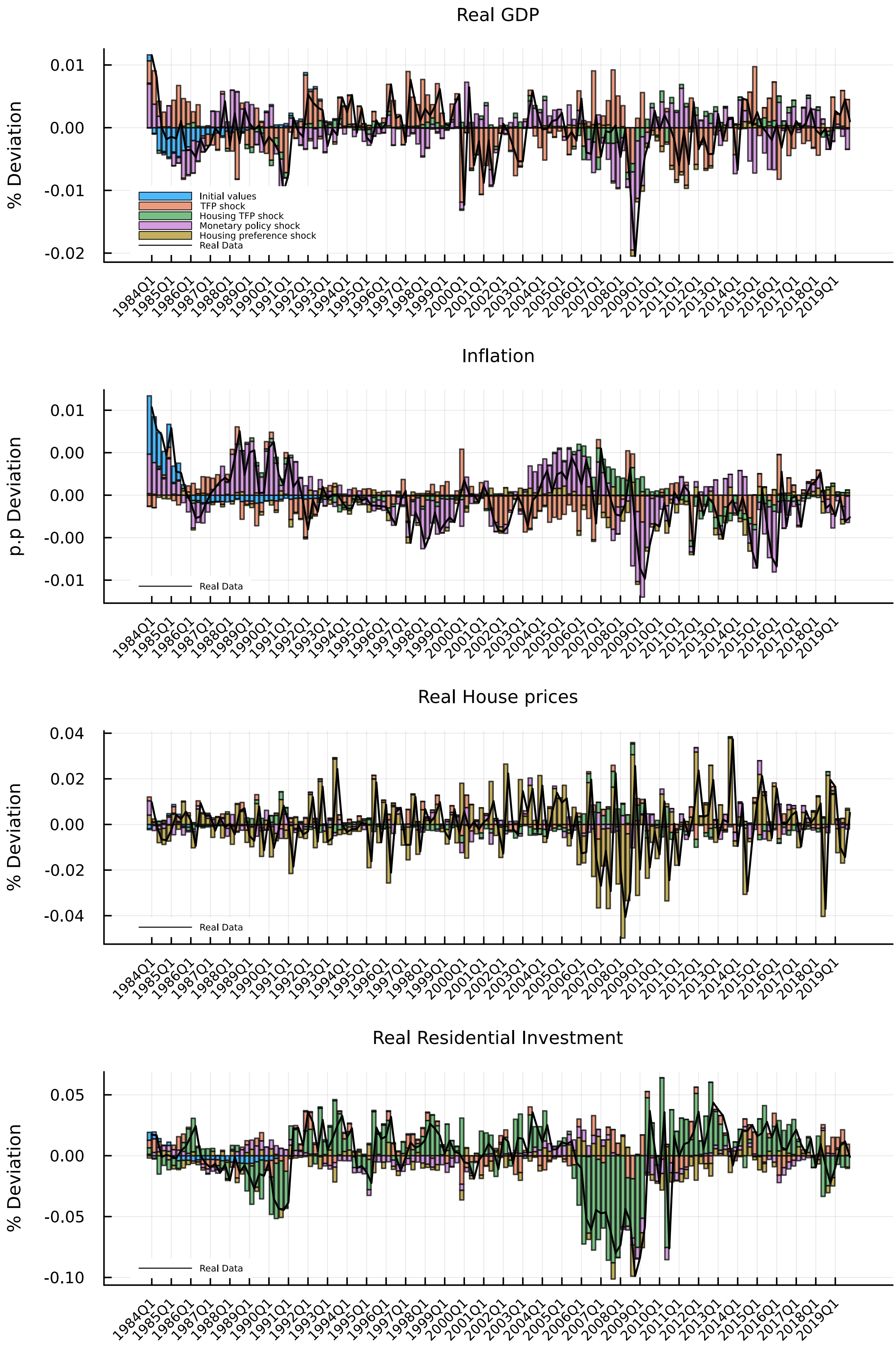


Figure 13: Historical shock decomposition under DE model with twelve-quarters reference.

Output

Data: Real Gross Domestic Product (Billions of chained 2012 Dollars, seasonally adjusted annual rate). The series is adjusted by the civilian non-institutional population. The result is log transformed, detrended using the first difference and demeaned. Source: Board of Governors of the Federal Reserve System.

Inflation

Data: Implicit Price Deflator (Index 2012 = 100, seasonally adjusted annual rate). The series is in quarter-on-quarter log differences and demeaned. Source: Board of Governors of the Federal Reserve System.

Nominal short-term interest rate

Data: Federal funds rate. Quarterly average of the monthly series. During the zero lower bound period, the Wu-Xia shadow federal funds rate is used. Source: Board of Governors of the Federal Reserve System and Wu and Xia (2016).

House prices

Data: Census Bureau House Price Index (Index 2012 = 100, quarterly new one-family houses sold including value of lot). This series is deflated using the Implicit Price Deflator. The result is log-transformed, detrended using the first difference, and also demeaned. Source: Census Bureau.

Loans to households

Data: Households and Non-profit Organisations, one-to-four-family residential mortgages (Billions of Dollars, seasonally adjusted), and Households and Non-profit Organisations, Consumer credit (Billions of Dollars, seasonally adjusted). The total amount of loans to households equals the sum of the two series, which is adjusted by the population level and converted in real terms using the Implicit Price Deflator. The result is log-transformed, detrended using the first difference and also demeaned. Source: Bureau of Economic Analysis.

Nonresidential investment

Data: Private Nonresidential Fixed Investment (Billions of Dollars, seasonally adjusted annual rate). The series is adjusted by the population level and converted in real terms using the Implicit Price Deflator. The result is log-transformed, detrended using the first difference and also demeaned. Source: Board of Governors of the Federal Reserve System.

Residential investment

Data: Private Residential Fixed Investment (Billions of Dollars, seasonally adjusted annual rate). The series is adjusted by the population level and converted in real terms using the Implicit Price Deflator. The result is log-transformed, detrended using the first difference and also demeaned. Source: Board of Governors of the Federal Reserve System.

Housing wealth

Data: Households and Non-profit Organisations, Real Estate at Market Value (Billions of Dollars, not seasonally adjusted). The series is adjusted by the population level and converted in real terms using the Implicit Price Deflator. The result is log-transformed, detrended using the first difference and also demeaned. Source: Bureau of Economic Analysis.

Population level

Thousands of Persons. Quarterly average of the monthly series. Not seasonally adjusted. I transformed this series into an index as in Smets and Wouters (2007) but with base year 2012:3.