Layered-SQT usage

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1. Introduction

This document describes layered-sqt, a command-line tool for simulating the Bidirectional Scattering Distribution Function (BSDF) which emerges from a layered assembly. In this context, a *layered assembly* is a theoretical construction consisting of N layers separated by N+1 participating media for $N \geq 1$. A layer is an infinite plane which is offset along (and normal to) the z-axis, and which is associated with a constituent BSDF that describes how light is scattered upon intersection. A medium is thought to occupy the space between adjacent layers or, in boundary cases, the spaces above and below the top and bottom layers respectively. See figure 1 for clarification.

The *emergent BSDF* is the BSDF observed in the limit as one backs infinitely far away from the assembly or, identically, as the assembly shrinks to an infinitesmial point. To ensure that the emergent BSDF is well-defined, layered-sqt requires that participating media be homogeneous, i.e., that scattering properties be independent of spatial location. Furthermore, for simplicity/tractability, layered-sqt does not account for wavelength-dependence.

1.1. Basics of LSQT format

The structure of a layered assembly is easy enough to convey in plain-text with rudimentary syntax, so this is the format layered-sqt accepts as input. By convention, we refer to this format as "LSQT format", and we suffix associated filenames with the extension .lsqt (though this suffix is not strictly required for the program to run).

An LSQT file is therefore a line-by-line plain-text description of a layered assembly from top to bottom. So, the first line describes the top medium, the second line describes the top layer beneath the top medium, the third line describes the medium beneath the top layer, and so on until the bottom medium. That being the case, odd-numbered lines describe media and even-numbered lines describe layers. For media as well as layers, the general syntax is

Name key1=val1 key2=val2

where Name is described by keyword arguments key1 and key2—importantly, this syntax is whitespace-delimited, so keyword arguments of the form key=val must not contain

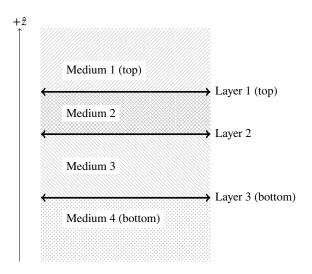


Figure 1: An example diagram of a layered assembly with 3 layers and 4 participating media.

whitespace. It is also worth mentioning that keyword arguments may appear in any order.

1.2. Basics of program usage

As layered-sqt is a command-line tool, it runs on the command line, whereby it scans command-line arguments for (optional) configuration flags and a (required) input filename appearing somewhere as a positional argument, i.e., an argument not consumed by a flag. It then parses the input file, simulates the emergent BSDF, and writes the results to a RAW-format file ready for conversion to SQT-format via raw2sqt. For instance,

simulates a layered assembly as described in example.lsqt with 50,000 paths, and writes the emergent BSDF in plaintext RAW-format to example.raw. To convert the BSDF to SQT-format, which is necessary for use in DIRSIG, run

\$./raw2sqt example.raw

which will write a new file example.sqt.

Two of the most common flags appear above, namely -p (or --path-count) to specify the number of paths used in the simulation and -o (or --output) to specify the output filename. To see a list of all acceptable flags with brief descriptions and default values, pass -h (or --help), or simply run layered-sqt with no filename. As an aside, layered-sqt verifies parameters specified in command-line flags as well as keyword arguments specified in the input LSQT file. In the event that something has an unreasonable value, the program issues an error and fails in a controlled manner.

Lastly, layered-sqt recognizes the single dash filename "-" as standard input. So, it is possible to pipe the (presumably LSQT format) output of a script into layered-sqt directly, if this happens to be convenient. As a trivial example,

```
$ cat example.lsqt | \
   ./layered-sqt -p 50000 -o example.raw -
```

is equivalent to just passing example.lsqt.

2. Tutorial

This section provides a series of progressively more interesting "learn-by-doing" tutorials, which should naturally introduce the concepts and features in layered-sqt. To get started, create a working directory lsqt-tutorial and link the relevant programs for easy access. From one of the Carlson CIS servers, type the following commands:

```
$ mkdir lsqt-tutorial & cd lsqt-tutorial
$ TMP_PATH=/cis/phd/mgs8033/layered-sqt/bin
$ ln -s $TMP_PATH/layered-sqt
$ ln -s $TMP_PATH/layered-sqt-lssview
$ ln -s /dirs/pkg/dirsig/bin/raw2sqt
```

2.1. Hello, world

Let's start with something mind-numbingly simple—a 60% reflective Lambertian surface. First, create and enter a sub-directory tutorial1.

```
$ mkdir tutorial1 & cd tutorial1
```

Using whatever text-editing program you prefer, create a plain-text LSQT file Lambertian.lsqt with three lines:

```
Medium
Layer z=0 LambertianBsdf fR=0.6 fT=0
Medium
```

As an aside, LSQT format input is syntax-highlighted in this document. However, syntax-highlighting files/add-ons are not available (yet) for any text editor.

Now, run layered-sqt on Lambertian.lsqt,

\$../layered-sqt Lambertian.lsqt

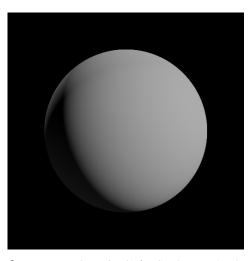


Figure 2: Image preview of a 60% reflective Lambertian BRDF output by layered-sqt-lssview, as in tutorial §2.1.

which should write two new files: Lambertian.lsqt.lss and Lambertian.lsqt.raw. As stated in the introduction, we may convert the plain-text RAW file to a binary SQT file for use with DIRSIG by running raw2sqt,

\$../raw2sqt Lambertian.lsqt.raw

which generates the final SQT file Lambertian.lsqt.sqt suitable for use in a DIRSIG scene.

Now, what is the LSS file? *LSS* is an initialism for *LSQT-Slice*, which is the name of the internal file format layered-sqt uses to store simulation data, and it is useful 1) for previewing the layered BSDF *without setting up and rendering a DIRSIG scene*, and 2) for simulating a layered BSDF progressively, with multiple runs of layered-sqt.

Run layered-sqt-lssview to preview the Lambertian BRDF.

\$../layered-sqt-lssview Lambertian.lsqt.lss

This should write a new file, Lambertian.lsqt.lss.png, which is a 512x512 rendering of the BSDF applied to a ball, shown in figure 2. When connected to the CIS server with X-window support (from a Linux machine, log-in with ssh-X), this image may be viewed by running eog.

\$ eog Lambertian.lsqt.lss.png

Importantly, layered-sqt-lssview is *not* a full-blown path-tracer, and may not perfectly represent how the BSDF will appear in DIRSIG. It only accounts for the direct (first bounce) contributions of a few directional light sources, and it further uses tone-mapping and sRGB correction, such that the output image is not suitable for any radiometric analysis. The intended use of this preview image is to determine if a simulated BSDF is suitably convergent/noise-free.

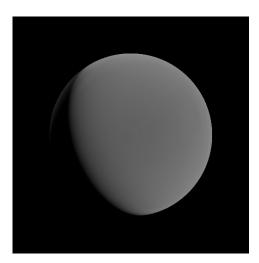


Figure 3: Image preview of a 60% reflective Lambertian BRDF with a rough microsurface output by layered-sqt-lssview, as in tutorial §2.2.

For more information on the Lambertian BSDF, refer to the documentation in §3.2.2.

2.2. Hello, world, with roughness

Let's expand on §2.1 by simulating a 60% reflective Lambertian BRDF with a rough microsurface—we assume that, microscopically, the surface geometry is not perfectly smooth, but is instead characterized by a distribution of normals about the primary surface normal. See the documentation in §3.2.3 for a more detailed explanation.

First, return to the lsqt-tutorial directory, and create and enter a new subdirectory tutorial2.

```
$ cd ..
$ mkdir tutorial2 & cd tutorial2
```

As before, create a new plain-text LSQT file Rough.lsqt with three lines:

```
Medium

Layer z=0 MicrosurfaceLambertianBrdf fR=0.6
alpha=2.4

Medium
```

Above, fR=0.6 specifies that the Lambertian BRDF is 60% reflective, and alpha=2.4 sets the roughness parameter α to be 2.4—this is very rough. Note that setting alpha=0 is effectively equivalent to using LambertianBsdf with fR=0.6 and fT=0.

Now, run layered-sqt on Rough.lsqt to generate files Rough.lsqt.lss and Rough.lsqt.raw. This should take longer to simulate than before, due to the computational complexity of the microsurface model.

```
$ ../layered-sqt Rough.lsqt
```

When this is complete, use layered-sqt-lssview to generate Rough.lsqt.lss.png, shown in figure 3.

\$../layered-sqt-lssview Rough.lsqt.lss

Notice that the microsurface roughness causes increased reflectance at grazing angles, and decreased reflectance elsewhere.

2.3. A simple substrate

Let's now consider a simple two-layer assembly to model a substrate. We imagine a 60% reflective diffuse Lambertian surface with a glossy dielectric coating. Before continuing, again return to the lsqt-tutorial directory, and create and enter a new subdirectory tutorial3.

```
$ cd ..
$ mkdir tutorial3 & cd tutorial3
```

Then create a new plain-text LSQT file Substr.lsqt with five lines as below:

```
Medium
Layer z=1 MicrosurfaceDielectricBsdf
alpha=0.2 use_multiple_scattering=false
Medium eta=1.4
Layer z=0 LambertianBsdf fR=0.6 fT=0
Medium
```

This assembly has two layers, appearing on lines 2 and 4. The first layer is positioned at *z*-height 1 with a Microsurface-leatricBsdf—this is similar to Microsurface-LambertianBrdf, but it applies the microsurface ideology to the dielectric Fresnel mirror BSDF instead of the Lambertian BRDF. This layer forms the surface of our glossy dielectric coating. The second layer is positioned at *z*-height 0 with a LambertianBsdf as in tutorial §2.1.

The medium between these layers (on line 3) is attributed a refractive index η of 1.4. Note that *every* medium in an LSQT file has a refractive index, which defaults to 1 if left unspecfied. Note also that MicrosurfaceDielectricBsdf depends on the refractive indices of the media above and below the layer to which it is assigned, as it generalizes the dielectric Fresnel mirror BSDF.

Run layered-sqt as usual and preview the resulting LSS file with layered-sqt-lssview.

```
$ ../layered-sqt Substr.lsqt
$ ../layered-sqt-lssview Substr.lsqt.lss
```

Now, upon viewing Substr.lsqt.lss.png, we might suspect that something has gone wrong. As shown in figure 4, there is quite a bit of low frequency noise and artifacting. What is going on?

Behind the scenes, layered-sqt estimates values of the emergent BSDF at a finite number of sample directions using Monte Carlo integration, i.e., integration by averaging

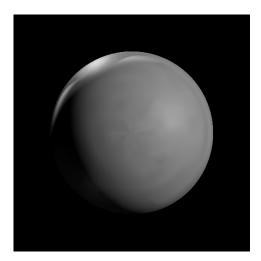


Figure 4: Image preview of the simple substrate BRDF in tutorial §2.3, as output from running layered-sqt with default settings. As is evident, the BRDF would benefit from more samples (higher resolution), and more simulated paths (less noise).

random evaluations. So, two central parameters controlling simulation fidelity are 1) the number of sample directions and 2) the number of random evaluations used to estimate the BSDF at each sample direction. By default, layered-sqt uses 80 sample directions and 10,000 random evaluations of potential light paths to estimate the emergent BSDF.

We can change the number of directions ω_i on the command line using the flag -wi (or --wi-count), and we can change the number of paths using the flag -p (or --path-count). In general, we should think to increase the number of directions to increase *resolution*, and we should think to increase the number of paths to reduce *noise*.

In the case of our simple substrate BRDF, let's increase the number of directions from 80 to 200, and the number of paths from 10,000 to 100,000. Enter the following three shell commands:

```
$ rm Substr.lsqt.lss
$ ../layered-sqt \
  -wi=200 \
  -p=100000 -rp=0.2 Substr.lsqt
$ ../layered-sqt-lssview Substr.lsqt.lss
```

We first remove the existing LSS file with rm. This is necessary to regenerate sample directions—so long as this file exists, layered-sqt will ignore all flags except -p/--path-count, and add this number of paths to the existing simulation data. Then we run layered-sqt with flags -wi=200, -p=100000, and -rp=0.2 (more on this flag in a moment). This should take a minute or two to compute. Upon previewing the BRDF with layered-sqt-lssview, we see something much more sensible, as shown in figure 5.

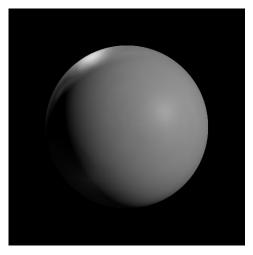


Figure 5: Image preview of the simple substrate BRDF in tutorial §2.3, as output from running layered-sqt with higher settings. In particular, this uses 200 incident directions (instead of the default 80) and 100,000 paths (instead of the default 10,000).

We have specified the number of sample directions with -wi, but it remains a non-trivial problem to determine what these directions ought to be—layered-sqt implements a presumably novel algorithm to form a so-called *Redundancy Reduced Sample Set (RRSS)* of directions by initially sampling *more* directions than the user requests, then choosing the subset of those directions that minimizes a quantitative measure of "redundancy" with respect to a desirable density, being the normalization of the non-cosine-weighted BSDF.

The flag -rp (or --rrss-path-frac) thus specifies the fraction of paths (as specified by -p) used to initially estimate the emergent BSDF in forming the RRSS of directions ω_i . There is also the flag -rx (or --rrss-oversampling), which specifies the oversampling multiplier. For example,

```
$ layered-sqt \
  -wi=100 -p=20000 -rp=0.25 -rx=5 File.lsqt
```

samples $100 \times 5 = 500$ directions initially, and uses $20000 \times 0.25 = 5000$ paths to estimate the emergent BSDF at each of these 500 directions, then forms an RRSS containing 100 of these 500 directions, and lastly uses 20000 - 5000 = 15000 paths to improve the estimates of the emergent BSDF at each of these 100 directions (such that the total number of paths is 20000).

In general, increasing -rp and -rx leads to better sample placement, at the (potentially extreme) cost of computation time. We discuss practical simulation strategies using these flags, including incremental simulation with multiple runs of layered-sqt, over the remaining tutorial sections.

TODO

3. Documentation

This section documents the properties of participating media and scattering layers in more detail, and enumerates the available scattering models for media and layers, and how to specify them in LSQT format. Note that "phase function", "BRDF", and "BSDF" are specific types of scattering models. In particular,

- a *phase function* is a scattering model which accounts for volume-scattering within a participating medium,
- a BRDF, i.e., a Bidirectional Reflectance Distribution Function, is a scattering model which accounts for only reflection at a surface, and
- a BSDF, i.e., a Bidirectional Scattering Distribution Function, is a scattering model which accounts for reflection and transmission at a surface.

Note that phase functions are normalized by definition, as volume-absorbtion is accounted for by a separate parameter. BRDFs and BSDFs, however, account for absorption as well as scattering, and so are only normalized if the model is non-absorbing (in other words, perfectly energy conserving). To state all of this more rigorously, a phase function p is normalized with respect to integration over incident directions ω_i , such that

$$\int_{\mathcal{S}^2} p(\omega_o \to \omega_i) \, \mathrm{d}\omega_i = 1.$$

However, a BRDF/BSDF f is only normalized as such if it is non-absorbing,

$$\int_{\mathcal{S}^2} f(\omega_o \to \omega_i) \, d\omega_i = 1 \iff f \text{ is non-absorbing.}$$

As this ought to suggest—this document follows the convention that BRDFs and BSDFs contain an implicit cosine-weighting with respect to incident direction. This is often written explicitly elsewhere in the literature, such that the above normalization condition is written as

$$\int_{\mathcal{S}^2} f(\omega_o \to \omega_i) |\cos \theta_i| \, \mathrm{d}\omega_i = 1.$$

We effectively make the substitution $f \leftarrow f |\cos \theta_i|$.

3.1. Participating media

A medium is characterized by its absolute index of refraction $\eta > 0$, its absorption coefficient $\mu_a \ge 0$, its scattering coefficient $\mu_s \ge 0$, and a local phase function model.

To specify a medium, simply type the name Medium followed by (optional) keywords arguments eta, mua, and mus for η , μ_a , and μ_s respectively. Keyword arguments for a medium have default values eta=1, mua=0, mus=0, such that any may be omitted for brevity. If $\mu_s > 0$, the keyword arguments must be followed by the name of the desired phase

function, followed by any keyword arguments for the phase function. If $\mu_s = 0$, it is not necessary to specify a phase function since no scattering occurs.

As the defaults may suggest, layered-sqt considers that vacuum is just a medium with refractive index 1 which neither absorbs nor scatters. So,

Medium

specifies vacuum.

3.1.1. Henyey-Greenstein phase

The Henyey-Greenstein phase function is given by

$$p(\omega_o \to \omega_i) = \frac{1}{4\pi} \frac{1 - g^2}{(1 + g^2 + 2g\omega_o \cdot \omega_i)^{3/2}}$$

with shape parameter $g \in (-1, 1)$. It may be helpful to note that g is the mean cosine of the scattered direction with respect to the incident direction, such that p becomes forward scattering as $g \to 1$, backward scattering as $g \to -1$, and uniformly/isotropically scattering as $g \to 0$.

To specify the Henyey-Greenstein phase in LSQT format, use the name HenyeyGreensteinPhase followed by the (optional) keyword argument g for g, which is zero by default. For example,

Medium mus=1 HenyeyGreensteinPhase g=-0.22

specifies a moderately back-scattering medium.

3.1.2. Rayleigh phase

The general form of the Rayleigh phase function is given by

$$p(\omega_o \to \omega_i) = \frac{3}{16\pi} \left[\frac{1+3\gamma}{1+2\gamma} + \frac{1-\gamma}{1+2\gamma} (\omega_o \cdot \omega_i)^2 \right]$$

where

$$\gamma = \frac{\rho}{2 - \rho}$$

where, in turn, $\rho \in [-1, 1]$ is a shaping parameter known as the *depolarization factor*. The simpler, more common form of the Rayleigh phase function is given by

$$p(\omega_o \to \omega_i) = \frac{3}{16\pi} (1 + (\omega_o \cdot \omega_i)^2),$$

which is obtained by setting $\rho = 0$.

The Rayleigh phase function prefers parallel scattering to perpendicular scattering. That is to say, scattering in directions perpendicular to the direction of propagation is diminished, and scattering is symmetric in terms of forward-versus back-scattering. This effect is maximized as $\rho \to -1$ and minimized as $\rho \to +1$. Note that ρ becomes uniformly/isotropically scattering in the limiting case $\rho = +1$.

To specify the Rayleigh phase function in LSQT format, use the name RayleighPhase followed by the (optional)

keyword argument rho for ρ , which is zero by default. For example,

```
Medium mus=1 RayleighPhase rho=-0.5
```

specifies a medium with an exaggerated "Rayleigh effect" relative to the common form where $\rho = 0$.

3.2. Layers

A layer is characterized by its z-height and a local BSDF model. As different BSDF models require different parameters, there is a different "type" of layer for each BSDF model implemented in layered-sqt.

Every layer accepts *z*-height as a parameter, so every line describing a layer must begin with the name Layer followed by the (required) keyword argument z. This is followed in turn by the name of the desired BSDF, followed by any keyword arguments for the BSDF. For example,

```
Layer z=1 NullBsdf
```

specifies a layer at *z*-height 1 with a null BSDF. As explained in the next sub-section, a null BSDF is a special case which accepts no additional keyword arguments.

The implementation requires that layers appear in top-tobottom order, such that the z-heights of subsequent layers in an LSQT file are strictly decreasing. If this condition is violated, the implementation issues an error and exits.

3.2.1. Null BSDF

At times, it is desirable to separate participating media without scattering at a layer. This is possible in layered-sqt by assigning a null BSDF with name NullBsdf, which may be interpreted as a 100% transmissive directional delta function in the direction a ray is already traveling. This is useful, for example, to model a layer of dust on top of a surface. To do so, we might specify the following LSQT file

```
Medium
Layer z=1 NullBsdf
Medium mus=1 HenyeyGreensteinPhase g=0.2
Layer z=0 LambertianBsdf fR=1 fT=0
Medium
```

which describes a layer of forward-scattering "dust" on top of a 100% reflective Lambertian surface.

3.2.2. Lambertian BSDF

The Lambertian BSDF models a uniformly scattering surface which may reflect, transmit, or both. That is, the (cosine-weighted) scattering function is

$$f(\omega_o \to \omega_i) = \frac{|\cos \theta_i|}{\pi} \begin{cases} f_R & (\omega_o, \omega_i) \text{ in same hemisphere} \\ f_T & \text{otherwise} \end{cases}$$

where f_R and f_T specify the amount of incident energy reflected and transmitted respectively. Both f_R and f_T should be non-negative numbers such that $f_R + f_T \le 1$ for this to be physically plausible. In the event that $f_R + f_T = 1$, this is perfectly energy-conserving.

To specify the Lambertian BSDF in LSQT format, use the name LambertianBsdf followed by keyword arguments fR and fT for f_R and f_T respectively. The implementation requires that the values of fR and fT be physically plausible, i.e., non-negative numbers whose sum does not exceed 1. For example,

```
Layer z=2.2 LambertianBsdf fR=0.8 fT=0.1
```

specifies a Lambertian surface at *z*-height 2.2 which is 80% reflective, 10% transmissive, and 10% absorptive.

3.2.3. Microsurface Lambertian BRDF

A microsurface is thought to be an infinitesimally thin cloud of microfacets, where each facet is thought to scatter light according to another, simpler BSDF. The cloud is characterized geometrically by 1) a distribution of the slopes of the facets and 2) a distribution of the heights of the facets, which are assumed to uncorrelated, such that the facets are discontinuous. The distribution of slopes is parameterized by its so-called roughness α (more generally, anisotropic roughness α_x , α_y). As $\alpha \to \infty$, the distribution of slopes widens and the emergent BSDF appears rougher. As $\alpha \to 0$, the distribution of slopes collapses, recovering the initial, simpler BSDF in the limiting case $\alpha = 0$.

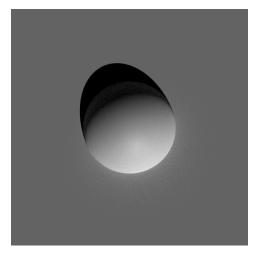
As given in [3], the microsurface Lambertian BRDF is a multiple scattering (stochastic) microfacet model wherein facets are assumed to be Lambertian reflectors. This is particularly useful for representing rough diffuse surfaces. The implementation in layered-sqt is parameterized by roughness $\alpha > 0$, the Lambertian BRDF coefficient $f_R \in [0, 1]$, and the number of local stochastic process iterations $n_{\text{iter}} \geq 1$. For rougher microsurfaces (say, $\alpha > 0.4$), it is typically more efficient to increase n_{iter} rather than the global path count

To specify the microsurface Lambertian BRDF in LSQT format, use the name

MicrosurfaceLambertianBrdf

followed by (optional) keyword arguments alpha, fR, and iter_count for $\alpha,\,f_R,$ and $n_{\rm iter}$ respectively. By default, alpha=0.5, fR=1, and iter_count is chosen to be 1, 2, 4, or 6 depending on alpha. For reference, $\alpha<0.1$ is very smooth, $0.1<\alpha<0.8$ is somewhat rough, and $0.8<\alpha$ is very rough. Furthermore, multiple scattering may be disabled by the keyword argument

```
use_multiple_scattering=false
```



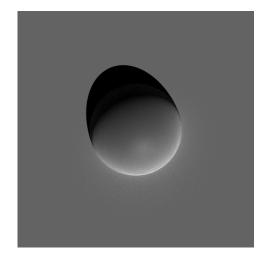


Figure 6: A sphere with an LSQT BRDF on a Lambertian ground plane rendered from above with DIRSIG5. The LSQT BRDF models a dusty dielectric substrate—from top to bottom, we have a vacuum medium, a null BSDF layer, a slightly back-scattering medium with g = -0.2, a decently smooth microsurface dielectric BSDF layer, a transparent medium with $\eta = 1.5$, and a Lambertian BRDF layer. On the left, the Lambertian layer has $f_R = 0.8$. On the right, the Lambertian layer has $f_R = 0.4$, such that the backscattering due to "dust" is more obvious. All other parameters are the same in both images.

in which case only the single scattering term is computed. It is important to note that, if multiple-scattering interactions are ignored, then setting fR=1 does not guarantee that the BRDF is perfectly energy conserving. For small roughness values (say, $\alpha < 0.1$), the energy carried by multiple scattering interations is insignificant, and so this may not be a concern. For large roughness values however, multiple scattering is important to prevent energy loss.

3.2.4. Microsurface dielectric BSDF

The microsurface dielectric BSDF follows the same logical construction as the microsurface Lambertian BRDF, except the constituent BSDF assigned to the facets is the delta Fresnel mirror BSDF. The implementation in layered-sqt is parameterized by roughness $\alpha>0$, a scaling factor for the Fresnel mirror BRDF $k_R\in[0,1]$, a scaling factor for the Fresnel mirror BTDF $k_T\in[0,1]$, and the number of local stochastic process iterations $n_{\text{iter}}\geq 1$.

To specify the microsurface dielectric BSDF in LSQT format, use the name

MicrosurfaceDielectricBsdf

followed by (optional) keyword arguments alpha, kR, kT, and iter_count for α , k_R , k_T , and $n_{\rm iter}$ respectively. By default, alpha=0.5, kR=1, kT=1, and iter_count is chosen to be 1, 2, 4, or 6 depending on alpha. As in the Lambertian case, multiple-scattering may be disabled by the keyword argument

use multiple scattering=false

so that only the single-scattering term is computed. Unlike the Lambertian case however, the single-scattering term is directly computable (without a stochastic process) due to the delta function in the Fresnel mirror BSDF—such that disabling multiple-scattering makes iter_count irrelevant. This leads to noticeably faster simulations, though still at the cost of significant energy loss for rough surfaces.

3.2.5. Oren-Nayar diffuse BRDF

TODO

References

- [1] Adam A. Goodenough and Scott D. Brown. "DIRSIG5: Next-Generation Remote Sensing Data and Image Simulation Framework". In: *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing* 10.11 (Nov. 2017), pp. 4818–4833.
- [2] E. Heitz. "Understanding the masking-shadowing function in microfacet-based BRDFs". In: *Journal of Computer Graphics Techniques (JCGT)* 3.2 (June 2014), pp. 48–107. ISSN: 2331-7418.
- [3] E. Heitz et al. "Multiple-scattering microfacet BSDFs with the Smith model". In: *ACM Transactions on Graphics* 35.4 (July 2016), pp. 1–58. ISSN: 0730-0301.