Sensitivity analysis of recent GIQEs

M. Grady Saunders
mgs8033@rit.edu

IMGS-765 Remote Sensing/Image Analysis
Chester F. Carlson Center for Imaging Science
Rochester Institute of Technology

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Review [1/3]

- GIQEs are *General Image Quality Equations* given by the National Geospatial-intelligence Agency to systematically predict NIIRS ratings
- NIIRS is the *National Image Interpretability Rating Scale*
- NIIRS is a subjective 10 level scale
  - 0 - not interpretable
  - 9 - highly interpretable (in the visible, identify screws, bolts, etc.)

Review [2/3]

- GIQEv4 depends on:
  - GSD, ground sample distance in inches
  - RER, relative edge response
  - G, noise gain due to sharpening
  - H, edge overshoot due to sharpening
  - SNR, signal/noise ratio
- GIQEv5, which does not consider sharpening, only depends on:
  - GSD, ground sample distance in inches
  - RER, relative edge response
  - SNR, signal/noise ratio

Review [3/3]

- GIQEv4 is given by

$$g_4 = 10.251 - a\log_{10}(\mathrm{GSD}) + b\log_{10}(\mathrm{RER}) - 0.656H + 0.344\frac{G}{\mathrm{SNR}}$$

  where

$$a = 3.160 + 0.160\,\theta(\mathrm{RER} - 0.9)$$
$$b = 2.817 - 1.258\,\theta(\mathrm{RER} - 0.9)$$

- GIQEv5 is given by

$$g_5 = 9.57 - 3.32\log_{10}(\mathrm{GSD}) + 3.32\left(1 - \exp\left[-\frac{1.9}{\mathrm{SNR}}\right]\right)\log_{10}^4(\mathrm{RER}) - \frac{1.8}{\mathrm{SNR}}$$

Expectation and variance in d dimensions

- Consider $Y = f(\mathbf{X})$ for $\mathbf{X} \in M \subset \mathbb{R}^d$
- Abstractly, the global expectation $E(Y)$ and variance $V(Y)$ are functionals

$$E(Y) = \int_M f(\mathbf{X}) d\mu(\mathbf{X})$$
$$V(Y) = E((Y - E(Y))^2)$$

- For mutually exclusive index sets I and J such that $I \cup J = \{1, 2, \dots, d\}$, conditional expectation and variance are given by

$$E_{\mathbf{X}_I}(Y|\mathbf{X}_J) = \int_{M_I} f(\mathbf{X}_I|\mathbf{X}_J) d\mu(\mathbf{X}_I)$$
$$V_{\mathbf{X}_I}(Y|\mathbf{X}_J) = E_{\mathbf{X}_I}((Y - E_{\mathbf{X}_I}(Y|\mathbf{X}_J))^2|\mathbf{X}_J)$$

Global sensitivity analysis via Sobol indices [1/3]

- Consider $Y = f(\mathbf{X})$ for $\mathbf{X} \in M \subset \mathbb{R}^d$
- For each $i = 1, 2, \ldots, d$, we may decompose the total variance $V(Y)$ as the sum of
  - $V_{X_i}(E_{\mathbf{X}_{\sim i}}(Y|X_i))$
  - $E_{X_i}(V_{\mathbf{X}_{\sim i}}(Y|X_i))$
- Normalizing yields first order sensitivity indices S_i

$$1 = \frac{V_{X_i}(E_{\mathbf{X}_{\sim i}}(Y|X_i))}{V(Y)} + \frac{E_{X_i}(V_{\mathbf{X}_{\sim i}}(Y|X_i))}{V(Y)}$$

$$\rightarrow S_i \equiv \frac{V_{X_i}(E_{\mathbf{X}_{\sim i}}(Y|X_i))}{V(Y)}$$

Global sensitivity analysis via Sobol indices [2/3]

- First-order indices S_i characterize variance in Y due directly to X_i

$$S_i = \frac{V_{X_i}(E_{\mathbf{X}_{\sim i}}(Y|X_i))}{V(Y)} \implies \sum_i S_i \leq 1$$

- A slight manipulation yields total-order indices S_{T_i}, which characterize variance in Y due directly and indirectly to X_i (e.g., through covariance with other X_j, $j \neq i$)

$$S_{T_i} = \frac{E_{\mathbf{X}_{\sim i}}(V_{X_i}(Y|\mathbf{X}_{\sim i}))}{V(Y)} \implies \sum_i S_{T_i} \geq 1$$

Global sensitivity analysis via Sobol indices [3/3]

- Estimate Sobol indices by Monte Carlo integration
- Given N random samples $(\mathbf{A}_k, \mathbf{B}_k) \in M^2$

$$V_{X_i}(E_{\mathbf{X}_{\sim i}}(Y|X_i)) \approx \frac{1}{N}\sum_{k=1}^{N} f(\mathbf{B}_k)(f(\mathbf{C}_k) - f(\mathbf{A}_k))$$

$$E_{\mathbf{X}_{\sim i}}(V_{X_i}(Y|\mathbf{X}_{\sim i})) \approx \frac{1}{2N}\sum_{j=1}^{N}(f(\mathbf{A}_k) - f(\mathbf{C}_k))^2$$

where

$$\mathbf{C}_k \implies C_{k,\ell} = \begin{cases} A_{k,\ell} & \ell \neq i \\ B_{k,\ell} & \ell = i \end{cases}$$

Change prediction via derivative expectations

- For some change in the input ΔX_i, we crudely predict the change in the output ΔY as

$$\Delta Y \approx E\left(\frac{\partial Y}{\partial X_i}\right)\Delta X_i$$

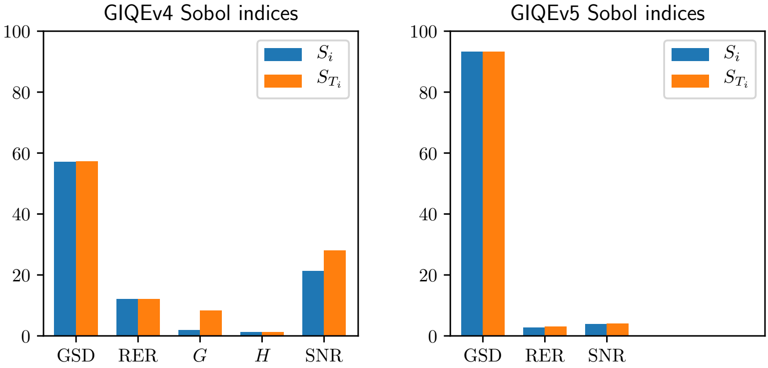
- We carry out this calculation analytically with the help of a computer algebra system (Sympy/WolframAlpha)
- We estimate the average ΔX_i necessary to produce a change of 1 NIIRS by setting $\Delta Y = 1$

$$\Delta Y = 1 \implies \Delta X_i \approx 1/E\left(\frac{\partial Y}{\partial X_i}\right)$$

Domain and sampling parameters

- Domain M is a Cartesian product of uniform intervals (hyperrectangle or d-orthotope):
  - $\mathrm{GSD} \in [0.01\,\mathrm{m}, 5\,\mathrm{m})$ (in inches)
  - $\mathrm{RER} \in [0.15, 0.95)$
  - $G \in [1, 50)$ (only for GIQEv4)
  - $H \in [1, 2)$ (only for GIQEv4)
  - $\mathrm{SNR} \in [1, 100)$
- Sampling for Monte Carlo integration uses Latin Hypercube Sampling, i.e., stratification and permutation in each subdomain
- Sampling parameters:
  - $N_{\text{samps}} = 4096$ samples per iteration
  - $N_{\text{iters}} = 128$ iterations

Global sensitivity analysis results



Change prediction results

- Average change necessary to improve GIQEv4 by 1:
  - $\Delta\text{GSD} = 1/E(\partial g_4/\partial\text{GSD}) = -10.8$ inches
  - $\Delta\text{RER} = 1/E(\partial g_4/\partial\text{RER}) = 0.390$
  - $\Delta G = 1/E(\partial g_4/\partial G) = -62.5$ (not feasible)
  - $\Delta H = 1/E(\partial g_4/\partial H) = -1.52$ (not feasible)
  - $\Delta\text{SNR} = 1/E(\partial g_4/\partial\text{SNR}) = 11.4$
- Average change necessary to improve GIQEv5 by 1:
  - $\Delta\text{GSD} = 1/E(\partial g_5/\partial\text{GSD}) = -10.3$ inches
  - $\Delta\text{RER} = 1/E(\partial g_5/\partial\text{RER}) = 1.11$ (not feasible)
  - $\Delta\text{SNR} = 1/E(\partial g_5/\partial\text{SNR}) = 54.1$

Implementation notes

- Implementation at `github.com/mgradysaunders/giqe-vsa`
  - Single-file C++ implementation
  - Formula sheet for derivative expections
  - This slide-deck
- After compiling, the default domain and sampling parameters may be overridden on the command line
- Note, requires C++17 (g++-7 or newer) because one of the derivative expectation formulas involves an exponential integral