

# Sensitivity analysis of recent GIQEs

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# Review [1/3]

- ▶ GIQEs are *General Image Quality Equations* given by the National Geospatial-intelligence Agency to systematically predict NIIRS ratings
- ▶ NIIRS is the *National Image Interpretability Rating Scale*
- ▶ NIIRS is a subjective 10 level scale
  - ▶ 0 - not interpretable
  - ▶ 9 - highly interpretable (in the visible, identify screws, bolts, etc.)

## Review [2/3]

- ▶ GIQEv4 depends on:
  - ▶ GSD, ground sample distance in inches
  - ▶ RER, relative edge response
  - ▶  $G$ , noise gain due to sharpening
  - ▶  $H$ , edge overshoot due to sharpening
  - ▶ SNR, signal/noise ratio
- ▶ GIQEv5, which does not consider sharpening, only depends on:
  - ▶ GSD, ground sample distance in inches
  - ▶ RER, relative edge response
  - ▶ SNR, signal/noise ratio

## Review [3/3]

- ▶ GIQEv4 is given by

$$g_4 = 10.251 - a \log_{10}(\text{GSD}) \\ + b \log_{10}(\text{RER}) - 0.656H + 0.344 \frac{G}{\text{SNR}}$$

where

$$a = 3.160 + 0.160 \theta(\text{RER} - 0.9)$$

$$b = 2.817 - 1.258 \theta(\text{RER} - 0.9)$$

- ▶ GIQEv5 is given by

$$g_5 = 9.57 - 3.32 \log_{10}(\text{GSD}) \\ + 3.32 \left( 1 - \exp \left[ -\frac{1.9}{\text{SNR}} \right] \right) \log_{10}^4(\text{RER}) - \frac{1.8}{\text{SNR}}$$

# Expectation and variance in $d$ dimensions

- ▶ Consider  $Y = f(\mathbf{X})$  for  $\mathbf{X} \in M \subset \mathbb{R}^d$
- ▶ Abstractly, the global expectation  $E(Y)$  and variance  $V(Y)$  are functionals

$$E(Y) = \int_M f(\mathbf{X}) d\mu(\mathbf{X})$$

$$V(Y) = E((Y - E(Y))^2)$$

- ▶ For mutually exclusive index sets  $I$  and  $J$  such that  $I \cup J = \{1, 2, \dots, d\}$ , conditional expectation and variance are given by

$$E_{\mathbf{X}_I}(Y|\mathbf{X}_J) = \int_{M_I} f(\mathbf{X}_I|\mathbf{X}_J) d\mu(\mathbf{X}_I)$$

$$V_{\mathbf{X}_I}(Y|\mathbf{X}_J) = E_{\mathbf{X}_I}((Y - E_{\mathbf{X}_I}(Y|\mathbf{X}_J))^2|\mathbf{X}_J)$$

# Global sensitivity analysis via Sobol indices [1/3]

- ▶ Consider  $Y = f(\mathbf{X})$  for  $\mathbf{X} \in M \subset \mathbb{R}^d$
- ▶ For each  $i = 1, 2, \dots, d$ , we may decompose the total variance  $V(Y)$  as the sum of
  - ▶  $V_{X_i}(E_{\mathbf{X}_{\sim i}}(Y|X_i))$
  - ▶  $E_{X_i}(V_{\mathbf{X}_{\sim i}}(Y|X_i))$
- ▶ Normalizing yields first order sensitivity indices  $S_i$

$$1 = \frac{V_{X_i}(E_{\mathbf{X}_{\sim i}}(Y|X_i))}{V(Y)} + \frac{E_{X_i}(V_{\mathbf{X}_{\sim i}}(Y|X_i))}{V(Y)}$$
$$\rightarrow S_i \equiv \frac{V_{X_i}(E_{\mathbf{X}_{\sim i}}(Y|X_i))}{V(Y)}$$

# Global sensitivity analysis via Sobol indices [2/3]

- First-order indices  $S_i$  characterize variance in  $Y$  due directly to  $X_i$

$$S_i = \frac{V_{X_i}(E_{\mathbf{X}_{\sim i}}(Y|X_i))}{V(Y)} \implies \sum_i S_i \leq 1$$

- A slight manipulation yields total-order indices  $S_{T_i}$ , which characterize variance in  $Y$  due directly and indirectly to  $X_i$  (e.g., through covariance with other  $X_j$ ,  $j \neq i$ )

$$S_{T_i} = \frac{E_{\mathbf{X}_{\sim i}}(V_{X_i}(Y|\mathbf{X}_{\sim i}))}{V(Y)} \implies \sum_i S_{T_i} \geq 1$$

# Global sensitivity analysis via Sobol indices [3/3]

- ▶ Estimate Sobol indices by Monte Carlo integration
- ▶ Given  $N$  random samples  $(\mathbf{A}_k, \mathbf{B}_k) \in M^2$

$$V_{X_i}(E_{\mathbf{X}_{\sim i}}(Y|X_i)) \approx \frac{1}{N} \sum_{k=1}^N f(\mathbf{B}_k)(f(\mathbf{C}_k) - f(\mathbf{A}_k))$$

$$E_{\mathbf{X}_{\sim i}}(V_{X_i}(Y|\mathbf{X}_{\sim i})) \approx \frac{1}{2N} \sum_{j=1}^N (f(\mathbf{A}_k) - f(\mathbf{C}_k))^2$$

where

$$\mathbf{C}_k \Rightarrow C_{k,\ell} = \begin{cases} A_{k,\ell} & \ell \neq i \\ B_{k,\ell} & \ell = i \end{cases}$$



# Change prediction via derivative expectations

- ▶ For some change in the input  $\Delta X_i$ , we crudely predict the change in the output  $\Delta Y$  as

$$\Delta Y \approx E \left( \frac{\partial Y}{\partial X_i} \right) \Delta X_i$$

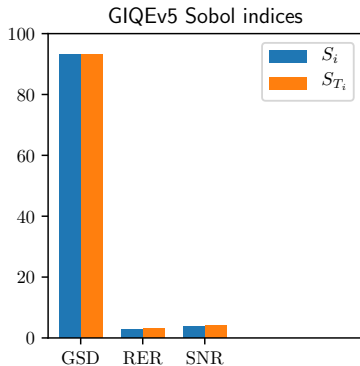
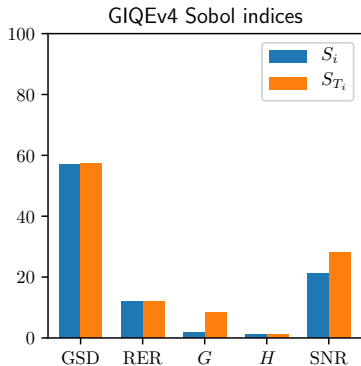
- ▶ We carry out this calculation analytically with the help of a computer algebra system (SymPy/WolframAlpha)
- ▶ We estimate the average  $\Delta X_i$  necessary to produce a change of 1 NIIRS by setting  $\Delta Y = 1$

$$\Delta Y = 1 \implies \Delta X_i \approx 1/E \left( \frac{\partial Y}{\partial X_i} \right)$$

# Domain and sampling parameters

- ▶ Domain  $M$  is a Cartesian product of uniform intervals (hyperrectangle or  $d$ -orthotope):
  - ▶  $GSD \in [0.01 \text{ m}, 5 \text{ m})$  (in inches)
  - ▶  $RER \in [0.15, 0.95)$
  - ▶  $G \in [1, 50)$  (only for GIQEv4)
  - ▶  $H \in [1, 2)$  (only for GIQEv4)
  - ▶  $SNR \in [1, 100)$
- ▶ Sampling for Monte Carlo integration uses Latin Hypercube Sampling, i.e., stratification and permutation in each subdomain
- ▶ Sampling parameters:
  - ▶  $N_{\text{samps}} = 4096$  samples per iteration
  - ▶  $N_{\text{iters}} = 128$  iterations

# Global sensitivity analysis results



# Change prediction results

- ▶ Average change necessary to improve GIQEv4 by 1:
  - ▶  $\Delta\text{GSD} = 1/E(\partial g_4/\partial\text{GSD}) = -10.8$  inches
  - ▶  $\Delta\text{RER} = 1/E(\partial g_4/\partial\text{RER}) = 0.390$
  - ▶  $\Delta G = 1/E(\partial g_4/\partial G) = -62.5$  (not feasible)
  - ▶  $\Delta H = 1/E(\partial g_4/\partial H) = -1.52$  (not feasible)
  - ▶  $\Delta\text{SNR} = 1/E(\partial g_4/\partial\text{SNR}) = 11.4$
- ▶ Average change necessary to improve GIQEv5 by 1:
  - ▶  $\Delta\text{GSD} = 1/E(\partial g_5/\partial\text{GSD}) = -10.3$  inches
  - ▶  $\Delta\text{RER} = 1/E(\partial g_5/\partial\text{RER}) = 1.11$  (not feasible)
  - ▶  $\Delta\text{SNR} = 1/E(\partial g_5/\partial\text{SNR}) = 54.1$

# Implementation notes

- ▶ Implementation at [github.com/mgradysaunders/giqe-vsa](https://github.com/mgradysaunders/giqe-vsa)
  - ▶ Single-file C++ implementation
  - ▶ Formula sheet for derivative expectations
  - ▶ This slide-deck
- ▶ After compiling, the default domain and sampling parameters may be overridden on the command line
- ▶ Note, requires C++17 (g++-7 or newer) because one of the derivative expectation formulas involves an exponential integral

# Takeaways and room for improvement

- ▶ Takeaways:
  - ▶ When in doubt, focus on improving GSD.
  - ▶ If GIQEv5 is supposed to be better suited to  $Q = 2$  systems, it makes sense that it is less sensitive than GIQEv4 to RER/SNR.
- ▶ Room for improvement:
  - ▶ Do not assume independent/uniform distributions.
  - ▶ Perhaps consider the distribution of system parameters in real-world systems?
  - ▶ In GIQEv4, there may be strong correlation between RER,  $G$ ,  $H$ , and SNR. Characterize this somehow?