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# Reinforcement Learning
# Deep Reinforcement Learning

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**(Deep) Reinforcement Learning**

**Lesson**

Slides are available **here**

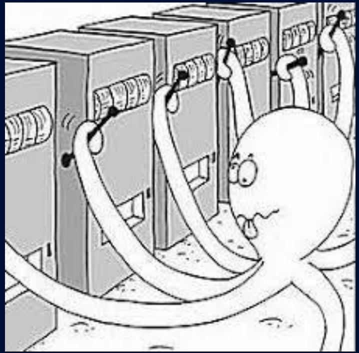
1. Reinforcement Learning, part I
    1. Multi-armed Bandits
    2. Action, reward, action-value, estimated action-value
    3. Policies
    4. Your turn ☺
2. Renforcement Learning, part II
    1. Classic RL problem and Markov Decision Process
    2. Return, state-value, action-value
    3. Temporal Difference Learning
    4. Your turn ☺
3. Deep Reinforcement Leaning
    1. Q-network
    2. Experience replay
    3. Target Network
    4. Your turn ☺

# Reinforcement Learning, Part I
# Multi-armed Bandit

## Problem formulation:

- You have a Wi-Fi network with 7 different channels
- You need to transmit 10.000 packets

→ Which channel do you choose?



## Framework:

- For each attempt, you choose a channel (action)
- If the packet was sent, you got a reward : 1
- If there was a collision, you got no reward : 0

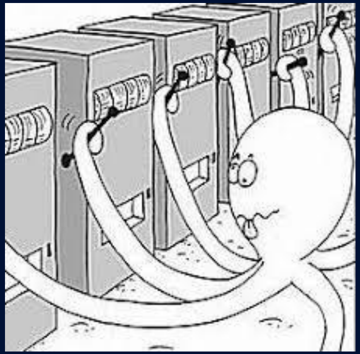
## Goal: Online training

→ Find the best action to maximize the total number of transmitted packets (received rewards)

## Reinforcement Learning, Part I
## Action, reward, action-value, estimated action-value

Notations:

- The action and reward at times step $t$ denoted in capital letters: $A_t, R_t$
- Their possible values are denoted in lower case : $a, r$



We evaluate each action with its action-value function :

- What is the expected return if we choose this channel (action)?

$$q_*(a) = \mathrm{E}[R_t | A_t = a]$$

Problem : We don’t know it ☹



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# Reinforcement Learning, Part I
## Action, reward, action-value, estimated action-value

We need to estimate the action-value function :

- Test each channel many times
- Compute an average for each channel

$$Q_t(a) = \frac{\text{sum of rewards when we took the action } a}{\text{number of time we took the action } a}$$
$$= \frac{\sum_{i=1}^{N_t(a)} R_i^a}{N_t(a)}$$

Notations:

- The true action-value function is denoted in lower case : $q_*(a)$
- Its estimate at time $t$ is denoted in capital letters : $Q_t(a)$
- $N_t(a)$ denotes the number of times action $a$ has been selected at time $t$
- $\left[R_1^a, R_2^a, \dots, R_{N_t(a)}^a\right]$ denotes the rewards we got when taking the action $a$

# Reinforcement Learning, Part I
## Action, reward, action-value, estimated action-value

We can then choose the best action at time $t$ :

$$A_t = \arg\max_a Q_t(a)$$

We chose the action that will most probably give the best reward

Tradeoff:

- We need to test each channel many times to have the best estimate $Q_t(a)$ → Exploration
- We need to take the best action to maximize the rewards → Exploitation

# Reinforcement Learning, Part I
## Policies

How to both explore and exploit ? We follow a policy $\pi$

- The policy define how we chose the action at each time step $t$

The most basic policy is the $\epsilon$-greedy policy:

At each time step $t$:

- With a probability $\epsilon$, take a random action
  → Refine the action-value estimation for that action
- With a probability (1- $\epsilon$), take the best action $A_t = \arg\max_a Q_t(a)$
  → Maximize the reward

$\epsilon$ defines the tradeoff exploration/exploitation

# Policies

Another one is the Upper-Confidence-Bound (UCB) policy:

$N_t(a)$ denotes the number of times action $a$ has been selected at time $t$

Select the best action according to :

$$A_t = \arg\max_a \left[ Q_t(a) + \sqrt{\frac{\ln(t)}{N_t(a)}} \right]$$

Estimate of the average return
for the action $a$

Measure of uncertainty
of the estimate $Q_t(a)$

Often takes the best action but still refine the estimate $Q_t(a)$

# Let’s play ☺

**Exercise**

Multiple Access Channel with Reinforcement Learning

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    1. Multi-armed Bandits
    2. Action, reward, action-value, estimated action-value
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    4. Your turn ☺
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# Reinforcement Learning, Part II
## Classic RL problem

Problem formulation:

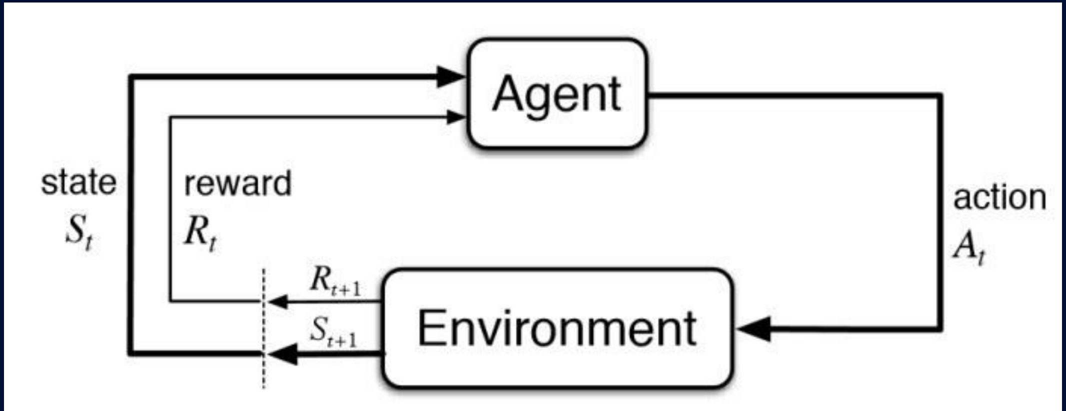
- You are in an industrial environment with 7 different channels
- All existing machines transmit with a certain periodicity
- You install a new IoT sensor, and need to find on which channel to transmit 5 packets
- You can sense the channels before sending a packet(!)

→ Which channels do you choose?

Framework: At each time step

- You sense the channels (state)
- You see if the previously transmitted packet has been correctly received (reward)
- You decide on which channel you transmit next (action)

# Reinforcement Learning, Part II
## Classic RL problem




The IoT sensor is a agent which interacts with an environment

At each time step $t$ :

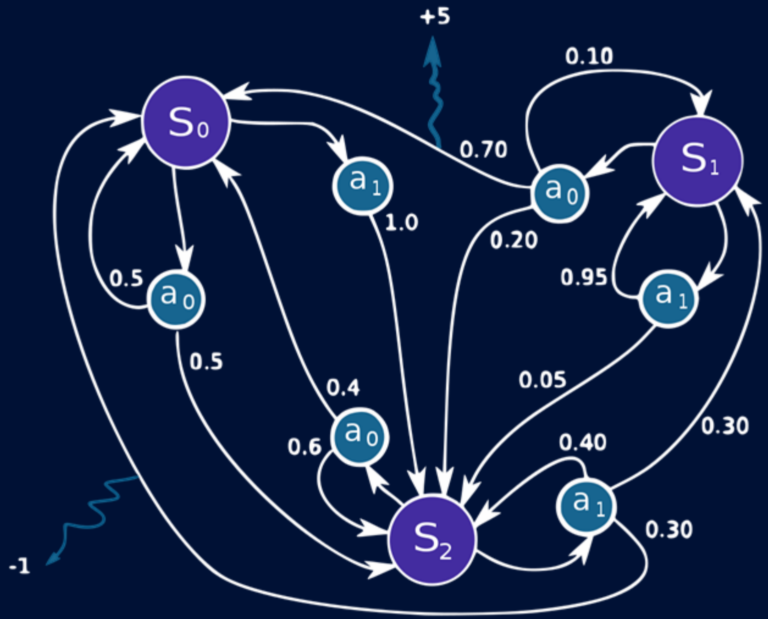
- The agent receives a state (sensed channels) and a reward (previous tx successful?)
- The agent takes an action (choose the next channel)

The sequence is defined by $S_0, A_0, \mathrm{R}_1, \mathrm{S}_1, \mathrm{A}_1, \mathrm{R}_2, \mathrm{S}_2, \mathrm{A}_2, \mathrm{R}_3, \ldots$

# Reinforcement Learning, Part II
# Markov Decision Process

A Markov Decision Process (MDP) is a 4-tuple $(\mathcal{S}, \mathcal{A}_s, \mathcal{P}_a, \mathcal{R}_a)$ :

- $\mathcal{S}$ is a finite set of states
- $\mathcal{A}_s$ is the finite set of actions available from state $s$
- $\mathcal{P}_a(s,s') = \Pr(s_{t+1} = s' | s_t = s, a_t = a)$ is the probability that action $a$ in state $s$ at time $t$ will lead to state $s'$ at time $t+1$
- $\mathcal{R}_a(s,s')$ is the reward received after transitioning from state $s$ to state $s'$, due to action $a$



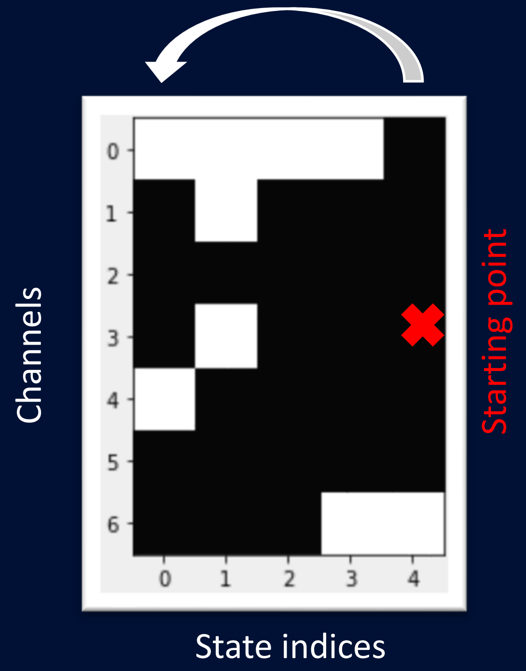
# Reinforcement Learning, Part II
## Classic RL problem

For simplicity, let's have a known deterministic environment:

- The white boxes are free channels
- The black boxes are already used channels
- You have 5 different channel states
- You start at the "starting point"

Goal : Find the best channels for the 5 transmissions

The transmission of 5 packets is called an episode



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# Reinforcement Learning, Part II
## Classic RL problem

The state consists of:

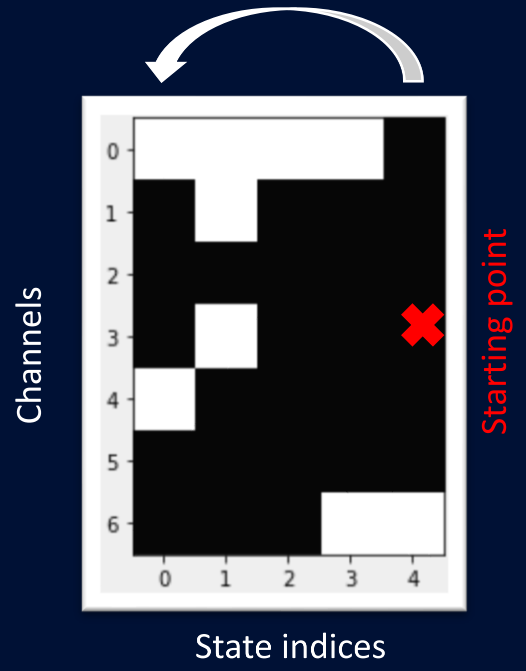
- The channel state index we sense (5 possibilities)
- The channel in which we transmitted (7 possibilities)
- A Boolean to indicate if we reached the last state (5 transmissions)

The rewards are :

- 1 if the transmission was successful
- 0 otherwise

The possible actions are (!):

- Transmit in the channel above (mod 7)
- Transmit in the same channel
- Transmit in the channel below (mod 7)



# Return, state-value, action-value

The discounted return is the sum of rewards after a time step $t$ :

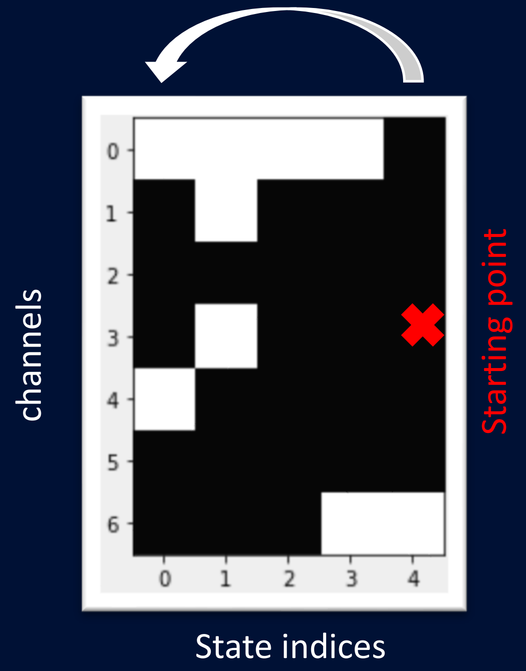
$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

$$= \mathrm{R}_{t+1} + \gamma \, G_{t+1}$$

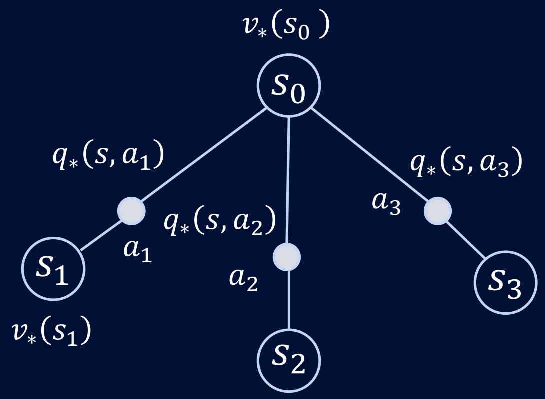
The discount factor $\gamma$ , with $0 < \gamma < 1$, is used for:

- Having a finite return even if the number of future time steps $k$ is infinite
- Maximizing short-term ($\gamma = 0$) or long-term ($\gamma = 1$) reward

Note that in our problem, the maximum time step is 5



# Reinforcement Learning, Part II
# Return, state-value, action-value




For a given state $s$ :

- The policy $\pi(a|s)$ is the probability of choosing an action $a$
- The state-value function is the expected return

$$v_\pi(s) = \mathrm{E}[G_t(\pi)|\, S_t = s]$$

- The action-value function is the expected return if we choose action $a$

$$q_\pi(s,a) = \mathrm{E}[G_t(\pi)|\, S_t = s, A_t = a]$$

We want to find a policy that choose the best actions according to the $q_\pi(s,a_i)$

But how do we estimate those $q_\pi(s,a_i)$ ?

# Reinforcement Learning, Part II
## Return, state-value, action-value

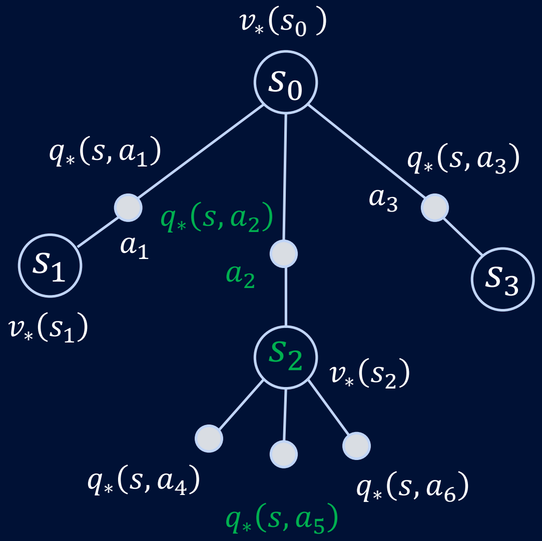
The optimal policy is the one that maximizes $v_\pi(s)$ and $q_\pi(s)$:

- The optimal action-value function is

$$q_*(s,a) = \max_\pi q_\pi(s,a)$$

- The optimal state-value function is

$$v_*(s) = \max_\pi v_\pi(s)$$
$$= \max_a q_*(s,a)$$




Bellman optimality equation :

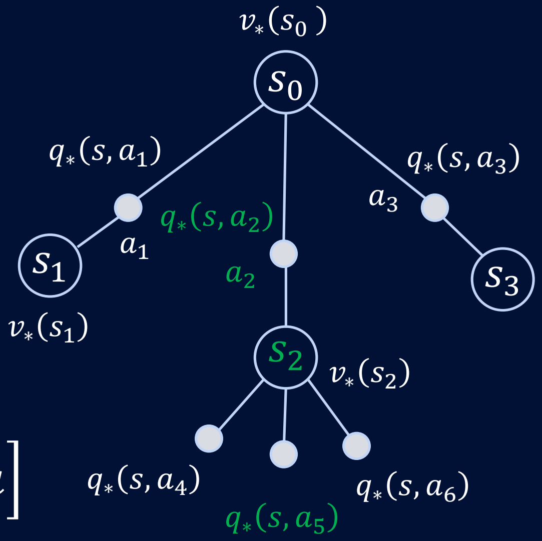
$$q_*(s,a) = \max_\pi \mathrm{E}[G_t(\pi) |\ S_t = s, A_t = a]$$
$$= \max_\pi \mathrm{E}[\mathrm{R}_{t+1} + \gamma G_{t+1}(\pi) |\ S_t = s, A_t = a]$$
$$= \mathrm{E}[\mathrm{R}_{t+1} + \gamma v_*(S_{t+1}) |\ S_t = s, A_t = a]$$

$$= \mathrm{E}\left[\mathrm{R}_{t+1} + \gamma \max_{a'} q_*(S_{t+1}, a') |\ S_t = s, A_t = a\right]$$

# Reinforcement Learning, Part II
# Return, state-value, action-value

## Bellman optimality equation

Trick : when the latest reward is received, there is no more state
"End" Boolean $\xi$: 1 if we reached the last state, 0 otherwise

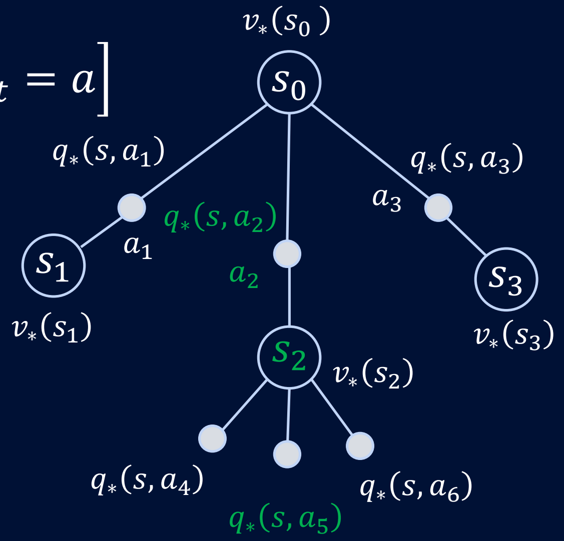



$$q_*(s,a) = \mathrm{E}\left[\mathrm{R}_{t+1} + \gamma(1-\xi)\max_{a'} q_*(S_{t+1},a') \,|\, S_t = s, A_t = a\right]$$

Next goal : find an estimate $Q_*(s,a)$ of $q_*(s,a)$

# Reinforcement Learning, Part II
# Temporal Difference Learning

Estimate $q_*(s,a) = \mathrm{E}\left[\mathrm{R}_{t+1} + \gamma(1-\xi)\max_{a'} q_*(S_{t+1},a') | S_t = s, A_t = a\right]$




Q-Learning : wait to finish a $S_t, A_t, R_{t+1}, S_{t+1}$ (with a policy $\pi$)

(0. Initialize all $\mathrm{Q}_*(s,a)$ randomly)

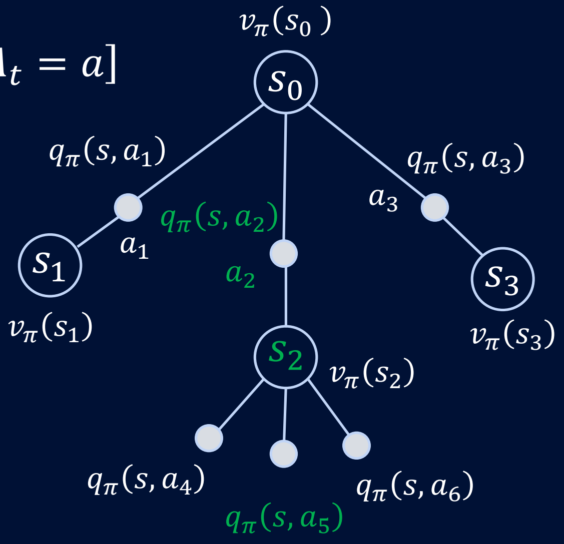
1. Take the $\mathrm{Q}_*(S_t, A_t)$ associated with your state and action
2. When in $S_{t+1}$, take the best q-value: $\max_a Q_*(S_{t+1}, a)$
3. Compute a better estimate $Q'_*(S_t, A_t) = R_{t+1} + \gamma(1-\xi)\max_a Q_*(S_{t+1}, a)$
4. Compute an error $Q'_*(S_t, A_t) - Q_*(S_t, A_t)$
5. Update $\mathrm{Q}_*(S_t, A_t) \leftarrow Q_*(S_t, A_t) + \alpha\,[Q'_*(S_t, A_t) - Q_*(S_t, A_t)]$

$\alpha$ is the learning rate

Temporal Difference (TD): use the time step $t$+1 to refine the time step $t$

# Reinforcement Learning, Part II
# Temporal Difference Learning

With a policy $\pi$: $q_\pi(s,a) = \mathrm{E}[\mathrm{R}_{t+1} + \gamma(1-\xi)q_\pi(S_{t+1},A_{t+1})|\, S_t = s, A_t = a]$




SARSA : wait to finish a $S_t, A_t, R_{t+1}, S_{t+1}, A_{t+1}$

(0. Initialize all $\mathrm{Q}(s,a)$ randomly)

1. Take the $\mathrm{Q}_\pi(S_t,A_t)$ associated with your state and action
2. Compute a better estimate $Q'_\pi(S_t,A_t) = R_{t+1} + \gamma(1-\xi)Q_\pi(S_{t+1},A_{t+1})$
3. Compute an error $Q'_\pi(S_t,A_t) - Q_\pi(S_t,A_t)$
4. Update $\mathrm{Q}_\pi(S_t,A_t) \leftarrow Q_\pi(S_t,A_t) + \alpha\,[Q'_\pi(S_t,A_t) - Q_\pi(S_t,A_t)]$

$\alpha$ is the learning rate

Here we estimated the $q_\pi(s,a)$ according to another policy $\pi$

# Reinforcement Learning, Part II
# Temporal Difference Learning

With a policy $\pi$, record $S_t, A_t, R_{t+1}, S_{t+1}, A_{t+1}$

- SARSA: On-policy method

$$Q_\pi(S_t, A_t) \leftarrow Q_\pi(S_t, A_t) + \alpha[R_{t+1} + \gamma(1-\xi)Q_\pi(S_{t+1}, A_{t+1}) - Q_\pi(S_t, A_t)]$$

Behavior policy
i.e., $\epsilon$-greedy

Behavior policy
i.e., $\epsilon$-greedy

You estimate the $Q_\pi$ according to the policy you are using

- Q-Learning : Off-policy method

$$Q_*(S_t, A_t) \leftarrow Q_*(S_t, A_t) + \alpha\left[R_{t+1} + \gamma(1-\xi)\max_a Q_*(S_{t+1}, a) - Q_*(S_t, A_t)\right]$$

Behavior policy
i.e., $\epsilon$-greedy

Optimal policy

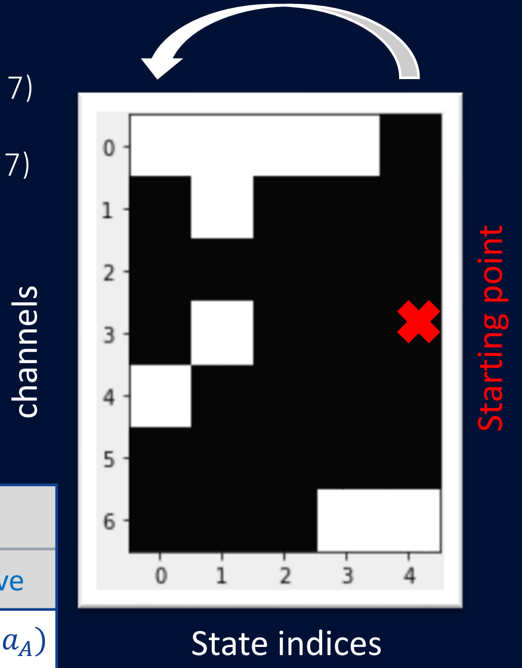
You use the policy $\pi$ to explore and estimate the optimal policy

# Reinforcement Learning, Part II
## Classic RL problem

The state comprises both:
- The channel in which we transmitted (7 possibilities)
- The channel state index we sense (5 possibilities)
- The $\xi$ Boolean (=1 if it's the end, =0 otherwise)

The rewards are :
- 1 if the transmission was successful
- 0 otherwise

The possible actions are :
- Tx in channel above (mod 7)
- Tx in same chanel
- Tx in channel below (mod 7)




Q-Learning : fill the Q-table following a $\epsilon$-greedy policy

| | **State index 0** | | | **…** | **State index 4** | | |
|---|---|---|---|---|---|---|---|
| | $a_B$: Below | $a_S$ : Same | $a_A$ : Above | … | $a_B$: Below | $a_S$ : Stay | $a_A$ : Above |
| **Ch 0** | $Q_*([0,0],a_B)$ | $Q_*([0,0],a_S)$ | $Q_*([0,0],a_A)$ | … | $Q_*([0,4],a_B)$ | $Q_*([0,4],a_S)$ | $Q_*([0,4],a_A)$ |
| **…** | … | … | … | … | … | … | … |
| **Ch 6** | $Q_*([6,0],a_B)$ | $Q_*([6,0],a_S)$ | $Q_*([6,0],a_A)$ | … | $Q_*([6,4],a_B)$ | $Q_*([6,4],a_S)$ | $Q_*([6,4],a_A)$ |

# Reinforcement Learning, Part II

## Let’s play ☺

**Exercise**

Multiple Access Channel with Reinforcement Learning

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    1. Multi-armed Bandits
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# Deep Reinforcement Learning
# Q-Network

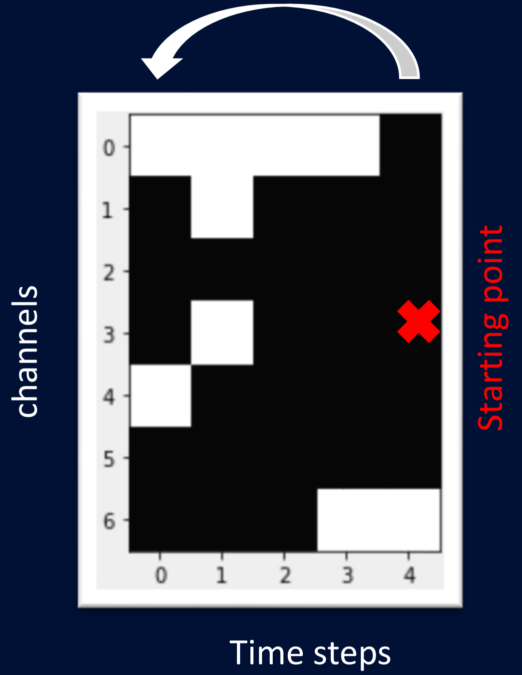
Previously, we knew that there were only 5 channel states
→ What if we don't know that?

One state is now defined by the state vector :

$$\underbrace{0\ 0\ 0\ 0\ 0\ 0\ 1}_{\text{Channel state}}\ \underbrace{0\ 0\ 0\ 1\ 0\ 0\ 0}_{\text{Current channel}}$$

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |

Free channels are set to 1 — The channel we are in is set to 1



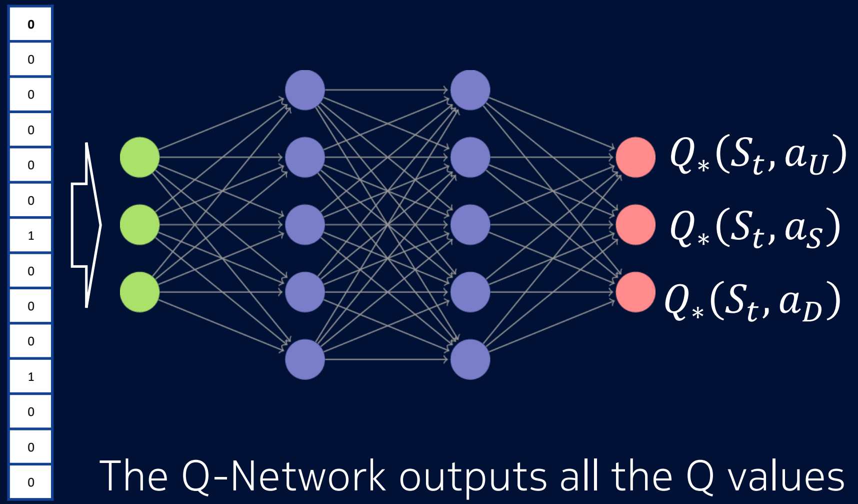

And the $\xi$ boolean

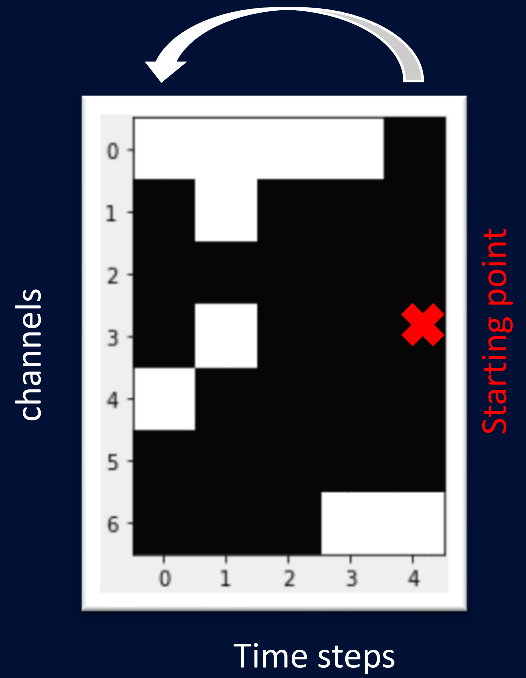
# Deep Reinforcement Learning
# Q-Network

We don't want to store a huge Q-table.

We can use a Q-Network instead :

$S_t$ vector




The Q-Network outputs all the Q values for a given states

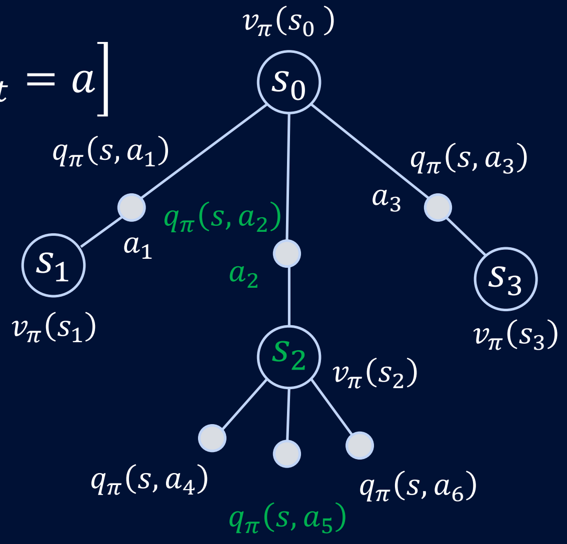
# Deep Reinforcement Learning
## Q-Network

Estimate $q_*(s,a) = \mathrm{E}\left[\mathrm{R}_{t+1} + \gamma(1-\xi)\max_{a'} q_*(S_{t+1}, a') | S_t = s, A_t = a\right]$

Q-Learning : wait to finish a $S_t, A_t, R_{t+1}, S_{t+1}$

(0. Initialize all $\mathrm{Q}_*(s,a)$ randomly)

1. Take the $\mathrm{Q}_*(S_t, A_t)$ associated with your state and action
2. When in $S_{t+1}$, take the best q-value : $\max_a Q_*(S_{t+1}, a)$
3. Compute a better estimate $Q'_*(S_t, A_t) = R_{t+1} + \gamma \max_a Q_*(S_{t+1}, a)$
4. Compute loss : $\mathrm{MSE}\big(Q'_*(S_t, A_t),\ \ Q_*(S_t, A_t)\big)$
5. Update parameters of the Q-Network by SGD to minimize the loss



# Deep Reinforcement Learning
## Q-Network

$$loss = MSE\left(\overbrace{R_{t+1} + \gamma(1-\xi)\max_a Q_*(S_{t+1}, a)}^{\text{Target}} \overbrace{- Q_*(S_t, A_t)}^{\text{Prediction}}\right)$$

Each time we update for one prediction, every parameters in the NN changes!
Two problems arises :

- *Correlation* : when you follow the trajectory, your NN will be optimized only for the last few $(s, a)$ that you took

- *Nonstationary target* : each time we update the NN, the target change as well
→ not stable

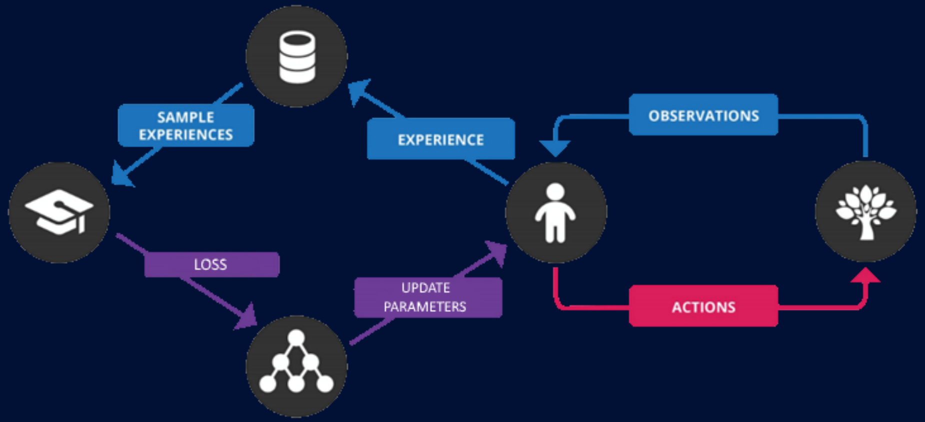
# Deep Reinforcement Learning
## Experience Replay

To remove *correlation*, we store all samples $(S_t, A_t, R_{t+1}, S_{t+1})$ in a dataset.

Then, at each iteration, we perform experience replay:

- We take a random batch of samples
- We compute the predictions and targets
- We evaluate the loss and update the Q-Network

$$loss = MSE(\text{predictions}, \text{targets})$$

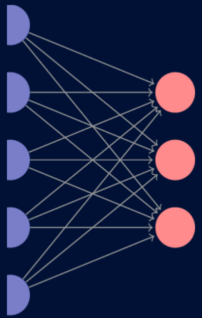


# Deep Reinforcement Learning
## Experience Replay

To remove *correlation*, we store all samples $(S_t, A_t, R_{t+1}, S_{t+1})$ in a dataset.

$$loss = MSE(\text{predictions}, \text{targets})$$

The targets can only reflect the action chosen in the sample
If at state $S_t$, the action taken was $a_U$:

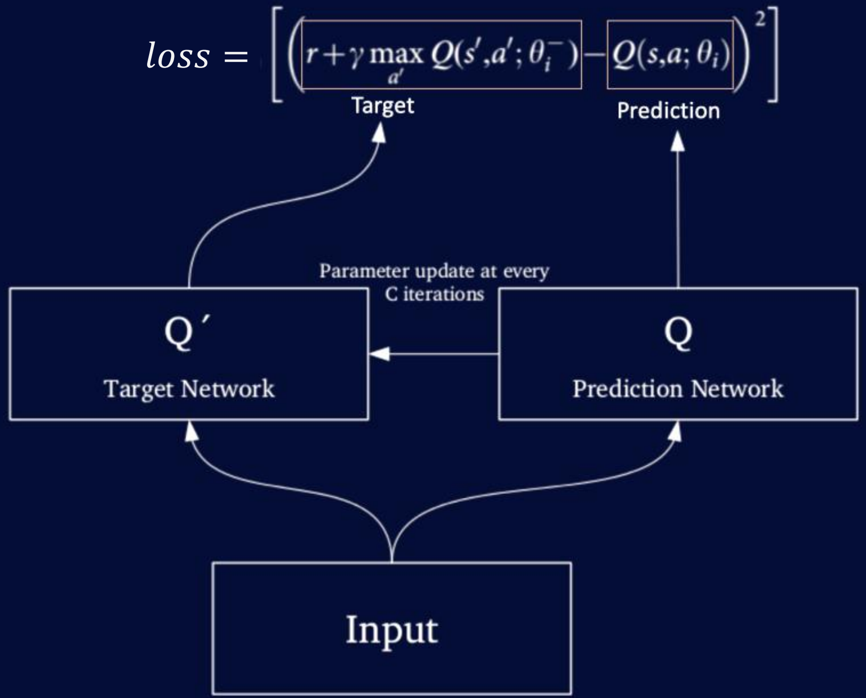


$$MSE\begin{bmatrix} \overbrace{Q_*(S_t, a_U)}^{\text{Prediction}} & - & \overbrace{R_{t+1} + \gamma(1-\xi)\max_a Q_*(S_{t+1}, a_U)}^{\text{Target}} \\ Q_*(S_t, a_S) & - & Q_*(S_t, a_S) \\ Q_*(S_t, a_D) & - & Q_*(S_t, a_D) \end{bmatrix} = MSE\begin{bmatrix} pred - target \\ 0 \\ 0 \end{bmatrix}$$

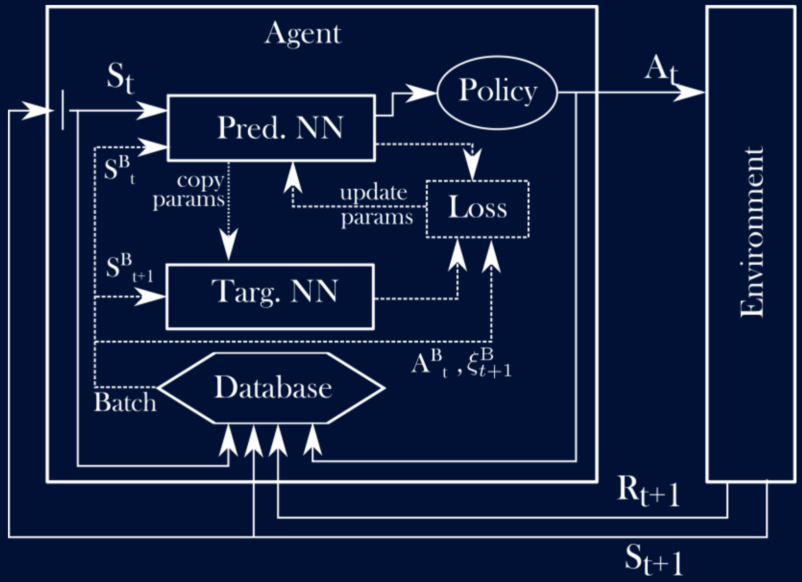
# Deep Reinforcement Learning
## Target Network

To alleviate the *nonstationary target*, we maintain a target Q-Network :

- The targets are computed according to the target network
- The parameters of the target network are updated every $C$ iterations



# Deep Reinforcement Learning
## Target Network




Main algorithm :

Initialize the pred.NN and the targ.NN with the same params

Play actions according to a policy $\pi$ to populate the datasets

For a given number of episodes :

- While $\xi \neq 1$ :
  - Choose an action $A_t$ according to the state $S_t$ and the policy $\pi$
  - Receive $R_{t+1}, S_{t+1}$ and store $(A_t, S_t, R_{t+1}, S_{t+1})$ in the database
  - Take a random batch from the database (B = batch size)
  - Compute the loss using targets from the target network
  - Update the (prediction) Q-Network
- Every $C$ iterations, copy the parameters of the pred.NN to the targ.NN

# Let's play ☺

## Exercise

Multiple Access Channel with Reinforcement Learning

# Thank you

Everything is available on
mgoutay.github.io