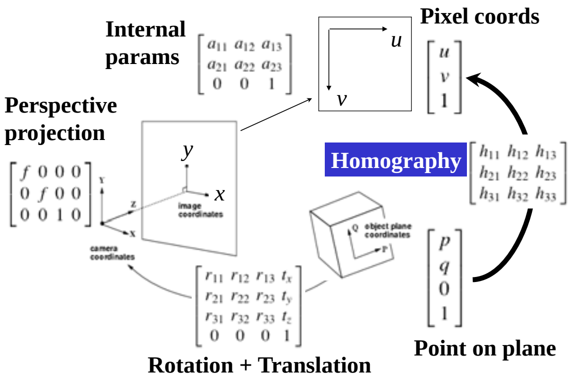
Lecture 06

Estimating Homography Parameters and Warping Images

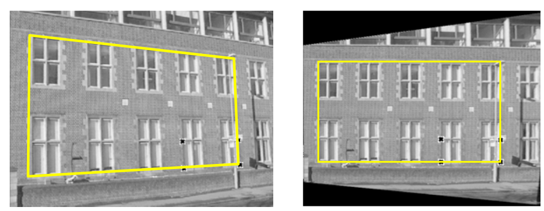
Today's Objectives

- Apply least squares to estimate the homography transformation given four or more point correspondences
- Develop an image mapping technique to (un)warp image pixel values to produce a new picture

Recall: Planar Projection



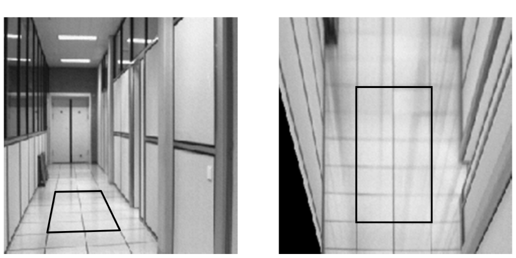
Applying Homographies to Remove Perspective Distortion



from Hartley & Zisserman

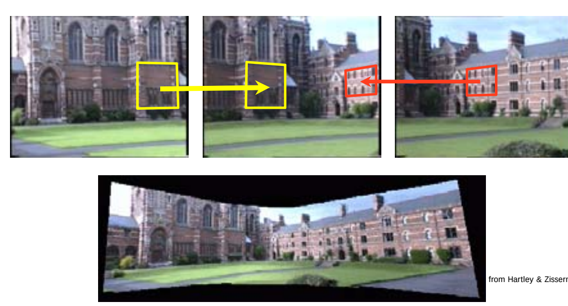
4 point correspondences suffice for the planar building facade

Homographies for Bird's-eye Views



from Hartley & Zisserman

Homographies for Mosaicing



from Hartley & Zisserman

Estimating a Homography

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Matrix Form:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \sim \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Equations:

$$x' = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + h_{33}}$$

$$y' = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + h_{33}}$$

Degrees of Freedom?

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$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \sim \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

There are 9 numbers $h_{11},\ldots,h_{33}$, so are there 9 DOF?

No. Note that we can multiply all h_{ij} by nonzero k without changing the equations:

$$x' = \frac{kh_{11}x + kh_{12}y + kh_{13}}{kh_{31}x + kh_{32}y + kh_{33}} \qquad y' = \frac{kh_{21}x + kh_{22}y + kh_{23}}{kh_{31}x + kh_{32}y + kh_{33}}$$

$$\Rightarrow$$

$$x' = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + h_{33}} \qquad y' = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + h_{33}}$$

Enforcing 8 DOF

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One approach: Set h_{33} = 1.

$$x' = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + 1}$$

$$y' = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + 1}$$

Second approach: Impose unit vector constraint

$$x' = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + h_{33}}$$

$$y' = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + h_{33}}$$

Subject to the constraint: $h_{11}^2 + h_{12}^2 + h_{13}^2 + h_{21}^2 + h_{22}^2 + h_{23}^2 + h_{31}^2 + h_{32}^2 + h_{33}^2 = 1$

L.S. using Algebraic Distance

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Setting h_{33} = 1

$$x' = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + 1}$$

$$y' = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + 1}$$

Multiplying through by denominator

$$(h_{31}x + h_{32}y + 1)x' = h_{11}x + h_{12}y + h_{13}$$

$$(h_{31}x + h_{32}y + 1)y' = h_{21}x + h_{22}y + h_{23}$$

Rearrange

$$h_{11}x + h_{12}y + h_{13} - h_{31}xx' - h_{32}yx' = x'$$

$$h_{21}x + h_{22}y + h_{23} - h_{31}xy' - h_{32}yy' = y'$$

Algebraic Distance, h_{33}=1 (cont)

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$$\begin{array}{c} \\ \text{Point 1} \\ \\ \text{Point 2} \\ \\ \text{Point 3} \\ \\ \text{Point 4} \\ \end{array} \overset{\textbf{2N x 8}}{\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' \\ x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2x_2' & -y_2x_2' \\ 0 & 0 & 0 & x_2 & y_2 & 1 & -x_2y_2' & -y_2y_2' \\ x_3 & y_3 & 1 & 0 & 0 & 0 & -x_3x_3' & -y_3x_3' \\ 0 & 0 & 0 & x_3 & y_3 & 1 & -x_3y_3' & -y_3y_3' \\ x_4 & y_4 & 1 & 0 & 0 & 0 & -x_4x_4' & -y_4x_4' \\ 0 & 0 & 0 & x_4 & y_4 & 1 & -x_4y_4' & -y_4y_4' \end{bmatrix}} \overset{\textbf{8 x 1}}{\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix}} = \overset{\textbf{2N x 1}}{\begin{bmatrix} x_1' \\ y_1' \\ x_2' \\ y_2' \\ x_3' \\ y_3' \\ x_4' \\ y_4' \end{bmatrix}}$$

additional points $\vdots$ $\vdots$

Algebraic Distance, h_{33}=1 (cont)

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Linear equations $\underset{}{\overset{2Nx8}{\mathbf{A}}} \; \overset{8x1}{\mathbf{h}} = \overset{2Nx1}{\mathbf{b}}$

Solve: $\overset{8x2N}{\mathbf{A^T}} \; \overset{2Nx8}{\mathbf{A}} \; \overset{8x1}{\mathbf{h}} = \overset{8x2N}{\mathbf{A^T}} \; \overset{2Nx1}{\mathbf{b}}$

$$\overbrace{(\mathbf{A^T} \; \mathbf{A})}^{8x8} \; \overset{8x1}{\mathbf{h}} = \overbrace{(\mathbf{A^T} \; \mathbf{b})}^{8x1}$$

$$\mathbf{h} = (\mathbf{A^T} \; \mathbf{A})^{-1} (\mathbf{A^T} \; \mathbf{b})$$

Matlab: $\mathbf{h} = \mathbf{A} \backslash \mathbf{b}$

Caution: Numeric Conditioning

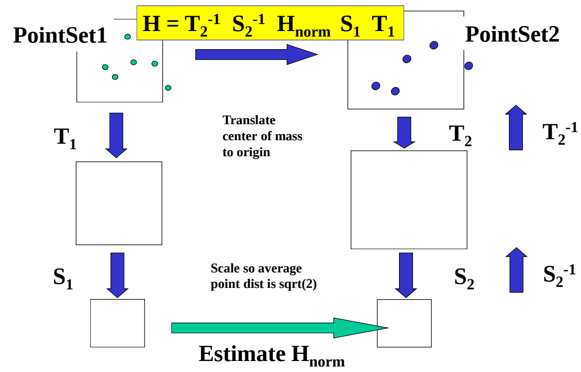
R.Hartley: “In Defense of the Eight Point Algorithm”

Observation: Linear estimation of projective transformation parameters from point correspondences often suffer from poor “conditioning” of the matrices involves. This means the solution is sensitive to noise in the points (<u>even if there are no outliers</u>).

To get better answers, precondition the matrices by performing a normalization of each point set by:

- translating center of mass to the origin
- scaling so that average distance of points from origin is sqrt(2).
- do this normalization to each point set independently

Hartley’s PreConditioning



A More General Approach

What might be wrong with setting $h_{33} = 1$?

If h_{33} actually = 0, we can’t get the right answer.

Algebraic Distance, ||h||=1

||h|| = 1

$$x' = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + h_{33}}$$

$$y' = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + h_{33}}$$

Multiplying through by denominator

$$(h_{31}x + h_{32}y + h_{33})x' = h_{11}x + h_{12}y + h_{13}$$

$$(h_{31}x + h_{32}y + h_{33})y' = h_{21}x + h_{22}y + h_{23}$$

Rearrange

$$h_{11}x + h_{12}y + h_{13} - h_{31}xx' - h_{32}yx' - h_{33}x' = 0$$

$$h_{21}x + h_{22}y + h_{23} - h_{31}xy' - h_{32}yy' - h_{33}y' = 0$$

Algebraic Distance, ||h||=1 (cont)

2N x 9   9 x 1   2N x 1

4 POINTS

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' & -x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' & -y_1' \\ & & & & \vdots & & & & \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \end{bmatrix}$$

additional points

Algebraic Distance, ||h||=1 (cont)

Homogeneous equations $\underset{2N\times 9}{\mathbf{A}} \; \underset{9\times 1}{\mathbf{h}} = \underset{2N\times 1}{\mathbf{0}}$

Solve: $\underset{9\times 2N}{\mathbf{A}^T} \; \underset{2N\times 9}{\mathbf{A}} \; \underset{9\times 1}{\mathbf{h}} = \underset{9\times 2N}{\mathbf{A}^T} \; \underset{2N\times 1}{\mathbf{0}}$

$$\overbrace{(\mathbf{A}^T \mathbf{A})}^{9\times 9} \; \underset{9\times 1}{\mathbf{h}} = \underset{9\times 1}{\mathbf{0}}$$

SVD of A^TA = U D V^T

**Let h be the column of V associated with the smallest singular value in D.
(if only 4 points, that singular value will be 0)**