1.

Machine	Details produced	Rental cost ($)	Unit production cost ($)	Total production capacity
1	1, 2, 3	2000	17	250
2	1, 3, 4	2500	15	200
3	2, 4	1500	20	100
4	3, 4	1400	18	300
5	1, 4	1700	14	150

Let D_i - set of details producible on i^{th} machine

R_i - rental cost of i^{th} machine

U_i - unit production cost for i^{th} machine

T_i - total production capacity of i^{th} machine

P_j - monthly production rate for j^{th} detail

m_i - indicator of using i^{th} machine

X_{ij} - number of j^{th} details produced on i^{th} machine

Target: $z = \sum_{i=1}^{5} m_i \cdot \left[R_i + U_i \cdot \sum_{j \in D_i} X_{ij} \right] \to min$

Constraints: $\forall j \sum_{i=1}^{5} m_i \cdot x_{ij} \geq P_j$ - monthly plan

$\forall i \sum_{j=1}^{4} m_i \cdot X_{ij} \leq T_i$ - production capacity

$\forall i \; m_i \in \{0, 1\} \quad \forall i \forall j \; X_{ij} \geq 0$ - common sense

$\forall i \; \forall j \notin D_i \; X_{ij} = 0$ - production possibility

Using AMPL: z = 11750 $

```
x :=
1 1   150
1 2   100
1 3     0
1 4     0
2 1    50
2 2     0
2 3    50
2 4   100
3 1     0
3 2     0
3 3     0
3 4     0
4 1     0
4 2     0
4 3     0
4 4     0
5 1     0
5 2     0
5 3     0
5 4     0
;
```

```
m [*] :=
1  1
2  1
3  0
4  0
5  0
;
```

That is machine_1 : $150 \cdot d_1 + 100 \cdot d_2$ (d_i - i^{th} detail)

machine_2 : $50 \cdot d_1 + 50 d_3 + 100 d_4$

$Z = 11750\$$

2.

Design	Required number of carpets				
	Month 1	Month 2	Month 3	Month 4	Month 5
Aladdin tale A_j	30	20	20	20	30
Persian night P_j	10	30	35	15	10

	Machine 1	Machine 2	Machine 3
Aladdin tale → Persian night			
Resetting cost (\$) ARC_i	100	90	110
Resetting time (hours) ART_i	20	15	18
Persian night → Aladdin tale			
Resetting cost (\$) PRC_i	150	180	120
Resetting time (hours) PRT_i	15	10	12

	Machine 1	Machine 2	Machine 3
Unit production time (hours)			
Aladdin tale APT_i	10	12	8
Persian night PPT_i	12	14	16
Unit production cost (\$)			
Aladdin tale APC_i	90	80	120
Persian night PPC_i	120	110	130

5\$/month - cost of storing overproduced carpet

$a_0 = 20$ - initial #Aladdin tale

$p_0 = 15$ - initial #persian night

160 working hours for each machine

Let S_{ij} - whether i^{th} machine produces Aladdin tale at j^{th} month

q_{ij} - quantity produced by i^{th} machine at j^{th} month

Then $a_j = a_0 + \sum_{k=1}^{j} \sum_{i=1}^{3} S_{ik} \cdot q_{ik} - \sum_{k=1}^{j} A_k$ is #Alladin after j^{th} month

analogically $p_j = p_0 + \sum_{k=1}^{j} \sum_{i=1}^{3} (1 - S_{ik}) \cdot q_{ik} - \sum_{k=1}^{j} P_k$ is #persian night

Target: $\sum_{j=0}^{4} (a_j + p_j) \cdot 5\$ + \sum_{j=1}^{5} \sum_{i=1}^{3} q_{ij} \cdot \left[S_{ij} \cdot APC_i + (1 - S_{ij}) \cdot PPC_i \right] +$

$+ \sum_{j=1}^{5} \sum_{i=1}^{3} (S_{ij} \neq S_{ij-1}) \cdot \left[S_{ij} \cdot PRC_i + (1 - S_{ij}) \cdot ARC_i \right] \to min$

Constraints: $\forall j \; a_j \geq 0 \quad \forall j \; p_j \geq 0$ - demand satisfaction

$\forall i \forall j \in \{1..5\} \; q_{ij} \cdot \left[S_{ij} \cdot APT_i + (1 - S_{ij}) \cdot PPT_i \right] +$

$+ (S_{ij} \neq S_{ij-1}) \cdot \left[S_{ij} \cdot PRT_i + (1 - S_{ij}) \cdot ART_i \right] \leq 160$ - working hours limit

$\forall i \forall j \; S_{ij} \in \{0, 1\} \quad \forall i \forall j \; q_{ij} \geq 0$ - common sense $\forall i \; S_{i0} = 1$ - initial state

Solving with AMPL:

```
z = 21150 $

:      s    q      :=
1 0    1     .
1 1    0    10
1 2    0    13
1 3    0    13
1 4    1     9
1 5    1    16
2 0    1     .
2 1    1    13
2 2    0    10
2 3    0    11
2 4    1    12
2 5    1    13
3 0    1     .
3 1    1    17
3 2    1    20
3 3    0     8
3 4    0    10
3 5    0    10
;

:    a     p      :=
0    20    15
1    20    15
2    20     8
3     0     5
4     1     0
5     0     0
;
```

3.

	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
Profit ($/ton) P_j	100	120	110	140	90	100

Accumulated amount of processed ore	I_s	Maintenance cost ($) M_s		Processing capacity decrease (%) PCD_s	
0 – 1500 tons	$I_0=0$	1000	$M_0=0$	20%	$PCD_0=0$
1500 – 2500 tons	$I_1=1500$	2000		30%	
2500 – 4000 tons	$I_2=2500$ $I_3=4000$	4000		50%	

Let a_{ij} - acc. amount of processed ore by i^{th} mill after j^{th} month

m_{ijs} – was i^{th} mill maintained at j^{th} month at s^{th} wear interval

X_{ij} – amount of processed ore by i^{th} mill at j^{th} month

It's reasonable to put $a_{10}=600$ $a_{20}=400$ $a_{30}=1000$

rep_{ij} - repaired resource of i^{th} mill at j^{th} month

Then: $a_{ij} = a_{ij-1} + X_{ij} - rep_{ij}$

Target: $\sum_{j=1}^{6}\sum_{i=1}^{3} P_j \cdot X_{ij} - \sum_{j=1}^{6}\sum_{i=1}^{3}\sum_{s=1}^{3} m_{ijs} \cdot M_s \Rightarrow max$

Constraints: $\forall j \quad \sum_{i=1}^{3} X_{ij} \in [2000, 3000]$ - next stage condition

$\forall i \forall j \quad X_{ij} \le 1000 \cdot \sum_{s=0}^{3} \left[m_{ijs} \cdot (1 - PCD_s) \right]$ - production restriction

$\forall i \forall j \quad \sum_{s=0}^{3} m_{ijs} = 1 \quad \forall i \forall j \forall s \; m_{ijs} \in \{0, 1\}$ - common sense

$\forall j \quad \sum_{i=1}^{3}\sum_{s=1}^{3} m_{ijs} \le 1$ - at most 1 machine maintenance per month

$\forall i,j,s \quad m_{ijs} \cdot I_{s-1} \le a_{ij-1} \le a_{ij-1} \cdot (1 - m_{ijs}) + m_{ijs} \cdot I_s$ - wear level condition

$\forall i \forall j \quad rep_{ij} = a_{ij-1} \cdot \sum_{s=1}^{3} m_{ijs}$ - repaired by maintenance

$\forall i,j \quad X_{ij} \ge 0 \quad \forall i,j \quad a_{ij} \ge 0$ - common sense

$\forall i,j \quad a_{ij} \le 4000$ - max without maintenance

Answer from AMPL:
(take $eps = 10^{-9}$)

```
z = 1801000$

m [1,*,*]
:   0   1   2   3   :=
1   1   0   0   0
2   1   0   0   0
3   1   0   0   0
4   0   0   0   1
5   1   0   0   0
6   1   0   0   0

 [2,*,*]
:   0   1   2   3   :=
1   1   0   0   0
2   1   0   0   0
3   0   0   1   0
4   1   0   0   0
5   1   0   0   0
6   1   0   0   0

 [3,*,*]
:   0   1   2    3  :=
1   0   1   0    0
2   1   0   0    0
3   1   0   0    0
4   1   0   0    0
5   0   0   0    1
6   1   0   0    0
;

:       a      x      rep    :=
1 0    600      .       .
1 1   1600   1000       0
1 2   2600   1000       0
1 3   3600   1000       0
1 4    500    500    3600
1 5   1500   1000       0
1 6   2500   1000       0
2 0    400      .       .
2 1   1400   1000       0
2 2   2400   1000       0
2 3    700    700    2400
2 4   1700   1000       0
2 5   2700   1000       0
2 6   3700   1000       0
3 0   1000      .       .
3 1    800    800    1000
3 2   1800   1000       0
3 3   2800   1000       0
3 4   3800   1000       0
3 5    500    500    3800
3 6   1500   1000       0
;
```