

# Reinforcement Learning

## Deep Reinforcement Learning

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**(Deep) Reinforcement Learning**

Lesson

Slides are available [here](#)

## 1. Reinforcement Learning, part I

1. Multi-armed Bandits
2. Action, reward, action-value, estimated action-value
3. Policies
4. Your turn 😊

## 2. Reinforcement Learning, part II

1. Classic RL problem and Markov Decision Process
2. Return, state-value, action-value
3. Temporal Difference Learning
4. Your turn 😊

## 3. Deep Reinforcement Learning

1. Q-network
2. Experience replay
3. Target Network
4. Your turn 😊

# Reinforcement Learning, Part I

## Multi-armed Bandit

Problem formulation:

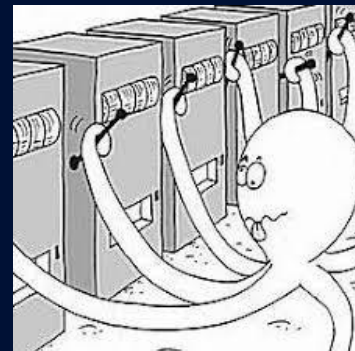
- You have a Wi-Fi network with 7 different channels
  - You need to transmit 10.000 packets
- Which channel do you choose?

Framework:

- For each attempt, you choose a channel (**action**)
- If the packet was sent, you got a **reward** : 1
- If there was a collision, you got no **reward** : 0

Goal: **Online training**

→ Find the best action to maximize the total number of transmitted packets (received rewards)



# Reinforcement Learning, Part I

## Action, reward, action-value, estimated action-value

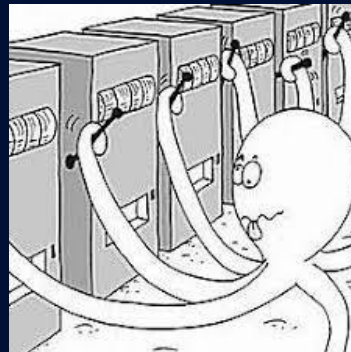
Notations:

- The action and reward at times step  $t$  denoted in capital letters:  $A_t, R_t$
- Their possible values are denoted in lower case :  $a, r$

We evaluate each action with its **action-value function** :

- What is the expected return if we choose this channel (action)?

$$q_*(a) = E[R_t | A_t = a]$$



Problem : We don't know it ☹

# Reinforcement Learning, Part I

## Action, reward, action-value, estimated action-value

We need to **estimate the action-value function** :

- Test each channel many times
- Compute an average for each channel

$$Q_t(a) = \frac{\text{sum of rewards when we took the action } a}{\text{number of time we took the action } a}$$
$$= \frac{\sum_{i=1}^{N_t(a)} R_i^a}{N_t(a)}$$

Notations:

- The true action-value function is denoted in lower case :  $q_*(a)$
- Its estimate at time  $t$  is denoted in capital letters :  $Q_t(a)$
- $N_t(a)$  denotes the number of times action  $a$  has been selected at time  $t$
- $[R_1^a, R_2^a, \dots, R_{N_t(a)}^a]$  denotes the rewards we got when taking the action  $a$

## Reinforcement Learning, Part I

Action, reward, action-value, estimated action-value

We can then choose the best action at time  $t$  :

$$A_t = \arg \max_a Q_t(a)$$

We chose the action that will most probably give the best reward

Tradeoff:

- We need to test each channel many times to have the best estimate  $Q_t(a) \rightarrow$  Exploration
- We need to take the best action to maximize the rewards  $\rightarrow$  Exploitation

# Reinforcement Learning, Part I

## Policies

How to both explore and exploit ? We follow a **policy  $\pi$**

- The policy define how we chose the action at each time step  $t$

The most basic policy is the  **$\epsilon$ -greedy policy**:

At each time step  $t$ :

- With a probability  $\epsilon$ , take a random action
  - Refine the action-value estimation for that action
- With a probability  $(1 - \epsilon)$ , take the best action  $A_t = \arg \max_a Q_t(a)$ 
  - Maximize the reward

$\epsilon$  defines the tradeoff exploration/exploitation



# Reinforcement Learning, Part I

## Policies

Another one is the **Upper-Confidence-Bound (UCB)** policy:

$N_t(a)$  denotes the number of times action  $a$  has been selected at time  $t$   
Select the best action according to :

$$A_t = \arg \max_a \left[ Q_t(a) + \sqrt{\frac{\ln(t)}{N_t(a)}} \right]$$

←  
Estimate of the average return  
for the action  $a$

←  
Measure of uncertainty  
of the estimate  $Q_t(a)$

Often takes the best action but still refine the estimate  $Q_t(a)$

Let's play 😊

## Exercise

Multiple Access Channel with Reinforcement Learning

## 1. Reinforcement Learning, part I

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4. Your turn 😊

## 2. Reinforcement Learning, part II

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# Reinforcement Learning, Part II

## Classic RL problem

Problem formulation:

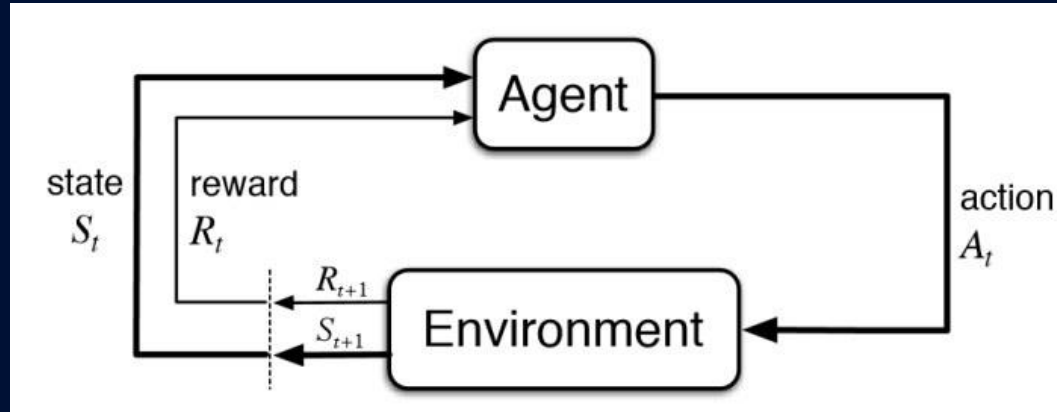
- You are in an industrial environment with 7 different channels
  - All existing machines transmit with a certain periodicity
  - You install a new IoT sensor, and need to find on which channel to transmit 5 packets
  - You can sense the channels before sending a packet(!)
- Which channels do you choose?

Framework: At each time step

- You sense the channels (**state**)
- You see if the previously transmitted packet has been correctly received (**reward**)
- You decide on which channel you transmit next (**action**)

## Reinforcement Learning, Part II

### Classic RL problem



The IoT sensor is a **agent** which interacts with an **environment**

At each time step  $t$  :

- The agent receives a state (sensed channels) and a reward (previous tx successful?)
- The agent takes an action (choose the next channel)

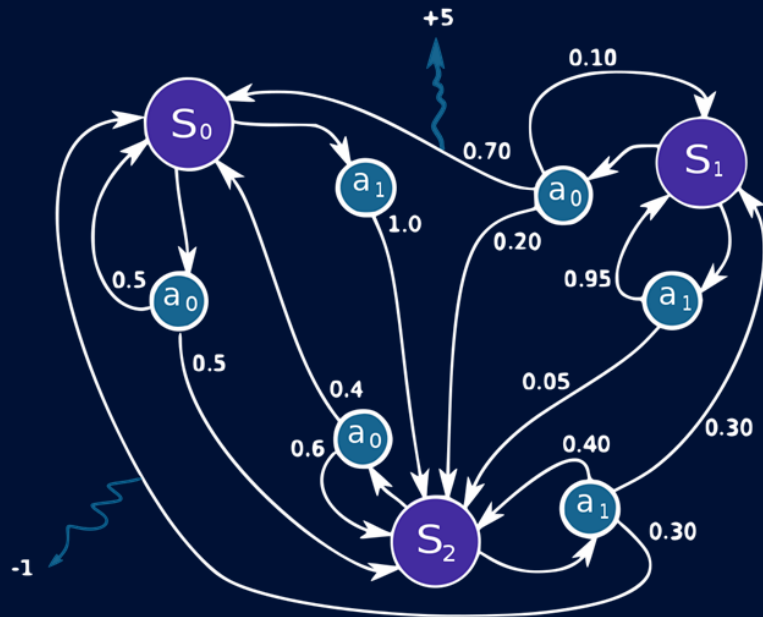
The sequence is defined by  $S_0, A_0, R_1, S_1, A_1, R_2, S_2, A_2, R_3, \dots$

# Reinforcement Learning, Part II

## Markov Decision Process

A **Markov Decision Process** (MDP) is a 4-tuple  $(\mathcal{S}, \mathcal{A}_s, \mathcal{P}_a, \mathcal{R}_a)$  :

- $\mathcal{S}$  is a finite set of states
- $\mathcal{A}_s$  is the finite set of actions available from state  $s$
- $\mathcal{P}_a(s, s') = \Pr(s_{t+1} = s' | s_t = s, a_t = a)$  is the probability that action  $a$  in state  $s$  at time  $t$  will lead to state  $s'$  at time  $t + 1$
- $\mathcal{R}_a(s, s')$  is the reward received after transitioning from state  $s$  to state  $s'$ , due to action  $a$



# Reinforcement Learning, Part II

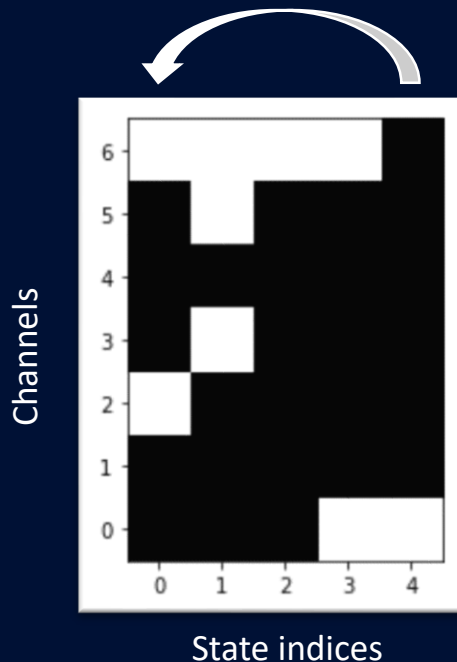
## Classic RL problem

For simplicity, let's have a known deterministic environment:

- The white boxes are free channels
- The black boxes are already used channels
- You have 5 different channel states
- You start at the “starting point”

Goal : Find the best channels for the 5 transmissions

The transmission of 5 packets is called an **episode**



# Reinforcement Learning, Part II

## Classic RL problem

The **state** consists of:

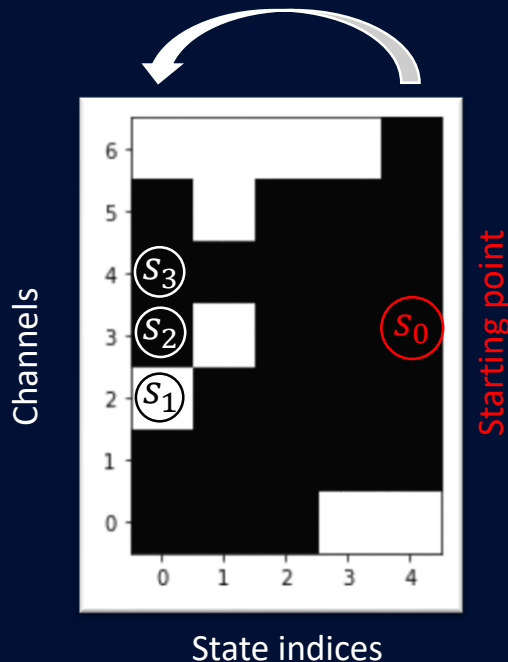
- The state index we sense (5 possibilities)
- The channel in which we transmitted (7 possibilities)
- A Boolean to indicate if we reached the last state (5 transmissions)

The **rewards** are :

- 1 if the transmission was successful
- 0 otherwise

The possible **actions** are:

- Transmit in the channel above (mod 7) :  $A$
- Transmit in the same channel :  $S$
- Transmit in the channel below (mod 7) :  $B$

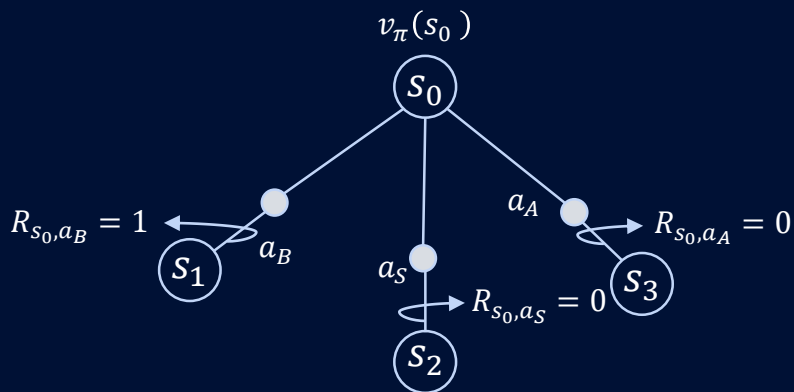




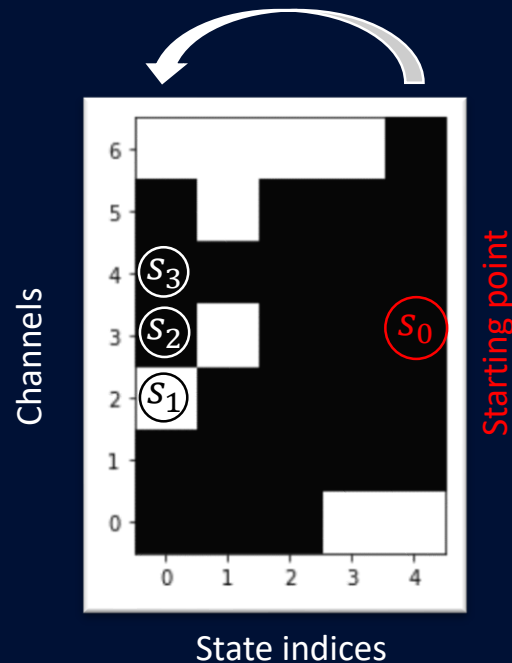
# Reinforcement Learning, Part II

## Classic RL problem

First state :  $S_0 = [state\_index, prev\_channel, bool] = [4, 3, 0]$



Possible second state :  $S_1 = [0, 2, 0]$ ,  $S_2 = [0, 3, 0]$ ,  $S_3 = [0, 4, 0]$

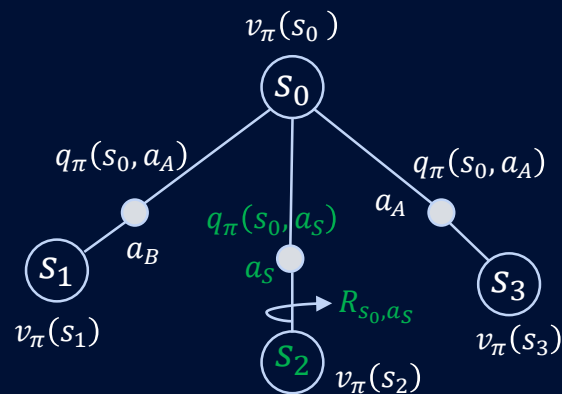


## Reinforcement Learning, Part II

### Return, state-value, action-value

The **discounted return** is the sum of rewards after a time step  $t$  :

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$
$$= R_{t+1} + \gamma G_{t+1}$$



The **discount factor**  $\gamma$ , with  $0 < \gamma < 1$ , is used for:

- Having a finite return even if the number of future time steps  $k$  is infinite
- Maximizing short-term ( $\gamma = 0$ ) or long-term ( $\gamma = 1$ ) reward

Note that in our problem, the maximum time step is 5

# Reinforcement Learning, Part II

## Return, state-value, action-value

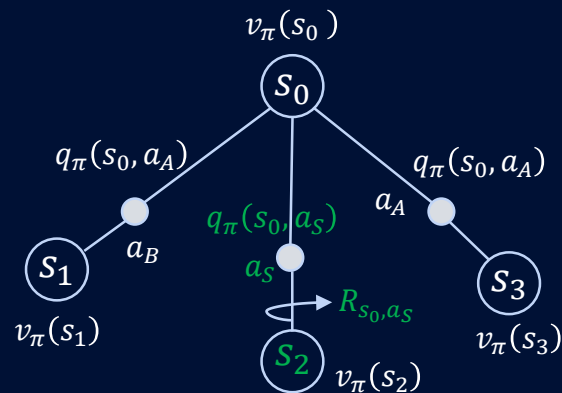
For a given state  $s$  :

- The **policy**  $\pi(a|s)$  is the probability of choosing an action  $a$
- The **state-value function** is the expected return

$$v_{\pi}(s) = E[G_t(\pi) | S_t = s]$$

- The **action-value function** is the expected return if we choose action  $a$

$$q_{\pi}(s, a) = E[G_t(\pi) | S_t = s, A_t = a]$$



We want to find a policy that choose the best actions according to the  $q_{\pi}(s, a_i)$

But how do we estimate those  $q_{\pi}(s, a_i)$  ?

# Reinforcement Learning, Part II

## Return, state-value, action-value

The optimal policy is the one that maximizes  $v_\pi(s)$  and  $q_\pi(s)$ :

- The optimal action-value function is

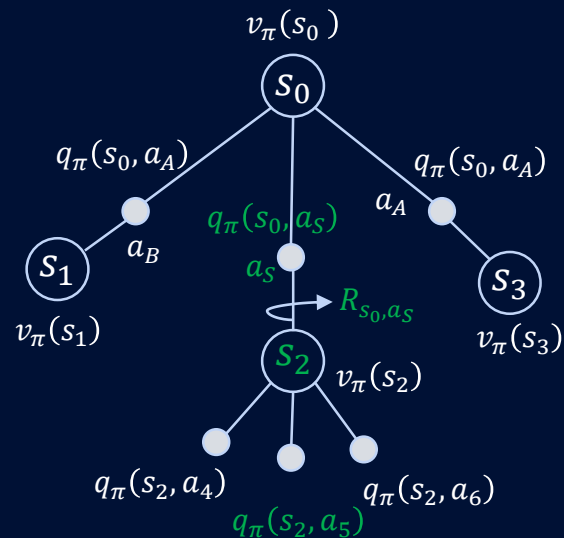
$$q_*(s, a) = \max_{\pi} q_\pi(s, a)$$

- The optimal state-value function is

$$\begin{aligned} v_*(s) &= \max_{\pi} v_\pi(s) \\ &= \max_a q_*(s, a) \end{aligned}$$

Bellman optimality equation :

$$\begin{aligned} q_*(s, a) &= \max_{\pi} \mathbb{E}[G_t(\pi) | S_t = s, A_t = a] \\ &= \max_{\pi} \mathbb{E}[R_{t+1} + \gamma G_{t+1}(\pi) | S_t = s, A_t = a] \\ &= \mathbb{E}[R_{t+1} + \gamma v_*(S_{t+1}) | S_t = s, A_t = a] \\ &= \mathbb{E} \left[ R_{t+1} + \gamma \max_{a'} q_*(S_{t+1}, a') | S_t = s, A_t = a \right] \end{aligned}$$



# Reinforcement Learning, Part II

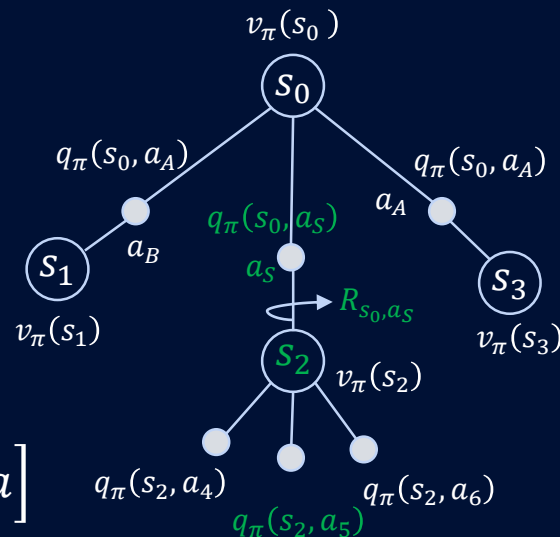
## Return, state-value, action-value

### Bellman optimality equation

Trick : when the latest reward is received, there is no more state  
“End” Boolean  $\xi$ : 1 if we reached the last state, 0 otherwise

$$q_*(s, a) = \mathbb{E} \left[ R_{t+1} + \gamma(1 - \xi) \max_{a'} q_*(S_{t+1}, a') \mid S_t = s, A_t = a \right]$$

Next goal : find an estimate  $Q_*(s, a)$  of  $q_*(s, a)$



# Reinforcement Learning, Part II

## Temporal Difference Learning

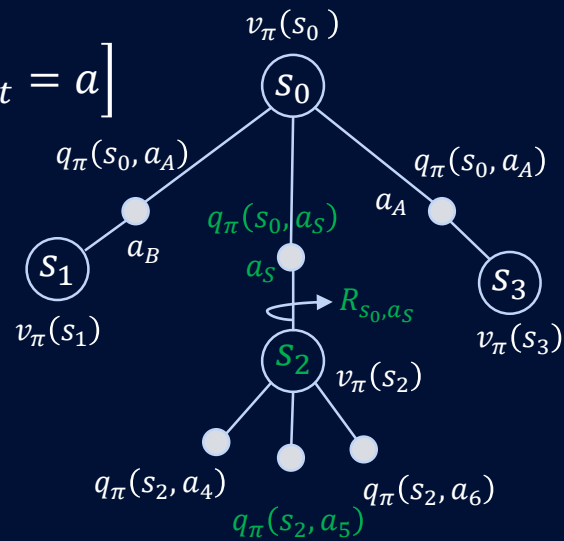
Estimate  $q_*(s, a) = \mathbb{E} \left[ R_{t+1} + \gamma(1 - \xi) \max_{a'} q_*(S_{t+1}, a') \mid S_t = s, A_t = a \right]$

**Q-Learning**: wait to finish a  $S_t, A_t, R_{t+1}, S_{t+1}$  (with a policy  $\pi$ )

(0. Initialize all  $Q_*(s, a)$  randomly)

1. Take the  $Q_*(S_t, A_t)$  associated with your state and action
2. When in  $S_{t+1}$ , take the best q-value:  $\max_a Q_*(S_{t+1}, a)$
3. Compute a better estimate  $Q'_*(S_t, A_t) = R_{t+1} + \gamma(1 - \xi) \max_a Q_*(S_{t+1}, a)$
4. Compute an error  $Q'_*(S_t, A_t) - Q_*(S_t, A_t)$
5. Update  $Q_*(S_t, A_t) \leftarrow Q_*(S_t, A_t) + \alpha [Q'_*(S_t, A_t) - Q_*(S_t, A_t)]$

$\alpha$  is the learning rate



**Temporal Difference** (TD): use the time step  $t+1$  to refine the time step  $t$

# Reinforcement Learning, Part II

## Temporal Difference Learning

With a policy  $\pi$ :  $q_{\pi}(s, a) = E[R_{t+1} + \gamma(1 - \xi)q_{\pi}(S_{t+1}, A_{t+1}) | S_t = s, A_t = a]$

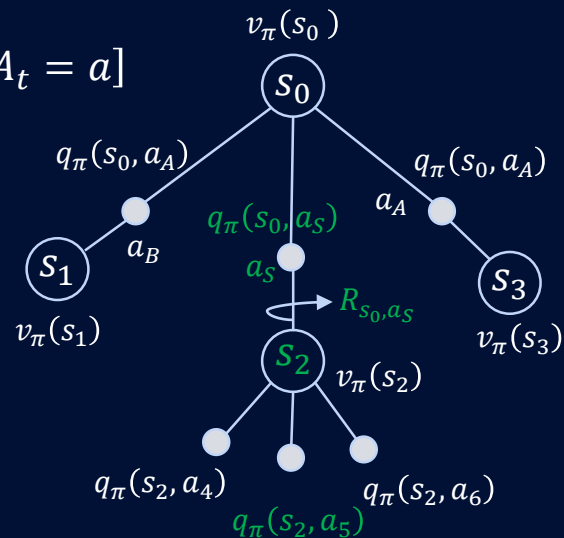
**SARSA**: wait to finish a  $S_t, A_t, R_{t+1}, S_{t+1}, A_{t+1}$

(0. Initialize all  $Q(s, a)$  randomly)

1. Take the  $Q_{\pi}(S_t, A_t)$  associated with your state and action
2. Compute a better estimate  $Q'_{\pi}(S_t, A_t) = R_{t+1} + \gamma(1 - \xi)Q_{\pi}(S_{t+1}, A_{t+1})$
3. Compute an error  $Q'_{\pi}(S_t, A_t) - Q_{\pi}(S_t, A_t)$
4. Update  $Q_{\pi}(S_t, A_t) \leftarrow Q_{\pi}(S_t, A_t) + \alpha [Q'_{\pi}(S_t, A_t) - Q_{\pi}(S_t, A_t)]$

$\alpha$  is the learning rate

Here we estimated the  $q_{\pi}(s, a)$  according to the same policy  $\pi$



# Reinforcement Learning, Part II

## Temporal Difference Learning

With a policy  $\pi$ , record  $S_t, A_t, R_{t+1}, S_{t+1}, A_{t+1}$

- SARSA: **On-policy** method

$$Q_{\pi}(S_t, A_t) \leftarrow Q_{\pi}(S_t, A_t) + \alpha [R_{t+1} + \gamma(1 - \xi) Q_{\pi}(S_{t+1}, A_{t+1}) - Q_{\pi}(S_t, A_t)]$$



Behavior policy  
i.e.,  $\epsilon$ -greedy



Behavior policy  
i.e.,  $\epsilon$ -greedy

You estimate the  $Q_{\pi}$  according to the policy you are using

- Q-Learning : **Off-policy** method

$$Q_*(S_t, A_t) \leftarrow Q_*(S_t, A_t) + \alpha \left[ R_{t+1} + \gamma(1 - \xi) \max_a Q_*(S_{t+1}, a) - Q_*(S_t, A_t) \right]$$

Optimal policy

Optimal policy

You use the policy  $\pi$  to explore and estimate the optimal policy



# Reinforcement Learning, Part II

## Classic RL problem

The **state** comprises both:

- The channel in which we transmitted (7 possibilities)
- The channel state index we sense (5 possibilities)
- The  $\xi$  Boolean (=1 if it's the end, =0 otherwise)

The **rewards** are :

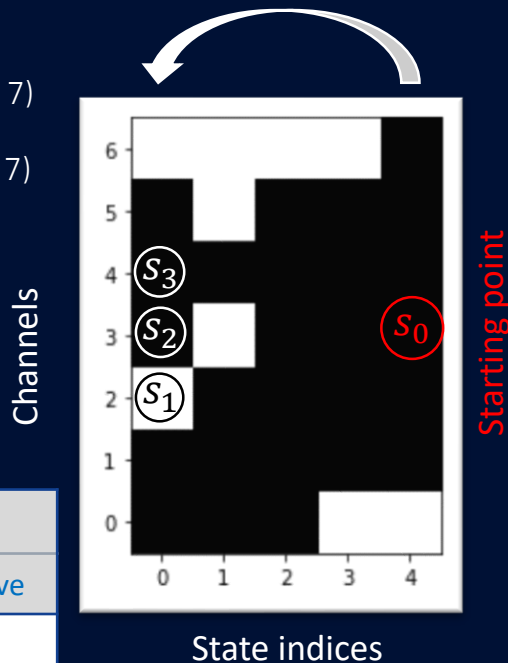
- 1 if the transmission was successful
- 0 otherwise

Q-Learning : fill the **Q-table** following a  $\epsilon$ -greedy policy

	State index 0			...	State index 4		
	$a_B$ : Below	$a_S$ : Same	$a_A$ : Above	...	$a_B$ : Below	$a_S$ : Stay	$a_A$ : Above
...	...	...	...	...	...	...	...
Ch 3	$Q_*([3, 0], a_B)$	$Q_*([3, 0], a_S)$	$Q_*([3, 0], a_A)$	...	$Q_*([3, 4], a_B)$	$Q_*([3, 4], a_S)$	$Q_*([3, 4], a_A)$
...	...	...	...	...	...	...	...

The possible **actions** are :

- Tx in channel above (mod 7)
- Tx in same channel
- Tx in channel below (mod 7)



Let's play 😊

## Exercise

Multiple Access Channel with Reinforcement Learning

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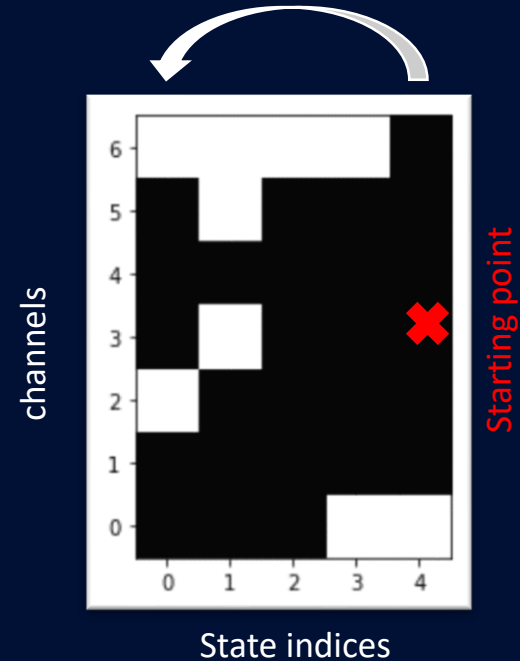
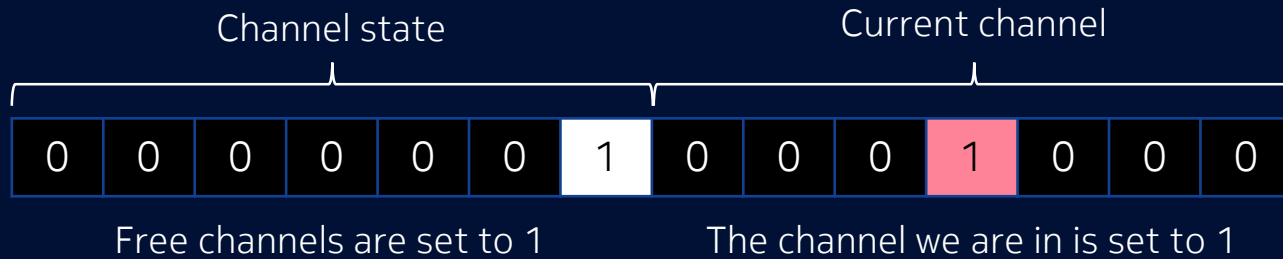
## 3. Deep Reinforcement Learning

1. Q-network
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# Deep Reinforcement Learning Q-Network

Previously, we knew that there were only 5 channel states  
→ What if we don't know that?

One state is now defined by the state vector :



And the  $\xi$  boolean

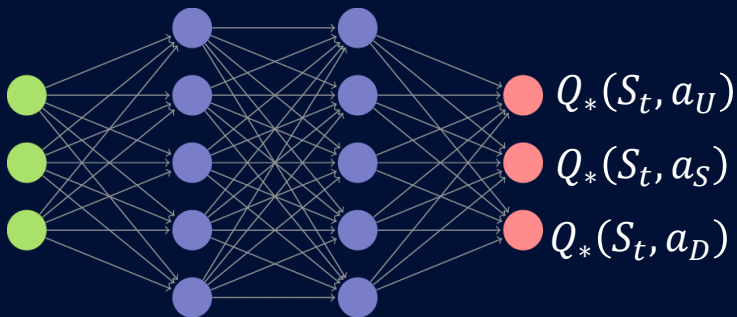
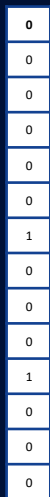
# Deep Reinforcement Learning

## Q-Network

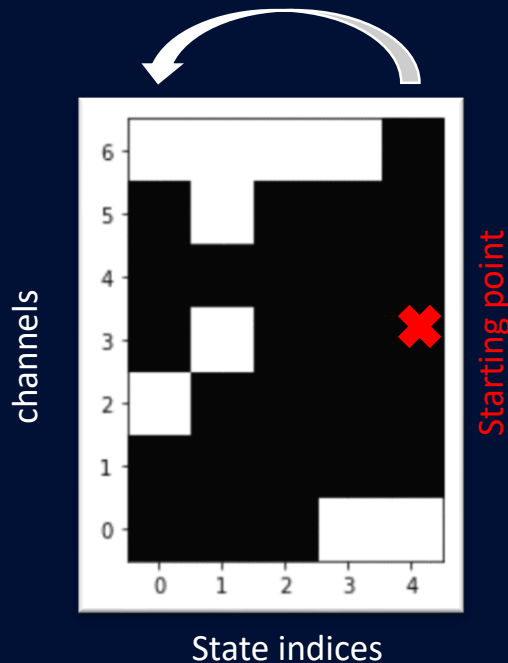
We don't want to store a huge Q-table.

We can use a **Q-Network** instead :

$S_t$  vector



The Q-Network outputs all the Q values for a given states



# Deep Reinforcement Learning

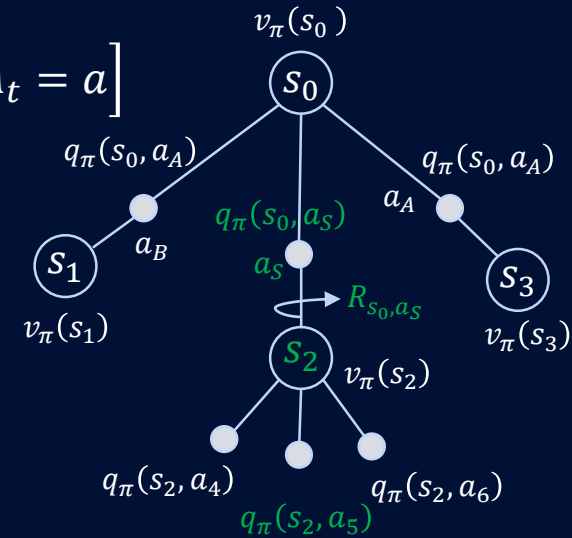
## Q-Network

Estimate  $q_*(s, a) = \mathbb{E} \left[ R_{t+1} + \gamma(1 - \xi) \max_{a'} q_*(S_{t+1}, a') | S_t = s, A_t = a \right]$

**Q-Learning** : wait to finish a  $S_t, A_t, R_{t+1}, S_{t+1}$

(0. Initialize all  $Q_*(s, a)$  randomly)

1. Take the  $Q_*(S_t, A_t)$  associated with your state and action
2. When in  $S_{t+1}$ , take the best q-value :  $\max_a Q_*(S_{t+1}, a)$
3. Compute a better estimate  $Q'_*(S_t, A_t) = R_{t+1} + \gamma \max_a Q_*(S_{t+1}, a)$
4. Compute loss :  $\text{MSE}(Q'_*(S_t, A_t), Q_*(S_t, A_t))$
5. Update parameters of the Q-Network by SGD to minimize the loss



# Deep Reinforcement Learning

## Q-Network

$$\text{loss} = \text{MSE} \left( \overbrace{R_{t+1} + \gamma(1 - \xi) \max_a Q_*(S_{t+1}, a)}^{\text{Target}} - \overbrace{Q_*(S_t, A_t)}^{\text{Prediction}} \right)$$

Each time we update for one prediction, every parameters in the NN changes!  
Two problems arises :

- *Correlation* : when you follow the trajectory, your NN will be optimized only for the last few  $(s, a)$  that you took
- *Nonstationary target* : each time we update the NN, the target change as well  
→ not stable

# Deep Reinforcement Learning

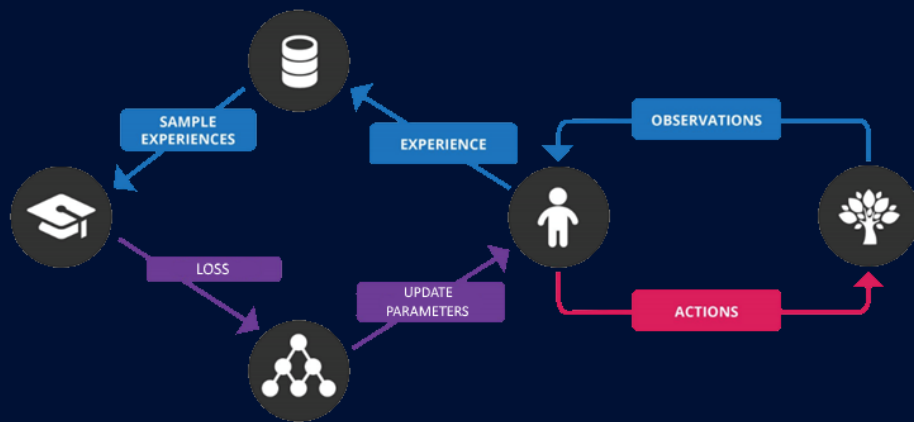
## Experience Replay

To remove *correlation*, we store all experiences  $(S_t, A_t, R_{t+1}, S_{t+1})$  in a dataset.

Then, at each iteration, we perform **experience replay**:

- We take a random batch of experiences
- We compute the predictions and targets
- We evaluate the loss and update the Q-Network

$$loss = MSE(\text{predictions}, \text{targets})$$



Circular buffer (FIFO)

$S_1, A_1, R_2, S_2$
$S_2, A_2, R_3, S_3$
$S_3, A_3, R_4, S_4$
$S_4, A_4, R_5, S_5$
...



# Deep Reinforcement Learning

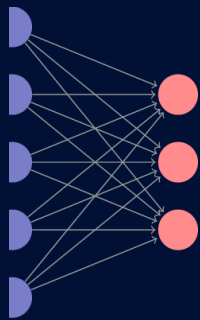
## Experience Replay

To remove *correlation*, we store all samples  $(S_t, A_t, R_{t+1}, S_{t+1})$  in a dataset.

$$loss = MSE(\text{predictions}, \text{targets})$$

The targets can only reflect the action chosen in the sample

If at state  $S_t$ , the action taken was  $a_A$ :



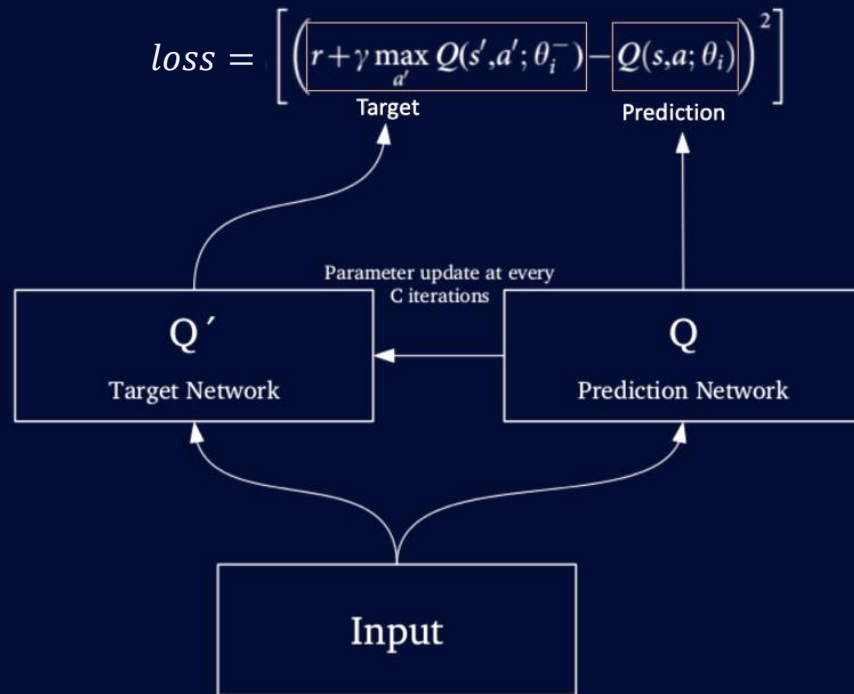
$$MSE \left[ \begin{array}{c} \text{Prediction} \\ Q_*(S_t, a_A) \\ Q_*(S_t, a_S) \\ Q_*(S_t, a_B) \end{array} - \begin{array}{c} \text{Target} \\ R_{t+1} + \gamma(1 - \xi) \max_a Q_*(S_{t+1}, a) \\ Q_*(S_t, a_S) \\ Q_*(S_t, a_D) \end{array} \right] = MSE \left[ \begin{array}{c} pred - target \\ 0 \\ 0 \end{array} \right]$$

# Deep Reinforcement Learning

## Target Network

To alleviate the *nonstationary target*, we maintain a **target Q-Network** :

- The targets are computed according to the target network
- The parameters of the target network are updated every  $C$  iterations



## Deep Reinforcement Learning

### Target Network

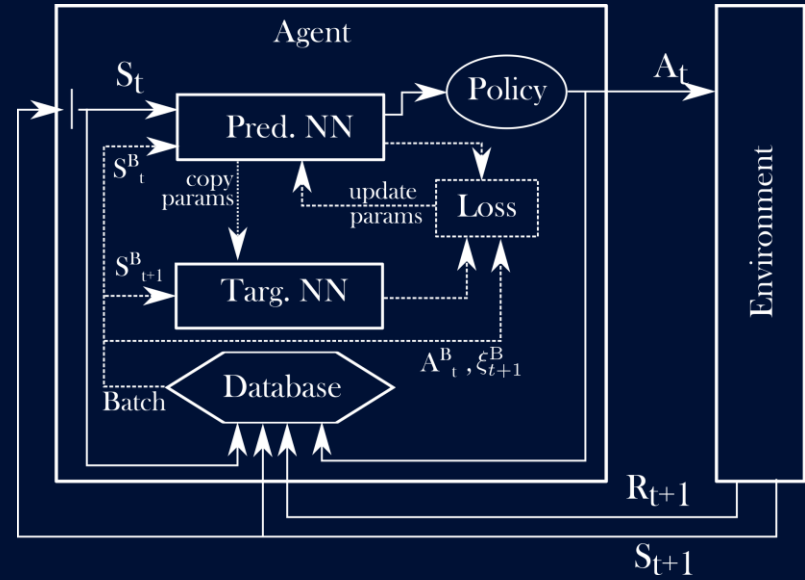
## Main algorithm :

Initialize the pred.NN and the targ.NN with the same params

Play actions according to a policy  $\pi$  to populate the datasets

For a given number of episodes :

- While  $\xi \neq 1$  :
  - Choose an action  $A_t$  according to the state  $S_t$  and the policy  $\pi$
  - Receive  $R_{t+1}, S_{t+1}$  and store  $(A_t, S_t, R_{t+1}, S_{t+1})$  in the database
  - Take a random batch from the database ( $B$  = batch size)
  - Compute the loss using targets from the target network
  - Update the (prediction) Q-Network
- Every  $C$  iterations, copy the parameters of the pred.NN to the targ.NN



Let's play 😊

## Exercise

Multiple Access Channel with Reinforcement Learning

# Thank you

Everything is available on  
[mgoutay.github.io](https://mgoutay.github.io)