Mathieu equation stability, δ-ϵ.

x'' + [$\delta + \epsilon$ Cos(2t)]x = 0 - Mathieu equation

In[ ]:=
```
ClearAll[A, B, n];
Aπ[n_] :=
 Module[{temp}, temp = DiagonalMatrix[Array[δ - 4 (# - 1)^2 &, n], 0] + DiagonalMatrix[
     Array[ϵ / 2 &, n - 1], 1] + DiagonalMatrix[Array[ϵ / 2 &, n - 1], -1];
  temp[[2, 1]] = ϵ;
  temp]


Bπ[n_] := DiagonalMatrix[Array[δ - 4 (#)^2 &, n], 0] + DiagonalMatrix[
    Array[ϵ / 2 &, n - 1], 1] + DiagonalMatrix[Array[ϵ / 2 &, n - 1], -1];


A2π[n_] :=
 Module[{temp}, temp = DiagonalMatrix[Array[δ - (2 # - 1)^2 &, n], 0] + DiagonalMatrix[
     Array[ϵ / 2 &, n - 1], 1] + DiagonalMatrix[Array[ϵ / 2 &, n - 1], -1];
  temp[[1, 1]] += ϵ / 2;
  temp]


B2π[n_] :=
 Module[{temp}, temp = DiagonalMatrix[Array[δ - (2 # - 1)^2 &, n], 0] + DiagonalMatrix[
     Array[ϵ / 2 &, n - 1], 1] + DiagonalMatrix[Array[ϵ / 2 &, n - 1], -1];
  temp[[1, 1]] -= ϵ / 2;
  temp]
```

In[ ]:=
```
MatrixForm@Aπ[10]
MatrixForm@Bπ[4]
MatrixForm@A2π[4]
MatrixForm@B2π[4]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \delta & \frac{\epsilon}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \epsilon & -4+\delta & \frac{\epsilon}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\epsilon}{2} & -16+\delta & \frac{\epsilon}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\epsilon}{2} & -36+\delta & \frac{\epsilon}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\epsilon}{2} & -64+\delta & \frac{\epsilon}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\epsilon}{2} & -100+\delta & \frac{\epsilon}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\epsilon}{2} & -144+\delta & \frac{\epsilon}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\epsilon}{2} & -196+\delta & \frac{\epsilon}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\epsilon}{2} & -256+\delta & \frac{\epsilon}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\epsilon}{2} & -324+\delta \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} -4+\delta & \frac{\epsilon}{2} & 0 & 0 \\ \frac{\epsilon}{2} & -16+\delta & \frac{\epsilon}{2} & 0 \\ 0 & \frac{\epsilon}{2} & -36+\delta & \frac{\epsilon}{2} \\ 0 & 0 & \frac{\epsilon}{2} & -64+\delta \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} -1+\delta+\frac{\epsilon}{2} & \frac{\epsilon}{2} & 0 & 0 \\ \frac{\epsilon}{2} & -9+\delta & \frac{\epsilon}{2} & 0 \\ 0 & \frac{\epsilon}{2} & -25+\delta & \frac{\epsilon}{2} \\ 0 & 0 & \frac{\epsilon}{2} & -49+\delta \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} -1+\delta-\frac{\epsilon}{2} & \frac{\epsilon}{2} & 0 & 0 \\ \frac{\epsilon}{2} & -9+\delta & \frac{\epsilon}{2} & 0 \\ 0 & \frac{\epsilon}{2} & -25+\delta & \frac{\epsilon}{2} \\ 0 & 0 & \frac{\epsilon}{2} & -49+\delta \end{pmatrix}$$

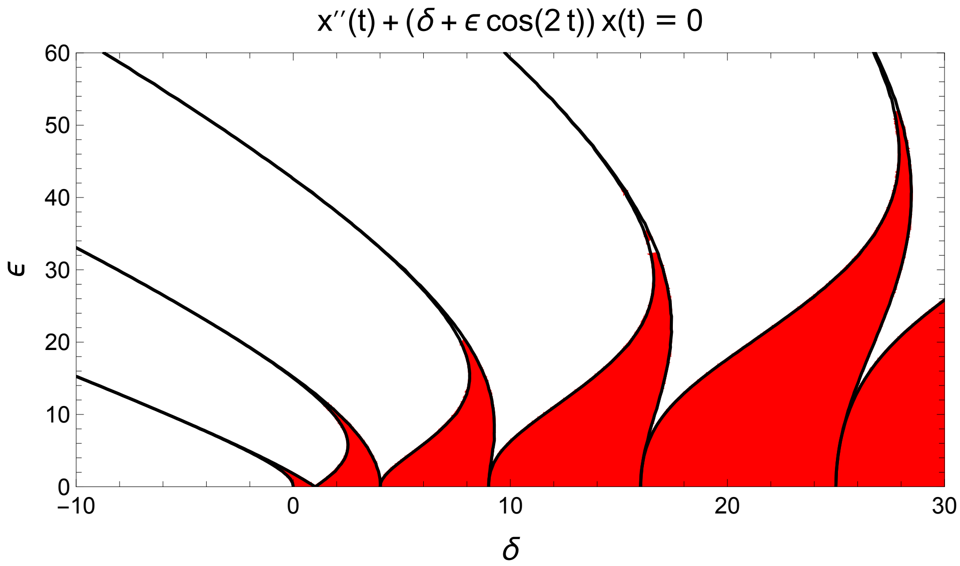
Mathieu stability diagram

In[∘]:=
```
Show[
 RegionPlot[
  {
   Det[A2π[15]] ≤ 0 && Det[Aπ[15]] ≥ 0,
   Det[Aπ[15]] ≤ 0 && Det[A2π[15]] ≥ 0,
   Det[B2π[15]] ≤ 0 && Det[Bπ[15]] ≥ 0,
   Det[Bπ[15]] ≤ 0 && Det[B2π[15]] ≥ 0
  },
  {δ, -10, 30}, {ϵ, 0, 60},

  MaxRecursion → 6,
  PlotStyle → Red,
  BoundaryStyle → None,
  PlotRangePadding → None,
  FrameTicksStyle → Directive[Black, 12],
  FrameLabel → {Style["δ", 16], Style["ϵ", 16]},
  PlotLabel →
   Style[Text@ToExpression["x''(t)+(\\delta + \\epsilon \\cos(2t))x(t)=0",
      TeXForm, HoldForm], 16, Black],
  ImageSize → {500, 300},
  AspectRatio → 1 / 2
 ],

 ContourPlot[{Det[Aπ[15]] == 0, Det[Bπ[15]] == 0, Det[A2π[15]] == 0,
   Det[B2π[15]] == 0}, {δ, -10, 30}, {ϵ, 0, 60}, ContourStyle → Black]
]
```

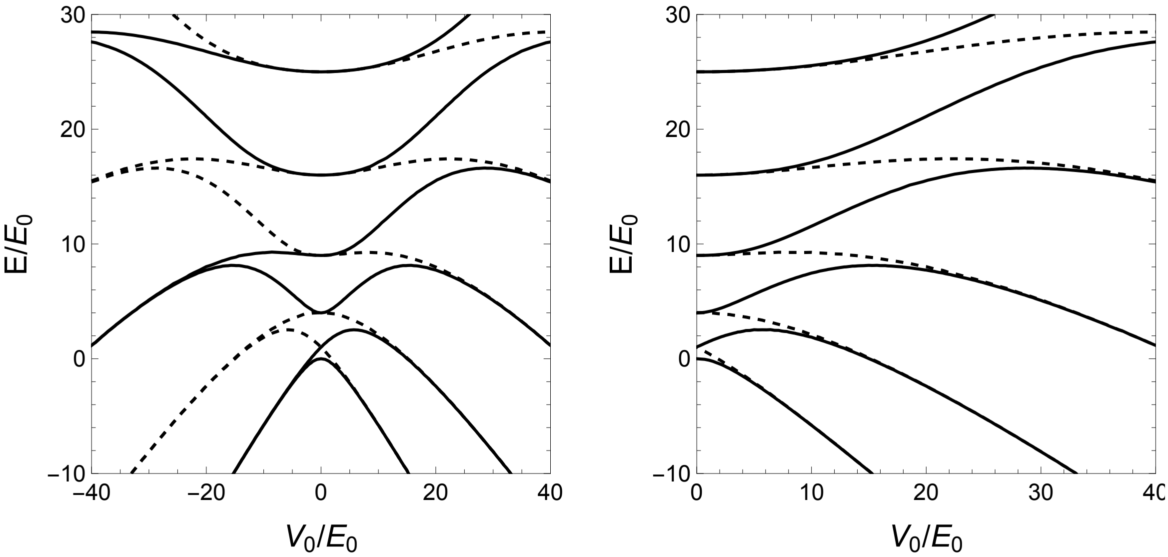
Out[∘]=



Spectrum of quantum particle in 1D cosine potential

```
GraphicsRow@{Show[
   ContourPlot[
    {Det[Aπ[15]] == 0, Det[B2π[15]]}, {ϵ, -40, 40} , {δ, -10, 30},
    PlotRangePadding → None,
    FrameTicksStyle → Directive[Black, 12],
    FrameLabel → {Style["V0/E0", 16], Style["E/E0", 16]},
    ContourStyle → Black
   ],
   ContourPlot[
    {Det[A2π[15]] == 0, Det[Bπ[15]]}, {ϵ, -40, 40} , {δ, -10, 30},
    PlotRangePadding → None,
    FrameTicksStyle → Directive[Black, 12],
    FrameLabel → {Style["V0/E0", 16], Style["E/E0", 16]},
    ContourStyle → {{Black, Dashed}}
   ]
  ],
  Show[
   ContourPlot[
    {Det[Aπ[15]] == 0, Det[B2π[15]]}, {ϵ, 0, 40} , {δ, -10, 30},
    PlotRangePadding → None,
    FrameTicksStyle → Directive[Black, 12],
    FrameLabel → {Style["V0/E0", 16], Style["E/E0", 16]},
    ContourStyle → Black
   ],
   ContourPlot[
    {Det[A2π[15]] == 0, Det[Bπ[15]]}, {ϵ, 0, 40} , {δ, -10, 30},
    PlotRangePadding → None,
    FrameTicksStyle → Directive[Black, 12],
    FrameLabel → {Style["V0/E0", 16], Style["E/E0", 16]},
    ContourStyle → {{Black, Dashed}}
   ]
  ]}
```

Out[ ]=



In[21]:=

```
Export["/Users/goloshch/Desktop/Materials/Mathematica/Mathieu/Mathieu.pdf",
 EvaluationNotebook[]]
```