73. Statistical Physics: Virus Transmission

I. Introduction

Statistical physics is a branch of physics that solves physical problems usually on a macroscopic scale using probabilistic theories or statistics. Its main purpose is to investigate the properties of the system that hold in the thermodynamic limit, starting from the microscopic behaviour of its components which are essentially governed by statistics. Many physically relevant questions belong to the class of percolation problems, for example, the study of how fashion, opinions and epidemics spread through a population, how trendy words become popularly used in a language, and how tightly ground coffee beans can be pressed but still allow a flow of water in order to make espresso. Thus, statistical physics is an important concept that is also prevalent in other fields of study like network science, epidemiology, sociology, chemistry, etc.

In this simulation experiment, you will investigate the threshold probability that dictates whether a virus that starts at the north border of country A is transmitted to country B, which shares the South border with country A. You will also assess the effectiveness of government quarantine policies in 'flattening the curve'.

II. Set-up of the problem

Country A is modelled as a squared grid and is populated by N x N individuals, where each individual occupies a spot in the grid. We start with Day 0 which marks the day that one person along the North border has been infected. Each person in the nation can have one of four possible status: uninfected, infected, recovered, dead.

Each day:

- an infected person could infect his/her close contacts. For each close contact, the probability of infection is p_{inf}.
- an infected person could recover (with probability p_{reco}), die (with probability p_{death}), or stay infected.

III. Preparation

In preparation for your tasks, you should prepare the following and discuss them with your teaching assistant during your first meeting.

1. Analytic part

- Without quarantine, each infected person can travel freely around the country: each person will have every day on average k close contacts. Write down the differential equation that governs the rate of total infections and solve it to obtain how it depends on time. For the sake of argument, we assume that rate of recovery and deaths are zero.
- With quarantine, however, each infected person cannot move from their position. In this situation, new infections can only happen among the nearest neighbours (that is, the individuals at distance of one square horizontally or vertically). Write down the differential equation that governs the rate of total infections and solve it in order to obtain the dependency on time. For the sake of argument, we assume again that the rate of recovery and death is zero.

How do the solutions in the two cases compare?

2. Simulation part

The simulation code is provided in the accompanying jupyter notebook. It implements both the time and geographical evolution of the epidemic, in the setting described in section II. Get yourself acquainted with the simulation code, by going through the code and by reading the documentation of the classes. Test your understanding by running exercise 0.

IV. Taks

Using the simulation code provided in the accompanying jupyter notebook, perform the exercises detailed there. Here we just provide a summary of the main tasks.

1. For country A of size $N = 10$, find the threshold probability p^*, such that if $p_{Inf} < p^*$, the virus will almost always not be transmitted to country B.

2. Repeat exercise 1 for different values of N. How does p^* depend on N?

3. Find how the number of total infections in country A varies with time in the case in the two cases where individual can or cannot move. How do the two curves compare?