COMPUTATIONAL MODEL OF HUMAN BEHAVIOR IN SECURITY GAMES WITH VARYING NUMBER OF TARGETS

by

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Abbreviations

ARMOR:	Assistant for Randomized Monitoring Over Routes
COBRA:	Combined OBservability and Rationality Assumption
DOBSS:	Decomposed Optimal Bayesian Stackelberg Solver
FAMS:	Federal Air Marshal Service
GUARDS:	Game theoretic Unpredictable And Randomly Deployed Security
IRIS:	Intelligent Randomization In Scheduling
MILP:	Mixed Integer Linear Program
MLE:	Maximum Likelihood Estimator
No.:	Number
PT:	Prospect Theory
QRE:	Quantal Response Equilibrium
TSA:	Transportation Security Administration

Abstract

Security is one of the biggest concerns all around the world. There is only limited number of resources that can be allocated in security coverage. Terrorists can exploit any pattern of monitoring deployed by the security personals. It becomes important to make the security pattern unpredictable and randomized. In such a scenario, the security forces can be randomized using algorithms based on Stackelberg games.

Stackelberg games have recently gained significant importance in deployment for real world security. Game-theoretic techniques make standard assumption that adversaries' actions are perfectly rational. It is a challenging task to account for human behavior in such circumstances.

What becomes more challenging in applying game-theoretic techniques to real-world security problem is the standard assumption that the adversary is perfectly rational in responding to security force's strategy, which can be unrealistic for human adversaries. Different models in the form of PT, PT-Attract, COBRA, DOBSS and QRE have already been proposed to address the scenario in settings with fixed number of targets. My work focuses on the evaluation of these models when the number of targets is varied, giving rise to an entirely new problem set.

Introduction

It is a well-known fact that the game-theoretic approaches make an assumption of perfect rationality, which induces errors in the prediction of human behavior for multi-agent decision making problems [3, 5]. Various models are being developed and studied in order to account for the variations in human behavior from the initial assumption of perfect rationality. Behavioral game theory and cognitive science are both fully devoted to this domain. Multi-agent systems community has shown growing interest in adopting these models for decision-making and providing advice to human decision-makers [6, 26]. There has been profound work in improving the computational models of human behavior, especially in the field of security games [27].

In Stackelberg games, one player, the leader, commits to a strategy publicly before the remaining players, the followers, make their decision [7]. These types of commitments are necessary for security agents in a number of domains, pertaining to attacker-defender scenarios [1, 2, 11, 15] and Stackelberg games are well-suited to appropriately model these commitments [14, 16].

Game-theoretic models are now being used for analyzing real-world security resource allocation problems [8, 20]. These models provide enough complexity to the system so as to not allow attackers to find any patterns in the allocation of security. ARMOR [16], IRIS [22] and GUARDS [19] are the best examples of the systems that use this approach for allocation of security in the real-world domain. In the past, such systems were designed under the assumption that the attackers are perfectly rational and would only work on maximizing their own benefit, from the system. Such a system would work best against a very intelligent attacker who can calculate his own proxemics in the best way possible. However, this is not true in all the cases. The attackers may be prone to human errors that the system may not be robust enough to handle. There has always been a need to design a system that can not only account for the human errors but also exploit it in the best way possible.

The challenges in moving beyond rationality have been very significant. Different models handle these challenges in different ways. There is, however, very little consensus on what model suits which domain, and if one model can outperform all the others. Thus, it becomes a very important research question to analyze all these models and determine which is more suited to one setting or the other. Another important problem that needs to be addressed in this respect is to compute the best strategy based on complex mathematical equations. In this context, some of the models have proven themselves to be more effective than the others.

Beside addressing the issue of perfect rationality and determining the best model that can account for irrational human behavior, the current works address the different domains individually. ARMOR uses DOBSS for randomizing the checkpoints on the roadways entering the airport and canine patrol routes within the airport terminals at Los Angeles International airport. IRIS is used in limited international sectors by FAMS. GUARDS is being used by TSA for scheduling airport security operations. It is noteworthy that in general, these domains have specific number of targets that the adversaries can attack.

My work further explains the effectiveness of these models and analyzes their performance in the cases where the attackers are encountered with settings pertaining to varying number of targets.

My analysis has been carried out based on the data collected via the online game *The Guards and The Treasure* designed to simulate different security scenarios. The payoff structures used in the games are based on classification techniques that ascertain separation of models from each-other spanning the game space as widely as possible.

Chapter One: Stackelberg Games

In a Stackelberg game, a leader first commits to a strategy followed by the follower, who can selfishly optimize his reward based on the action chosen by the leader. It is noteworthy, that in some game settings, the leader actually has an advantage of making the first move. Considering a simple example of Stackelberg game, the payoff structure may be shown as in Table 1.

	x	**y**
a	2, 1	4, 0
b	1, 0	3, 2

Table 1: Sample Stackelberg game

In this case, the leader's utilities are represented in the rows corresponding to *a* and *b* while the follower's utilities are represented in the columns corresponding to *x* and *y*. The pure-strategy Nash equilibrium for this game is denoted by the action <a, x>. When the leader plays *a*, the follower plays *x*. This gives leader, a pay-off of 2. In this case, the strategy *b* is strictly dominated by *a*. However, when we view this game as a Stackelberg game, the leader has a choice of committing to strategy *b*. In this case, the follower would respond by choosing strategy *y*, as this would give follower a reward of 2. Thus, committing to a strategy actually increases the reward of leader from 2 to 3. In addition to this, the leader can actually choose to play a mixed strategy of playing *a* and *b* with probabilities 0.5 each. In this case, the leader would fetch a utility of 3.5.

When expanding such settings to security games, the assumption of rationality implies that the defender makes its decision assuming that if it chooses *b*, the attacker would respond by choosing *d*. Such an assumption induces errors when the adversary happens to be irrational in its behavior. Another aspect of the game is that the follower may not be able to observe the applied strategy before making its choice. If the defender can determine the attackers' responses to such mixed-strategies, it still remains a hard question to design optimal strategy against such adversaries.

Chapter Two: Related Work

This work is motivated by the various algorithms developed to compute optimal defender strategies in Stackelberg games. These algorithms have been designed taking into consideration the different aspects of human decision making such as risk/loss aversion, non-linear weighing of probabilities and bounded rationality.

Prospect Theory

Prospect Theory (PT) describes a simple form of game involving alternatives that involve risk where the probabilities are known. The model tries to model real-life choices rather than optimal decisions. This theory describes the decision making process as maximization of the *prospect* which in general, has close proximity to what is referred as *expected utility* by other models. The theory describes how individuals evaluate potential losses and gains. The decision-making is based on the editing of the choices to determine what humans equate as similar outcomes followed by the evaluation of the choices obtained through the calculation of utility value. The general principles of probability do not hold good as the numbers are perceived lower than what they actually signify. An empirical form of the function is shown in Figure 1.

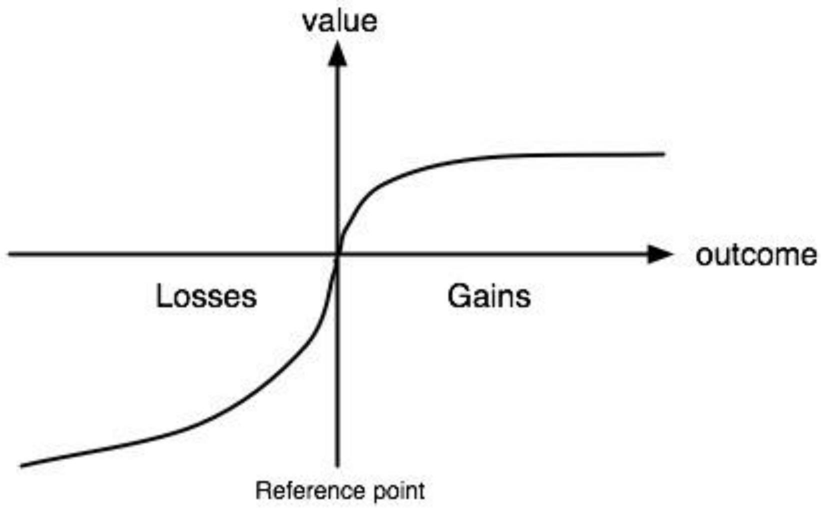

Figure 1: Empirical function for Prospect Theory

Any outcome which is lower than the reference point is considered to be a loss, while the one higher than the reference point is considered to be gain. There are many different forms of the value function which are reflexive and loss-averse [10].

PT-Attract

PT-Attract is a model proposed by Yang et al [27], which is a linear program based on PT, designed to improve the PT model by introducing new parameters addressing more number of constraints. The imprecision of certainty for human adversaries to make the optimal choices can affect the expected outcome of a defender strategy based on PT model. These losses can be very fatal when considering the real world security domains. In order to compensate for these losses, PT-Attract model introduces a new constraint that accounts for all the Ɛ-optimal strategies corresponding to any particular choice of the defender. It turns out that the defender reward as obtained by the PT model is smaller than the defender reward for the Ɛ-optimal adversarial strategies.

COBRA

COBRA is one of the leading works to account for human behavior [17]. This model is supported by experimental evidence over the human subjects. Whether or not, this model is as fast as some of the other models is questionable. This model defines the adversaries' behavior from two different prospects of bounded rationality on computing the optimal strategy and anchoring bias caused by attackers' inability to get the precise observation of mixed strategy that the defender is using for the purpose of security allocation. The calculations here are based on Mixed Integer Linear Program (MILP) with variables to account for selections of Ɛ-optimal adversarial strategies as in the case of PT-Attract model. The major problem with this model is that finding an optimal solution in a Bayesian Stackelberg game is NP-hard [4], thus COBRA happens to be a MILP facing an NP-hard problem. Alpha (α) parameter is varied to account for the limited number of observations that a human adversary may be able to use. The value α can vary from 0 to 1 to denote the ignorance prior while $1 - \alpha$ would denote the occurrences viewed by the adversaries. Thus, the value $\alpha = 0$ would mean that the adversaries have fully observed the system while $\alpha = 1$ would mean that the adversaries are completely ignorant of the system. In my experiments, the α parameter has been set to 0.15 and 0.5.

DOBSS

DOBSS happens to be the one of the most efficient general Stackelberg solver [14]. It is in currently being used in the ARMOR software that is deployed for security scheduling at the Los Angeles International airport. It directly uses the Bayesian representations for calculations and thus, does not require the normal-form Bayesian game to go through the Harsanyi transformation [9]. This gives a very significant speedup over the other multiple linear program methods [4]. It uses the property that instead of carrying out the Harsanyi transformation, the same result can be achieved by evaluating the payoffs against all the game matrices one-by-one and then carrying out a weighted sum.

QRE

Quantal Response Equilibrium (QRE) acts as a base for many different types of studies [21, 25]. It states that the response of individuals in Stackelberg games is quantal, and the chance of making an error increases as the cost of such an error decreases [13]. Thus, it does consider that the very costly errors are unlikely to occur. It is noteworthy that QRE, in general, can return results that are significantly different than Nash equilibrium. The equilibrium condition is based on the realization of the beliefs of the opponent player. In the base model, every strategy is used with nonzero probability. In modeling the error of the players, lambda (λ) parameter is used. $\lambda = 0$ denotes that the players can play any pure strategy with equal probability while $\lambda = \infty$ denotes that the quantal response is same as the best response. λ is very sensitive when it comes to the payoff strategies thus, a very crucial step is to determine the exact value of λ for deploying into the QRE model. My experiments are based on λ values of 0.45 and 0.76, pre-determined by Maximum Likelihood Estimator (MLE) used in Yang et al [27].

Chapter Three: Payoff Structure Classification

In order to justify the results, it becomes critically important to account for the entire state-space of the payoff structures. It is also important that the strategies generated from different models should be separated from each other. In order to span the entire state space, a very large number of payoff structures are randomly generated, as in Table 2.

No. of gates	No. of randomly generated payoff structures
3	1000
6	1000
9	1000
12	100
15	100

Table 2: Count of randomly generated payoff structures

The range of the rewards and the penalties are same for both the defender as well as the attacker. The rewards in each of these cases vary from 1 to 10, while the penalties vary from -1 to -10. This is in consistency with the previous work carried out in this domain [17, 27].

In order to separate the different models from each other, the randomly generated payoff structures are divided into four different groups using k-means clustering. There are 8 different features extracted in the same way as in Yang et al [27]. These features denote the adversaries' utilities, the defenders' utilities and the level of gain of one player over the other.

The difference of mixed strategies is calculated using Kullback-Leibler divergence [12], which gives a non-symmetric measure of the difference between the various payoff structures. Being a premetric, it generates a topology on the space of generalized distributions. Thereafter, from each group one payoff structure is selected such that it is closest to the group center.

Chapter Four: Experimental Setup

The tests on human participants were conducted via an online game *The Guards and The Treasure* designed specifically for this purpose. The interface of the game is shown in Figure 2.

Figure 2: *The Guards and The Treasure* game interface

The experiments were designed to study the human behavior and their perception of the different payoff structures in the best way possible. This is to account for the standard perfect rationality assumption that the various game models would otherwise make.

The role of leader is followed by the guard while the role of follower is followed by the attacker. The mixed strategy is presented to the attackers in the form of probabilities. For clarity we present both the probability of the presence of a guard as well as the probability of the absence of the guard. The leader strategies are decided on the basis of the various game models discussed earlier.

Each participant plays 35 rounds in all. Each of the rounds corresponds to one of the 5 gate settings and one of the 7 guard allocation strategies generated by the different game models. The various gate settings are as follows:

- 3 gates and 1 guard
- 6 gates and 2 guards
- 9 gates and 3 guards
- 12 gates and 4 guards
- 15 gates and 7 guards

The various guard allocation strategies are as follows:

- PT
- PT-Attract
- COBRA ($\alpha = 0.15$)

- COBRA ($\alpha = 0.5$)
- DOBSS
- QRE ($\lambda = 0.45$)
- QRE ($\lambda = 0.76$)

In each of the round, the participant gets to attack one of the gates. In each of the rounds, exactly one-third of the gates are protected by the guards based on the displayed probability distribution, marked as *Probability of Guard* in Figure 2). An attack is considered to be successful if the attacked gate was not being protected by a guard. An attack is considered to be a failure if the attacked gate was being protected by a guard. For a successful attack the player receives points corresponding to *Your Rewards* in Figure 2, while for a failure the player loses points corresponding to *Your Penalties* in Figure 2.

As described in Chapter Three: Payoff Structure Classification, there are four different payoff structures corresponding to four different groups decided using k-means clustering. Each of the participant plays only of these groups, but plays against all the payoff structures of the group. The order of the games is randomized to ensure that there is no learning and that any bias due to ordering of the games can be removed. Also, the intermediate results of the rounds are not presented to the participants. This is to ensure that their choices are not affected by the points they have already earned.

Each subject begins with base money of \$8.00. Thereafter, for every point scored, the participant wins \$0.10, while for every point lost, the participant loses \$0.10. The monetary data is included in Table 3.

Cluster	No. of Participants	Highest	Lowest	Average
1	21	\$22.20	\$5.80	\$12.03
2	25	\$23.00	\$12.50	\$17.65
3	23	\$22.70	\$12.20	\$17.79
4	22	\$25.30	\$11.80	\$19.38
Total	**91**	**\$25.30**	**\$5.80**	**\$16.81**

Table 3: Money paid to the participants

In the experimental setup, it was also ensured that the data is only recorded once the participant has played all the rounds of the cluster, in which he is participating.

Chapter Five: Experimental Results

In total, 91 participants played the game. The participation based on cluster is given in Table 3.

In order to evaluate the performances of different game models, defenders' utilities are calculated based on the participants' choices. These calculations are based on the following formula:

$$U(x) = \frac{1}{N} . \sum_{i=1}^{k} (N_i . U_i(x))$$

where, $U(x)$ denotes the average expected utility of the defender corresponding to the given number of gates in the specified cluster,

N is the total number of participants, who played that cluster,

k are the number of different gates chosen by the participant,

$U_i(x)$ is the defender's expected utility for gate i given x as the mixed strategy

Results Based on Clusters

The average expected utilities against no. of gates for various clusters are illustrated in Tables 4 to 7 and Figures 3 to 6.

Game Model	3 Gates	6 Gates	9 Gates	12 Gates	15 Gates
PT	-4.35	-1.122	-0.194	-1.949	0.29
PT-Attract	-4.57	-1.094	0.296	-0.03	-1.061
COBRA (Alpha = 0.15)	-4.113	-1.137	-0.651	-1.559	-0.162
COBRA (Alpha =0.5)	-3.965	-1.491	0.284	-1.801	-0.551
DOBSS	-4.408	-2.267	-0.342	-1.72	-0.3
QRE (Lambda = 0.45)	-3.511	-1.243	-0.32	-1.886	-1.561
QRE (Lambda = 0.76)	-3.696	-1.274	-0.999	-2.756	-0.663

Table 4: Average Expected Utilities of Game Models for Cluster 1

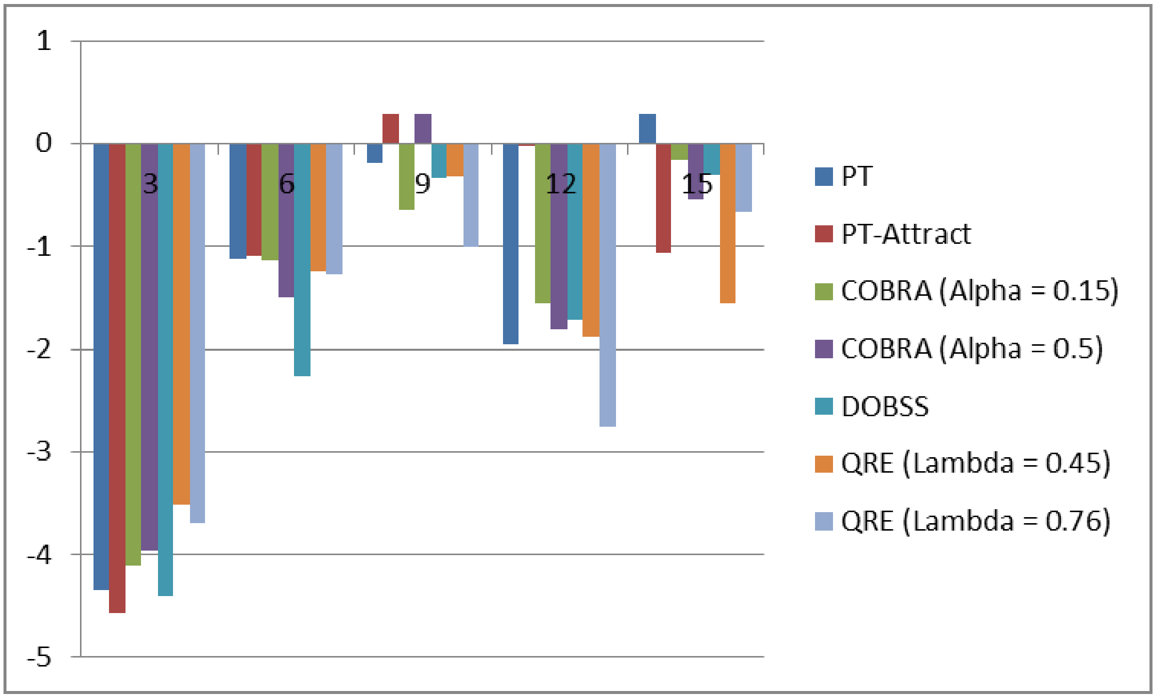

Figure 3: *Average Expected Utilities* against *No. of Gates* for Cluster 1

Game Model	3 Gates	6 Gates	9 Gates	12 Gates	15 Gates
PT	-0.405	-1.946	1.337	1.042	0.83
PT-Attract	-0.295	-0.036	-1.031	-1.084	-0.685
COBRA (Alpha = 0.15)	-0.476	-5.28	-0.768	-1.68	-0.263
COBRA (Alpha =0.5)	-0.593	-4.186	-0.546	-1.536	-0.385
DOBSS	-0.574	-3.225	-2.397	0.1	0.183
QRE (Lambda = 0.45)	-0.91	-0.242	-1.58	-1.215	-1.044
QRE (Lambda = 0.76)	-0.814	0.069	-2.167	-1.515	-0.339

Table 5: Average Expected Utilities of Game Models for Cluster 2

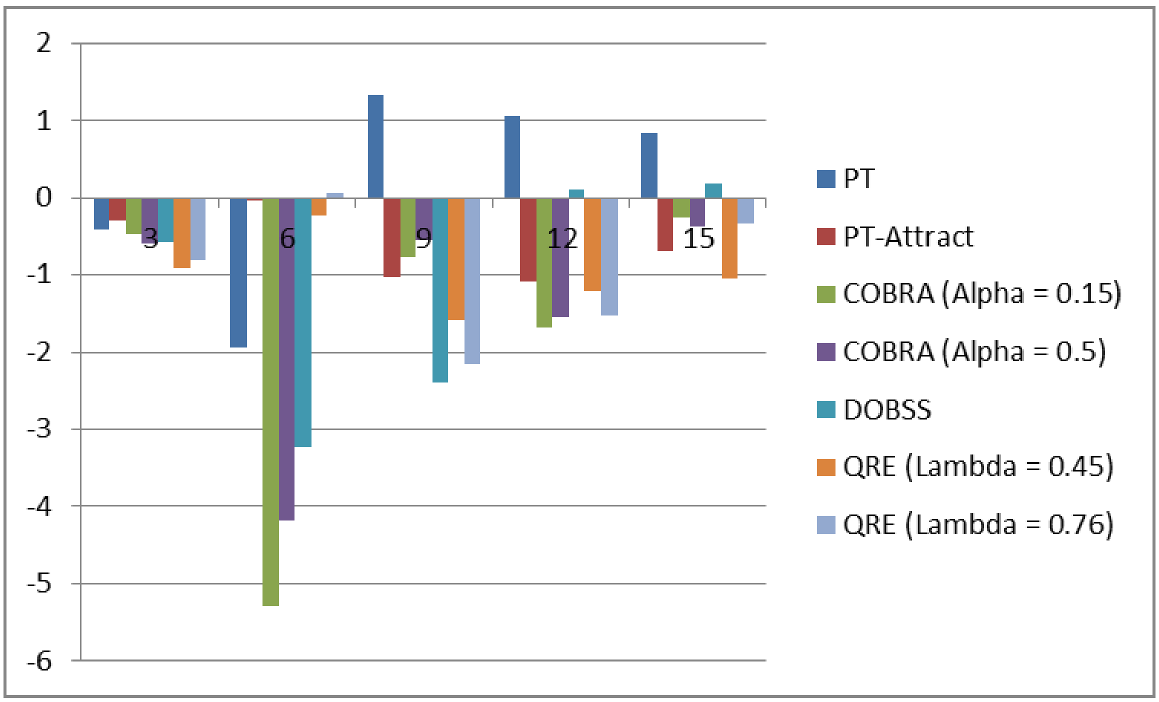

Figure 4: *Average Expected Utilities* against *No. of Gates* for Cluster 2

Game Model	3 Gates	6 Gates	9 Gates	12 Gates	15 Gates
PT	-2.523	-2.283	-2.713	-2.479	-3.341
PT-Attract	-3.536	-0.311	-1.918	-2.911	-1.374
COBRA (Alpha = 0.15)	-5.333	-1.149	-3.339	-2.606	-2.006
COBRA (Alpha =0.5)	-5.64	-0.193	-3.39	-2.754	-3.953
DOBSS	-4.462	-0.699	-1.618	-2.224	-2.446
QRE (Lambda = 0.45)	-3.909	-0.932	-2.123	-2.578	-1.712
QRE (Lambda = 0.76)	-4.213	-0.903	-2.594	-2.425	-2.021

Table 6: Average Expected Utilities of Game Models for Cluster 3

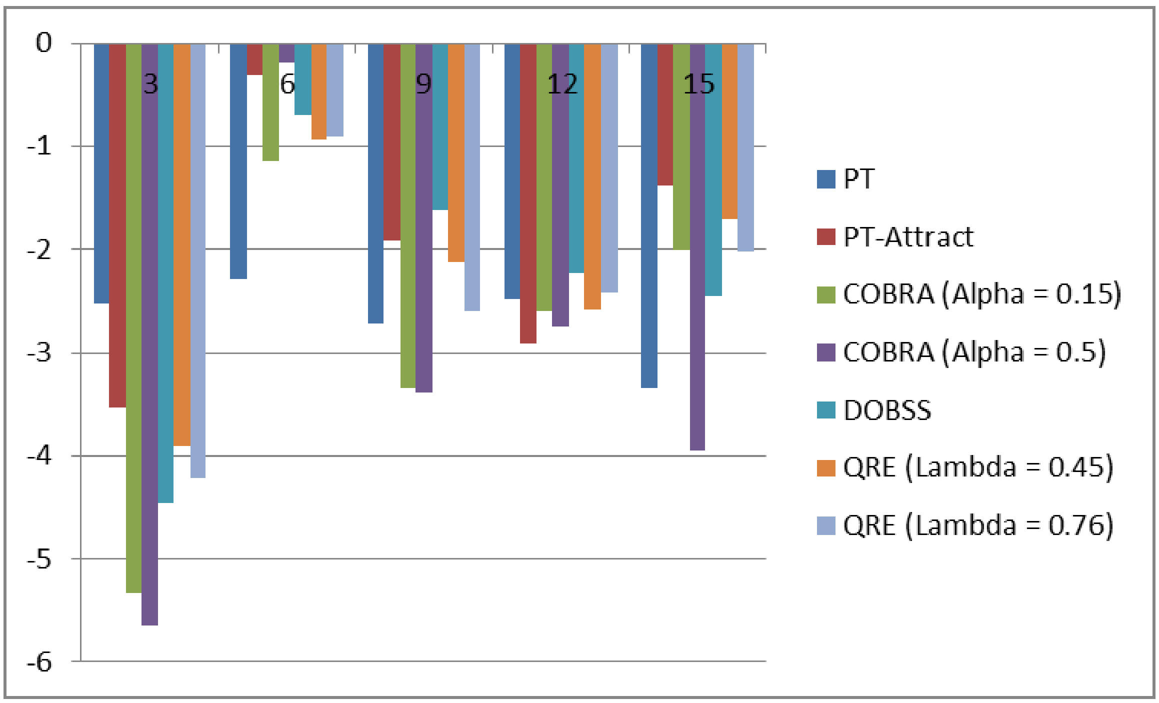

Figure 5: *Average Expected Utilities* against *No. of Gates* for Cluster 3

Game Model	3 Gates	6 Gates	9 Gates	12 Gates	15 Gates
PT	1.093	-2.341	-3.541	-2.956	-0.906
PT-Attract	1.412	-2.54	-2.99	-2.391	-1.062
COBRA (Alpha = 0.15)	1.999	-1.844	-1.31	-2.967	-0.224
COBRA (Alpha =0.5)	2.134	-2.091	-2.017	-4.419	0.104
DOBSS	2.198	-3.025	-2.697	-3.848	-0.586
QRE (Lambda = 0.45)	1.68	-3.164	-1.857	-2.858	-0.515
QRE (Lambda = 0.76)	1.433	-3.359	-2.344	-3.328	-0.241

Table 7: Average Expected Utilities of Game Models for Cluster 4

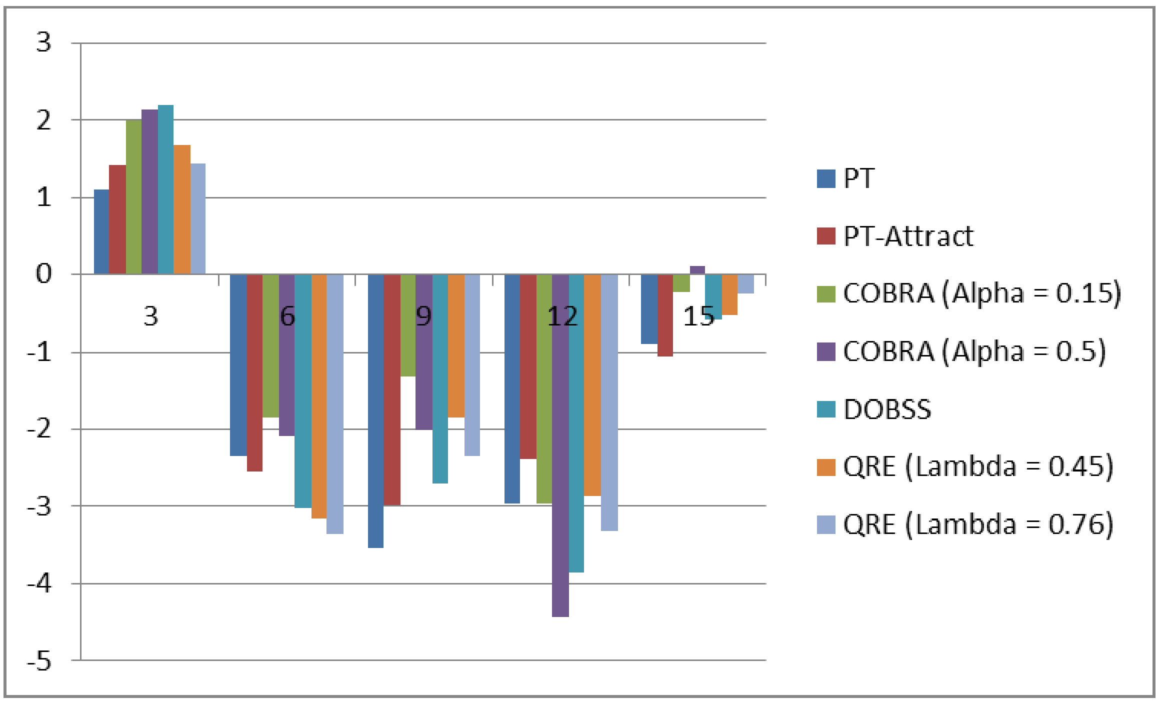

Figure 6: *Average Expected Utilities* against *No. of Gates* for Cluster 4

Results Based on Total Number of Gates

The average expected utilities against various clusters for different number of total gates are illustrated in Tables 8 to 12 and Figures 7 to 11.

Game Model	Cluster 1	Cluster 2	Cluster 3	Cluster 4
PT	-4.35	-0.405	-2.523	1.093
PT-Attract	-4.57	-0.295	-3.536	1.412
COBRA (Alpha = 0.15)	-4.113	-0.476	-5.333	1.999
COBRA (Alpha =0.5)	-3.965	-0.593	-5.64	2.134
DOBSS	-4.408	-0.574	-4.462	2.198
QRE (Lambda = 0.45)	-3.511	-0.91	-3.909	1.68
QRE (Lambda = 0.76)	-3.696	-0.814	-4.213	1.433

Table 8: Average Expected Utilities of Game Models for 3 Gates

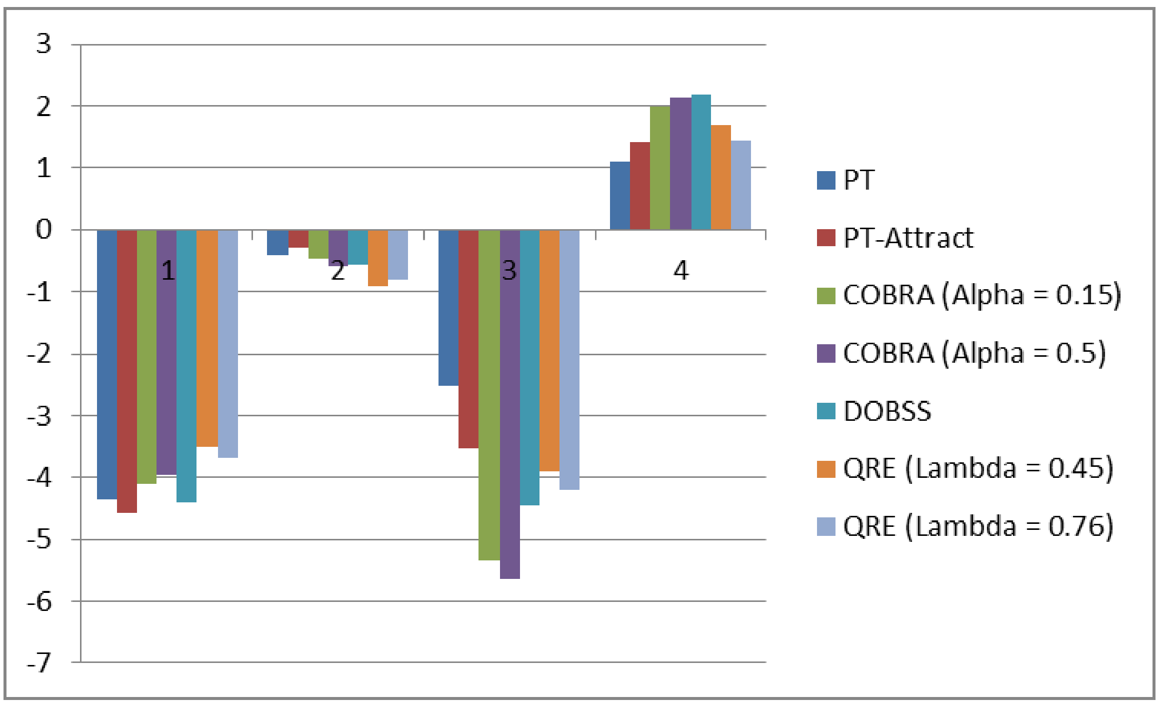

Figure 7: *Average Expected Utilities* against *Various Clusters* for 3 Gates

Game Model	Cluster 1	Cluster 2	Cluster 3	Cluster 4
PT	-1.122	-1.946	-2.283	-2.341
PT-Attract	-1.094	-0.036	-0.311	-2.54
COBRA (Alpha = 0.15)	-1.137	-5.28	-1.149	-1.844
COBRA (Alpha =0.5)	-1.491	-4.186	-0.193	-2.091
DOBSS	-2.267	-3.225	-0.699	-3.025
QRE (Lambda = 0.45)	-1.243	-0.242	-0.932	-3.164
QRE (Lambda = 0.76)	-1.274	0.069	-0.903	-3.359

Table 9: Average Expected Utilities of Game Models for 6 Gates

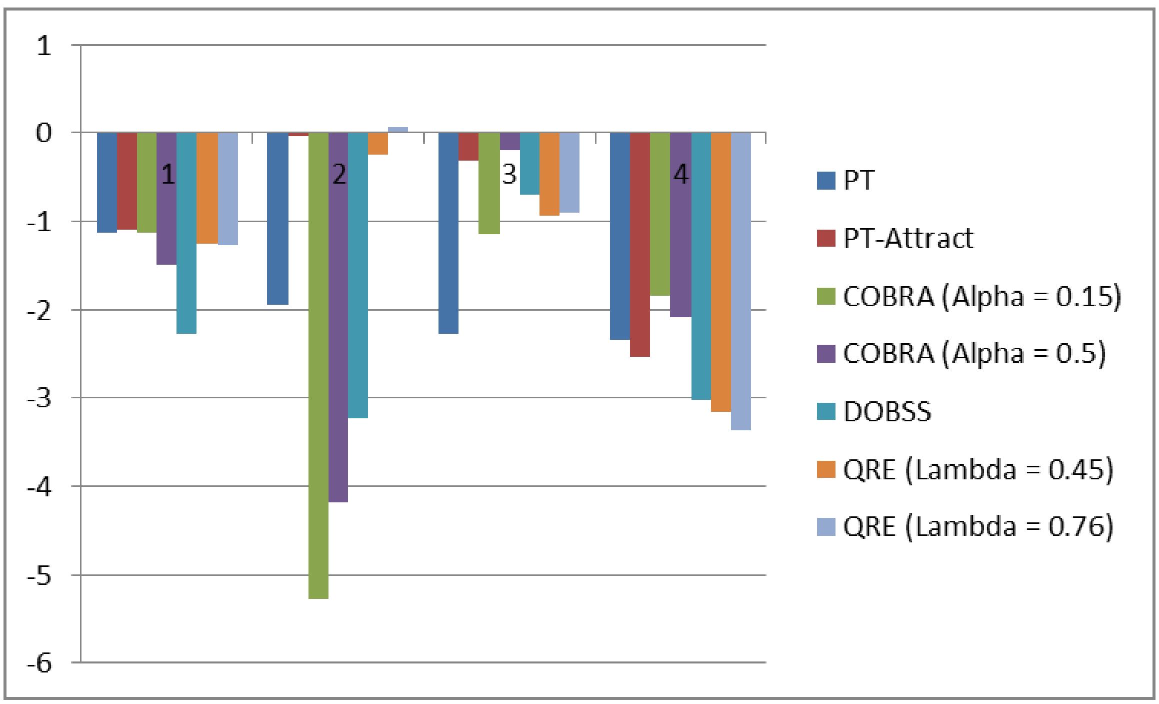

Figure 8: *Average Expected Utilities* against *Various Clusters* for 6 Gates

Game Model	Cluster 1	Cluster 2	Cluster 3	Cluster 4
PT	-0.194	1.337	-2.713	-3.541
PT-Attract	0.296	-1.031	-1.918	-2.99
COBRA (Alpha = 0.15)	-0.651	-0.768	-3.339	-1.31
COBRA (Alpha =0.5)	0.284	-0.546	-3.39	-2.017
DOBSS	-0.342	-2.397	-1.618	-2.697
QRE (Lambda = 0.45)	-0.32	-1.58	-2.123	-1.857
QRE (Lambda = 0.76)	-0.999	-2.167	-2.594	-2.344

Table 10: Average Expected Utilities of Game Models for 9 Gates

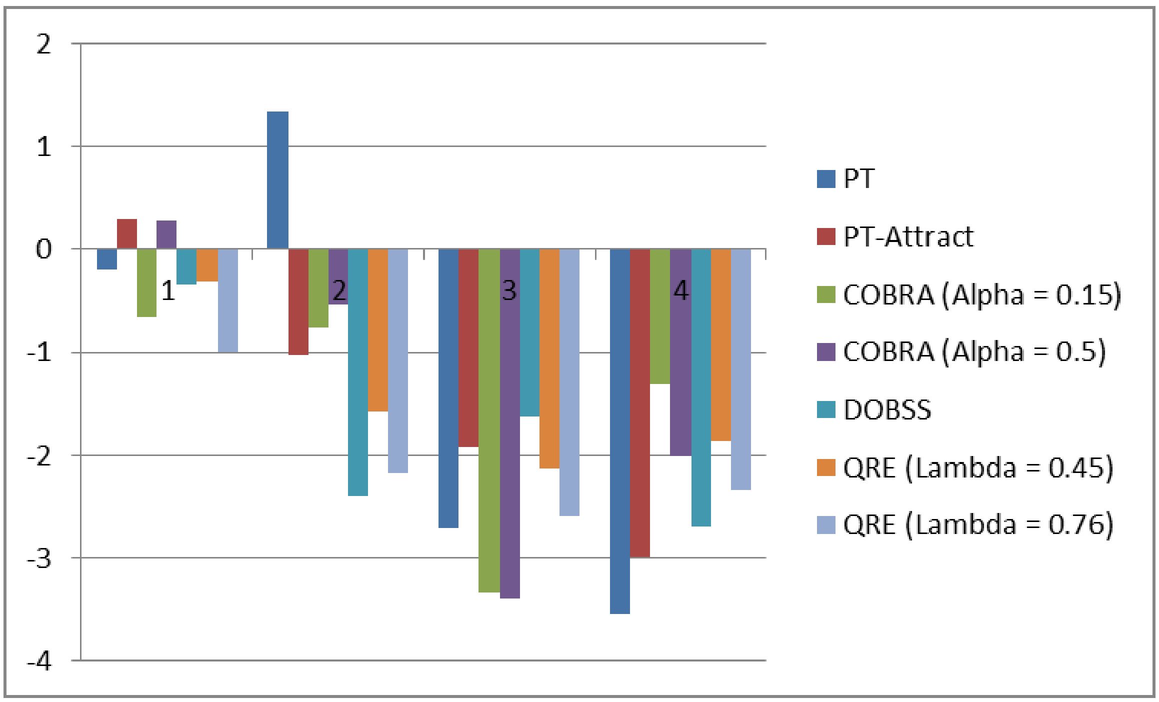

Figure 9: *Average Expected Utilities* against *Various Clusters* for 9 Gates

Game Model	Cluster 1	Cluster 2	Cluster 3	Cluster 4
PT	-1.949	1.042	-2.479	-2.956
PT-Attract	-0.03	-1.084	-2.911	-2.391
COBRA (Alpha = 0.15)	-1.559	-1.68	-2.606	-2.967
COBRA (Alpha =0.5)	-1.801	-1.536	-2.754	-4.419
DOBSS	-1.72	0.1	-2.224	-3.848
QRE (Lambda = 0.45)	-1.886	-1.215	-2.578	-2.858
QRE (Lambda = 0.76)	-2.756	-1.515	-2.425	-3.328

Table 11: Average Expected Utilities of Game Models for 12 Gates

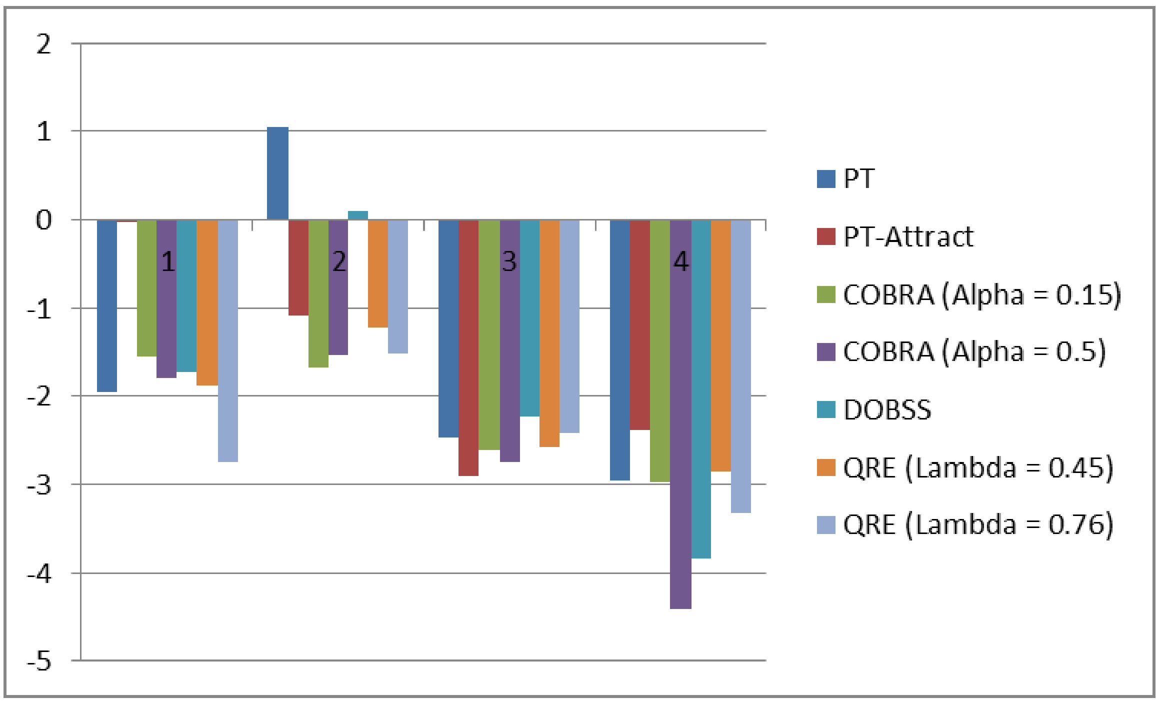

Figure 10: *Average Expected Utilities* against *Various Clusters* for 12 Gates

Game Model	Cluster 1	Cluster 2	Cluster 3	Cluster 4
PT	0.29	0.83	-3.341	-0.906
PT-Attract	-1.061	-0.685	-1.374	-1.062
COBRA (Alpha = 0.15)	-0.162	-0.263	-2.006	-0.224
COBRA (Alpha =0.5)	-0.551	-0.385	-3.953	0.104
DOBSS	-0.3	0.183	-2.446	-0.586
QRE (Lambda = 0.45)	-1.561	-1.044	-1.712	-0.515
QRE (Lambda = 0.76)	-0.663	-0.339	-2.021	-0.241

Table 12: Average Expected Utilities of Game Models for 15 Gates

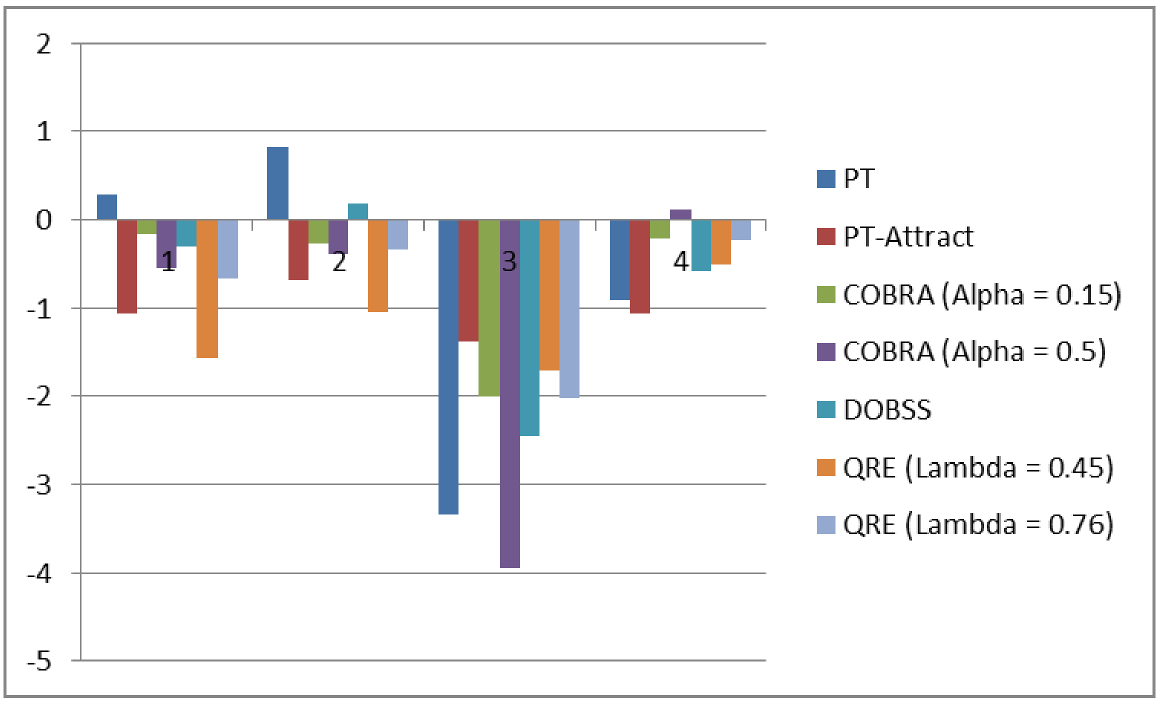

Figure 11: *Average Expected Utilities* against *Various Clusters* for 15 Gates

Results Based on Game Models

On the basis of the different game models, the average expected utilities against varying gates and clusters are illustrated in Tables 13 to 19 and Figures 12 to 18.

	3 Gates	6 Gates	9 Gates	12 Gates	15 Gates
Cluster 1	-4.35	-1.122	-0.194	-1.949	0.29
Cluster 2	-0.405	-1.946	1.337	1.042	0.83
Cluster 3	-2.523	-2.283	-2.713	-2.479	-3.341
Cluster 4	1.093	-2.341	-3.541	-2.956	-0.906

Table 13: Average Expected Utilities for PT Model

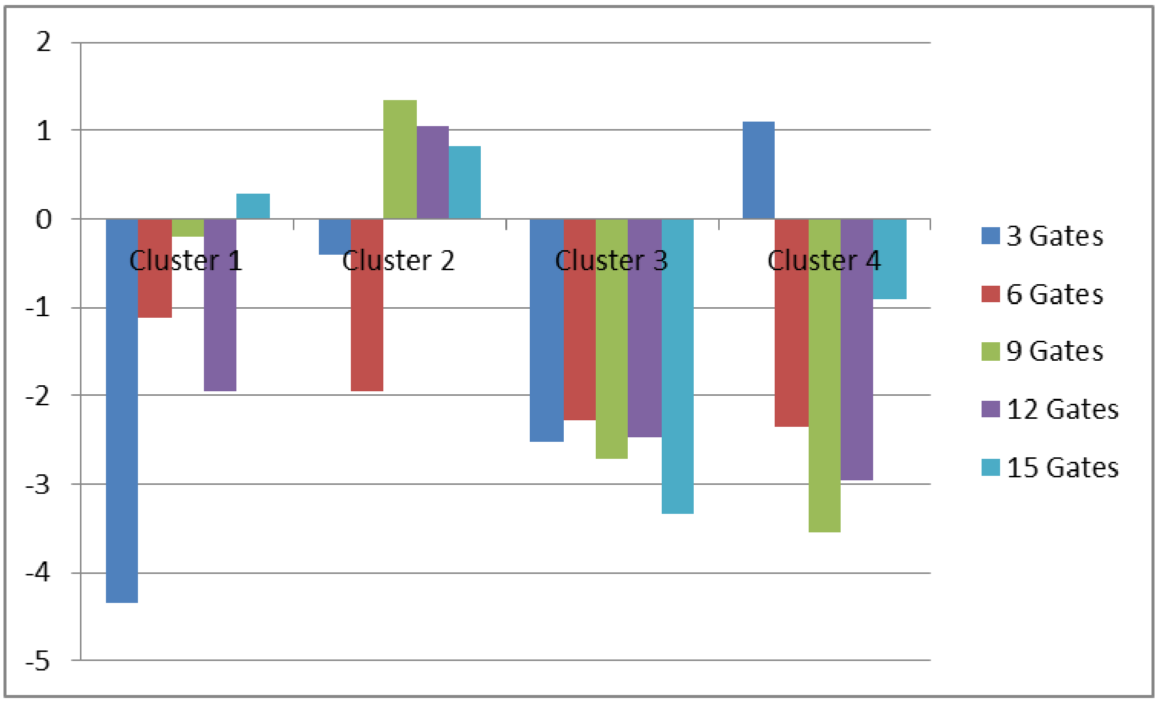

Figure 12: Average Expected Utilities for PT Model

	3 Gates	6 Gates	9 Gates	12 Gates	15 Gates
Cluster 1	-4.57	-1.094	0.296	-0.03	-1.061
Cluster 2	-0.295	-0.036	-1.031	-1.084	-0.685
Cluster 3	-3.536	-0.311	-1.918	-2.911	-1.374
Cluster 4	1.412	-2.54	-2.99	-2.391	-1.062

Table 14: Average Expected Utilities for PT-Attract Model

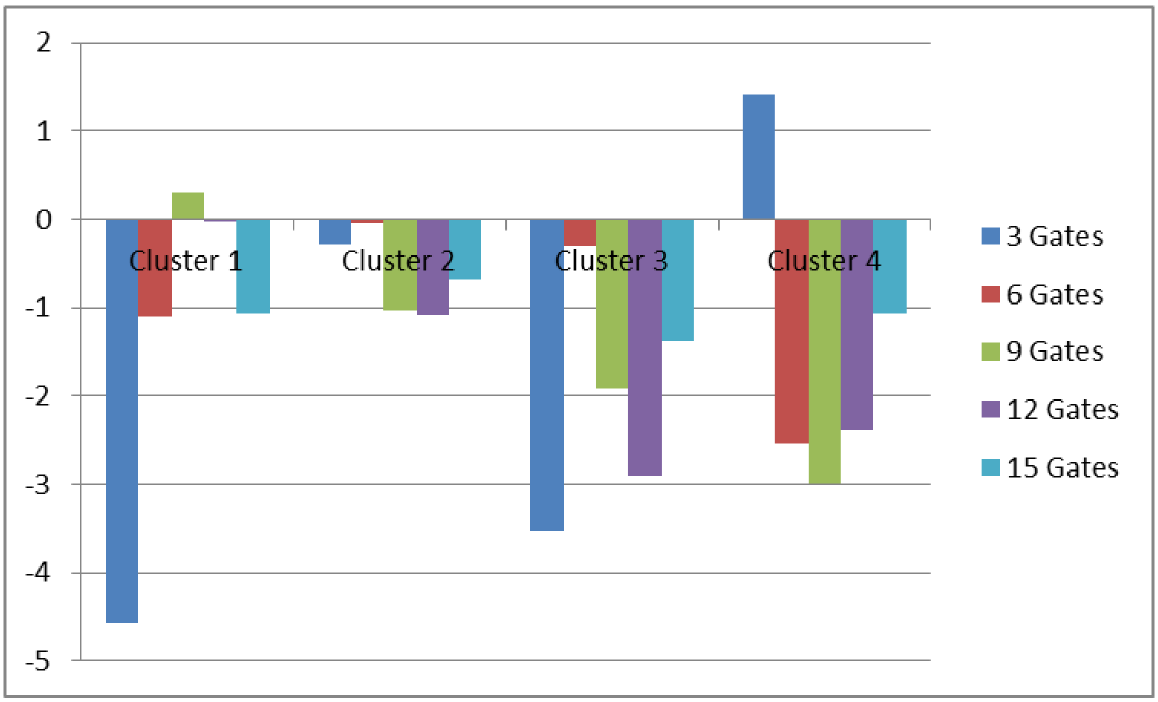

Figure 13: Average Expected Utilities for PT-Attract Model

	3 Gates	6 Gates	9 Gates	12 Gates	15 Gates
Cluster 1	-4.113	-1.137	-0.651	-1.559	-0.162
Cluster 2	-0.476	-5.28	-0.768	-1.68	-0.263
Cluster 3	-5.333	-1.149	-3.339	-2.606	-2.006
Cluster 4	1.999	-1.844	-1.31	-2.967	-0.224

Table 15: Average Expected Utilities for COBRA (α = 0.15) Model

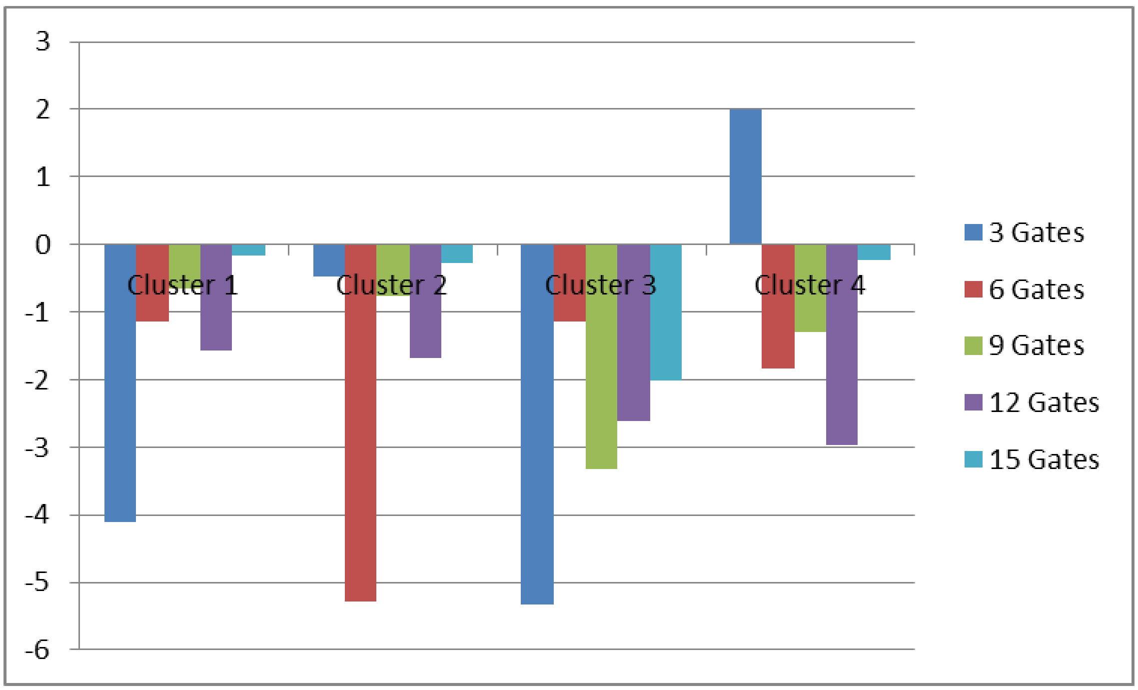

Figure 14: Average Expected Utilities for COBRA (α = 0.15) Model

	3 Gates	6 Gates	9 Gates	12 Gates	15 Gates
Cluster 1	-3.965	-1.491	0.284	-1.801	-0.551
Cluster 2	-0.593	-4.186	-0.546	-1.536	-0.385
Cluster 3	-5.64	-0.193	-3.39	-2.754	-3.953
Cluster 4	2.134	-2.091	-2.017	-4.419	0.104

Table 16: Average Expected Utilities for COBRA (α = 0.5) Model

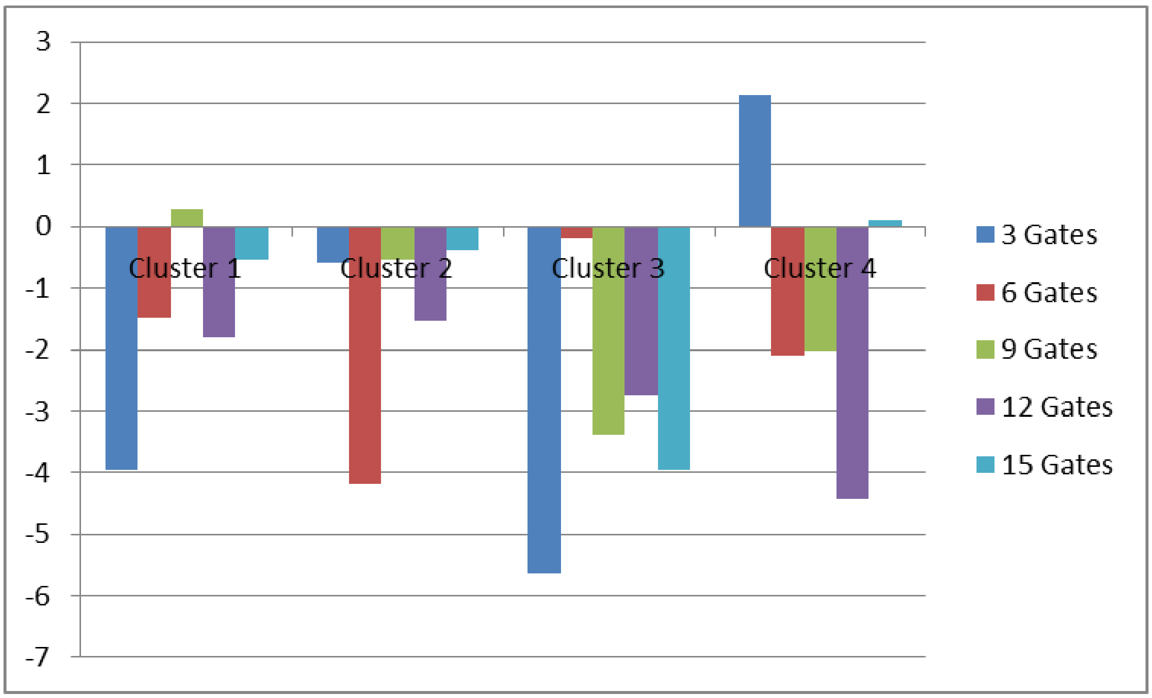

Figure 15: Average Expected Utilities for COBRA (α = 0.5) Model

	3 Gates	6 Gates	9 Gates	12 Gates	15 Gates
Cluster 1	-4.408	-2.267	-0.342	-1.72	-0.3
Cluster 2	-0.574	-3.225	-2.397	0.1	0.183
Cluster 3	-4.462	-0.699	-1.618	-2.224	-2.446
Cluster 4	2.198	-3.025	-2.697	-3.848	-0.586

Table 17: Average Expected Utilities for DOBSS Model

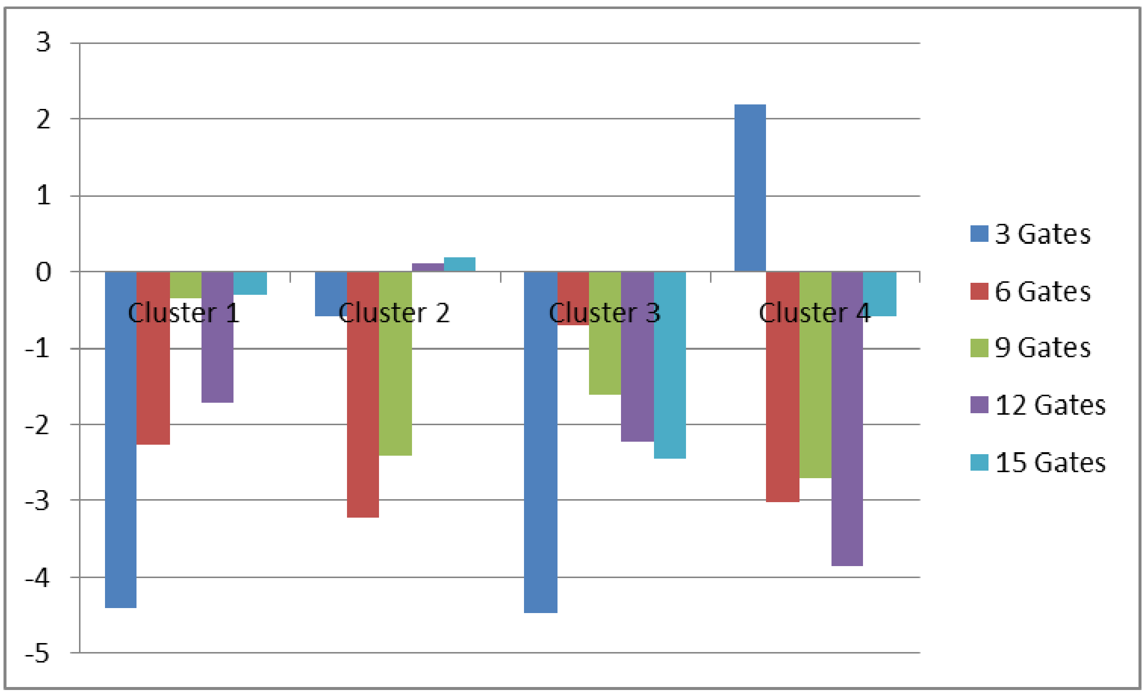

Figure 16: Average Expected Utilities for DOBSS Model

	3 Gates	6 Gates	9 Gates	12 Gates	15 Gates
Cluster 1	-3.511	-1.243	-0.32	-1.886	-1.561
Cluster 2	-0.91	-0.242	-1.58	-1.215	-1.044
Cluster 3	-3.909	-0.932	-2.123	-2.578	-1.712
Cluster 4	1.68	-3.164	-1.857	-2.858	-0.515

Table 18: Average Expected Utilities for QRE (λ = 0.45) Model

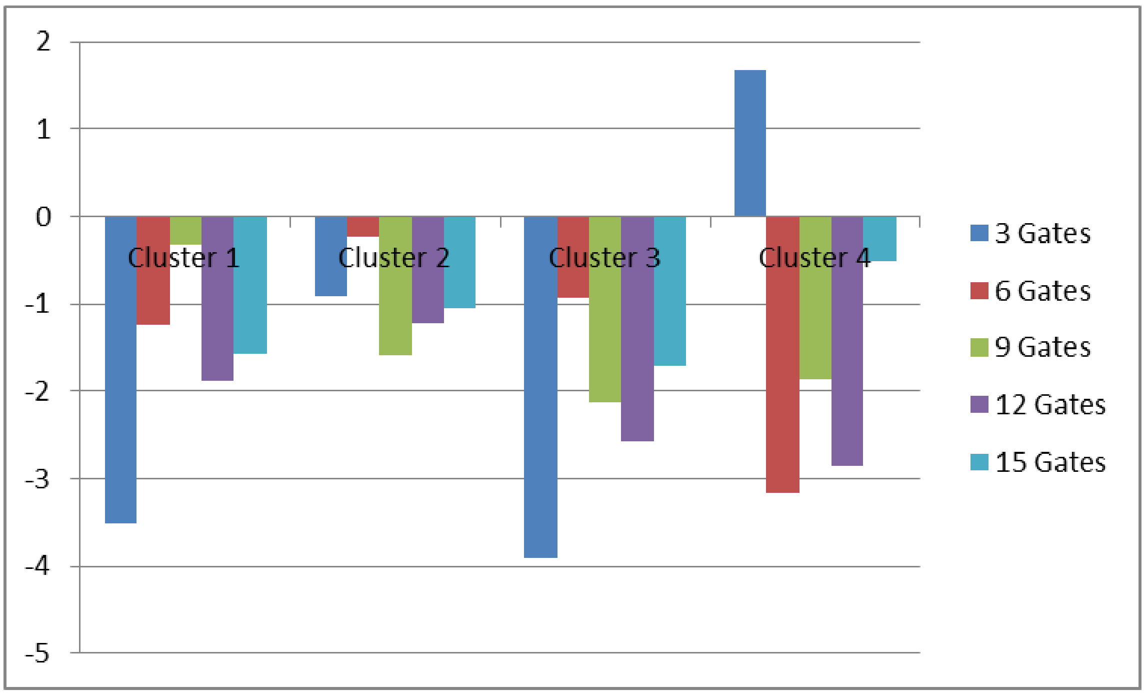

Figure 17: Average Expected Utilities for QRE (λ = 0.45) Model

	3 Gates	6 Gates	9 Gates	12 Gates	15 Gates
Cluster 1	-3.696	-1.274	-0.999	-2.756	-0.663
Cluster 2	-0.814	0.069	-2.167	-1.515	-0.339
Cluster 3	-4.213	-0.903	-2.594	-2.425	-2.021
Cluster 4	1.433	-3.359	-2.344	-3.328	-0.241

Table 19: Average Expected Utilities for QRE ($\lambda = 0.76$) Model

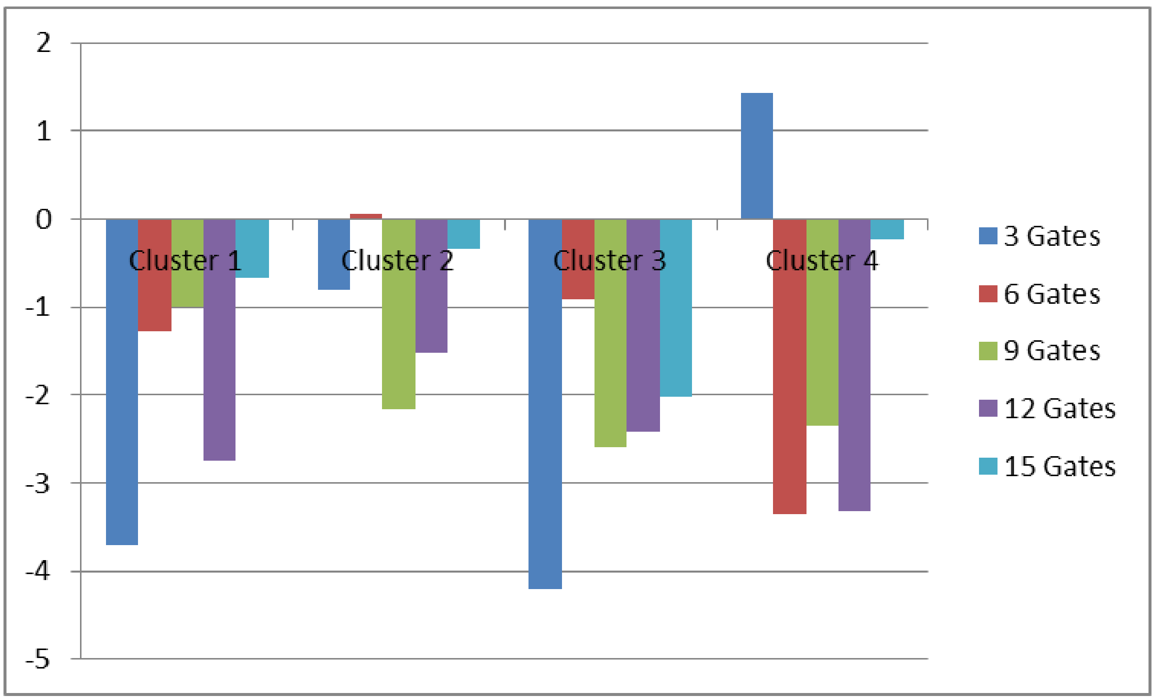

Figure 18: Average Expected Utilities for QRE ($\lambda = 0.76$) Model

Results Based on Time Taken by Participants

When considering the response of participants in an experiment with varying number of gates, it becomes critically important that the participants pay importance to the data presented to them. This is necessary to eradicate conditions where a participant may make choices based on instinct rather than analysis of the defenders' strategy. To assign more credits to participants who paid attention to such details, I carried out another calculation of average expected utilities weighted on the basis of the time taken by the participants. This calculation is based on the response time capped at 120 seconds, i.e. if any participant took more than 120 seconds to analyze a payoff structure; his response time was recorded as 120 seconds only. This was done to ensure that one or more participants do not get a very heavy weightage over other participants.

The calculation of time based average expected utility is carried out as follows:

$$U^t(x) = \frac{1}{T} . \sum_{i=1}^{k} (T_i . U_i^t(x))$$

where, $U^t(x)$ denotes the time based average expected utility of the defender corresponding to the given number of gates in the specified cluster,

T is the total time spent by the participants, who played that cluster,

k are the number of different gates chosen by the participant,

$U_i^t(x)$ is the defender's expected time based utility for gate i given x as the mixed strategy

The time weighted average expected utilities against no. of gates for various clusters are illustrated in Tables 20 to 23 and Figures 19 to 22.

Game Model	3 Gates	6 Gates	9 Gates	12 Gates	15 Gates
PT	-4.877	-1.135	-0.651	-1.961	0.033
PT-Attract	-4.102	-0.69	0.364	0.299	-1.141
COBRA (Alpha = 0.15)	-4.337	-1.82	-0.693	-1.122	-0.292
COBRA (Alpha =0.5)	-4.155	-0.903	0.092	-0.755	-0.92
DOBSS	-4.38	-2.069	0.075	-1.916	0.476
QRE (Lambda = 0.45)	-3.305	-1.083	-0.44	-1.952	-1.322
QRE (Lambda = 0.76)	-3.835	-1.114	-0.756	-3.014	-0.21

Table 20: Time Weighted Average Expected Utilities of Game Models for Cluster 1

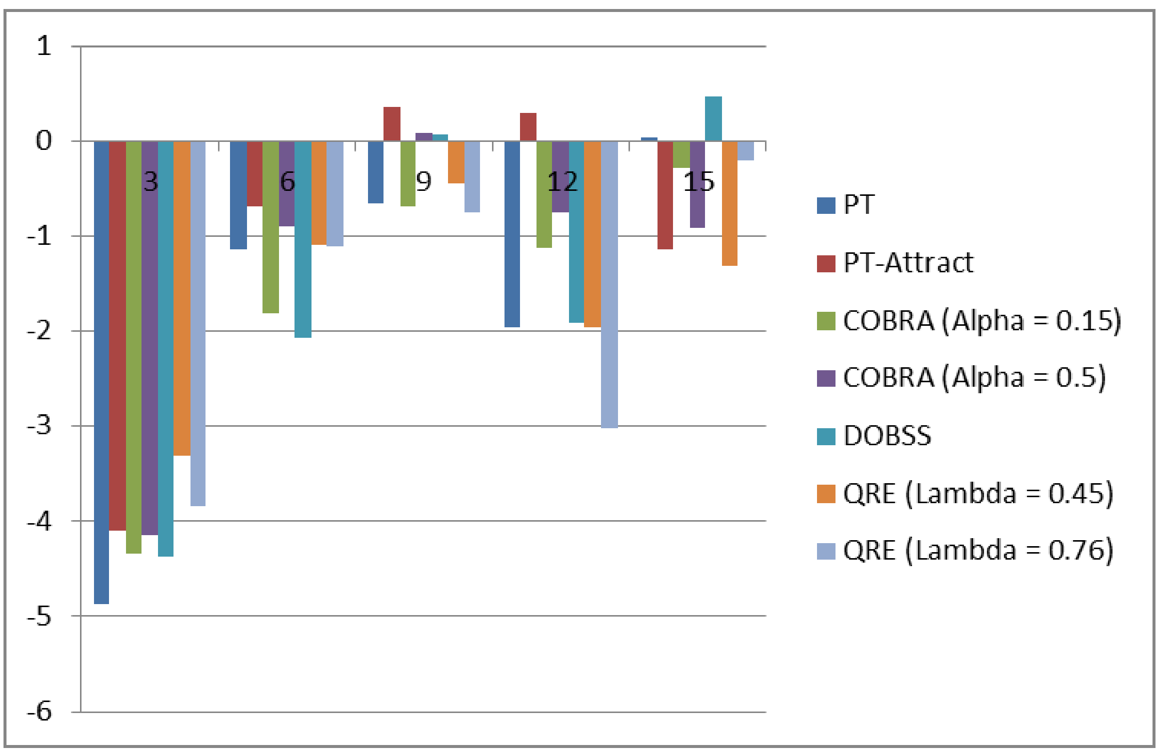

Figure 19: *Time Weighted Average Expected Utilities* against *No. of Gates* for Cluster 1

Game Model	3 Gates	6 Gates	9 Gates	12 Gates	15 Gates
PT	-0.331	-1.975	0.132	1.006	0.57
PT-Attract	-1.098	0.014	-0.798	-1.395	-0.411
COBRA (Alpha = 0.15)	-0.585	-4.908	-0.64	-1.949	-0.363
COBRA (Alpha =0.5)	-0.499	-3.427	-0.5	-2.172	-0.189
DOBSS	-0.933	-2.61	-2.529	0.444	0.867
QRE (Lambda = 0.45)	-0.915	-0.285	-1.607	-1.229	-0.802
QRE (Lambda = 0.76)	-0.822	-0.017	-2.129	-1.817	-0.351

Table 21: Time Weighted Average Expected Utilities of Game Models for Cluster 2

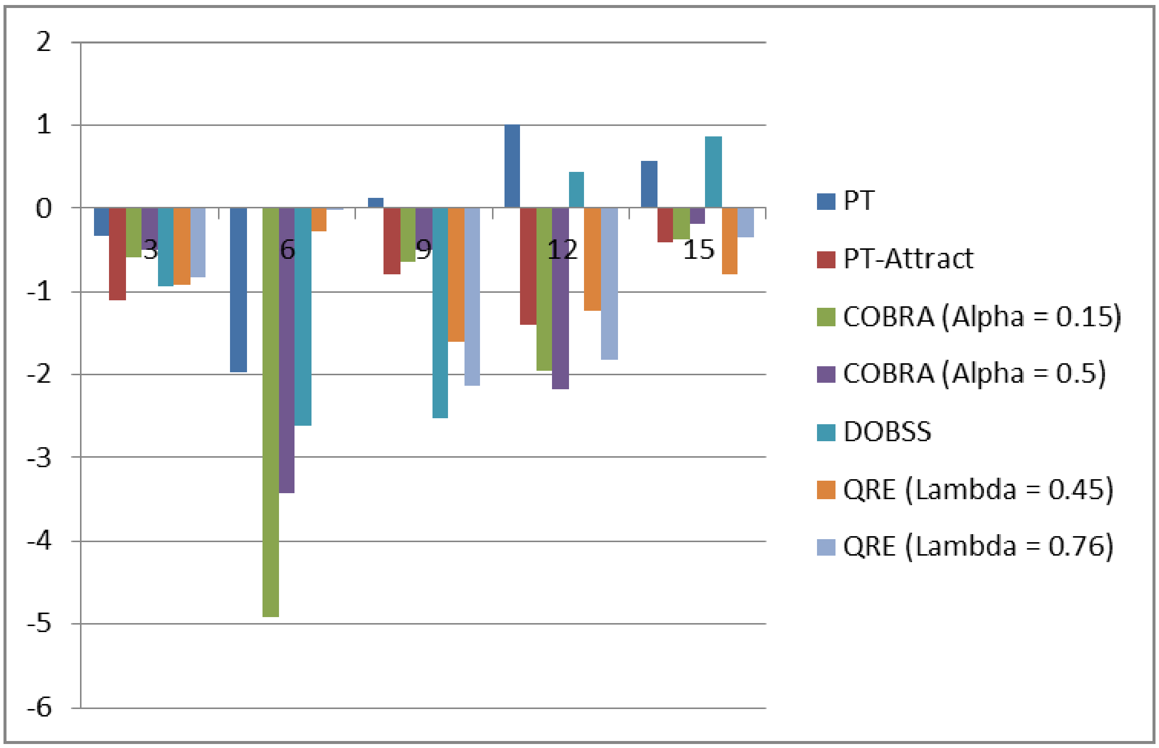

Figure 20: *Time Weighted Average Expected Utilities* against *No. of Gates* for Cluster 2

Game Model	3 Gates	6 Gates	9 Gates	12 Gates	15 Gates
PT	-1.514	-1.968	-2.46	-2.938	-3.003
PT-Attract	-2.889	-0.272	-2.194	-2.952	-1.118
COBRA (Alpha = 0.15)	-5.215	-1.681	-3.783	-2.538	-1.963
COBRA (Alpha =0.5)	-5.28	0.023	-3.548	-2.617	-3.905
DOBSS	-2.908	-1.025	-1.808	-1.183	-2.378
QRE (Lambda = 0.45)	-4.229	-0.907	-2.074	-2.465	-1.671
QRE (Lambda = 0.76)	-2.621	-0.82	-2.52	-1.764	-2.238

Table 22: Time Weighted Average Expected Utilities of Game Models for Cluster 3

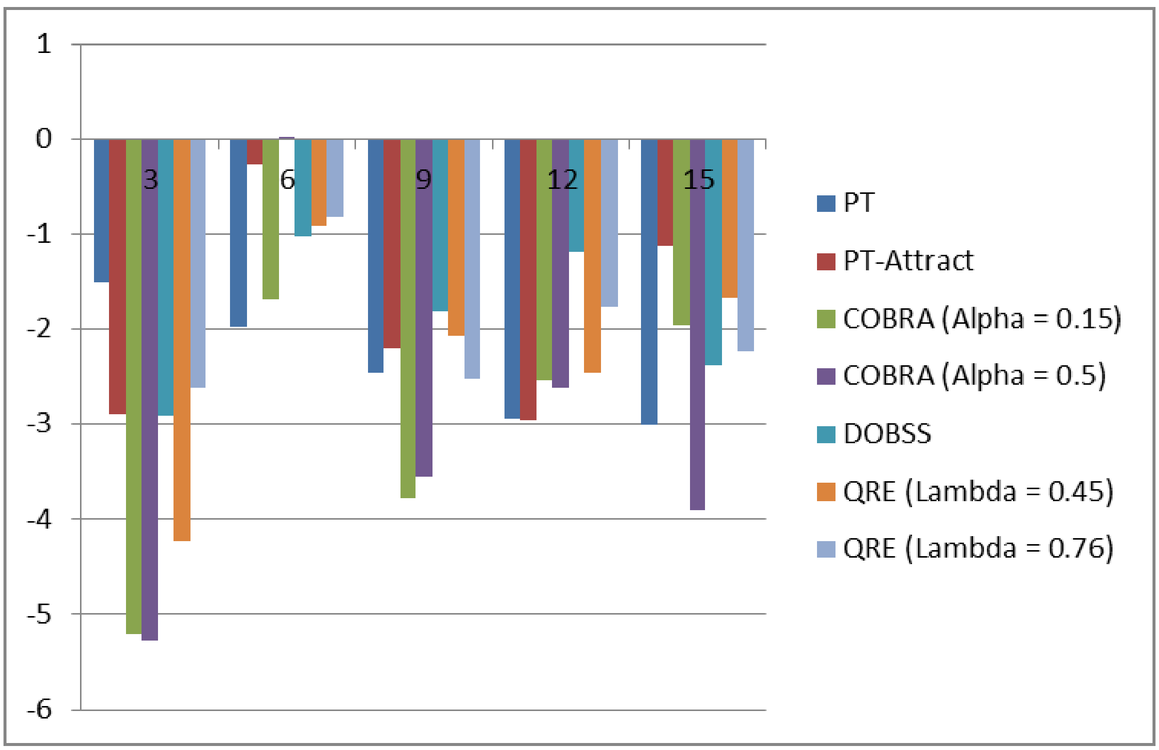

Figure 21: *Time Weighted Average Expected Utilities* against *No. of Gates* for Cluster 3

Game Model	3 Gates	6 Gates	9 Gates	12 Gates	15 Gates
PT	1.013	-1.738	-3.667	-2.624	-0.902
PT-Attract	1.291	-2.465	-3.298	-2.144	-1.441
COBRA (Alpha = 0.15)	2.165	-1.703	-2.681	-3.023	-0.576
COBRA (Alpha =0.5)	2.485	-1.493	-1.668	-5.065	0.505
DOBSS	1.296	-2.823	-2.557	-2.944	-0.558
QRE (Lambda = 0.45)	2.044	-3.386	-2.015	-2.992	-1.313
QRE (Lambda = 0.76)	1.828	-3.613	-2.367	-3.66	-0.263

Table 23: Time Weighted Average Expected Utilities of Game Models for Cluster 4

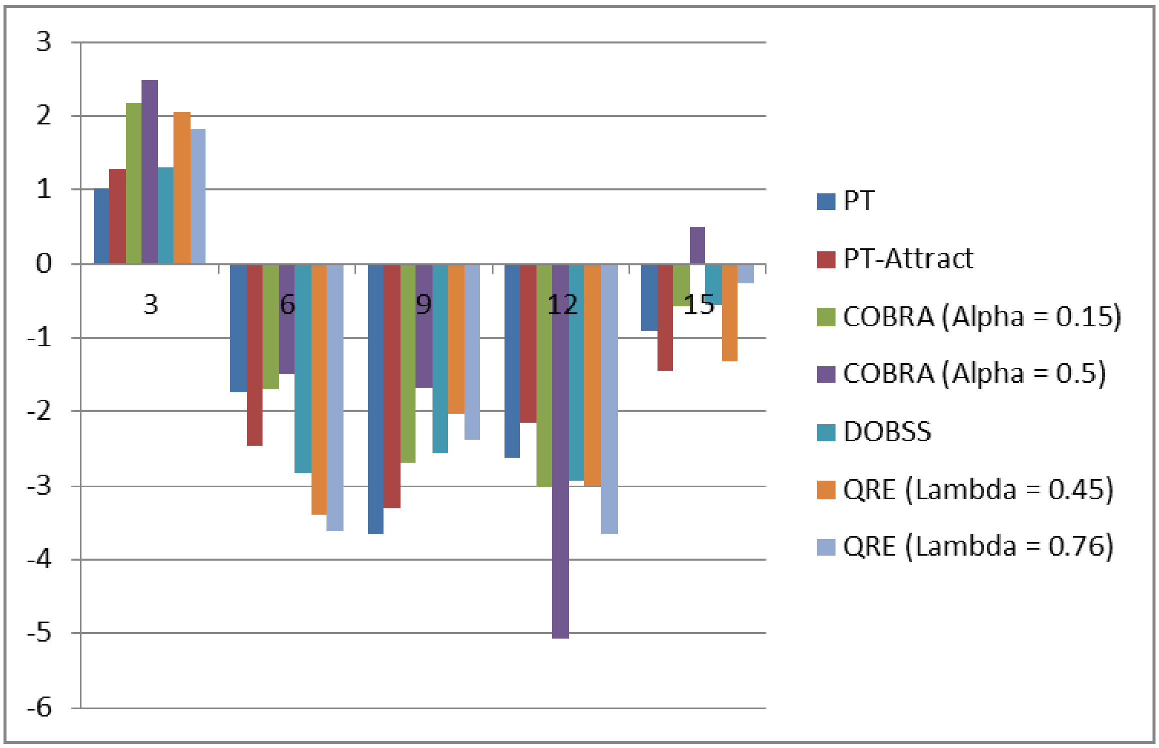

Figure 22: *Time Weighted Average Expected Utilities* against *No. of Gates* for Cluster 4

Rankings of Game Models

From the experiments, there does not seem to be a distinct game model that would be superior to all the other models in every case. Thus, to rank the game models, a different ranking scheme is deployed.

A table is designed to count the number of times a game model is ranked 1, considering all the game settings, then the number of times it is ranked 1 or 2, the number of times it is ranked 1, 2 or 3 and so on. Each position gives 1 point to the game model. Thus, the overall ranking can then be calculated based on this point system. The points allocated are shown in Table 24.

Rank	**1**	**1 or 2**	**1, 2 or 3**	**1, 2, 3 or 4**	**1, 2, 3, 4 or 5**	**1, 2, 3, 4, 5 or 6**	**Total**
PT	5	7	11	12	14	17	**66**
PT-Attract	6	10	11	13	14	17	**71**
COBRA (Alpha = 0.15)	2	5	11	13	14	18	**63**
COBRA (Alpha =0.5)	2	8	9	11	13	17	**60**
DOBSS	4	4	8	9	16	18	**59**
QRE (Lambda = 0.45)	0	3	6	12	15	16	**52**
QRE (Lambda = 0.76)	1	3	4	10	14	17	**49**

Table 24: Game Model Point Allocation

Based on this ranking system, PT-Attract Model outperforms all the other game models.

On similar lines, when Time Weighted Average Utilities are considered the point allocation obtained is as shown in Table 25.

Rank	1	1 or 2	1, 2 or 3	1, 2, 3 or 4	1, 2, 3, 4 or 5	1, 2, 3, 4, 5 or 6	Total
PT	4	7	8	10	14	16	**59**
PT-Attract	6	7	10	13	14	17	**67**
COBRA (Alpha = 0.15)	0	2	6	9	12	17	**46**
COBRA (Alpha =0.5)	5	11	12	13	15	17	**73**
DOBSS	4	4	7	10	15	18	**58**
QRE (Lambda = 0.45)	1	4	9	12	15	18	**59**
QRE (Lambda = 0.76)	0	5	8	13	15	17	**58**

Table 25: Game Model Point Allocation for Time Weighted Average Expected Utilities

Based on this point allocation, COBRA ($\alpha = 0.5$) outperforms the other models, however, PT-Attract follows as a close second.

From both the obtained tables, it can be derived that PT-Attract and COBRA ($\alpha = 0.5$) outperform all the other models in this set of experiments.

Chapter Six: Statistical Analysis

In order to carry out the statistical analysis over the data set, a robust method is needed. This is because of the non-normal distribution of the data. I chose to run Yuen's test for comparing trimmed means [28]. For my tests, the standard 20% trimmed mean was used. A trimmed mean refers to a situation where a certain proportion of the largest and smallest sample points are removed and the remaining sample points are averaged. This is typically done to help reduce variance in data collections that may have extreme outliers that can skew data sets [23, 24]. This method has been considered due to its use in some of the previous work [18].

Cluster 1

The statistical probability parameters (p-value) for various game models against each other in cluster 1 are shown in Tables 26 to 30.

	PT	**PT-Attract**	**COBRA (α = 0.15)**	**COBRA (α = 0.5)**	**DOBSS**	**QRE (λ = 0.45)**	**QRE (λ = 0.76)**
PT	1	0.6225	0.0962	0.1654	0.4407	0.0016	0.0215
PT-Attract	0.6225	1	0.0268	0.0489	0.1572	0.0004	0.0054
COBRA (α = 0.15)	0.0962	0.0268	1	0.0004	0	0	0.0423
COBRA (α = 0.5)	0.1654	0.0489	0.0004	1	0	0	0.0104
DOBSS	0.4407	0.1572	0	0	1	0	0.0005
QRE (λ = 0.45)	0.0016	0.0004	0	0	0	1	0.0094
QRE (λ = 0.76)	0.0215	0.0054	0.0423	0.0104	0.0005	0.0094	1

Table 26: p-values for 3 Gate settings in Cluster 1

	PT	PT-Attract	COBRA (α = 0.15)	COBRA (α = 0.5)	DOBSS	QRE (λ = 0.45)	QRE (λ = 0.76)
PT	1	0.2233	0.0003	0.1576	0.5006	0.918	0.5075
PT-Attract	0.2233	1	0.2351	0.5868	0.2245	0.2064	0.3092
COBRA (α = 0.15)	0.0003	0.2351	1	0.8687	0.0696	0	0
COBRA (α = 0.5)	0.1576	0.5868	0.8687	1	0.1449	0.1542	0.206
DOBSS	0.5006	0.2245	0.0696	0.1449	1	0.4736	0.3898
QRE (λ = 0.45)	0.918	0.2064	0	0.1542	0.4736	1	0.2686
QRE (λ = 0.76)	0.5075	0.3092	0	0.206	0.3898	0.2686	1

Table 27: p-values for 6 Gate settings in Cluster 1

	PT	PT-Attract	COBRA (α = 0.15)	COBRA (α = 0.5)	DOBSS	QRE (λ = 0.45)	QRE (λ = 0.76)
PT	1	0.3417	0	0.0003	0.0029	0.0079	0
PT-Attract	0.3417	1	0	0.2736	0.0013	0.0047	0
COBRA (α = 0.15)	0	0	1	0	0.9699	0.014	0.8738
COBRA (α = 0.5)	0.0003	0.2736	0	1	0.0003	0.0002	0
DOBSS	0.0029	0.0013	0.9699	0.0003	1	0.1535	0.9027
QRE (λ = 0.45)	0.0079	0.0047	0.014	0.0002	0.1535	1	0.052
QRE (λ = 0.76)	0	0	0.8738	0	0.9027	0.052	1

Table 28: p-values for 9 Gate settings in Cluster 1

	PT	PT-Attract	COBRA (α = 0.15)	COBRA (α = 0.5)	DOBSS	QRE (λ = 0.45)	QRE (λ = 0.76)
PT	1	0.0131	0.8214	0.187	0.187	0.187	0.0414
PT-Attract	0.0131	1	0.1722	0	0	0	0
COBRA (α = 0.15)	0.8214	0.1722	1	0.2994	0.2994	0.2944	0.1078
COBRA (α = 0.5)	0.187	0	0.2994	1	0	0	0.1098
DOBSS	0.187	0	0.2994	0	1	0	0.1098
QRE (λ = 0.45)	0.187	0	0.2944	0	0	1	0.1098
QRE (λ = 0.76)	0.0414	0	0.1078	0.1098	0.1098	0.1098	1

Table 29: p-values for 12 Gate settings in Cluster 1

	PT	PT-Attract	COBRA (α = 0.15)	COBRA (α = 0.5)	DOBSS	QRE (λ = 0.45)	QRE (λ = 0.76)
PT	1	0.0467	0.8915	0.512	0.4957	0.1525	0.4116
PT-Attract	0.0467	1	0.0057	0.0235	0.1293	0.9235	0.043
COBRA (α = 0.15)	0.8915	0.0057	1	0.006	0.3729	0.1061	0.0209
COBRA (α = 0.5)	0.512	0.0235	0.006	1	0.8053	0.2079	0.5527
DOBSS	0.4957	0.1293	0.3729	0.8053	1	0.3249	0.9875
QRE (λ = 0.45)	0.1525	0.9235	0.1061	0.2079	0.3249	1	0.2639
QRE (λ = 0.76)	0.4116	0.043	0.0209	0.5527	0.9875	0.2639	1

Table 30: p-values for 15 Gate settings in Cluster 1

Cluster 2

The statistical probability parameters (p-value) for various game models against each other in cluster 2 are shown in Tables 31 to 35.

	PT	PT-Attract	COBRA (α = 0.15)	COBRA (α = 0.5)	DOBSS	QRE (λ = 0.45)	QRE (λ = 0.76)
PT	1	0.4592	0.8231	0.6536	0.9574	0.1744	0.238
PT-Attract	0.4592	1	0.1867	0.1219	0.2582	0.0157	0.0243
COBRA (α = 0.15)	0.8231	0.1867	1	0.0213	0.3114	0	0
COBRA (α = 0.5)	0.6536	0.1219	0.0213	1	0.0348	0	0
DOBSS	0.9574	0.2582	0.3114	0.0348	1	0	0
QRE (λ = 0.45)	0.1744	0.0157	0	0	0	1	0
QRE (λ = 0.76)	0.238	0.0243	0	0	0	0	1

Table 31: p-values for 3 Gate settings in Cluster 2

	PT	PT-Attract	COBRA (α = 0.15)	COBRA (α = 0.5)	DOBSS	QRE (λ = 0.45)	QRE (λ = 0.76)
PT	1	0.0209	0.0025	0.0041	0.0876	0.2459	0.125
PT-Attract	0.0209	1	0.0004	0.0003	0.0086	0	0
COBRA (α = 0.15)	0.0025	0.0004	1	0.514	0.125	0.0009	0.0007
COBRA (α = 0.5)	0.0041	0.0003	0.514	1	0.3061	0.0011	0.0008
DOBSS	0.0876	0.0086	0.125	0.3061	1	0.0271	0.0192
QRE (λ = 0.45)	0.2459	0	0.0009	0.0011	0.0271	1	0.0001
QRE (λ = 0.76)	0.125	0	0.0007	0.0008	0.0192	0.0001	1

Table 32: p-values for 6 Gate settings in Cluster 2

	PT	PT-Attract	COBRA (α = 0.15)	COBRA (α = 0.5)	DOBSS	QRE (λ = 0.45)	QRE (λ = 0.76)
PT	1	0	0	0	0	0	0
PT-Attract	0	1	0	0	0.0755	0.0507	0.0249
COBRA (α = 0.15)	0	0	1	0	0.0347	0.0118	0.007
COBRA (α = 0.5)	0	0	0	1	0.0099	0.0012	0.001
DOBSS	0	0.0755	0.0347	0.0099	1	0.5268	0.8044
QRE (λ = 0.45)	0	0.0507	0.0118	0.0012	0.5268	1	0.6116
QRE (λ = 0.76)	0	0.0249	0.007	0.001	0.8044	0.6116	1

Table 33: p-values for 9 Gate settings in Cluster 2

	PT	PT-Attract	COBRA (α = 0.15)	COBRA (α = 0.5)	DOBSS	QRE (λ = 0.45)	QRE (λ = 0.76)
PT	1	0.0064	0.0076	0.0011	0.0051	0.0006	0.001
PT-Attract	0.0064	1	0.9348	0	0	0	0
COBRA (α = 0.15)	0.0076	0.9348	1	0.0571	0.8371	0.0114	0.0421
COBRA (α = 0.5)	0.0011	0	0.0571	1	0	0	0.0001
DOBSS	0.0051	0	0.8371	0	1	0	0
QRE (λ = 0.45)	0.0006	0	0.0114	0	0	1	0
QRE (λ = 0.76)	0.001	0	0.0421	0.0001	0	0	1

Table 34: p-values for 12 Gate settings in Cluster 2

	PT	PT-Attract	COBRA (α = 0.15)	COBRA (α = 0.5)	DOBSS	QRE (λ = 0.45)	QRE (λ = 0.76)
PT	1	0.0227	0	0	0.8253	0.0053	0
PT-Attract	0.0227	1	0.9865	0.5453	0.5967	0.2236	0.8147
COBRA (α = 0.15)	0	0.9865	1	0	0.5849	0.1482	0.1408
COBRA (α = 0.5)	0	0.5453	0	1	0.7142	0.0718	0.0001
DOBSS	0.8253	0.5967	0.5849	0.7142	1	0.2715	0.5337
QRE (λ = 0.45)	0.0053	0.2236	0.1482	0.0718	0.2715	1	0.2004
QRE (λ = 0.76)	0	0.8147	0.1408	0.0001	0.5337	0.2004	1

Table 35: p-values for 15 Gate settings in Cluster 2

Cluster 3

The statistical probability parameters (p-value) for various game models against each other in cluster 3 are shown in Tables 36 to 40.

	PT	PT-Attract	COBRA (α = 0.15)	COBRA (α = 0.5)	DOBSS	QRE (λ = 0.45)	QRE (λ = 0.76)
PT	1	0.2873	0.0319	0.0377	0.2037	0.2345	0.1579
PT-Attract	0.2873	1	0.0591	0.0975	0.5339	0.9172	0.5182
COBRA (α = 0.15)	0.0319	0.0591	1	0.7999	0.5924	0.0322	0.146
COBRA (α = 0.5)	0.0377	0.0975	0.7999	1	0.5072	0.0808	0.1851
DOBSS	0.2037	0.5339	0.5924	0.5072	1	0.5329	0.7543
QRE (λ = 0.45)	0.2345	0.9172	0.0322	0.0808	0.5329	1	0.4044
QRE (λ = 0.76)	0.1579	0.5182	0.146	0.1851	0.7543	0.4044	1

Table 36: p-values for 3 Gate settings in Cluster 3

	PT	PT-Attract	COBRA (α = 0.15)	COBRA (α = 0.5)	DOBSS	QRE (λ = 0.45)	QRE (λ = 0.76)
PT	1	0.0006	0.0363	0	0.0058	0.0065	0.0089
PT-Attract	0.0006	1	0.5601	0.0771	0.7168	0.0376	0.0553
COBRA (α = 0.15)	0.0363	0.5601	1	0.137	0.7861	0.7458	0.7208
COBRA (α = 0.5)	0	0.0771	0.137	1	0.118	0.0003	0.0005
DOBSS	0.0058	0.7168	0.7861	0.118	1	0.3775	0.3714
QRE (λ = 0.45)	0.0065	0.0376	0.7458	0.0003	0.3775	1	0.9049
QRE (λ = 0.76)	0.0089	0.0553	0.7208	0.0005	0.3714	0.9049	1

Table 37: p-values for 6 Gate settings in Cluster 3

	PT	PT-Attract	COBRA (α = 0.15)	COBRA (α = 0.5)	DOBSS	QRE (λ = 0.45)	QRE (λ = 0.76)
PT	1	0.0852	0.0238	0.2964	0.3587	0.2489	0.9153
PT-Attract	0.0852	1	0.0077	0.0528	0.8435	0.2677	0.104
COBRA (α = 0.15)	0.0238	0.0077	1	0.8758	0.0875	0.0098	0.0752
COBRA (α = 0.5)	0.2964	0.0528	0.8758	1	0.1672	0.1583	0.3518
DOBSS	0.3587	0.8435	0.0875	0.1672	1	0.5929	0.3615
QRE (λ = 0.45)	0.2489	0.2677	0.0098	0.1583	0.5929	1	0.3671
QRE (λ = 0.76)	0.9153	0.104	0.0752	0.3518	0.3615	0.3671	1

Table 38: p-values for 9 Gate settings in Cluster 3

	PT	PT-Attract	COBRA (α = 0.15)	COBRA (α = 0.5)	DOBSS	QRE (λ = 0.45)	QRE (λ = 0.76)
PT	1	0.3712	0.8314	0.756	0.8033	0.9002	0.8535
PT-Attract	0.3712	1	0.2687	0.2496	0.0477	0.0706	0.064
COBRA (α = 0.15)	0.8314	0.2687	1	0.8537	0.2994	0.4389	0.3899
COBRA (α = 0.5)	0.756	0.2496	0.8537	1	0.0285	0.0814	0.0945
DOBSS	0.8033	0.0477	0.2994	0.0285	1	0.5996	0.8256
QRE (λ = 0.45)	0.9002	0.0706	0.4389	0.0814	0.5996	1	0.8277
QRE (λ = 0.76)	0.8535	0.064	0.3899	0.0945	0.8256	0.8277	1

Table 39: p-values for 12 Gate settings in Cluster 3

	PT	PT-Attract	COBRA (α = 0.15)	COBRA (α = 0.5)	DOBSS	QRE (λ = 0.45)	QRE (λ = 0.76)
PT	1	0.0002	0.005	0.4105	0.04	0	0.0002
PT-Attract	0.0002	1	0.4761	0.0001	0.2256	0.4728	0.2194
COBRA (α = 0.15)	0.005	0.4761	1	0.0026	0.5992	0.7619	0.7551
COBRA (α = 0.5)	0.4105	0.0001	0.0026	1	0.0205	0	0.0003
DOBSS	0.04	0.2256	0.5992	0.0205	1	0.34	0.7371
QRE (λ = 0.45)	0	0.4728	0.7619	0	0.34	1	0.2799
QRE (λ = 0.76)	0.0002	0.2194	0.7551	0.0003	0.7371	0.2799	1

Table 40: p-values for 15 Gate settings in Cluster 3

Cluster 4

The statistical probability parameters (p-value) for various game models against each other in cluster 4 are shown in Tables 41 to 45.

	PT	PT-Attract	COBRA (α = 0.15)	COBRA (α = 0.5)	DOBSS	QRE (λ = 0.45)	QRE (λ = 0.76)
PT	1	0.5852	0.0577	0.0363	0.0524	0.1727	0.4684
PT-Attract	0.5852	1	0.1119	0.0646	0.1031	0.3786	0.8508
COBRA (α = 0.15)	0.0577	0.1119	1	0	0.7015	0	0.1415
COBRA (α = 0.5)	0.0363	0.0646	0	1	0.7566	0	0.0791
DOBSS	0.0524	0.1031	0.7015	0.7566	1	0.0515	0.1306
QRE (λ = 0.45)	0.1727	0.3786	0	0	0.0515	1	0.492
QRE (λ = 0.76)	0.4684	0.8508	0.1415	0.0791	0.1306	0.492	1

Table 41: p-values for 3 Gate settings in Cluster 4

	PT	PT-Attract	COBRA (α = 0.15)	COBRA (α = 0.5)	DOBSS	QRE (λ = 0.45)	QRE (λ = 0.76)
PT	1	0.934	0.0694	0.2381	0.76	0.2963	0.2311
PT-Attract	0.934	1	0.02	0.0913	0.693	0.0667	0.1191
COBRA (α = 0.15)	0.0694	0.02	1	0.4484	0.3063	0.0038	0.0048
COBRA (α = 0.5)	0.2381	0.0913	0.4484	1	0.6068	0.0169	0.0203
DOBSS	0.76	0.693	0.3063	0.6068	1	0.3304	0.259
QRE (λ = 0.45)	0.2963	0.0667	0.0038	0.0169	0.3304	1	0.6771
QRE (λ = 0.76)	0.2311	0.1191	0.0048	0.0203	0.259	0.6771	1

Table 42: p-values for 6 Gate settings in Cluster 4

	PT	PT-Attract	COBRA (α = 0.15)	COBRA (α = 0.5)	DOBSS	QRE (λ = 0.45)	QRE (λ = 0.76)
PT	1	0.0407	0	0	0	0	0
PT-Attract	0.0407	1	0.0011	0.0315	0.2394	0.0015	0.0034
COBRA (α = 0.15)	0	0.0011	1	0.0163	0.0037	0.232	0.1388
COBRA (α = 0.5)	0	0.0315	0.0163	1	0.0868	0	0.0021
DOBSS	0	0.2394	0.0037	0.0868	1	0.0004	0.0021
QRE (λ = 0.45)	0	0.0015	0.232	0	0.0004	1	0.2363
QRE (λ = 0.76)	0	0.0034	0.1388	0.0021	0.0021	0.2363	1

Table 43: p-values for 9 Gate settings in Cluster 4

	PT	PT-Attract	COBRA (α = 0.15)	COBRA (α = 0.5)	DOBSS	QRE (λ = 0.45)	QRE (λ = 0.76)
PT	1	0.733	0.7725	0.1651	0.1895	0.7197	0.9357
PT-Attract	0.733	1	0.9094	0.0752	0.0937	0.9342	0.889
COBRA (α = 0.15)	0.7725	0.9094	1	0.24	0.2539	0.8818	0.8492
COBRA (α = 0.5)	0.1651	0.0752	0.24	1	0.9808	0.0536	0.2609
DOBSS	0.1895	0.0937	0.2539	0.9808	1	0.0713	0.2793
QRE (λ = 0.45)	0.7197	0.9342	0.8818	0.0536	0.0713	1	0.9078
QRE (λ = 0.76)	0.9357	0.889	0.8492	0.2609	0.2793	0.9078	1

Table 44: p-values for 12 Gate settings in Cluster 4

	PT	PT-Attract	COBRA (α = 0.15)	COBRA (α = 0.5)	DOBSS	QRE (λ = 0.45)	QRE (λ = 0.76)
PT	1	0.0521	0	0	0.1005	0.0198	0
PT-Attract	0.0521	1	0.0155	0.002	0.6278	0.3629	0.0961
COBRA (α = 0.15)	0	0.0155	1	0.0552	0.3767	0.446	0.1636
COBRA (α = 0.5)	0	0.002	0.0552	1	0.1355	0.1268	0.0163
DOBSS	0.1005	0.6278	0.3767	0.1355	1	0.8081	0.643
QRE (λ = 0.45)	0.0198	0.3629	0.446	0.1268	0.8081	1	0.8274
QRE (λ = 0.76)	0	0.0961	0.1636	0.0163	0.643	0.8274	1

Table 45: p-values for 15 Gate settings in Cluster 4

Conclusion

This work expands the domain of game-theoretic techniques as a solution to security allocation problems. Most of the previous work has been done on specific security settings with pre-defined number of targets. With varying number of targets, the game settings can also be expanded over multiple security domains at the same time.

Among all the evaluated game models including PT, PT-Attract, COBRA, DOBSS and QRE; PT-ATTRACT and COBRA ($\alpha = 0.5$) seem to perform better than the others. The results, however, are very close to determine any particular game model to be dominating all the others.

More research needs to be done in this field, to determine new strategies that can be applied to this niche of game-theoretic techniques for security allocation. If used in the suitable manner, this work can help solve security allocation problems spanning an entirely new problem set.

Bibliography

[1] N. Agmon, V. Sadov, S. Kraus and G. Kaminka. The impact of adversarial knowledge on adversarial planning in perimeter patrol. *AAMAS, 2008.*

[2] G. Brown, M. Carlyle, J. Sameron and K. Wood. Defending critical infrastructure. *Interfaces 36(6):530-544, 2006.*

[3] C. F. Camerer, T.-H. Ho and J.-K. Chongn. A cognitive hierarchy model of games. *QJE, 119(3):861-898, 2004*.

[4] V. Conitzer and T. Sandholm. Computing the optimal strategy to commit to. *EC, 2006*.

[5] M. Costa-Gomes, V. P. Crawford, and B. Broseta. Cognition and behavior in normal-form games: An experimental study. *Econometrica, 69(5):1193-1235, 2001*.

[6] S. Ficici and A. Pfeffer. Simultaneously modeling humans' preferences and their beliefs about others' preferences. *AAMAS, 2008*.

[7] D. Fudenberg and J. Tirole. Game Theory. *MIT Press, 1991*.

[8] N. Gatti. Game theoretical insights in strategic patrolling: Model and algorithm in normal-form. *ECAI-08:403-407, 2008*.

[9] J. C. Harsanyi and R. Shelten. A generalized Nash solution for two-person bargaining games with incomplete information. *Management Science, 18(5):80-106, 1972*.

[10] D. Kahneman and A. Tvesky. Prospect Theory: An analysis of decision under risk. *Econometrica, 47(2):263-292, 1979.*

[11] C. Kiekintveld, M. Jain, J. Tsai, J. Pita, M. Tambe and F. Ordonez. Computing optimal randomized resource allocations for massive security games. *AAMAS, 2009*.

[12] S. Kullback and R. A. Leibler. On information and Sufficiency. *Annals of Mathematical Statistics, 22(1):79-86, 1951*.

[13] R. D. McKelvey and T. R. Palfrey. Quantal response equilibria for normal form games. *Games and Economic Behavior, 2:6-38, 1995.*

[14] P. Paruchuri, J. Marecki, J. Pearce, M. Tambe, F. Ordonez and S. Kraus. Playing games for security: An efficient exact algorithm for solving Bayesian Stackelberg games. *AAMAS, 2008*.

[15] P. Paruchuri, M. Tambe, F. Ordonez and S. Kraus. Security in multi-agent systems by policy randomization. *AAMAS, 2006*.

[16] J. Pita, M. Jain, J. Marecki, F. Ordonez, C. Portway, M. Tambe, C. Western, P. Paruchuri and S. Kraus. Deployed ARMOR protection: The application of a game theoretic model for security at the Los Angeles International Airport. *AAMAS, 2008*.

[17] J. Pita, M. Jain, F. Ordonez, M. Tambe, S. Kraus and R. Magori-Cohen. Effective solutions for real-world Stackelberg games: When agents must deal with human uncertainties. *AAMAS, 2009*.

[18] J. Pita, M. Jain, M. Tambe, F. Ordonez and S. Kraus. Robust Solutions to Stackelberg Games: Addressing Bounded Rationality and Limited Observations in Human Cognition. *Artificial Intelligence Journal, 174(15):1142-1171, 2010*.

[19] J. Pita, M. Tambe, C. Kienkintveld, S. Cullen and E. Steigerwald. GUARDS – Game theoretic security allocation on a national scale. *AAMAS, 2011*.

[20] T. Sandler and D. G. A. M. Terrorism and game theory. *Simulation and Gaming, 34(4):319-337, 2003*.

[21] D. O. Stahl and P. W. Wilson. Experimental evidence on players' models of other players. *JEBO, 25(3):309-327, 1994*.

[22] J. Tsai, S. Rathi, C. Kienkintveld, F. Ordonez and M. Tambe. IRIS - a tool for strategic security allocation in transportation networks. *AAMAS, 2009*.

[23] R. R. Wilcox. How many discoveries have been lost by ignoring modern statistical methods? *American Psychologist, 53(3):300-314, 1998*.

[24] R. R. Wilcox. Introduction to Robust Estimation and Hypothesis Testing. *Academic Press, 2005*.

[25] J. R. Wright and K. Leyton-Brown. Beyond equilibrium: Predicting human behavior in normal-form games. *AAAI, 2010*.

[26] A. P. Ya'akov Gal. Modeling reciprocal behavior in human bilateral negotiation. *AAMAS, 2007*.

[27] R. Yang, C. Kiekintveld, F. Ordonez, M. Tambe and R. John. Improved computational models of human behavior in security games. *AAMAS, 2011*.

[28] K. K. Yuen. The Two-Sample Trimmed T for Unequal Population Variances. *Biometrika, 61:165-170, 1974*.

Appendix A: Payoff Structures for 3 Gate Settings

The payoff structures for 3 gate settings are given in the Tables 46 to 49.

	Gate 1	Gate 2	Gate 3
Your Rewards	8	6	3
Your Penalties	-4	-8	-2
Guards' Rewards	8	5	1
Guards' Penalties	-5	-9	-7

Table 46: Payoff Structure for 3 Gate Settings in Cluster 1

	Gate 1	Gate 2	Gate 3
Your Rewards	7	6	8
Your Penalties	-5	-1	-5
Guards' Rewards	8	7	3
Guards' Penalties	-4	-4	-9

Table 47: Payoff Structure for 3 Gate Settings in Cluster 2

	Gate 1	Gate 2	Gate 3
Your Rewards	7	3	8
Your Penalties	-6	-3	-3
Guards' Rewards	3	6	7
Guards' Penalties	-5	-10	-2

Table 48: Payoff Structure for 3 Gate Settings in Cluster 3

	Gate 1	Gate 2	Gate 3
Your Rewards	2	7	6
Your Penalties	-4	-3	-3
Guards' Rewards	1	8	8
Guards' Penalties	-2	-1	-3

Table 49: Payoff Structure for 3 Gate Settings in Cluster 4

Appendix B: Payoff Structures for 6 Gate Settings

The payoff structures for 6 gate settings are given in the Tables 50 to 53.

	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6
Your Rewards	4	7	2	2	3	1
Your Penalties	-5	-3	-5	-2	-2	-2
Guards' Rewards	2	2	5	8	3	1
Guards' Penalties	-3	-5	-7	-6	-1	-4

Table 50: Payoff Structure for 6 Gate Settings in Cluster 1

	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6
Your Rewards	10	6	3	10	4	9
Your Penalties	-3	-8	-2	-7	-7	-8
Guards' Rewards	4	5	5	9	5	9
Guards' Penalties	-6	-1	-9	-2	-10	-6

Table 51: Payoff Structure for 6 Gate Settings in Cluster 2

	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6
Your Rewards	7	9	10	6	2	6
Your Penalties	-6	-6	-10	-5	-2	-1
Guards' Rewards	8	9	3	8	10	7
Guards' Penalties	-9	-9	-2	-5	-4	-1

Table 52: Payoff Structure for 6 Gate Settings in Cluster 3

	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6
Your Rewards	3	3	10	6	8	8
Your Penalties	-1	-8	-6	-9	-4	-1
Guards' Rewards	4	3	6	3	4	7
Guards' Penalties	-9	-1	-7	-4	-3	-9

Table 53: Payoff Structure for 6 Gate Settings in Cluster 4

Appendix C: Payoff Structures for 9 Gate Settings

The payoff structures for 9 gate settings are given in the Tables 54 to 57.

	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8	Gate 9
Your Rewards	9	8	7	3	7	7	7	1	10
Your Penalties	-4	-6	-10	-3	-2	-8	-1	-1	-5
Guards' Rewards	8	2	8	6	8	10	4	5	4
Guards' Penalties	-1	-4	-4	-5	-9	-3	-8	-6	-3

Table 54: Payoff Structure for 9 Gate Settings in Cluster 1

	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8	Gate 9
Your Rewards	2	6	5	7	8	4	9	4	3
Your Penalties	-4	-8	-6	-10	-8	-3	-2	-8	-3
Guards' Rewards	7	7	1	3	4	9	6	10	2
Guards' Penalties	-5	-2	-10	-2	-9	-8	-10	-5	-6

Table 55: Payoff Structure for 9 Gate Settings in Cluster 2

	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8	Gate 9
Your Rewards	2	10	6	8	9	4	2	9	8
Your Penalties	-9	-2	-1	-7	-9	-6	-3	-8	-2
Guards' Rewards	4	1	8	2	10	2	9	9	8
Guards' Penalties	-7	-8	-3	-2	-7	-8	-4	-6	-7

Table 56: Payoff Structure for 9 Gate Settings in Cluster 3

	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8	Gate 9
Your Rewards	4	2	1	9	7	10	6	1	7
Your Penalties	-9	-4	-2	-9	-2	-8	-5	-6	-4
Guards' Rewards	5	2	10	7	1	6	5	9	6
Guards' Penalties	-6	-10	-3	-3	-7	-2	-10	-3	-1

Table 57: Payoff Structure for 9 Gate Settings in Cluster 4

Appendix D: Payoff Structures for 12 Gate Settings

The payoff structures for 12 gate settings are given in the Tables 58 to 61.

	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8	Gate 9	Gate 10	Gate 11	Gate 12
Your Rewards	6	3	10	9	4	6	3	9	6	7	2	10
Your Penalties	-1	-8	-5	-4	-10	-4	-6	-10	-3	-1	-2	-9
Guards' Rewards	8	1	6	1	6	4	2	5	10	1	3	7
Guards' Penalties	-1	-7	-3	-2	-9	-1	-8	-9	-2	-8	-2	-7

Table 58: Payoff Structure for 12 Gate Settings in Cluster 1

	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8	Gate 9	Gate 10	Gate 11	Gate 12
Your Rewards	3	3	7	3	4	9	7	5	9	4	10	2
Your Penalties	-2	-10	-8	-4	-10	-1	-7	-6	-8	-9	-3	-1
Guards' Rewards	1	2	2	7	4	6	3	9	2	10	1	2
Guards' Penalties	-2	-2	-9	-5	-9	-9	-10	-8	-8	-4	-3	-1

Table 59: Payoff Structure for 12 Gate Settings in Cluster 2

	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8	Gate 9	Gate 10	Gate 11	Gate 12
Your Rewards	8	6	5	9	2	3	6	10	4	6	4	1
Your Penalties	-9	-2	-5	-3	-5	-6	-10	-8	-1	-8	-9	-2
Guards' Rewards	1	7	1	3	7	3	2	7	9	8	3	6
Guards' Penalties	-7	-3	-8	-10	-5	-6	-4	-6	-6	-3	-4	-2

Table 60: Payoff Structure for 12 Gate Settings in Cluster 3

	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8	Gate 9	Gate 10	Gate 11	Gate 12
Your Rewards	7	8	9	5	3	1	7	7	8	8	5	2
Your Penalties	-4	-8	-9	-8	-1	-1	-2	-10	-10	-3	-2	-5
Guards' Rewards	1	8	9	9	4	2	8	7	10	5	1	2
Guards' Penalties	-6	-5	-6	-5	-1	-8	-1	-8	-4	-5	-10	-2

Table 61: Payoff Structure for 12 Gate Settings in Cluster 4

Appendix E: Payoff Structures for 15 Gate Settings

The payoff structures for 15 gate settings are given in the Tables 62 to 65.

	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8
Your Rewards	7	1	4	7	5	7	3	6
Your Penalties	-2	-2	-3	-10	-6	-3	-3	-6
Guards' Rewards	3	2	2	10	10	6	10	1
Guards' Penalties	-2	-6	-5	-7	-4	-7	-9	-2
	Gate 9	**Gate 10**	**Gate 11**	**Gate 12**	**Gate 13**	**Gate 14**	**Gate 15**	
Your Rewards	9	10	2	5	3	3	6	
Your Penalties	-4	-10	-1	-4	-1	-2	-5	
Guards' Rewards	8	10	1	7	5	1	9	
Guards' Penalties	-10	-1	-7	-5	-2	-8	-3	

Table 62: Payoff Structure for 15 Gate Settings in Cluster 1

	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8
Your Rewards	1	9	4	5	7	4	1	4
Your Penalties	-2	-9	-3	-4	-1	-2	-3	-6
Guards' Rewards	2	3	3	9	6	9	10	8
Guards' Penalties	-5	-4	-9	-10	-4	-4	-10	-5
	Gate 9	**Gate 10**	**Gate 11**	**Gate 12**	**Gate 13**	**Gate 14**	**Gate 15**	
Your Rewards	3	7	4	2	8	3	7	
Your Penalties	-8	-2	-9	-3	-4	-1	-9	
Guards' Rewards	1	1	9	2	7	7	9	
Guards' Penalties	-3	-8	-2	-10	-4	-8	-9	

Table 63: Payoff Structure for 15 Gate Settings in Cluster 2

	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8
Your Rewards	4	4	3	1	7	6	10	5
Your Penalties	-10	-4	-1	-9	-10	-7	-8	-4
Guards' Rewards	2	6	5	9	8	10	1	6
Guards' Penalties	-7	-6	-6	-8	-8	-2	-5	-5
	Gate 9	Gate 10	Gate 11	Gate 12	Gate 13	Gate 14	Gate 15	
Your Rewards	1	7	10	2	1	6	8	
Your Penalties	-4	-6	-5	-5	-8	-8	-8	
Guards' Rewards	7	8	3	5	4	6	1	
Guards' Penalties	-5	-9	-6	-5	-4	-4	-4	

Table 64: Payoff Structure for 15 Gate Settings in Cluster 3

	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8
Your Rewards	7	3	3	8	10	8	1	2
Your Penalties	-3	-1	-2	-10	-1	-3	-5	-9
Guards' Rewards	3	4	1	9	7	5	8	7
Guards' Penalties	-3	-1	-5	-1	-7	-9	-5	-8
	Gate 9	Gate 10	Gate 11	Gate 12	Gate 13	Gate 14	Gate 15	
Your Rewards	4	10	1	9	8	6	6	
Your Penalties	-9	-10	-2	-3	-5	-9	-5	
Guards' Rewards	7	3	4	9	8	4	10	
Guards' Penalties	-1	-4	-9	-6	-8	-8	-8	

Table 65: Payoff Structure for 15 Gate Settings in Cluster 4

Appendix F: Defender Strategies for 3 Gate Settings

The defender strategies for 3 gate settings are given in the tables 66 to 69.

Game Model	Gate 1	Gate 2	Gate 3
PT	0.44	0.19	0.37
PT-Attract	0.43	0.19	0.38
COBRA (α = 0.15)	0.38	0.32	0.3
COBRA (α = 0.5)	0.42	0.31	0.27
DOBSS	0.5	0.29	0.21
QRE (λ = 0.45)	0.34	0.39	0.27
QRE (λ = 0.76)	0.35	0.37	0.27

Table 66: Defender Strategies for 3 Gate Settings in Cluster 1

Game Model	Gate 1	Gate 2	Gate 3
PT	0.22	0.51	0.27
PT-Attract	0.23	0.49	0.28
COBRA (α = 0.15)	0.3	0.34	0.36
COBRA (α = 0.5)	0.28	0.33	0.39
DOBSS	0.29	0.36	0.35
QRE (λ = 0.45)	0.25	0.28	0.46
QRE (λ = 0.76)	0.26	0.29	0.45

Table 67: Defender Strategies for 3 Gate Settings in Cluster 2

Game Model	Gate 1	Gate 2	Gate 3
PT	0.27	0.22	0.5
PT-Attract	0.28	0.24	0.49
COBRA (α = 0.15)	0.42	0.19	0.4
COBRA (α = 0.5)	0.48	0.09	0.44
DOBSS	0.36	0.12	0.52
QRE (λ = 0.45)	0.38	0.35	0.27
QRE (λ = 0.76)	0.4	0.29	0.31

Table 68: Defender Strategies for 3 Gate Settings in Cluster 3

Game Model	Gate 1	Gate 2	Gate 3
PT	0.1	0.47	0.43
PT-Attract	0.11	0.45	0.44
COBRA (α = 0.15)	0.04	0.43	0.53
COBRA (α = 0.5)	0	0.45	0.55
DOBSS	0.03	0.52	0.46
QRE (λ = 0.45)	0.14	0.39	0.47
QRE (λ = 0.76)	0.15	0.39	0.46

Table 69: Defender Strategies for 3 Gate Settings in Cluster 4

Appendix G: Defender Strategies for 6 Gate Settings

The defender strategies for 6 gate settings are given in the tables 70 to 73.

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6
PT	0.25	0.54	0.13	0.37	0.46	0.24
PT-Attract	0.28	0.58	0.16	0.43	0.24	0.31
COBRA (α = 0.15)	0.43	0.65	0.22	0.4	0.14	0.16
COBRA (α = 0.5)	0.49	0.66	0.16	0.4	0.15	0.14
DOBSS	0.36	0.62	0.18	0.31	0.45	0.08
QRE (λ = 0.45)	0.31	0.58	0.41	0.5	0.06	0.14
QRE (λ = 0.76)	0.34	0.59	0.36	0.42	0.11	0.18

Table 70: Defender Strategies for 6 Gate Settings in Cluster 1

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6
PT	0.6	0.2	0.4	0.36	0.15	0.29
PT-Attract	0.63	0.22	0.46	0.21	0.17	0.31
COBRA (α = 0.15)	0.76	0.25	0.11	0.33	0.05	0.5
COBRA (α = 0.5)	0.66	0.28	0.18	0.29	0.16	0.43
DOBSS	0.59	0.27	0.14	0.45	0.16	0.39
QRE (λ = 0.45)	0.56	0.12	0.34	0.25	0.34	0.39
QRE (λ = 0.76)	0.58	0.18	0.28	0.28	0.29	0.39

Table 71: Defender Strategies for 6 Gate Settings in Cluster 2

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6
PT	0.27	0.34	0.23	0.27	0.2	0.7
PT-Attract	0.33	0.39	0.27	0.34	0.37	0.3
COBRA (α = 0.15)	0.43	0.52	0.45	0.41	0.01	0.18
COBRA (α = 0.5)	0.44	0.52	0.47	0.4	0	0.17
DOBSS	0.35	0.44	0.38	0.32	0	0.51
QRE (λ = 0.45)	0.46	0.5	0.32	0.36	0.12	0.23
QRE (λ = 0.76)	0.45	0.49	0.36	0.36	0.1	0.26

Table 72: Defender Strategies for 6 Gate Settings in Cluster 3

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6
PT	0.39	0.06	0.34	0.13	0.38	0.72
PT-Attract	0.48	0.06	0.36	0.15	0.4	0.55
COBRA (α = 0.15)	0.31	0.08	0.49	0.28	0.33	0.52
COBRA (α = 0.5)	0.36	0	0.5	0.25	0.36	0.53
DOBSS	0.15	0.05	0.47	0.24	0.47	0.62
QRE (λ = 0.45)	0.38	0	0.47	0.26	0.31	0.58
QRE (λ = 0.76)	0.35	0	0.47	0.28	0.34	0.57

Table 73: Defender Strategies for 6 Gate Settings in Cluster 4

Appendix H: Defender Strategies for 9 Gate Settings

The defender strategies for 9 gate settings are given in the tables 74 to 77.

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8	Gate 9
PT	0.43	0.29	0.15	0.19	0.54	0.19	0.71	0.11	0.4
PT-Attract	0.19	0.31	0.16	0.22	0.57	0.2	0.73	0.19	0.42
COBRA (α = 0.15)	0.27	0.42	0.27	0.03	0.54	0.31	0.61	0	0.55
COBRA (α = 0.5)	0.25	0.42	0.17	0	0.6	0.24	0.73	0	0.6
DOBSS	0.49	0.38	0.26	0.06	0.49	0.29	0.55	0	0.49
QRE (λ = 0.45)	0.23	0.42	0.29	0.22	0.55	0.26	0.61	0	0.42
QRE (λ = 0.76)	0.25	0.43	0.29	0.18	0.53	0.27	0.59	0	0.46

Table 74: Defender Strategies for 9 Gate Settings in Cluster 1

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8	Gate 9
PT	0.21	0.24	0.28	0.21	0.3	0.44	0.77	0.17	0.38
PT-Attract	0.29	0.14	0.33	0.24	0.34	0.42	0.58	0.2	0.46
COBRA (α = 0.15)	0.13	0.17	0.36	0.3	0.45	0.44	0.6	0.23	0.32
COBRA (α = 0.5)	0	0.23	0.37	0.35	0.52	0.48	0.63	0.14	0.28
DOBSS	0.09	0.32	0.32	0.33	0.41	0.37	0.69	0.21	0.26
QRE (λ = 0.45)	0.17	0.15	0.48	0.15	0.48	0.42	0.62	0.24	0.29
QRE (λ = 0.76)	0.16	0.16	0.45	0.2	0.47	0.39	0.61	0.24	0.32

Table 75: Defender Strategies for 9 Gate Settings in Cluster 2

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8	Gate 9
PT	0.04	0.67	0.69	0.26	0.23	0.14	0.12	0.26	0.6
PT-Attract	0.07	0.76	0.31	0.32	0.28	0.21	0.25	0.31	0.49
COBRA (α = 0.15)	0	0.78	0.27	0.46	0.41	0.23	0.01	0.4	0.46
COBRA (α = 0.5)	0	0.92	0.3	0.44	0.42	0.02	0	0.42	0.48
DOBSS	0	0.63	0.52	0.37	0.37	0.16	0	0.39	0.56
QRE (λ = 0.45)	0.16	0.66	0.31	0.22	0.38	0.33	0.08	0.37	0.49
QRE (λ = 0.76)	0.12	0.68	0.32	0.3	0.38	0.31	0.05	0.37	0.48

Table 76: Defender Strategies for 9 Gate Settings in Cluster 3

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8	Gate 9
PT	0.15	0.22	0.33	0.3	0.72	0.36	0.38	0.08	0.49
PT-Attract	0.18	0.29	0.29	0.33	0.8	0.34	0.43	0.11	0.25
COBRA (α = 0.15)	0.32	0	0	0.41	1	0.39	0.58	0	0.3
COBRA (α = 0.5)	0.24	0.2	0.06	0.44	0.77	0.43	0.51	0	0.35
DOBSS	0.23	0.16	0	0.44	0.67	0.5	0.45	0	0.54
QRE (λ = 0.45)	0.33	0.4	0.09	0.33	0.67	0.3	0.56	0.09	0.23
QRE (λ = 0.76)	0.31	0.35	0.05	0.36	0.69	0.34	0.54	0.07	0.28

Table 77: Defender Strategies for 9 Gate Settings in Cluster 4

Appendix I: Defender Strategies for 12 Gate Settings

The defender strategies for 12 gate settings are given in the tables 78 to 81.

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8	Gate 9	Gate 10	Gate 11	Gate 12
PT	0.71	0.07	0.42	0.45	0.08	0.33	0.1	0.21	0.41	0.75	0.21	0.26
PT-Attract	0.28	0.1	0.47	0.51	0.1	0.4	0.15	0.24	0.29	0.84	0.34	0.3
COBRA (α = 0.15)	0.19	0.12	0.42	0.64	0.17	0.35	0.15	0.43	0.23	0.76	0.06	0.5
COBRA (α = 0.5)	0.21	0	0.43	0.8	0	0.37	0	0.44	0.24	0.98	0	0.54
DOBSS	0.57	0.07	0.52	0.52	0.13	0.38	0.08	0.36	0.42	0.59	0	0.41
QRE (λ = 0.45)	0.24	0.28	0.38	0.41	0.33	0.17	0.32	0.48	0.27	0.68	0	0.46
QRE (λ = 0.76)	0.26	0.23	0.4	0.49	0.28	0.24	0.26	0.45	0.27	0.67	0	0.45

Table 78: Defender Strategies for 12 Gate Settings in Cluster 1

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6
PT	0.42	0.07	0.24	0.22	0.1	0.87
PT-Attract	0.55	0.1	0.29	0.32	0.14	0.58
COBRA (α = 0.15)	0.36	0	0.43	0.16	0.05	0.61
COBRA (α = 0.5)	0.34	0.13	0.42	0.29	0.2	0.54
DOBSS	0.29	0.11	0.36	0.21	0.17	0.74
QRE (λ = 0.45)	0	0.06	0.51	0.29	0.36	0.64
QRE (λ = 0.76)	0.09	0.09	0.48	0.27	0.32	0.62

Game Model	Gate 7	Gate 8	Gate 9	Gate 10	Gate 11	Gate 12
PT	0.27	0.23	0.3	0.11	0.62	0.55
PT-Attract	0.33	0.3	0.35	0.16	0.66	0.22
COBRA (α = 0.15)	0.48	0.34	0.57	0.08	0.78	0.14
COBRA (α = 0.5)	0.45	0.38	0.5	0.22	0.51	0.02
DOBSS	0.39	0.31	0.44	0.19	0.65	0.14
QRE (λ = 0.45)	0.53	0.41	0.53	0.22	0.45	0
QRE (λ = 0.76)	0.51	0.39	0.52	0.21	0.5	0

Table 79: Defender Strategies for 12 Gate Settings in Cluster 2

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6
PT	0.25	0.64	0.31	0.62	0.14	0.16
PT-Attract	0.31	0.34	0.39	0.7	0.23	0.24
COBRA (α = 0.15)	0.46	0.28	0.45	0.75	0.14	0.24
COBRA (α = 0.5)	0.51	0.34	0.5	0.8	0.01	0.15
DOBSS	0.39	0.57	0.36	0.63	0.08	0.17
QRE (λ = 0.45)	0.47	0.32	0.47	0.69	0.19	0.28
QRE (λ = 0.76)	0.46	0.33	0.46	0.66	0.17	0.27
Game Model	**Gate 7**	**Gate 8**	**Gate 9**	**Gate 10**	**Gate 11**	**Gate 12**
PT	0.17	0.34	0.78	0.22	0.13	0.25
PT-Attract	0.22	0.39	0.43	0.28	0.18	0.3
COBRA (α = 0.15)	0.35	0.45	0.39	0.26	0.25	0
COBRA (α = 0.5)	0.32	0.49	0.43	0.31	0.16	0
DOBSS	0.29	0.48	0.51	0.33	0.2	0
QRE (λ = 0.45)	0.3	0.42	0.45	0.23	0.22	0
QRE (λ = 0.76)	0.32	0.43	0.43	0.24	0.24	0

Table 80: Defender Strategies for 12 Gate Settings in Cluster 3

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6
PT	0.4	0.25	0.25	0.15	0.57	0.28
PT-Attract	0.44	0.27	0.27	0.17	0.51	0.41
COBRA (α = 0.15)	0.51	0.41	0.42	0.24	0.34	0
COBRA (α = 0.5)	0.62	0.45	0.46	0.16	0.28	0
DOBSS	0.47	0.38	0.4	0.24	0.28	0
QRE (λ = 0.45)	0.56	0.39	0.41	0.32	0.02	0.14
QRE (λ = 0.76)	0.56	0.4	0.41	0.31	0.11	0.01
Game Model	**Gate 7**	**Gate 8**	**Gate 9**	**Gate 10**	**Gate 11**	**Gate 12**
PT	0.61	0.17	0.2	0.53	0.51	0.1
PT-Attract	0.28	0.19	0.21	0.56	0.56	0.12
COBRA (α = 0.15)	0.28	0.31	0.36	0.62	0.5	0
COBRA (α = 0.5)	0.22	0.28	0.35	0.6	0.59	0
DOBSS	0.57	0.3	0.34	0.56	0.45	0.02
QRE (λ = 0.45)	0.24	0.43	0.31	0.51	0.66	0
QRE (λ = 0.76)	0.26	0.4	0.32	0.53	0.62	0.06

Table 81: Defender Strategies for 12 Gate Settings in Cluster 4

Appendix J: Defender Strategies for 15 Gate Settings

The defender strategies for 15 gate settings are given in the tables 82 to 85.

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8
PT	0.6	0.11	0.32	0.17	0.2	0.48	0.24	0.24
PT-Attract	0.47	0.17	0.36	0.12	0.23	0.51	0.29	0.27
COBRA (α = 0.15)	0.65	0	0.33	0.34	0.31	0.58	0.19	0.38
COBRA (α = 0.5)	0.66	0	0.36	0.3	0.29	0.64	0.14	0.4
DOBSS	0.59	0	0.32	0.31	0.3	0.53	0.21	0.36
QRE (λ = 0.45)	0.36	0.01	0.38	0.38	0.3	0.54	0.43	0.28
QRE (λ = 0.76)	0.45	0	0.38	0.37	0.3	0.53	0.36	0.35

Game Model	Gate 9	Gate 10	Gate 11	Gate 12	Gate 13	Gate 14	Gate 15
PT	0.47	0.24	0.43	0.3	0.55	0.35	0.29
PT-Attract	0.5	0.12	0.53	0.34	0.34	0.4	0.28
COBRA (α = 0.15)	0.62	0.28	0	0.38	0.29	0.23	0.42
COBRA (α = 0.5)	0.63	0.21	0	0.43	0.37	0.23	0.36
DOBSS	0.56	0.41	0.09	0.36	0.32	0.25	0.39
QRE (λ = 0.45)	0.59	0.16	0.27	0.39	0.15	0.45	0.3
QRE (λ = 0.76)	0.58	0.19	0.21	0.38	0.18	0.4	0.32

Table 82: Defender Strategies for 15 Gate Settings in Cluster 1

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8
PT	0.16	0.26	0.36	0.33	0.82	0.48	0.1	0.18
PT-Attract	0.28	0.29	0.42	0.39	0.44	0.35	0.17	0.22
COBRA (α = 0.15)	0	0.46	0.45	0.47	0.42	0.33	0	0.3
COBRA (α = 0.5)	0	0.54	0.48	0.53	0.44	0.34	0	0.24
DOBSS	0	0.43	0.38	0.4	0.7	0.44	0	0.26
QRE (λ = 0.45)	0	0.36	0.48	0.47	0.41	0.31	0.28	0.27
QRE (λ = 0.76)	0	0.4	0.47	0.46	0.43	0.31	0.19	0.28

Game Model	Gate 9	Gate 10	Gate 11	Gate 12	Gate 13	Gate 14	Gate 15	
PT	0.09	0.63	0.11	0.2	0.46	0.62	0.2	
PT-Attract	0.12	0.7	0.14	0.28	0.41	0.56	0.23	
COBRA (α = 0.15)	0.16	0.73	0.2	0.18	0.38	0.54	0.38	
COBRA (α = 0.5)	0.01	0.94	0.11	0.01	0.4	0.56	0.4	
DOBSS	0.15	0.62	0.2	0.12	0.55	0.4	0.35	
QRE (λ = 0.45)	0.1	0.63	0.11	0.36	0.35	0.46	0.41	
QRE (λ = 0.76)	0.16	0.65	0.13	0.32	0.36	0.44	0.4	

Table 83: Defender Strategies for 15 Gate Settings in Cluster 2

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8
PT	0.15	0.41	0.9	0.07	0.24	0.31	0.39	0.46
PT-Attract	0.19	0.48	0.53	0.1	0.28	0.15	0.43	0.44
COBRA (α = 0.15)	0.23	0.42	0.54	0	0.4	0.16	0.57	0.45
COBRA (α = 0.5)	0.02	0.18	0.06	0	0.33	0.02	0.54	0.3
DOBSS	0.23	0.4	0.54	0.02	0.36	0.4	0.51	0.46
QRE (λ = 0.45)	0.34	0.38	0.45	0.21	0.4	0.17	0.5	0.38
QRE (λ = 0.76)	0.32	0.38	0.44	0.17	0.4	0.17	0.52	0.39

Game Model	Gate 9	Gate 10	Gate 11	Gate 12	Gate 13	Gate 14	Gate 15	
PT	0.21	0.4	0.54	0.22	0.08	0.28	0.34	
PT-Attract	0.3	0.45	0.59	0.29	0.11	0.32	0.38	
COBRA (α = 0.15)	0	0.53	0.66	0.14	0	0.4	0.5	
COBRA (α = 0.5)	0	0.43	0.47	0	0	0.22	0.45	
DOBSS	0.03	0.47	0.61	0.16	0.02	0.37	0.45	
QRE (λ = 0.45)	0.15	0.49	0.54	0.22	0.08	0.3	0.41	
QRE (λ = 0.76)	0.12	0.48	0.56	0.21	0.08	0.31	0.44	

Table 84: Defender Strategies for 15 Gate Settings in Cluster 3

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8
PT	0.49	0.58	0.37	0.2	0.86	0.53	0.04	0.05
PT-Attract	0.56	0.36	0.49	0.18	0.56	0.59	0.08	0.07
COBRA (α = 0.15)	0.6	0.35	0.32	0.18	0.56	0.65	0	0.01
COBRA (α = 0.5)	0.71	0.27	0.15	0.26	0.61	0.76	0	0
DOBSS	0.52	0.29	0.23	0.34	0.74	0.56	0	0.02
QRE (λ = 0.45)	0.41	0	0.29	0.16	0.6	0.61	0.11	0.24
QRE (λ = 0.76)	0.46	0.09	0.32	0.18	0.6	0.61	0.06	0.19
Game Model	**Gate 9**	**Gate 10**	**Gate 11**	**Gate 12**	**Gate 13**	**Gate 14**	**Gate 15**	
PT	0.1	0.25	0.13	0.56	0.38	0.17	0.3	
PT-Attract	0.13	0.28	0.27	0.45	0.43	0.2	0.36	
COBRA (α = 0.15)	0.18	0.45	0	0.45	0.54	0.3	0.43	
COBRA (α = 0.5)	0.01	0.49	0	0.51	0.6	0.23	0.43	
DOBSS	0.17	0.41	0	0.6	0.47	0.28	0.38	
QRE (λ = 0.45)	0.09	0.43	0.2	0.49	0.51	0.42	0.45	
QRE (λ = 0.76)	0.13	0.44	0.1	0.49	0.51	0.4	0.43	

Table 85: Defender Strategies for 15 Gate Settings in Cluster 4

Appendix K: Participants' Choices for Cluster 1

The participants' choices for cluster 1 are illustrated in the Tables 86 to 90 and Figures 23 to 27.

Game Model	Gate 1	Gate 2	Gate 3
PT	0.19	0.52	0.29
PT-Attract	0.19	0.62	0.19
COBRA (Alpha = 0.15)	0.1	0.71	0.19
COBRA (Alpha = 0.5)	0.14	0.62	0.24
DOBSS	0.1	0.71	0.19
QRE (Lambda = 0.45)	0.1	0.71	0.19
QRE (Lambda = 0.76)	0.14	0.48	0.38

Table 86: Participants' Choices (in fraction) for 3 Gate Settings in Cluster 1

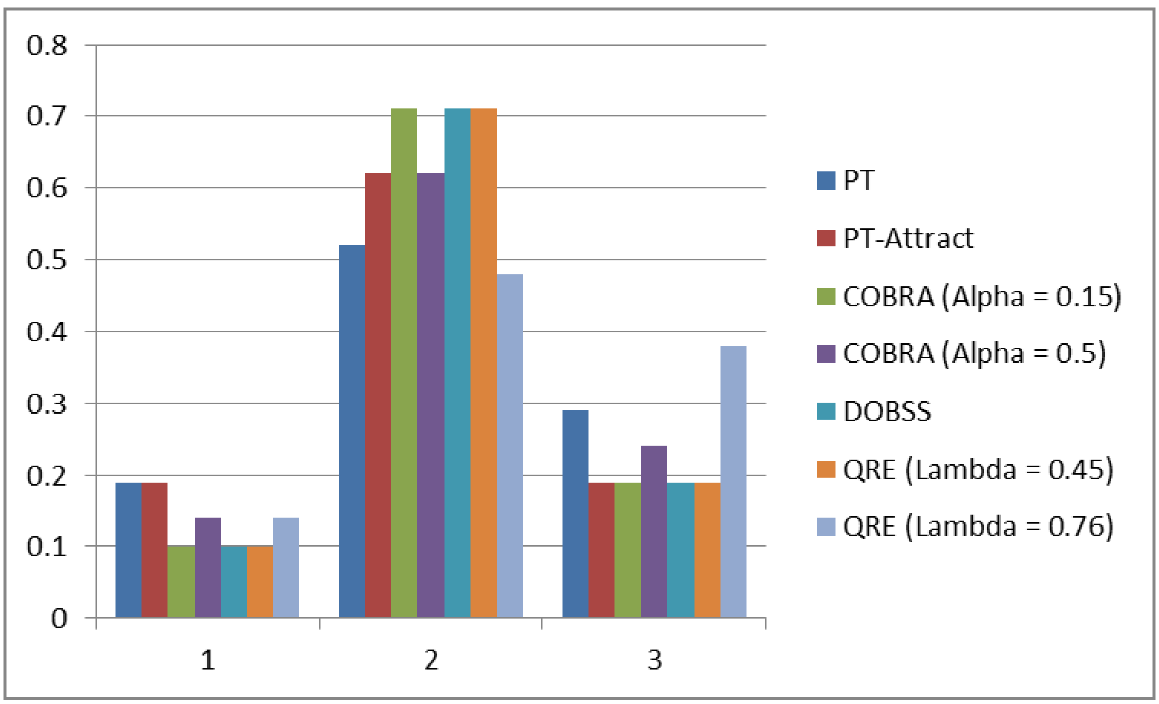

Figure 23: *Participants' Choices* against *Gate Number* for 3 Gate Settings in Cluster 1

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6
PT	0.52	0.14	0	0.19	0.14	0
PT-Attract	0.52	0	0.05	0.33	0.1	0
COBRA (Alpha = 0.15)	0.33	0	0.14	0.38	0.14	0
COBRA (Alpha = 0.5)	0.38	0	0.19	0.29	0.1	0.05
DOBSS	0.57	0.05	0.29	0.1	0	0
QRE (Lambda = 0.45)	0.62	0.05	0.05	0.14	0.05	0.1
QRE (Lambda = 0.76)	0.71	0.1	0.1	0.1	0	0

Table 87: Participants' Choices (in fraction) for 6 Gate Settings in Cluster 1

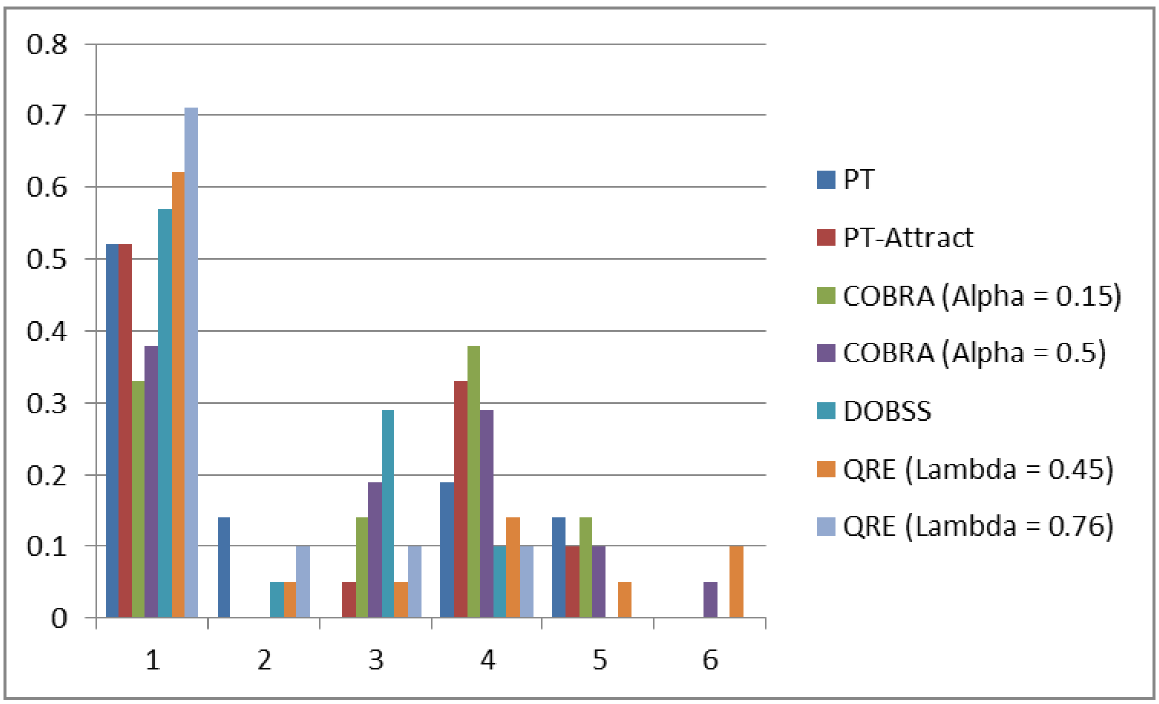

Figure 24: *Participants' Choices* against *Gate Number* for 6 Gate Settings in Cluster 1

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8	Gate 9
PT	0	0.05	0.05	0.05	0.14	0	0.67	0.05	0
PT-Attract	0	0.1	0	0	0.05	0.14	0.71	0	0
COBRA (Alpha = 0.15)	0.1	0.14	0	0.05	0	0.1	0.62	0	0
COBRA (Alpha = 0.5)	0.19	0	0	0.1	0.05	0.1	0.52	0	0.05
DOBSS	0.1	0.05	0.29	0	0.14	0.14	0.24	0	0.05
QRE (Lambda = 0.45)	0.1	0.05	0.1	0.1	0.38	0	0.29	0	0
QRE (Lambda = 0.76)	0	0.29	0.05	0.05	0.29	0.1	0.19	0.05	0

Table 88: Participants' Choices (in fraction) for 9 Gate Settings in Cluster 1

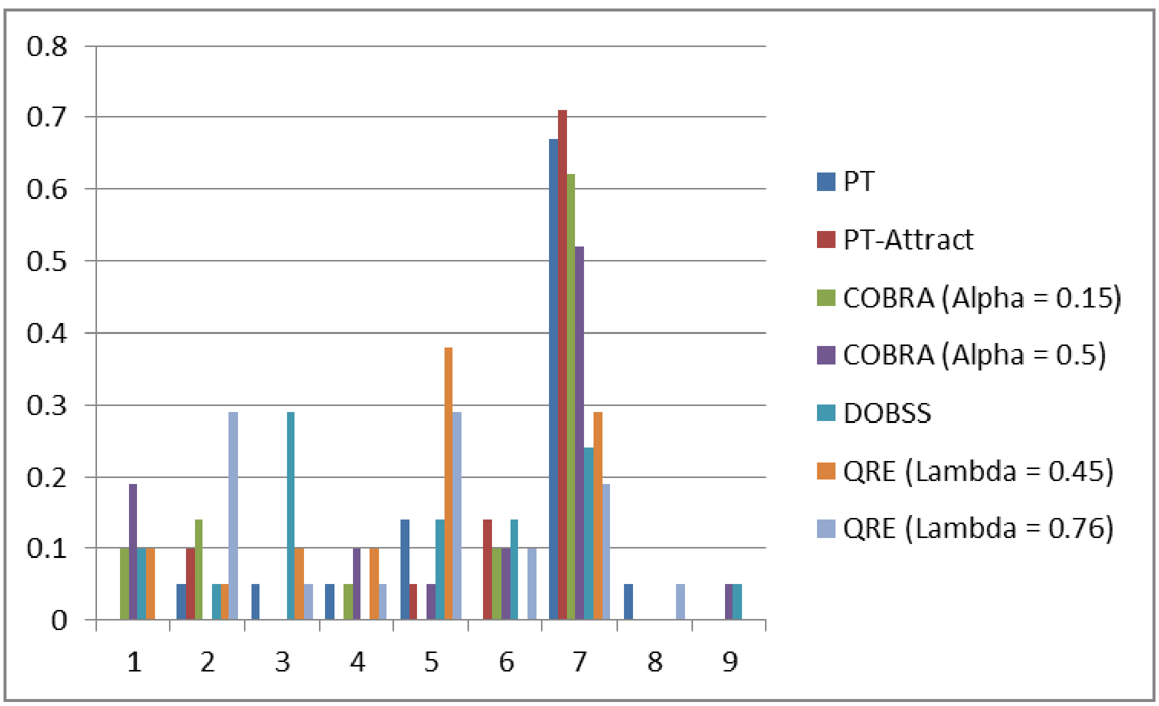

Figure 25: *Participants' Choices* against *Gate Number* for 9 Gate Settings in Cluster 1

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6
PT	0	0.05	0.1	0	0.14	0.33
PT-Attract	0	0	0	0	0.05	0.76
COBRA (Alpha = 0.15)	0	0.05	0.14	0	0.24	0.33
COBRA (Alpha = 0.5)	0	0	0.05	0	0.05	0.1
DOBSS	0	0	0.05	0	0	0.05
QRE (Lambda = 0.45)	0	0	0.05	0	0	0
QRE (Lambda = 0.76)	0	0	0.05	0	0.38	0.1
Game Model	**Gate 7**	**Gate 8**	**Gate 9**	**Gate 10**	**Gate 11**	**Gate 12**
PT	0	0	0	0	0.19	0.19
PT-Attract	0	0	0	0	0.05	0.14
COBRA (Alpha = 0.15)	0	0	0	0	0.05	0.19
COBRA (Alpha = 0.5)	0	0	0	0	0.76	0.05
DOBSS	0	0	0	0.05	0.86	0
QRE (Lambda = 0.45)	0	0	0	0	0.95	0
QRE (Lambda = 0.76)	0	0	0	0	0.48	0

Table 89: Participants' Choices (in fraction) for 12 Gate Settings in Cluster 1

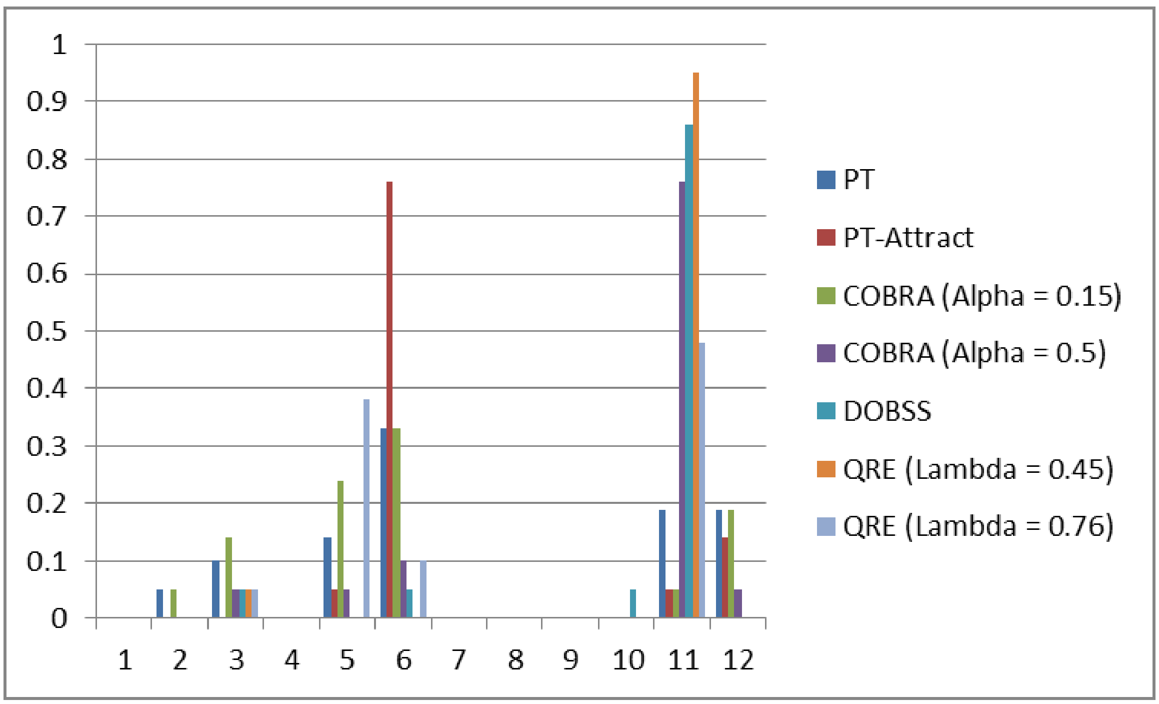

Figure 26: *Participants' Choices* against *Gate Number* for 12 Gate Settings in Cluster 1

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8
PT	0	0	0	0	0.43	0	0	0
PT-Attract	0	0	0.24	0	0.38	0	0	0
COBRA (Alpha = 0.15)	0	0	0.05	0	0.81	0	0	0
COBRA (Alpha = 0.5)	0	0.05	0.05	0	0.62	0	0	0
DOBSS	0	0	0.19	0	0.48	0	0.05	0
QRE (Lambda = 0.45)	0	0.1	0.1	0	0.43	0.05	0	0
QRE (Lambda = 0.76)	0	0.05	0.1	0	0.52	0.05	0	0

Game Model	Gate 9	Gate 10	Gate 11	Gate 12	Gate 13	Gate 14	Gate 15
PT	0	0	0.05	0	0.52	0	0
PT-Attract	0	0	0.1	0	0.29	0	0
COBRA (Alpha = 0.15)	0	0	0.05	0	0.1	0	0
COBRA (Alpha = 0.5)	0	0.05	0.05	0	0.19	0	0
DOBSS	0	0.1	0	0	0.19	0	0
QRE (Lambda = 0.45)	0	0	0.14	0	0.19	0	0
QRE (Lambda = 0.76)	0	0.1	0.05	0	0.14	0	0

Table 90: Participants' Choices (in fraction) for 15 Gate Settings in Cluster 1

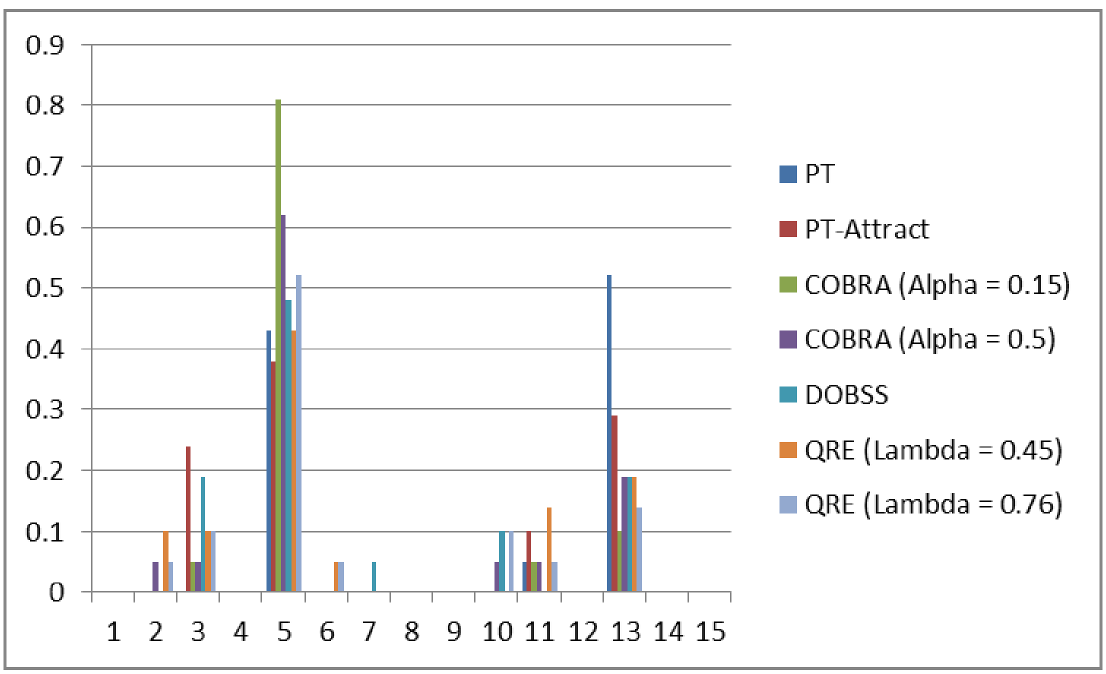

Figure 27: *Participants' Choices* against *Gate Number* for 15 Gate Settings in Cluster 1

Appendix L: Participants' Choices for Cluster 2

The participants' choices for cluster 1 are illustrated in the Tables 91 to 95 and Figures 28 to 32.

Game Model	Gate 1	Gate 2	Gate 3
PT	0.48	0.44	0.08
PT-Attract	0.32	0.56	0.12
COBRA (Alpha = 0.15)	0.28	0.68	0.04
COBRA (Alpha = 0.5)	0.24	0.72	0.04
DOBSS	0.32	0.6	0.08
QRE (Lambda = 0.45)	0.24	0.76	0
QRE (Lambda = 0.76)	0.16	0.84	0

Table 91: Participants' Choices (in fraction) for 3 Gate Settings in Cluster 2

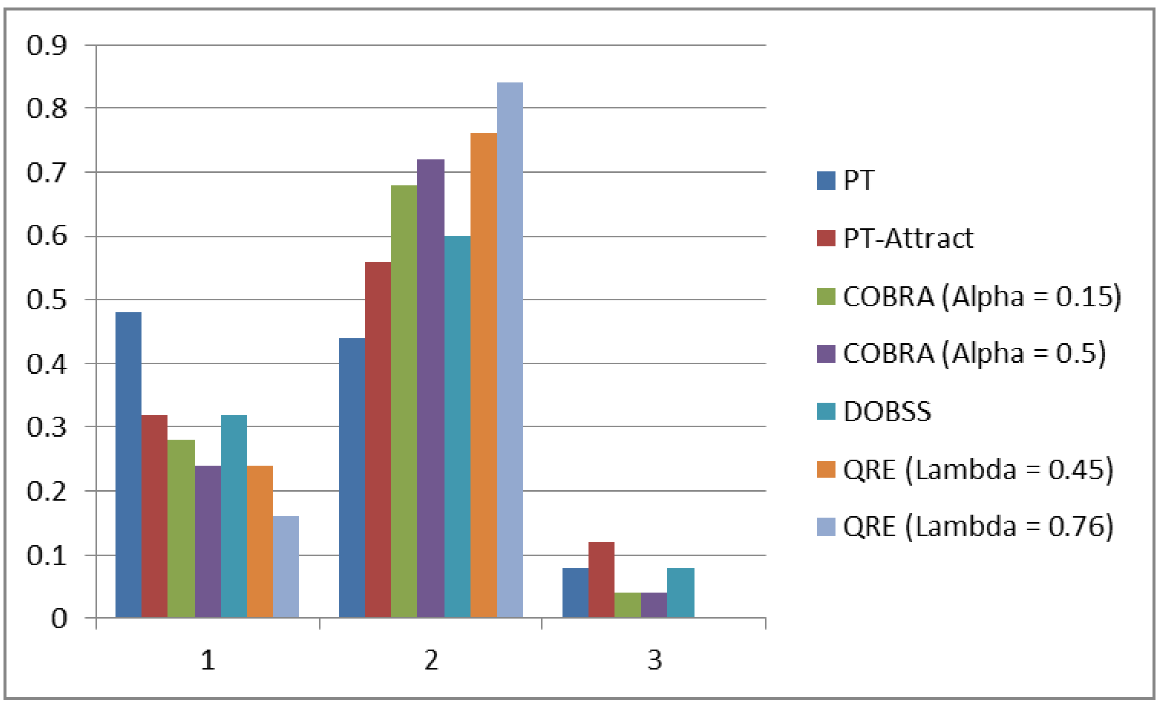

Figure 28: *Participants' Choices* against *Gate Number* for 3 Gate Settings in Cluster 2

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6
PT	0.36	0.16	0.12	0	0.16	0.2
PT-Attract	0.16	0	0.12	0.72	0	0
COBRA (Alpha = 0.15)	0.08	0	0.52	0.16	0.2	0.04
COBRA (Alpha = 0.5)	0.12	0	0.6	0.2	0.08	0
DOBSS	0.32	0.12	0.48	0.04	0	0.04
QRE (Lambda = 0.45)	0.76	0.08	0	0.12	0	0.04
QRE (Lambda = 0.76)	0.72	0.04	0	0.2	0	0.04

Table 92: Participants' Choices (in fraction) for 6 Gate Settings in Cluster 2

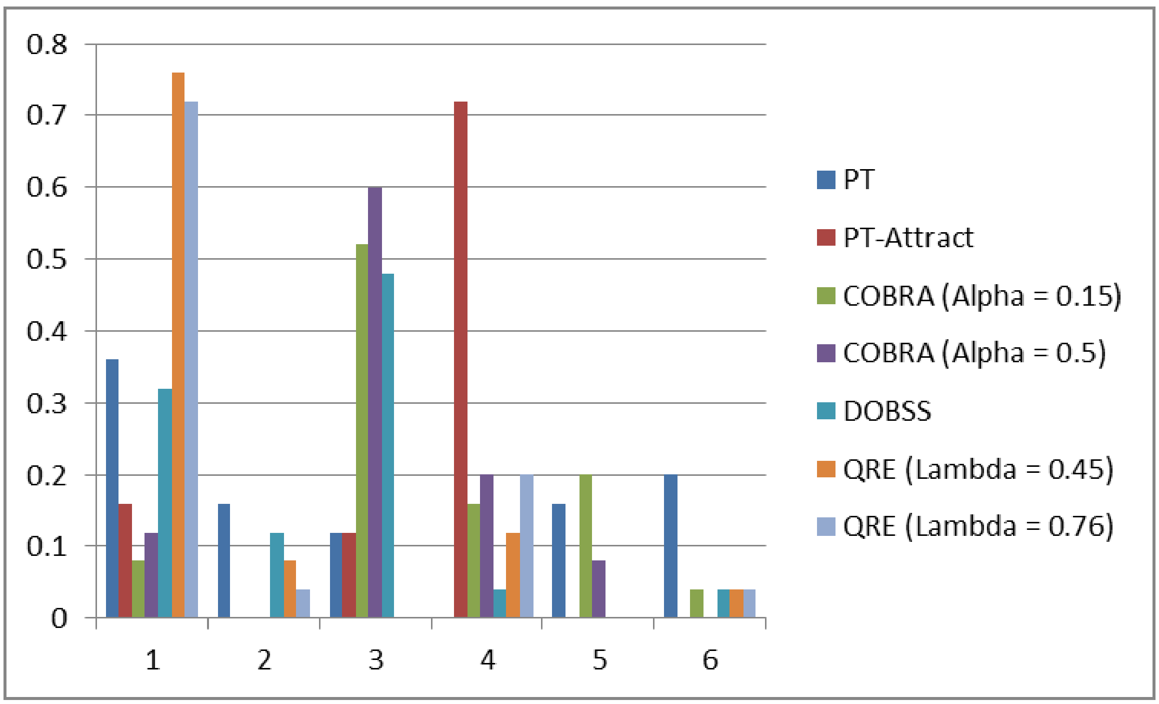

Figure 29: *Participants' Choices* against *Gate Number* for 6 Gate Settings in Cluster 2

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8	Gate 9
PT	0.04	0.08	0	0.04	0.04	0	0.76	0.04	0
PT-Attract	0	0.04	0.04	0.04	0	0.12	0.72	0	0.04
COBRA (Alpha = 0.15)	0.08	0.08	0	0.04	0.04	0.04	0.72	0	0
COBRA (Alpha = 0.5)	0	0.04	0.04	0	0	0.08	0.72	0.08	0.04
DOBSS	0	0.04	0.2	0	0.36	0	0.32	0.08	0
QRE (Lambda = 0.45)	0	0.08	0	0.04	0.44	0.04	0.32	0.04	0.04
QRE (Lambda = 0.76)	0.08	0.12	0.16	0	0.24	0	0.32	0	0.08

Table 93: Participants' Choices (in fraction) for 9 Gate Settings in Cluster 2

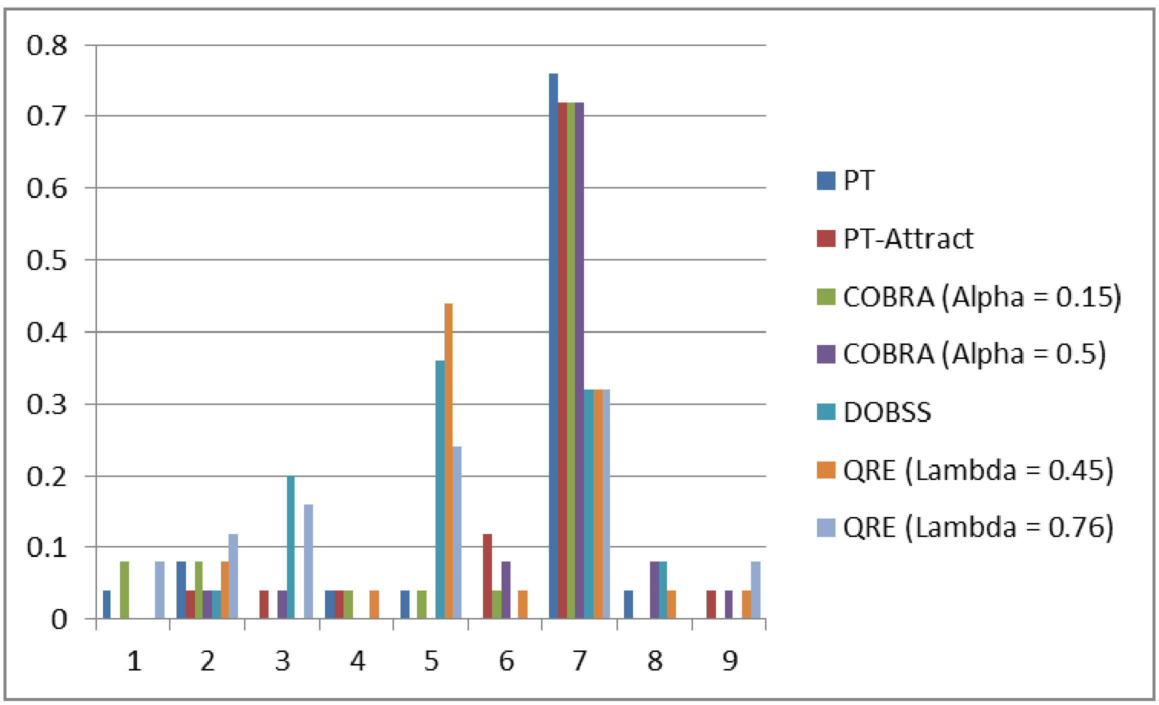

Figure 30: *Participants' Choices* against *Gate Number* for 9 Gate Settings in Cluster 2

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6
PT	0	0	0	0	0.04	0.48
PT-Attract	0	0	0.04	0	0.08	0.6
COBRA (Alpha = 0.15)	0	0.04	0.04	0	0.16	0.4
COBRA (Alpha = 0.5)	0	0.04	0.04	0	0.08	0.28
DOBSS	0	0	0	0	0	0.2
QRE (Lambda = 0.45)	0.08	0	0.08	0	0	0.12
QRE (Lambda = 0.76)	0	0	0.04	0	0.16	0.16
Game Model	**Gate 7**	**Gate 8**	**Gate 9**	**Gate 10**	**Gate 11**	**Gate 12**
PT	0	0.04	0.08	0.04	0.12	0.2
PT-Attract	0	0	0	0	0.08	0.2
COBRA (Alpha = 0.15)	0	0	0	0	0.08	0.28
COBRA (Alpha = 0.5)	0	0	0	0	0.56	0
DOBSS	0	0	0	0	0.8	0
QRE (Lambda = 0.45)	0	0	0	0	0.68	0.04
QRE (Lambda = 0.76)	0	0	0	0	0.6	0.04

Table 94: Participants' Choices (in fraction) for 12 Gate Settings in Cluster 2

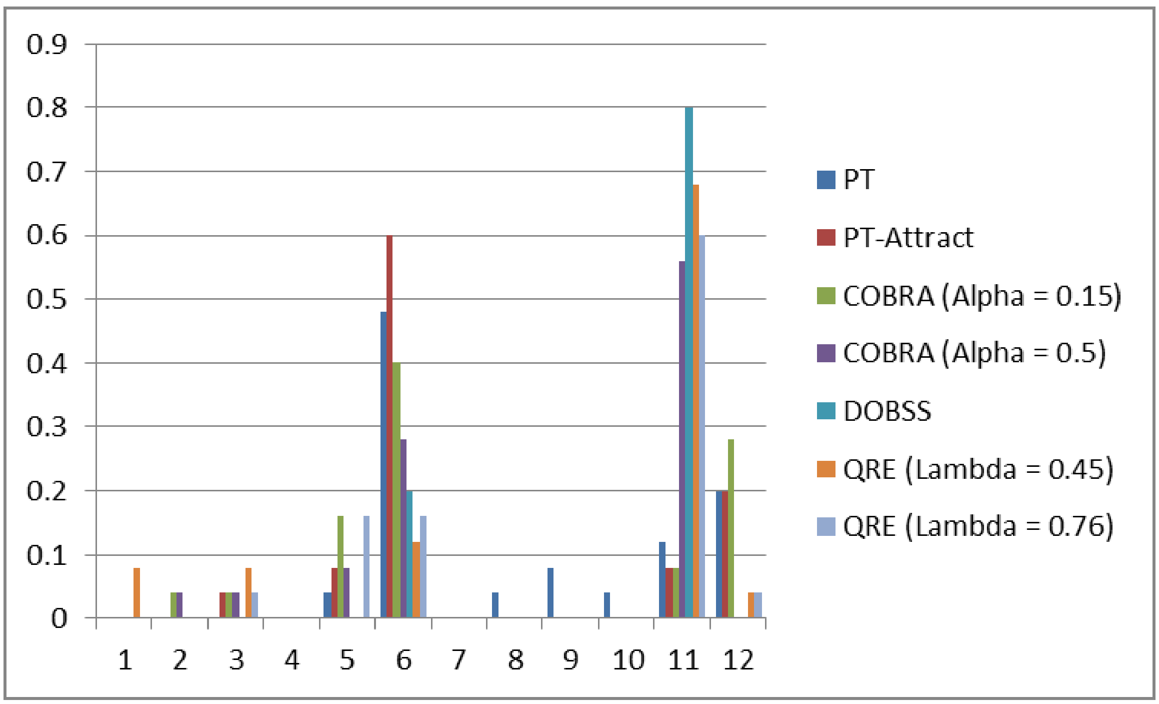

Figure 31: *Participants' Choices* against *Gate Number* for 12 Gate Settings in Cluster 2

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8
PT	0.08	0	0	0	0.16	0.04	0	0
PT-Attract	0	0.04	0.16	0	0.52	0	0.04	0
COBRA (Alpha = 0.15)	0	0	0.12	0	0.48	0	0	0
COBRA (Alpha = 0.5)	0.04	0	0.12	0	0.44	0	0	0
DOBSS	0.04	0	0.28	0	0.4	0	0	0
QRE (Lambda = 0.45)	0.04	0.04	0.24	0	0.48	0	0	0
QRE (Lambda = 0.76)	0	0	0.12	0	0.44	0.04	0	0.04
Game Model	**Gate 9**	**Gate 10**	**Gate 11**	**Gate 12**	**Gate 13**	**Gate 14**	**Gate 15**	
PT	0	0.04	0	0	0.6	0.04	0.04	
PT-Attract	0	0	0.04	0	0.2	0	0	
COBRA (Alpha = 0.15)	0	0	0.04	0	0.36	0	0	
COBRA (Alpha = 0.5)	0.04	0.04	0	0	0.32	0	0	
DOBSS	0	0	0.08	0	0.2	0	0	
QRE (Lambda = 0.45)	0	0	0.04	0	0.16	0	0	
QRE (Lambda = 0.76)	0	0	0	0	0.36	0	0	

Table 95: Participants’ Choices (in fraction) for 15 Gate Settings in Cluster 2

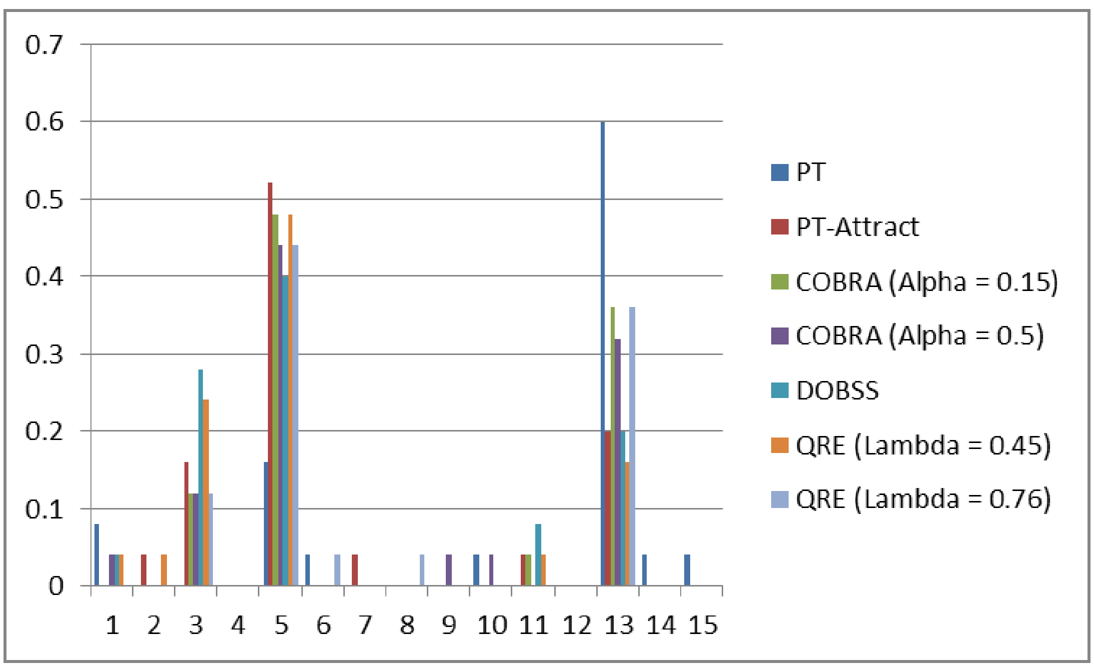

Figure 32: *Participants’ Choices* against *Gate Number* for 15 Gate Settings in Cluster 2

Appendix M: Participants' Choices for Cluster 3

The participants' choices for cluster 1 are illustrated in the Tables 96 to 100 and Figures 33 to 37.

Game Model	Gate 1	Gate 2	Gate 3
PT	0.22	0.43	0.35
PT-Attract	0.35	0.48	0.17
COBRA (Alpha = 0.15)	0.17	0.74	0.09
COBRA (Alpha = 0.5)	0.22	0.65	0.13
DOBSS	0.22	0.57	0.22
QRE (Lambda = 0.45)	0.13	0.83	0.04
QRE (Lambda = 0.76)	0.17	0.74	0.09

Table 96: Participants' Choices (in fraction) for 3 Gate Settings in Cluster 3

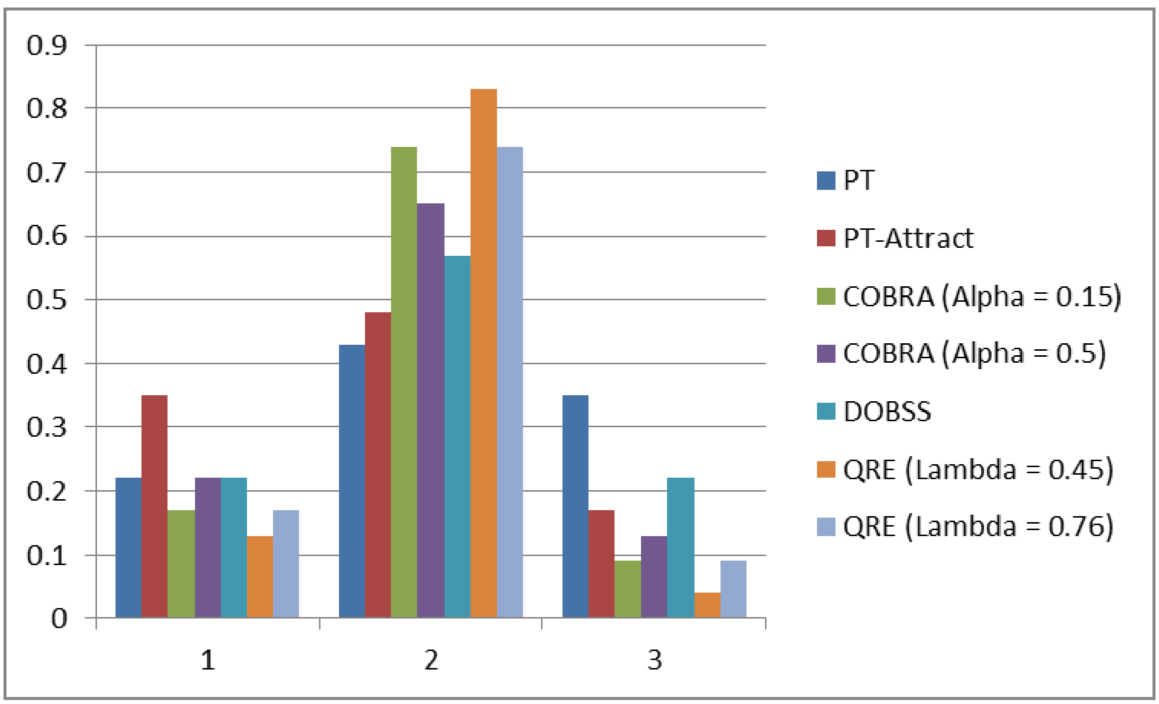

Figure 33: *Participants' Choices* against *Gate Number* for 3 Gate Settings in Cluster 3

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6
PT	0.26	0.26	0.09	0.13	0.22	0.04
PT-Attract	0.04	0	0.17	0.57	0.17	0.04
COBRA (Alpha = 0.15)	0.17	0	0.43	0.13	0.26	0
COBRA (Alpha = 0.5)	0.17	0.09	0.39	0.3	0.04	0
DOBSS	0.09	0.04	0.52	0.04	0.17	0.13
QRE (Lambda = 0.45)	0.7	0.09	0	0.17	0.04	0
QRE (Lambda = 0.76)	0.57	0.13	0.09	0.09	0.04	0.09

Table 97: Participants' Choices (in fraction) for 6 Gate Settings in Cluster 3

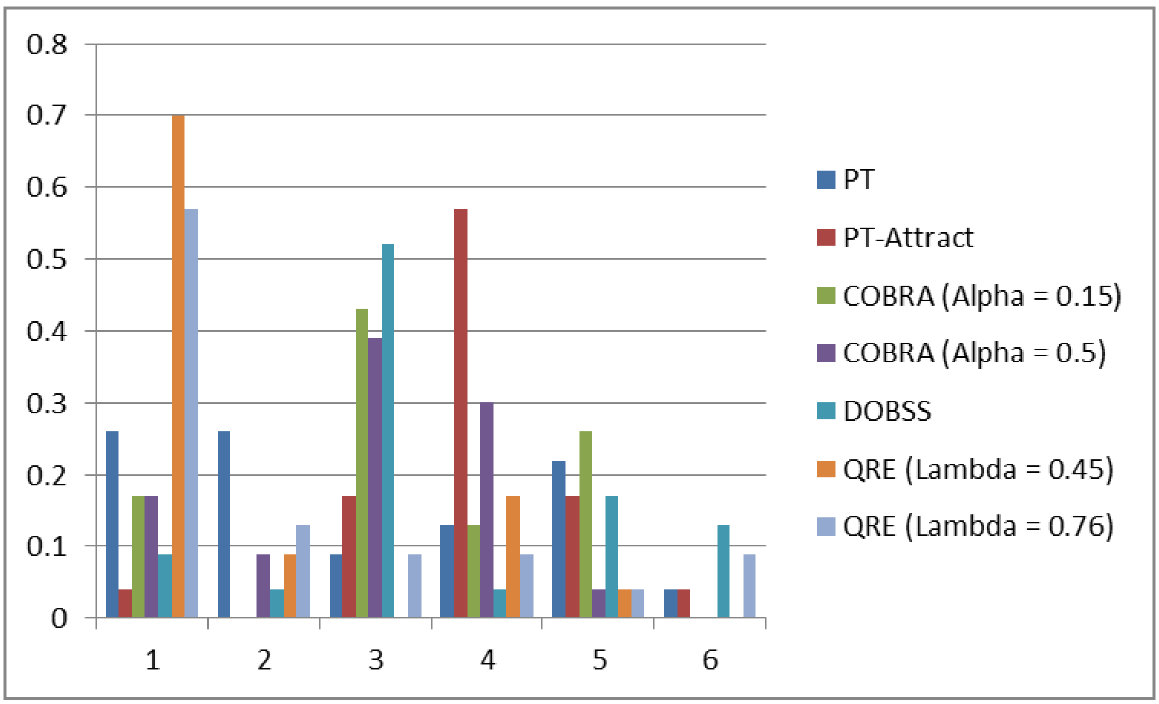

Figure 34: *Participants' Choices* against *Gate Number* for 6 Gate Settings in Cluster 3

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8	Gate 9
PT	0.04	0.04	0	0.09	0.13	0.04	0.61	0.04	0
PT-Attract	0.04	0.13	0	0.13	0	0.17	0.52	0	0
COBRA (Alpha = 0.15)	0.04	0.13	0.04	0	0.04	0.09	0.61	0.04	0
COBRA (Alpha = 0.5)	0.22	0.04	0.04	0.04	0.13	0	0.48	0.04	0
DOBSS	0.09	0	0.26	0.04	0.17	0.13	0.17	0.13	0
QRE (Lambda = 0.45)	0.04	0.17	0	0.13	0.26	0.04	0.35	0	0
QRE (Lambda = 0.76)	0.13	0.22	0.09	0.04	0.13	0.04	0.35	0	0

Table 98: Participants' Choices (in fraction) for 9 Gate Settings in Cluster 3

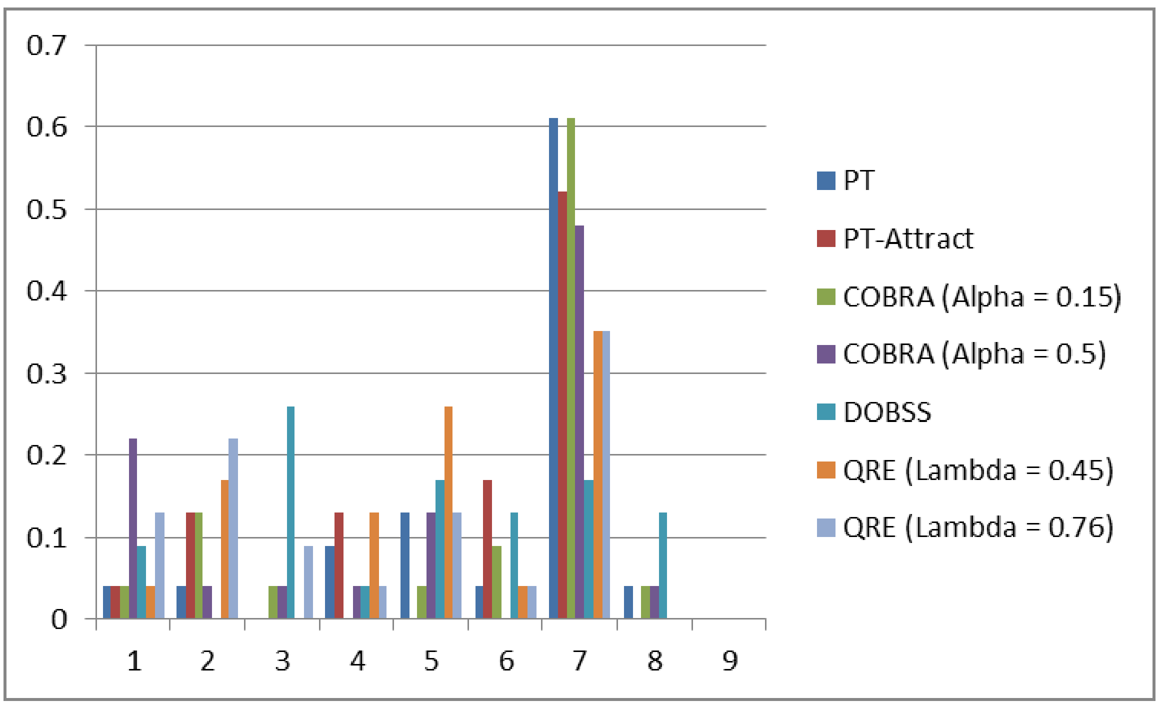

Figure 35: *Participants' Choices* against *Gate Number* for 9 Gate Settings in Cluster 3

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6
PT	0	0.04	0.17	0	0.09	0.26
PT-Attract	0	0.04	0.17	0.04	0	0.43
COBRA (Alpha = 0.15)	0	0.04	0	0	0.13	0.35
COBRA (Alpha = 0.5)	0	0	0.04	0	0	0.13
DOBSS	0	0	0.04	0.04	0	0.09
QRE (Lambda = 0.45)	0.17	0	0.04	0	0	0.04
QRE (Lambda = 0.76)	0.04	0	0	0	0.13	0.17
Game Model	**Gate 7**	**Gate 8**	**Gate 9**	**Gate 10**	**Gate 11**	**Gate 12**
PT	0	0	0	0.17	0.04	0.22
PT-Attract	0.09	0	0	0.04	0.09	0.09
COBRA (Alpha = 0.15)	0	0	0	0.09	0.04	0.35
COBRA (Alpha = 0.5)	0	0.04	0.04	0	0.61	0.13
DOBSS	0.04	0.04	0	0.13	0.52	0.09
QRE (Lambda = 0.45)	0	0.04	0	0	0.65	0.04
QRE (Lambda = 0.76)	0	0	0.04	0.04	0.43	0.13

Table 99: Participants' Choices (in fraction) for 12 Gate Settings in Cluster 3

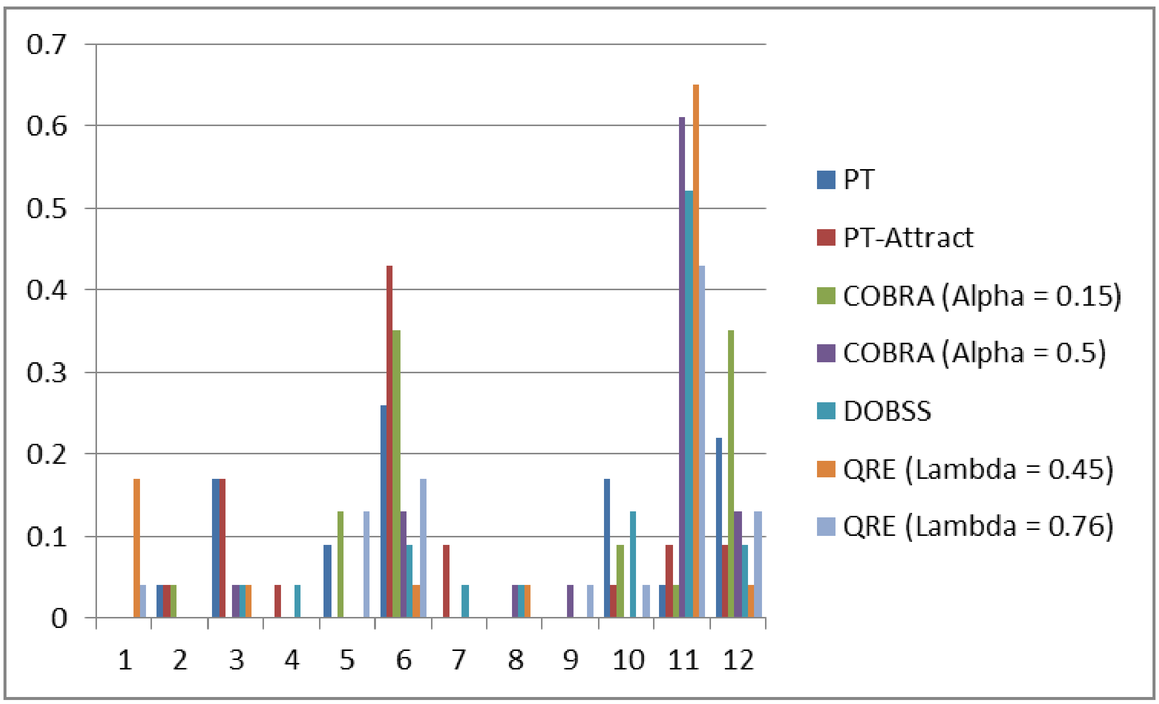

Figure 36: *Participants' Choices* against *Gate Number* for 12 Gate Settings in Cluster 3

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8
PT	0.04	0.04	0	0	0.26	0	0.09	0
PT-Attract	0	0.17	0.3	0	0.17	0.04	0	0.04
COBRA (Alpha = 0.15)	0.09	0.04	0.3	0.04	0.3	0	0	0
COBRA (Alpha = 0.5)	0.09	0.09	0.17	0	0.39	0	0	0
DOBSS	0.13	0.04	0.26	0.09	0.35	0	0	0
QRE (Lambda = 0.45)	0.04	0.04	0.17	0.04	0.48	0	0	0
QRE (Lambda = 0.76)	0.09	0.04	0.22	0.04	0.26	0.04	0	0

Game Model	Gate 9	Gate 10	Gate 11	Gate 12	Gate 13	Gate 14	Gate 15
PT	0.04	0.04	0	0.04	0.39	0	0.04
PT-Attract	0.04	0	0.04	0	0.17	0	0
COBRA (Alpha = 0.15)	0	0	0.04	0.04	0.13	0	0
COBRA (Alpha = 0.5)	0	0	0	0	0.26	0	0
DOBSS	0	0	0.04	0	0.04	0	0.04
QRE (Lambda = 0.45)	0	0.04	0.09	0	0.04	0	0.04
QRE (Lambda = 0.76)	0	0	0.09	0	0.17	0.04	0

Table 100: Participants' Choices (in fraction) for 15 Gate Settings in Cluster 3

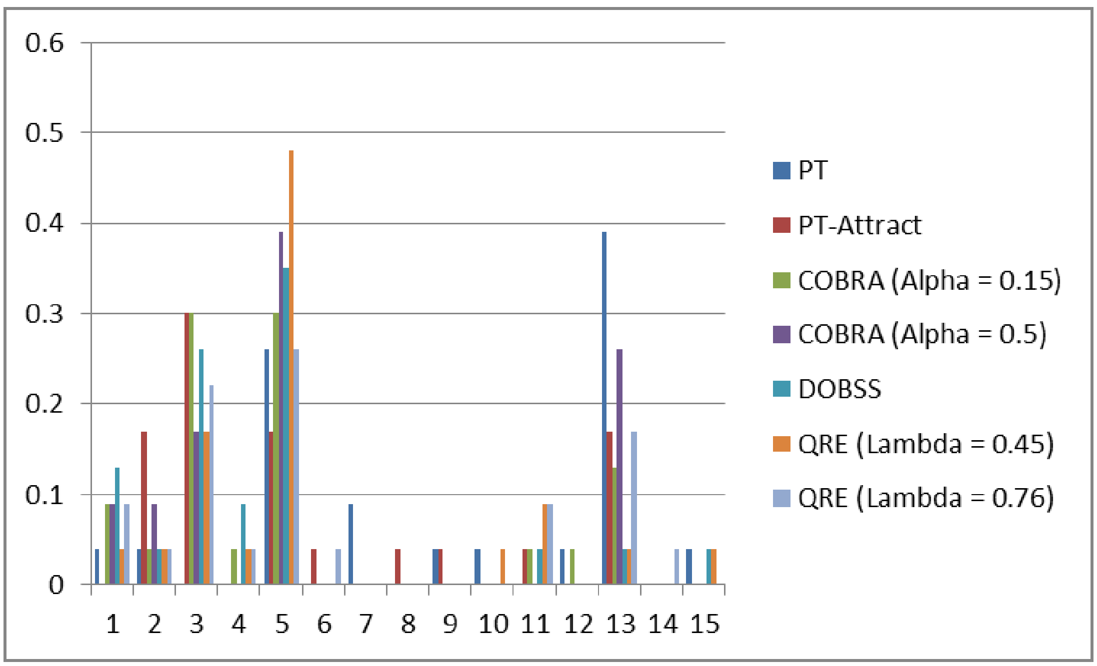

Figure 37: *Participants' Choices* against *Gate Number* for 15 Gate Settings in Cluster 3

Appendix N: Participants' Choices for Cluster 4

The participants' choices for cluster 1 are illustrated in the Tables 101 to 105 and Figures 38 to 42.

Game Model	Gate 1	Gate 2	Gate 3
PT	0.36	0.41	0.23
PT-Attract	0.23	0.32	0.45
COBRA (Alpha = 0.15)	0.18	0.64	0.18
COBRA (Alpha = 0.5)	0.18	0.55	0.27
DOBSS	0.18	0.55	0.27
QRE (Lambda = 0.45)	0.18	0.59	0.23
QRE (Lambda = 0.76)	0.23	0.45	0.32

Table 101: Participants' Choices (in fraction) for 3 Gate Settings in Cluster 4

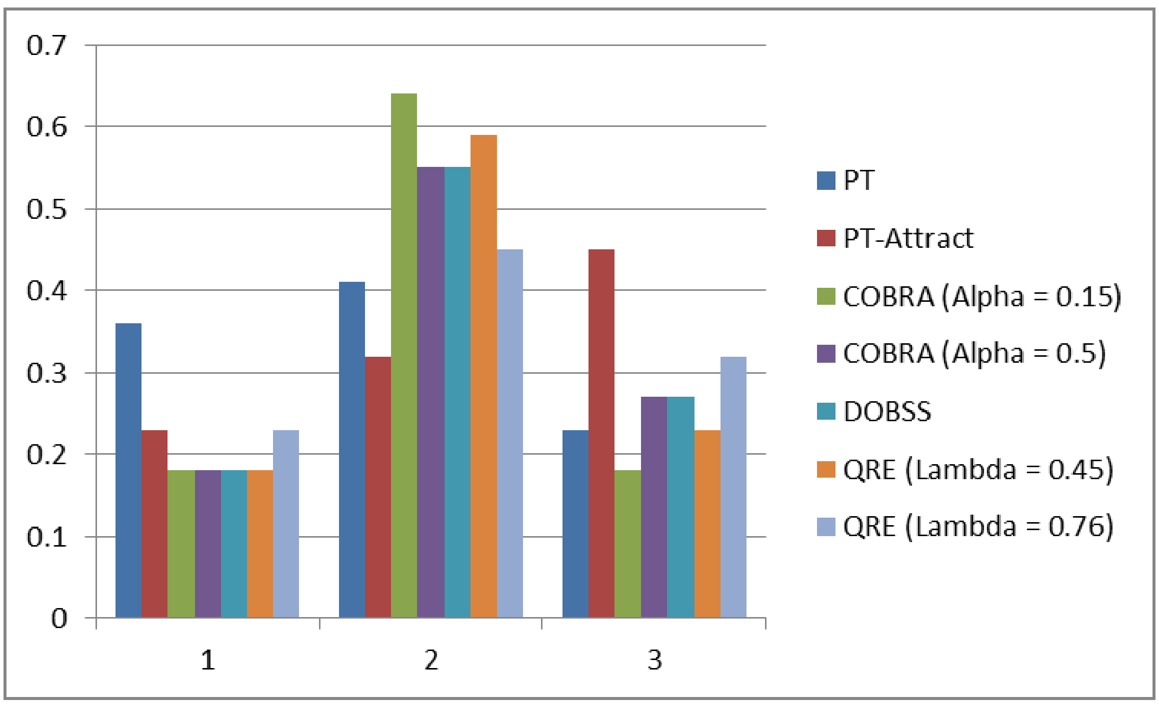

Figure 38: *Participants' Choices* against *Gate Number* for 3 Gate Settings in Cluster 4

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6
PT	0.5	0.14	0	0.14	0.14	0.09
PT-Attract	0.32	0.05	0.05	0.5	0.09	0
COBRA (Alpha = 0.15)	0.23	0.09	0.23	0.14	0.27	0.05
COBRA (Alpha = 0.5)	0.27	0	0.32	0.32	0.09	0
DOBSS	0.32	0.09	0.45	0.14	0	0
QRE (Lambda = 0.45)	0.64	0.09	0	0.23	0	0.05
QRE (Lambda = 0.76)	0.68	0.23	0	0.05	0	0.05

Table 102: Participants' Choices (in fraction) for 6 Gate Settings in Cluster 4

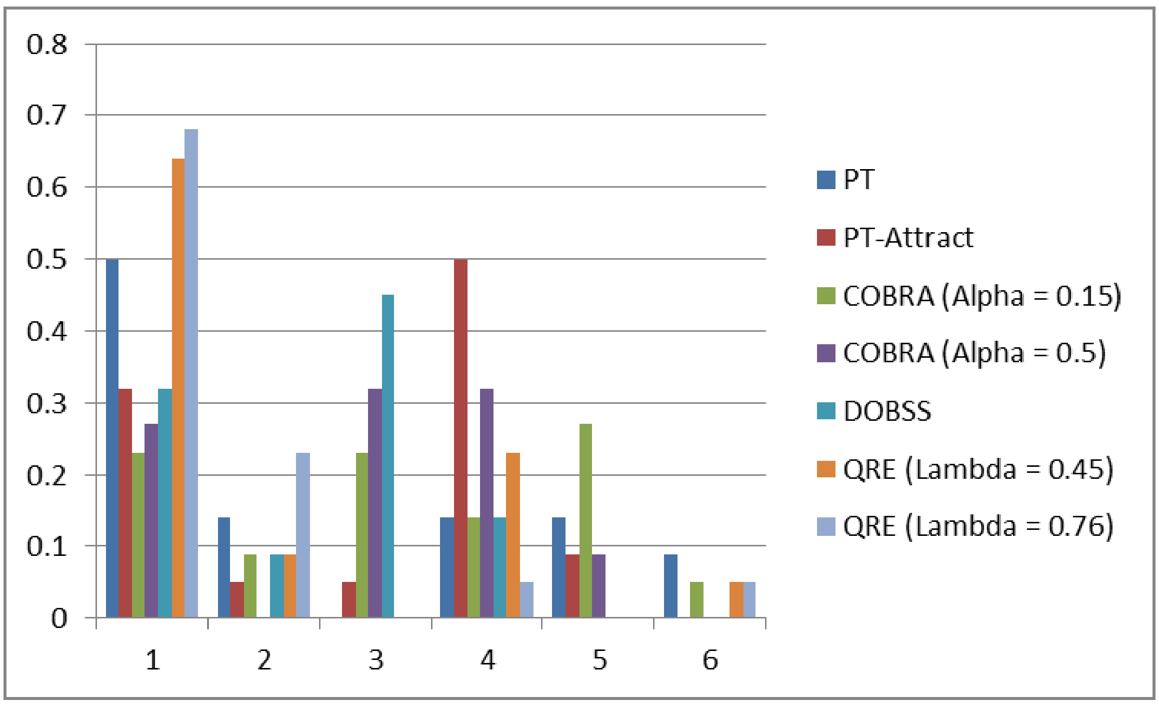

Figure 39: *Participants' Choices* against *Gate Number* for 6 Gate Settings in Cluster 4

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8	Gate 9
PT	0.14	0.05	0.05	0	0	0	0.64	0.05	0.09
PT-Attract	0.09	0.09	0.05	0	0.05	0.14	0.59	0	0
COBRA (Alpha = 0.15)	0.14	0.05	0.05	0	0.05	0.18	0.5	0	0.05
COBRA (Alpha = 0.5)	0.18	0	0.05	0	0	0.09	0.64	0	0.05
DOBSS	0.09	0	0.18	0	0.32	0	0.36	0.05	0
QRE (Lambda = 0.45)	0.09	0.09	0.14	0.05	0.45	0	0.14	0	0.05
QRE (Lambda = 0.76)	0.05	0.14	0.14	0	0.45	0	0.18	0.05	0

Table 103: Participants' Choices (in fraction) for 9 Gate Settings in Cluster 4

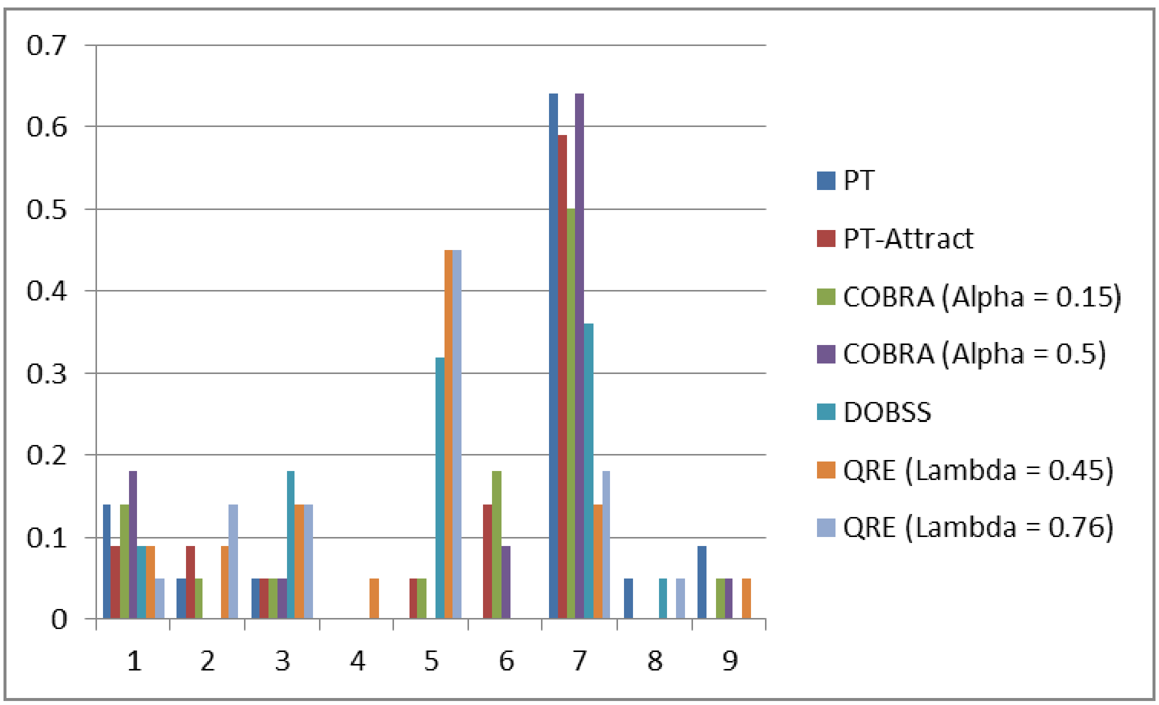

Figure 40: *Participants' Choices* against *Gate Number* for 9 Gate Settings in Cluster 4

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6
PT	0	0.05	0.14	0.09	0.05	0.32
PT-Attract	0	0.05	0	0	0.09	0.36
COBRA (Alpha = 0.15)	0	0.14	0.09	0	0.14	0.27
COBRA (Alpha = 0.5)	0	0	0.05	0	0	0.27
DOBSS	0.09	0.05	0.18	0	0	0.09
QRE (Lambda = 0.45)	0.09	0.05	0	0	0	0.09
QRE (Lambda = 0.76)	0.05	0	0	0	0.27	0.23
Game Model	**Gate 7**	**Gate 8**	**Gate 9**	**Gate 10**	**Gate 11**	**Gate 12**
PT	0	0	0	0	0.05	0.32
PT-Attract	0	0	0.05	0	0.14	0.32
COBRA (Alpha = 0.15)	0	0	0	0	0.09	0.27
COBRA (Alpha = 0.5)	0	0.05	0	0	0.55	0.09
DOBSS	0	0	0	0	0.55	0.05
QRE (Lambda = 0.45)	0	0	0	0	0.73	0.05
QRE (Lambda = 0.76)	0	0	0	0	0.36	0.09

Table 104: Participants’ Choices (in fraction) for 12 Gate Settings in Cluster 4

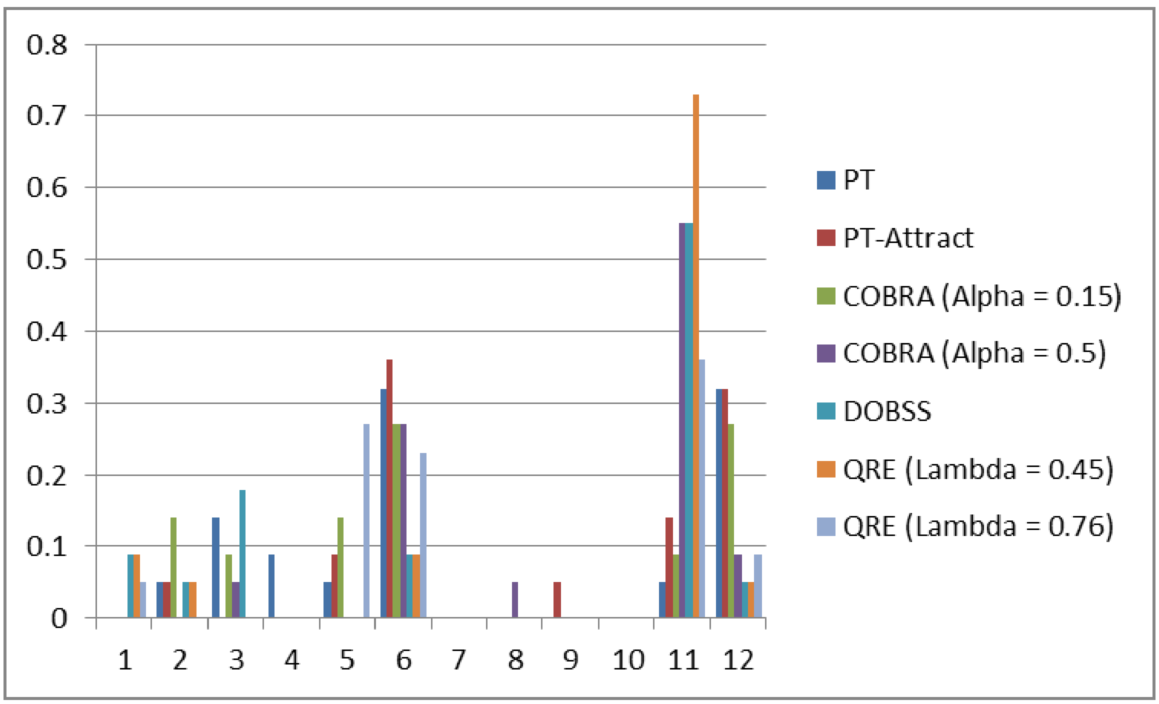

Figure 41: *Participants’ Choices* against *Gate Number* for 12 Gate Settings in Cluster 4

Game Model	Gate 1	Gate 2	Gate 3	Gate 4	Gate 5	Gate 6	Gate 7	Gate 8
PT	0.05	0.05	0.05	0	0.14	0	0	0
PT-Attract	0	0.09	0.18	0	0.32	0	0	0
COBRA (Alpha = 0.15)	0.05	0	0.05	0	0.55	0	0.05	0
COBRA (Alpha = 0.5)	0.09	0	0.05	0	0.45	0.05	0.09	0
DOBSS	0.14	0.05	0.14	0	0.27	0	0	0
QRE (Lambda = 0.45)	0.09	0	0.14	0	0.5	0	0	0
QRE (Lambda = 0.76)	0.18	0	0.05	0	0.36	0.05	0.05	0

Game Model	Gate 9	Gate 10	Gate 11	Gate 12	Gate 13	Gate 14	Gate 15
PT	0.05	0.09	0	0	0.55	0.05	0
PT-Attract	0	0.05	0.14	0.05	0.18	0	0
COBRA (Alpha = 0.15)	0	0.05	0.05	0.05	0.18	0	0
COBRA (Alpha = 0.5)	0	0.05	0.05	0	0.18	0	0
DOBSS	0	0.09	0.09	0	0.18	0	0.05
QRE (Lambda = 0.45)	0	0	0.09	0	0.14	0.05	0
QRE (Lambda = 0.76)	0	0	0.05	0	0.27	0	0

Table 105: Participants' Choices (in fraction) for 15 Gate Settings in Cluster 4

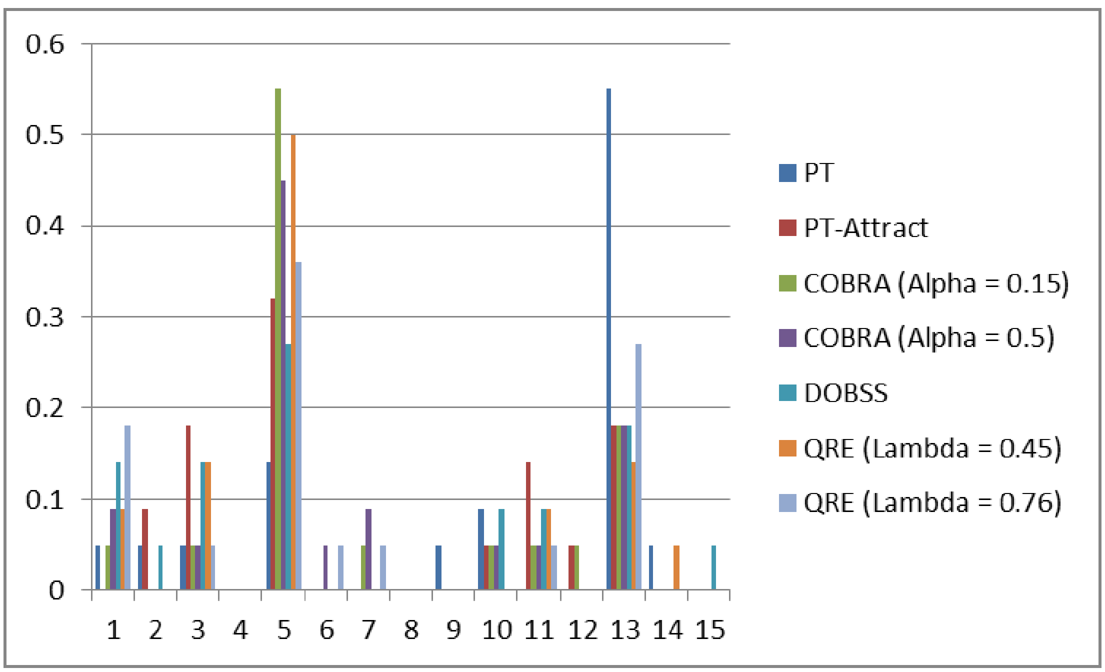

Figure 42: *Participants' Choices* against *Gate Number* for 15 Gate Settings in Cluster 4

Appendix O: Average Response Times (Uncapped)

The average response times of the participants for various game models are illustrated in Tables 106 to 109 and Figures 43 to 46.

Game Model	3 Gates	6 Gates	9 Gates	12 Gates	15 Gates
PT	11.476	11.905	12.381	19.333	9.619
PT-Attract	9.238	16.952	15.429	15.571	17
COBRA (Alpha = 0.15)	8.952	13	16.048	16.429	19.476
COBRA (Alpha = 0.5)	7.619	16.381	14.905	21.905	13.905
DOBSS	11.857	11.286	19.714	10.19	16.619
QRE (Lambda = 0.45)	7.524	11.81	16.667	11.905	15.619
QRE (Lambda = 0.76)	9.571	10	31	17.81	14.476

Table 106: Average Response Times (Uncapped) for Cluster 1

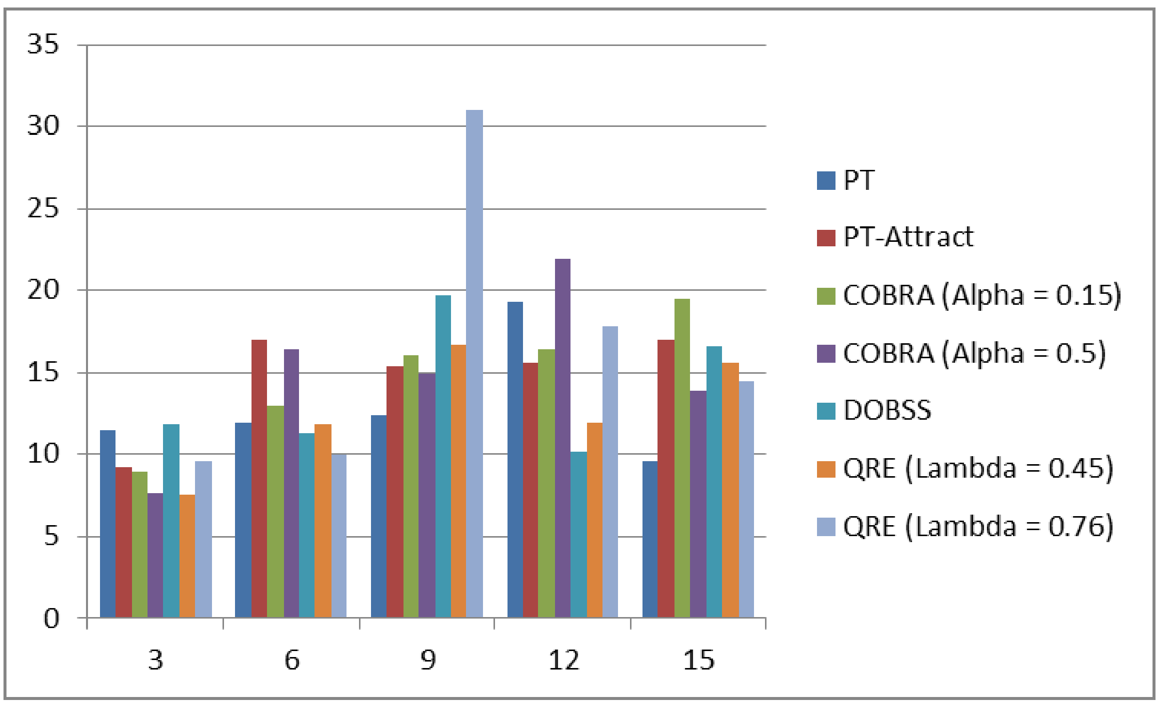

Figure 43: *Average Response Times (Uncapped)* against *Number of Gates* for Cluster 1

Game Model	3 Gates	6 Gates	9 Gates	12 Gates	15 Gates
PT	15.08	19.08	15.2	22.08	17.12
PT-Attract	10.32	16.76	14.96	13	17.08
COBRA (Alpha = 0.15)	10.08	18.44	16.04	18.56	21.88
COBRA (Alpha = 0.5)	10.6	15.8	16.48	13.72	19.96
DOBSS	11.52	17.08	22.52	15.16	19.68
QRE (Lambda = 0.45)	8.16	12.2	20.64	12.84	17.12
QRE (Lambda = 0.76)	11.48	11.76	24.96	14.2	17.76

Table 107: Average Response Times (Uncapped) for Cluster 2

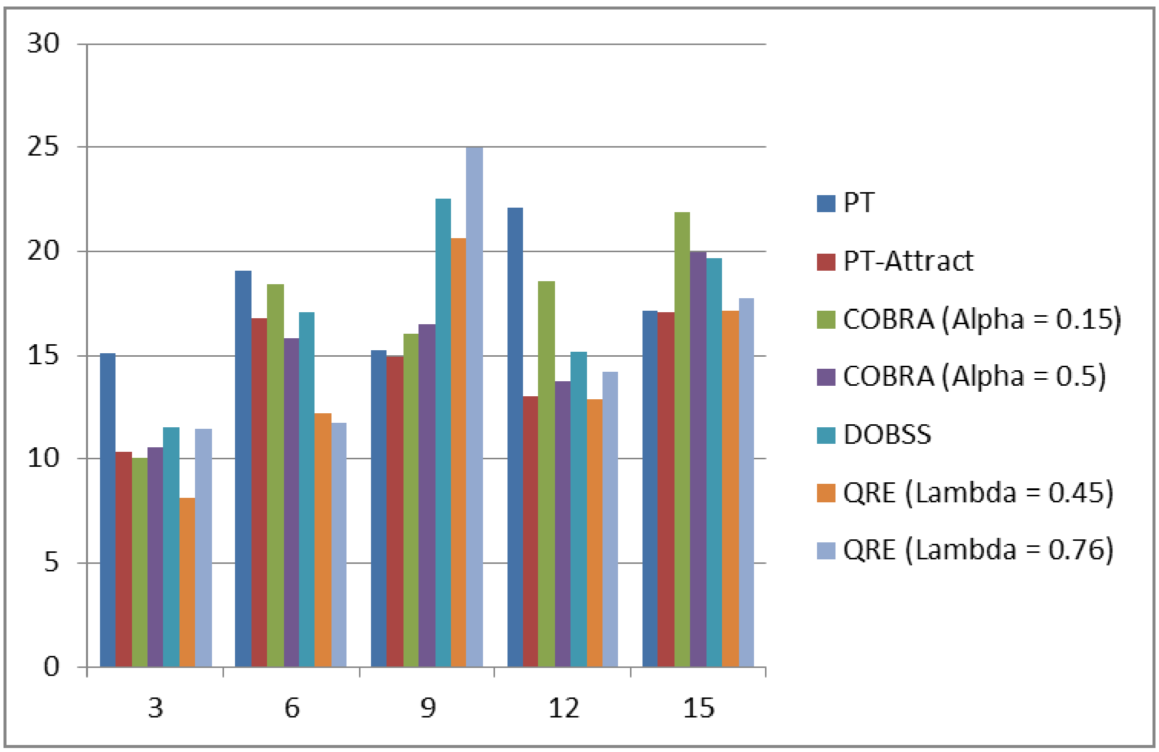

Figure 44: *Average Response Times (Uncapped)* against *Number of Gates* for Cluster 2

Game Model	3 Gates	6 Gates	9 Gates	12 Gates	15 Gates
PT	12.783	18.261	24.13	21.522	21.304
PT-Attract	12.261	20.087	20.783	22.783	26.217
COBRA (Alpha = 0.15)	11.652	20.696	24.609	24.087	20.913
COBRA (Alpha = 0.5)	11.957	22.652	13.304	23.087	19.652
DOBSS	10.435	18.391	28.261	17.043	23.478
QRE (Lambda = 0.45)	10.783	12.304	24.696	14.826	36.435
QRE (Lambda = 0.76)	11	14.043	25.609	21.435	19.609

Table 108: Average Response Times (Uncapped) for Cluster 3

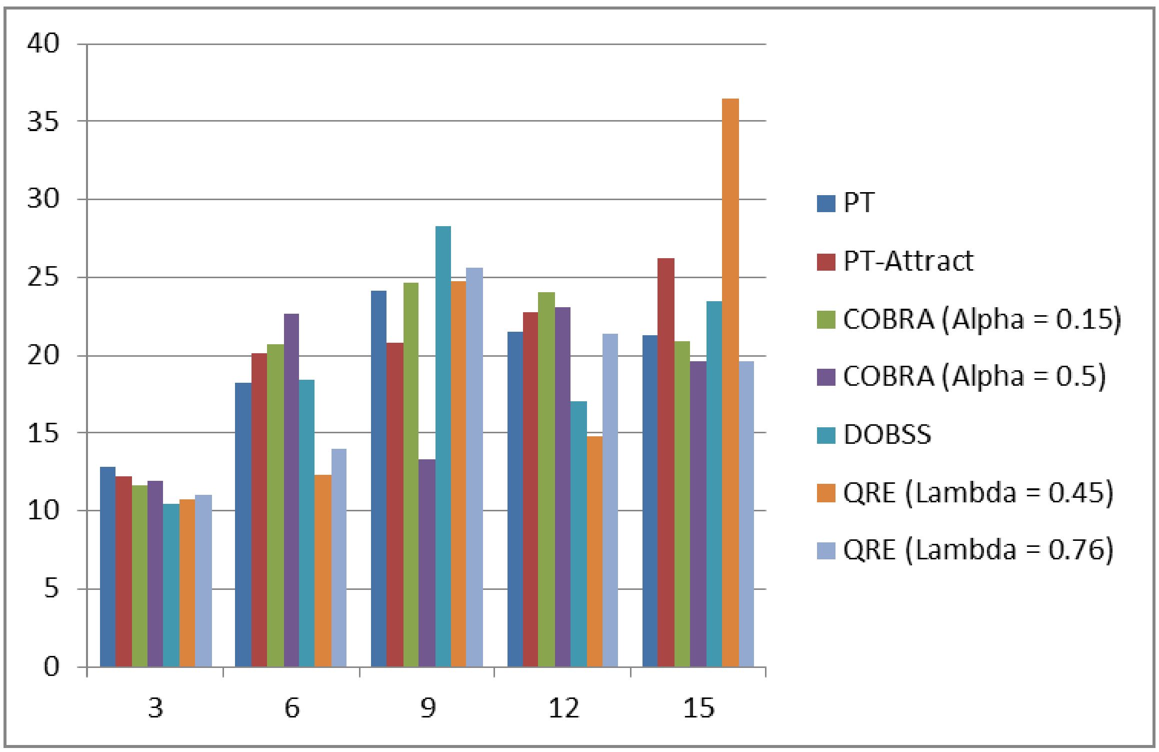

Figure 45: *Average Response Times (Uncapped)* against *Number of Gates* for Cluster 3

Game Model	3 Gates	6 Gates	9 Gates	12 Gates	15 Gates
PT	11.409	21.5	19.227	19.955	14.545
PT-Attract	10	17	14.545	12.636	17.955
COBRA (Alpha = 0.15)	10.455	19	63.227	15.773	16.727
COBRA (Alpha = 0.5)	10.591	14.591	18.409	18.5	20.136
DOBSS	8.545	11.364	22.091	16.227	12.682
QRE (Lambda = 0.45)	9.591	12.273	23.182	12.682	13.682
QRE (Lambda = 0.76)	11.727	18.955	20.182	14.273	17.5

Table 109: Average Response Times (Uncapped) for Cluster 4

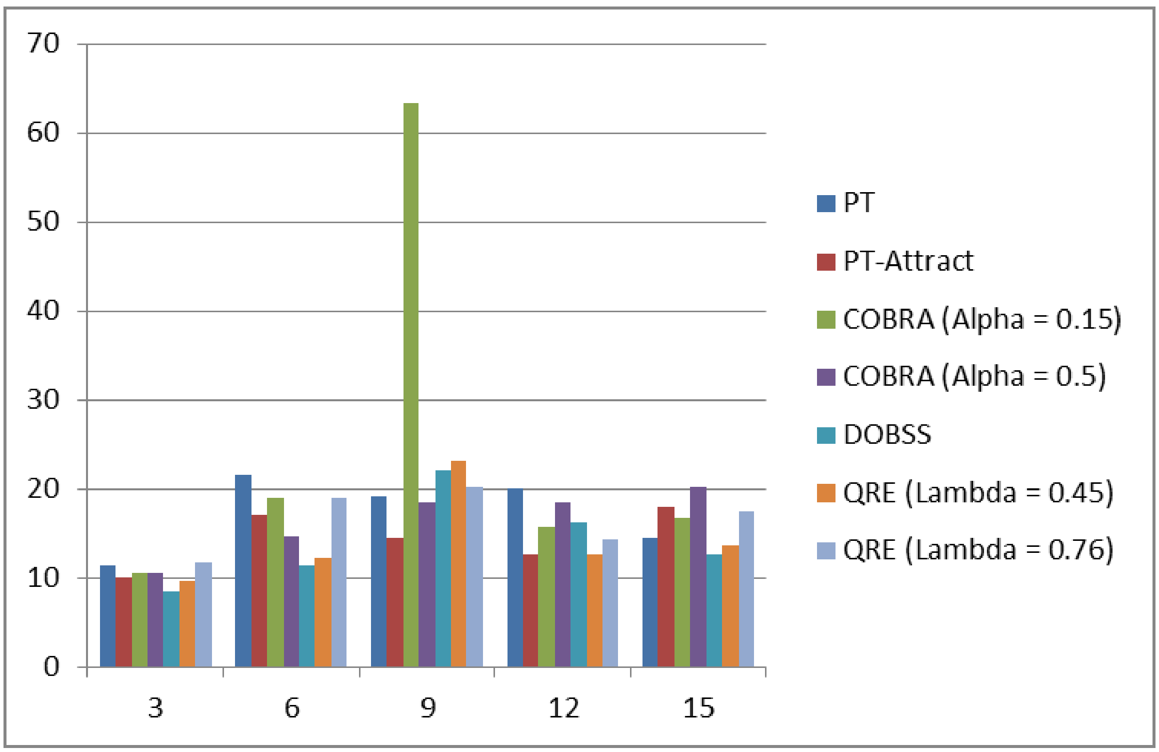

Figure 46: *Average Response Times (Uncapped)* against *Number of Gates* for Cluster 4

Appendix P: Average Response Times (Capped)

The average response times of the participants for various game models with a constraint of 120 seconds are illustrated in Tables 110 to 113 and Figures 47 to 50.

Game Model	3 Gates	6 Gates	9 Gates	12 Gates	15 Gates
PT	11.476	11.905	12.381	19.333	9.619
PT-Attract	9.238	16.952	15.429	15.571	17
COBRA (Alpha = 0.15)	8.952	13	16.048	16.429	19.476
COBRA (Alpha = 0.5)	7.619	16.381	14.905	21.905	13.905
DOBSS	11.857	11.286	19.714	10.19	16.619
QRE (Lambda = 0.45)	7.524	11.81	16.667	11.905	15.619
QRE (Lambda = 0.76)	9.571	10	31	17.81	14.476

Table 110: Average Response Times (Capped) for Cluster 1

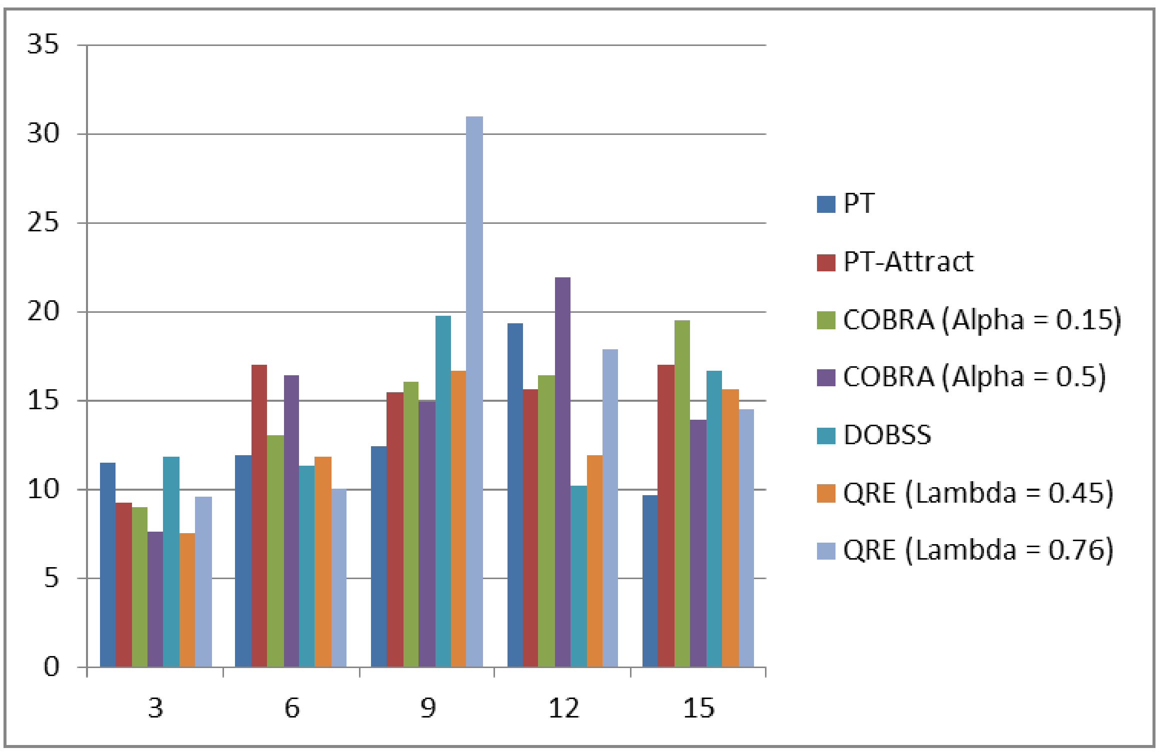

Figure 47: *Average Response Times (Capped)* against *Number of Gates* for Cluster 1

Game Model	3 Gates	6 Gates	9 Gates	12 Gates	15 Gates
PT	15.08	19.08	15.2	22.08	17.12
PT-Attract	10.32	16.76	14.96	13	17.08
COBRA (Alpha = 0.15)	10.08	18.44	16.04	18.56	21.88
COBRA (Alpha = 0.5)	10.6	15.8	16.48	13.72	19.96
DOBSS	11.52	17.08	22.52	15.16	19.68
QRE (Lambda = 0.45)	8.16	12.2	20.64	12.84	17.12
QRE (Lambda = 0.76)	11.48	11.76	24.96	14.2	17.76

Table 111: Average Response Times (Capped) for Cluster 2

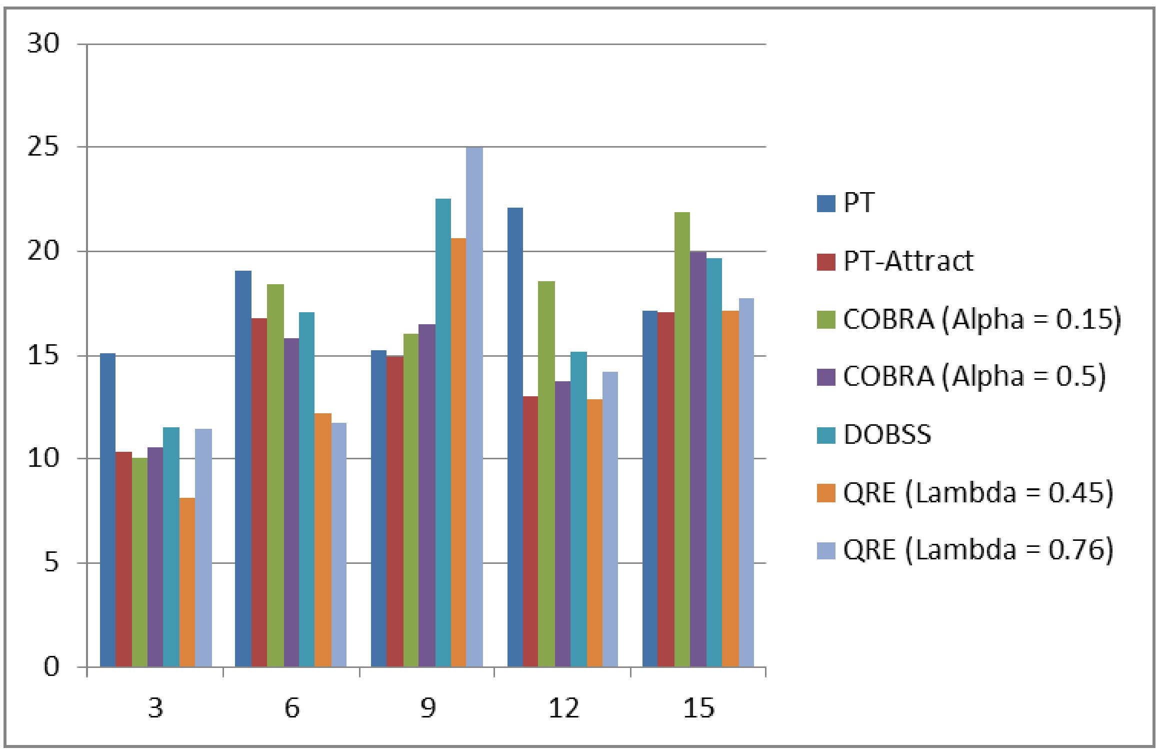

Figure 48: *Average Response Times (Capped)* against *Number of Gates* for Cluster 2

Game Model	3 Gates	6 Gates	9 Gates	12 Gates	15 Gates
PT	12.783	18.261	24.13	21.522	21.304
PT-Attract	12.261	20.087	20.783	22.783	26.217
COBRA (Alpha = 0.15)	11.652	20.696	22.565	24.087	20.913
COBRA (Alpha = 0.5)	11.957	22.652	13.304	20.783	19.652
DOBSS	10.435	18.391	28.261	17.043	20.174
QRE (Lambda = 0.45)	10.783	12.304	24.696	14.826	28.696
QRE (Lambda = 0.76)	11	14.043	25.609	21.435	19.609

Table 112: Average Response Times (Capped) for Cluster 3

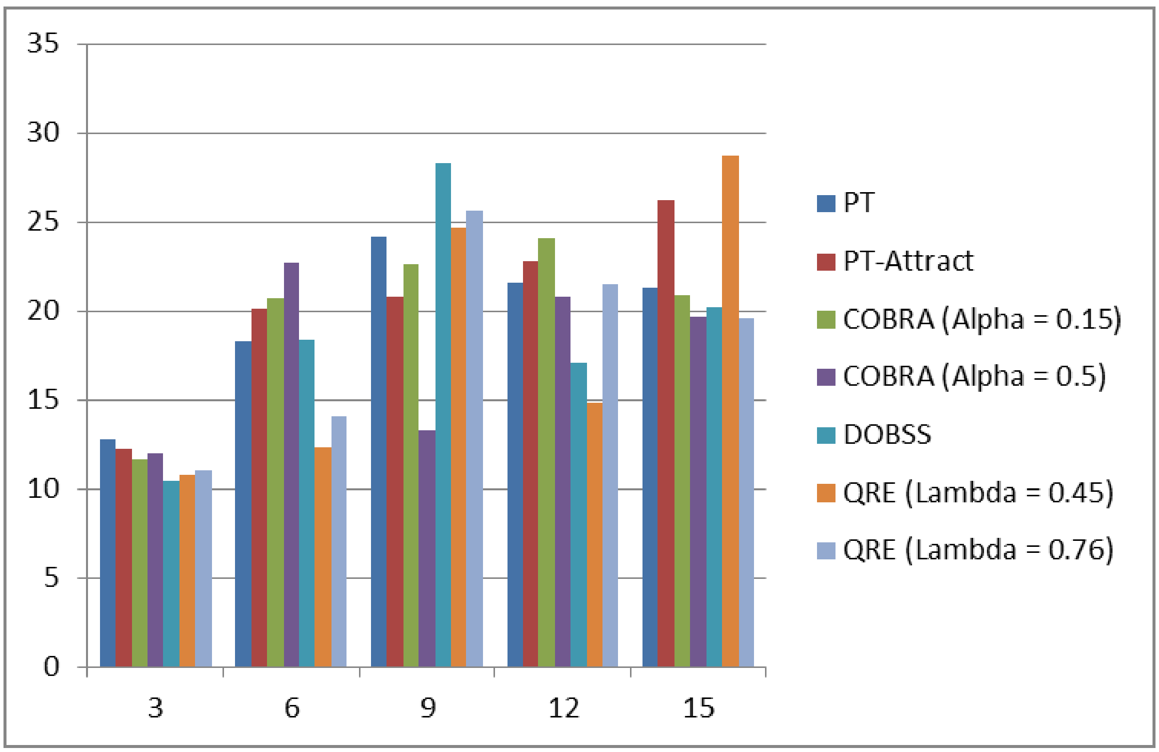

Figure 49: *Average Response Times (Capped)* against *Number of Gates* for Cluster 3

Game Model	3 Gates	6 Gates	9 Gates	12 Gates	15 Gates
PT	11.409	18.227	19.227	19.955	14.545
PT-Attract	10	17	14.545	12.636	17.955
COBRA (Alpha = 0.15)	10.455	19	27.864	15.773	16.727
COBRA (Alpha = 0.5)	10.591	14.591	18.409	18.5	20.136
DOBSS	8.545	11.364	22.091	16.227	12.682
QRE (Lambda = 0.45)	9.591	12.273	23.182	12.682	13.682
QRE (Lambda = 0.76)	11.727	18.955	20.182	14.273	17.5

Table 113: Average Response Times (Capped) for Cluster 4

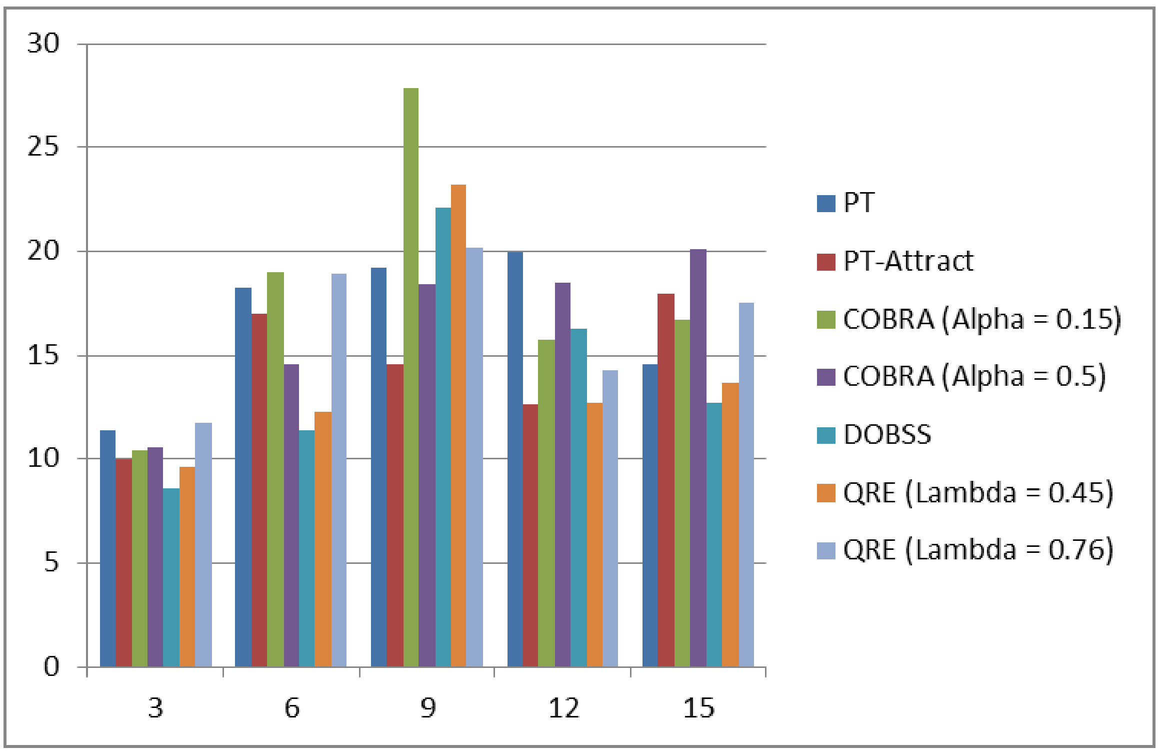

Figure 50: *Average Response Times (Capped)* against *Number of Gates* for Cluster 4