Yin, MST
Spring 2012 1

Lecture 07

Estimating Homography Parameters and Warping Images (cont.)

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Spring 2012 2

Recall: Algebraic Distance, h_{33}=1

$$
\begin{array}{l}
\\
\text{Point 1} \\
\\
\text{Point 2} \\
\\
\text{Point 3} \\
\\
\text{Point 4} \\
\end{array}
\overset{\textbf{2N x 8}}{\begin{bmatrix}
x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x'_1 & -y_1x'_1 \\
0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y'_1 & -y_1y'_1 \\
x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2x'_2 & -y_2x'_2 \\
0 & 0 & 0 & x_2 & y_2 & 1 & -x_2y'_2 & -y_2y'_2 \\
x_3 & y_3 & 1 & 0 & 0 & 0 & -x_3x'_3 & -y_3x'_3 \\
0 & 0 & 0 & x_3 & y_3 & 1 & -x_3y'_3 & -y_3y'_3 \\
x_4 & y_4 & 1 & 0 & 0 & 0 & -x_4x'_4 & -y_4x'_4 \\
0 & 0 & 0 & x_4 & y_4 & 1 & -x_4y'_4 & -y_4y'_4
\end{bmatrix}}
\overset{\textbf{8 x 1}}{\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix}}
=
\overset{\textbf{2N x 1}}{\begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ x'_3 \\ y'_3 \\ x'_4 \\ y'_4 \end{bmatrix}}
$$

additional points

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Recall: Algebraic Distance, h_{33}=1

Linear equations

$$\underset{}{\overset{2Nx8}{\mathbf{A}}}\ \overset{8x1}{\mathbf{h}} = \overset{2Nx1}{\mathbf{b}}$$

Solve:

$$\overset{8x2N}{\mathbf{A}^T}\ \overset{2Nx8}{\mathbf{A}}\ \overset{8x1}{\mathbf{h}} = \overset{8x2N}{\mathbf{A}^T}\ \overset{2Nx1}{\mathbf{b}}$$

$$\overbrace{(\mathbf{A}^T\ \mathbf{A})}^{8x8}\ \overset{8x1}{\mathbf{h}} = \overbrace{(\mathbf{A}^T\ \mathbf{b})}^{8x1}$$

$$\mathbf{h} = (\mathbf{A}^T\ \mathbf{A})^{-1}\ (\mathbf{A}^T\ \mathbf{b})$$

Matlab: **h = A \ b**

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Recall: Algebraic Distance, ||h||=1

$$
\begin{array}{c}
4 \\ \text{P} \\ \text{O} \\ \text{I} \\ \text{N} \\ \text{T} \\ \text{S}
\end{array}
\overset{\textbf{2N x 9}}{\begin{bmatrix}
x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x'_1 & -y_1x'_1 & -x'_1 \\
0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y'_1 & -y_1y'_1 & -y'_1 \\
 & & & & & & & & \\
 & & & & & & & &
\end{bmatrix}}
\overset{\textbf{9 x 1}}{\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix}}
=
\overset{\textbf{2N x 1}}{\begin{bmatrix} 0 \\ 0 \\ \\ \\ \end{bmatrix}}
$$

additional points

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Recall: Algebraic Distance, ||h||=1

Homogeneous equations

$$\overset{2Nx9}{\mathbf{A}}\ \overset{9x1}{\mathbf{h}} = \overset{2Nx1}{\mathbf{0}}$$

Solve:

$$\overset{9x2N}{\mathbf{A}^T}\ \overset{2Nx9}{\mathbf{A}}\ \overset{9x1}{\mathbf{h}} = \overset{9x2N}{\mathbf{A}^T}\ \overset{2Nx1}{\mathbf{0}}$$

$$\overbrace{(\mathbf{A}^T\ \mathbf{A})}^{9x9}\ \overset{9x1}{\mathbf{h}} = \overset{9x1}{\mathbf{0}}$$

SVD of A^TA = U D V^T

Let h be the column of V associated with the smallest singular value in D.
(if only 4 points, that singular value will be 0)

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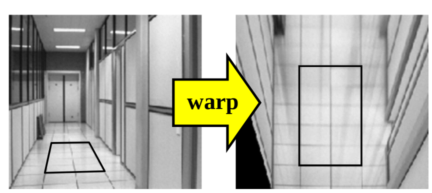
Image Warping

Once we have estimated a transformation, how can we (un)warp image pixel values to produce a new picture.



from Hartley & Zisserman

Warping & Bilinear Interpolation

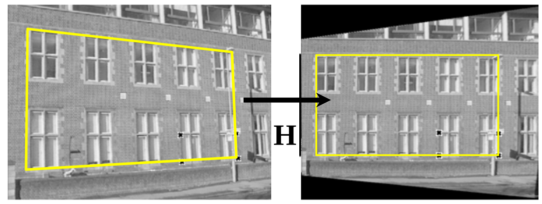
Given a transformation between two images, (coordinate systems) we want to “warp” one image into the coordinate system of the other.

We will call the coordinate system where we are mapping from the “source” image

We will call the coordinate system we are mapping to the “destination” image.

Warping Example

Transformation in this case is a projective transformation (general 3x3 matrix, operating on homogeneous coords)



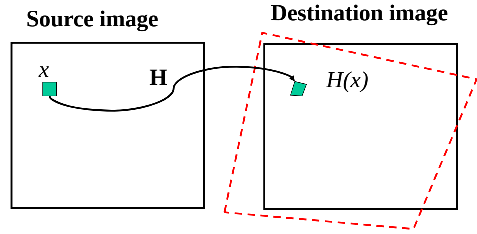
from Hartley & Zisserman

Source Image **Destination image**

Matlab Demo

Forward Warping

Source image **Destination image**

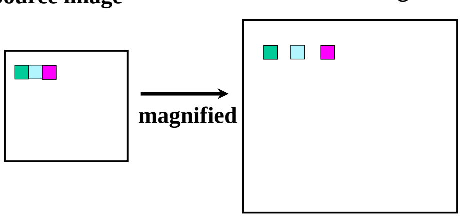


- **For each pixel *x* in the source image**
- **Determine where it goes as *H(x)***
- **Color the destination pixel**

Problems?

Forward Warping Problem

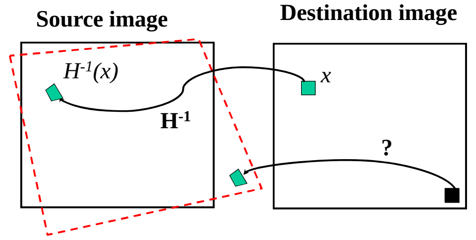
Source image **Destination image**



Can leave gaps!

Backward Warping (No gaps)

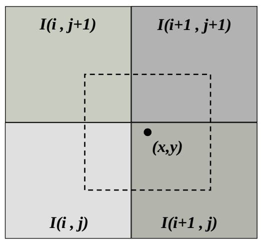
Source image **Destination image**



- **For each pixel *x* in the destination image**
- **Determine where it comes from as $H^{-1}(x)$**
- **Get color from that location**

Interpolation

What do we mean by “get color from that location”?
Consider grey values. What is intensity at (x,y)?

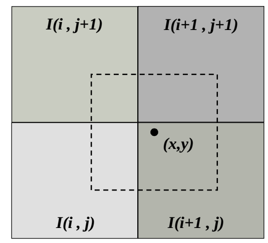


Nearest Neighbor:
Take color of pixel with closest center.

I(x,y) = I(i+1,j)

Bilinear interpolation

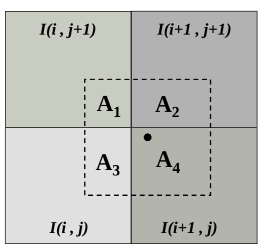
What do we mean by “get color from that location”?
Consider grey values. What is intensity at (x,y)?



Bilinear Interpolation:
Weighted average

Bilinear interpolation

What do we mean by “get color from that location”?
Consider grey values. What is intensity at (x,y)?

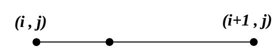


Bilinear Interpolation:
Weighted average

I(x,y) = A3*I(i,j)
+ A4*I(i+1,j)
+ A2*I(i+1,j+1)
+ A1*I(i,j+1)

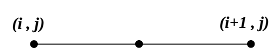
Bilinear Interpolation, Math

First, consider linear interpolation



Inituition: Given two pixel values, what should the value be at some intermediate point between them?

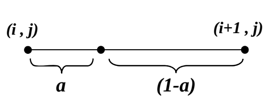
If close to (i,j), should be intensity similar to I(i,j)

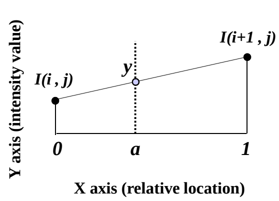


If equidistant from both, should be average of the two intensities

If close to (i+1,j), should be intensity similar to I(i+1,j)

Linear Interpolation





Recall:

$$y - y_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$$

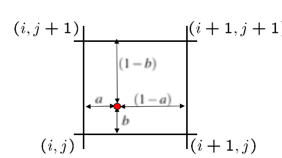
Instantiate

$$y - I(i,j) = \frac{(I(i+1,j) - I(i,j))}{(1-0)}(a-0)$$

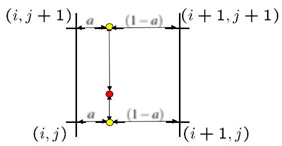
Solve

$$y = (1-a)\, I(i,j) + a\ I(i+1,j)$$

Bilinear Interpolation, Math

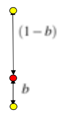


Bilinear Interpolation, Math

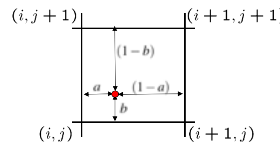


$$(1-b)\left[\,(1-a)\,I(i,j) + a\;I(i+1,j)\,\right]$$

$$+\;\; b\;\left[\,(1-a)\,I(i,j+1) + a\;I(i+1,j+1)\,\right]$$



Bilinear Interpolation, Math



$$\begin{aligned}\mathbf{I} = \; & (1-a)(1-b)\;I(i,j) \\ & + a\;(1-b)\;I(i+1,j) \\ & + (1-a)\;b\;\;I(i,j+1) \\ & + a\;b\;\;I(i+1,j+1)\end{aligned}$$