CPNS 35600 / STAT 42600

Problem Set 3

Bussgang's Theorem: In class we derived the optimal filter, $D(\tau)$, for estimating the firing rate of a neuron, r_{est}. We found that this optimal filter was proportional to the STA. This filter minimized the mean squared error of the linear estimate, $r_{\text{est}} = r_0 + L(t)$. We later added a static nonlinearity, F, to our estimate such that $r_{\text{est}} = r_0 + F(L(t))$. A theorem due to Julian Bussgang asserts that the optimal filter for the linear estimate still provides a reasonable (though not necessarily optimal) estimate of the firing rate under these conditions, provided that the white noise stimulus used is Gaussian. Bussgang showed that the optimal linear filter is self-consistent when a nonlinearity is present. In other words, if we swap $F(L)$ for L in our r_{est}, the correlation between the stimulus and the firing rate should remain the same, i.e.

$$D(\tau) = \frac{Q_{rs}(-\tau)}{\sigma_s^2} = \frac{1}{\sigma_s^2 T} = \int_0^T dt r_{\text{est}}(t) s(\tau - t)$$

with $r_{\text{est}} = r_0 + F(L(t))$ still holds for Gaussian white noise stimuli. This result is based on the following identity which, again, only holds for Gaussian white noise stimuli:

$$\frac{1}{\sigma_s^2 T} \int_0^T dt F(L(t)) s(\tau - t) = \frac{D(\tau)}{T} \int_0^T dt \frac{dF(L(t))}{dL} = \frac{D(\tau)}{T} \cdot 1.$$

1. Prove the key part of this identity for a simple one-dimensional random variable. Consider a Gaussian random variable, x, with zero mean and standard deviation σ. Show that

$$\langle xF(\alpha x)\rangle = \alpha\sigma^2 \langle F'(\alpha x)\rangle,$$

where α is a constant, F is any function, F' is its derivative,

$$\langle xF(\alpha x)\rangle = \int dx \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-x^2}{2\sigma^2}\right) xF(\alpha x),$$

with a similar equation for $\langle F'(\alpha x)\rangle$. *(Based on a problem in Dayan and Abbott.)*

LNP models: You have been provided with a MATLAB dataset from the fly H1 neuron, called `fly_data.mat` recorded by Rob de Ruyter van Steveninck. Data were collected for 20 minutes at a sampling rate of 500 Hz. In the file, `rho` is a vector that gives the sequence of spiking events or nonevents at the sampled times (every 2 ms). When an element of `rho` is one, this indicates the presence of a spike at the corresponding time, whereas a zero value indicates no spike. The variable `stim` gives the sequence of stimulus values at the sampled times.

2. Using the first half of the data, compute the spike-triggered-average stimulus (STA) that preceded a spike from this neuron. Plot the STA for this neuron. Label your axes.

3. Next, estimate its nonlinearity using the histogram method. Do this in parts. First compute the distribution of overlap between the STA and the stimulus at a random sample (or all) of the stimulus frames in the first half of the data, $P(L)$. Next, condition this distribution on the presence of a spike, $P(L|spike)$ (again use the first half of the data). Use the ratio of these two distributions to estimate the nonlinearity $P(spike|L) \propto P(L|spike)/P(L)$. Plot each of these three quantities.

4. Now take the second half of the stimulus and generate spikes from the LNP model you just constructed. Compare your results to the spike times of the real neuron. You may do this by simply putting both spike times, real and estimated, on the same plot. You may also create a moving average of the instantaneous firing rate in each spike train. Quote your window size. Label your axes.

5. Use the data provided from Nicole Rust's 2005 Neuron paper on receptive field properties of V1 cells in the macaque cortex in files `5331006.p05_stc.mat` and `5431021.p07_stc.mat` to compute the STA and spike-triggered covariance for these two cells. The variables `stim` and `spikes_per_frm` contain the data you need for this task. Plot the STA's. Plot the eigenvalue spectrum of the covariance matrix and determine the number of significant eigenvectors for each cell. Plot these eigenvectors, along with the eigenvector corresponding to the smallest eigenvalue. How does this STC analysis enhance your understanding of each cell's response properties, beyond what you learn from the STA? You may refer to the Rust et al. 2005 paper, for details on the experiment.

Bonus Problem Create an LNP model for the second V1 cell using the significant filters you found from the STC analysis, and assess its goodness of it. Be mindful of the dimensionality of $F(L)$.