CPNS 35600 / STAT 42600

Problem Set 2

1. Compute the variance of the interval, or ISI, distribution of a Poisson process ($\sigma_\tau^2 = \int_0^\infty d\tau \tau^2 r e^{-r\tau} - \langle\tau\rangle^2$). Compute the variance two ways: (1) do the integrals directly; (2) use the moment generating function for $P(\tau)$. Show your work.

2. ***Generating Poisson spike trains:***
From Dayan and Abbott, Chapter 1:

> Spike sequences can be simulated by using some estimate of the firing rate, $r_{\text{est}}(t)$, predicted from knowledge of the stimulus, to drive a Poisson process. A simple procedure for generating spikes in a computer program is based on the fact that the estimated probability of firing a spike during a short interval of duration Δt is $r_{\text{est}}(t)\Delta t$. The program progresses through time in small steps of size Δt and generates, at each time step, a random number x_{rand} chosen uniformly in the range between zero and one. If $r_{\text{est}}(t)\Delta t > x_{\text{rand}}$ at that time step, a spike is fired, otherwise it is not.

Generate spikes for 10 seconds (or longer if you want better statistics) using this Poisson spike generator with a constant rate of 100 spikes/second, and record their times of occurrence. Plot a histogram of the interspike intervals you obtain in these model data. Compute the coefficient of variation of the interspike intervals, and the Fano factor for spike counts obtained over counting intervals of 1, 10, 50, and 100 ms.

3. Add a refractory period to the Poisson spike generator by allowing the firing rate to depend on time. Initially, set the firing rate to a constant value, $r(t) = r_0$. After every spike, set $r(t)$ to 0, and then allow it to recover exponentially back to r_0 with a time constant τ_{ref} that controls the refractory recovery rate. In other words, have $r(t)$ obey the equation

$$\tau_{\text{ref}}\frac{dr}{dt} = r_0 - r$$

except immediately after a spike, when it is set to 0. Plot the coefficient of variation as a function of τ_{ref} over the range 1 ms $\leq \tau_{\text{ref}} \leq$ 20 ms, and plot interspike interval histograms for a few different values of τ_{ref} in this range. Compute the Fano factor for spike counts obtained over counting intervals ranging from 1 to 100 ms for the case $\tau_{\text{ref}} = 10$ ms.

4. For a constant rate Poisson process, every specific (up to a finite resolution) sequence of N spikes occurring over a given time interval is equally likely. This might seem paradoxical because we certainly do not expect to see all N spikes appearing within the first 1% of the time interval. Explain why we do not expect to see all of the spikes within the first 1% of the time interval (i.e. find the flaw in the logic that might lead to this false paradox).

5. Create a time-varying Poisson process to capture the spiking statistics of the neuron you will find on the Canvas site in the Matlab file
`cell14107_for_class.mat`
or in the CSV file with the same name. In the CSV file, the number at the end of the 'spike times' columns indicates the trial number. Stimuli were shown for about 1 second, and repeated for a total of 186 trials.

First compute the trial-averaged firing rate for this neuron, an MT neuron stimulated with a moving dot patch at 100% coherence at the cell's preferred direction. State what smoothing window or bin-size you use. (Something in the range 5-20ms will work best.) Create and plot the trial-averaged rate from your Poisson process. Compare both the mean and variance of the rate as a function of time in the trial for the real neuron and your Poisson model neuron. Is the neuron's variance well-captured by your model?

Bonus problem: Consider the negative-binomial distribution for a random variable, Y,

$$P(Y = y) = \binom{y + r - 1}{y} p^r (1 - p)^y$$

for integer $y \geq 0$. In this formulation, you may think of y as the number of failures before the r^{th} success in a series of independent Bernoulli trials, each with probability p of success. This formula extends to the case where r is a positive real number. This is a useful distribution to use to fit neural spike count data. If y is the number of spikes in a time window, T, describe what r and p might correspond to, conceptually. Show that $\text{var}(Y) \geq E(Y)$.