$N^2 = S(t) + I(t)$ $\qquad i(t) = \frac{I(t)}{N^2}$ $\qquad s(t) = \frac{S(t)}{N^2}$ ← fractions

$$\frac{dI}{dt} = b\, s(t)\, I(t)$$

$I(t)$ ← absolute number of infected people

$s(t)$ ← fraction of susceptible individuals

$b = k \times p_{inf}$ ← probability of infection at close contact

k ← average number of close contacts per person

→ Multiply by $\frac{1}{N^2}$

$$\frac{di}{dt} = b\,(1-i(t))\,i(t)$$

$$\int_{i_0}^{i} \frac{di}{i(1-i)} \int_0^t b\,dt$$

$$\frac{1}{i(1-i)} = \frac{1}{1-i} + \frac{1}{i}$$

$$\int_{i_0}^{i} \frac{di}{i(1-i)} = \int \frac{di}{1-i} + \frac{di}{i} = -\log(1-i)\Big|_{i_0}^{i} + \log(i)\Big|_{i_0}^{i} =$$

$$\log\left(\frac{i}{i_0}\,\frac{1-i_0}{1-i}\right) = b\,t$$

$$\frac{i(t)}{1-i(t)} = \frac{i_0}{1-i_0}\exp(b(t))$$

$$i(t) = (1-i(t))\,\frac{i_0}{1-i_0}\exp(bt)$$

$$i(t)\left(1 + \frac{i_0}{1-i_0}\exp(bt)\right) = \frac{i_0}{1-i_0}\exp(bt)$$

$$i(t) = \frac{\frac{i_0}{1-i_0}\exp(bt)}{1 + \frac{i_0}{1-i_0}\exp(bt)}$$

$$I(t) = N^2 i(t) = N^2\,\frac{\frac{I_0}{N^2-I_0}\exp(bt)}{1 + \frac{I_0}{N^2-I_0}\exp(bt)}$$

(logistic function)