CT 2

1. **1.** Nonlinear optimization problem is given in the following form

$$\min\{f(x) = 10\,{x_1}^2{x_2}^2 + 2\,{x_1}^2{x_3}^2 + {x_2}^4 - 10\,{x_2}^2{x_3}^2 - 3\,{x_3}^4\},$$

subject to

$$h_1(x) = 13\,{x_1}^2 + 8\,x_1x_2 - 2\,x_1x_3 + 1063\,x_1 + 3\,{x_2}^2 + 2\,x_2x_3 + 951\,x_2 + 10\,{x_3}^2 - 232\,x_3 - 120 = 0,$$

$$g_1(x) = 538\,x_1 + 479.5\,x_2 - 118\,x_3 - 58.5 \leqslant 0,$$

$$g_2(x) = 930.75\,x_1 + 755.25\,x_2 - 22.5\,x_3 - 657 \leqslant 0.$$

Check whether $\bar{x} = (1, -1, 0)^\top$, $\tilde{x} = (116.514008, -424.475095, 52.491911)^\top$, $\hat{x} = (4, -4, 2)^\top$ are stationary points. (1.0)

$$\vec{\nabla} f = \begin{bmatrix} f'_{x_1} \\ f'_{x_2} \\ f'_{x_3} \end{bmatrix} = \begin{bmatrix} 20x_1x_2^2 + 4x_1x_3^2 \\ 20x_1^2x_2 + 4x_2^3 - 20x_2x_3^2 \\ 4x_1^2x_3 - 20x_2^2x_3 - 12x_3^3 \end{bmatrix}$$

$$\vec{\nabla} h_1 = \begin{bmatrix} 26x_1 + 8x_2 - 2x_3 + 1063 \\ 8x_1 + 6x_2 + 2x_3 + 951 \\ -2x_1 + 2x_2 + 20x_3 - 232 \end{bmatrix}$$

$$\mathcal{L}(x, \lambda, \mu_1, \mu_2) = f(x) + \lambda h_1(x) + \mu_1 g_1(x) + \mu_2 g_2(x)$$

we need to check 4 conditions:

1) Primal feasibility
2) Complementarity
3) Stationarity
4) Dual feasibility

- Checking point $\bar{x}$

① $h_1(\bar{x})=0 \quad g_1(\bar{x})=0 \quad g_2(\bar{x})=-481.5$

② $\mu_1 g_1(\bar{x})=0 \qquad \mu_2 g_2(\bar{x})=0 \Rightarrow \mu_2=0$

③ $\mathcal{L}'_{x_i}=f'_{x_i}+\lambda h'_{1x_i}+\mu_1 g'_{1x_i}=0 \quad \forall i$

$$\begin{cases} 20+\lambda\cdot 1081+\mu_1 538=0 \\ -24+\lambda\cdot 353+\mu_1\cdot 479.5=0 \\ 0-\lambda\cdot 236-118\mu_1=0 \end{cases} \xrightarrow{RREF} \left[\begin{array}{ccc|c} 1 & 0 & -0.125 & 0 \\ 0 & 1 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\lambda=-4 \quad \mu_1=8$

④ $\mu_1=8\geq 0 \qquad \mu_2=0\geq 0$ $\qquad \underline{\bar{x} \text{ is stationary}}$

- Checking point $\tilde{x}$

① $h_1(\tilde{x})\approx 0\ (|h_1(\tilde{x})|<10^{-5}) \quad g_1(\tilde{x})=-147103.8172465$

$g_2(\tilde{x})=-213977.47055025$

② $\mu_1 g_1(\tilde{x})=0, \ \mu_2 g_2(\tilde{x})=0 \Rightarrow \mu_1=0$ and $\mu_2=0$

③ $\mathcal{L}'_{x_i}=f'_{x_i}+\lambda h'_{1x_i}=0 \quad \forall i$

$$\begin{cases} 421151967.72610205 + 591.579626 * \lambda = 0 \\ -397783545.7692034 + -558.754684 * \lambda = 0 \\ -188044128.50252962 + -264.139986 * \lambda = 0 \end{cases}$$

$$\begin{cases} \lambda = -711910.8725460098 \\ \lambda = -711910.8924896338 \\ \lambda = -711910.8747985233 \end{cases} \Rightarrow \lambda \approx -711910.9$$

④ $\mu_1=0\geq 0 \qquad \mu_2=0\geq 0$ $\qquad \underline{\tilde{x} \text{ is stationary}}$

- Checking point $\hat{x}$

① $h_1(\hat{x}) = 0 \quad g_1(\hat{x}) = -60.5 \quad g_2(\hat{x}) = 0$

② $\mu_1 g_1(\hat{x}) = 0 \Rightarrow \mu_1 = 0$

$\mu_2 g_2(\hat{x}) = 0$

③ $\mathcal{L}'_{x_i} = f'_{x_i} + \lambda h'_{1x_i} + \mu_2 g'_{2x_i} = 0 \quad \forall i$

$$\begin{cases} 1344 + 1131\lambda + 930.75\mu_2 = 0 \\ -1216 + 963\lambda + 755.25\mu_2 = 0 \\ -608 - 208\lambda - 22.5\mu_3 = 0 \end{cases} \xrightarrow{RREF} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \quad \text{No solutions}$$

$\hat{x}$ is not stationary