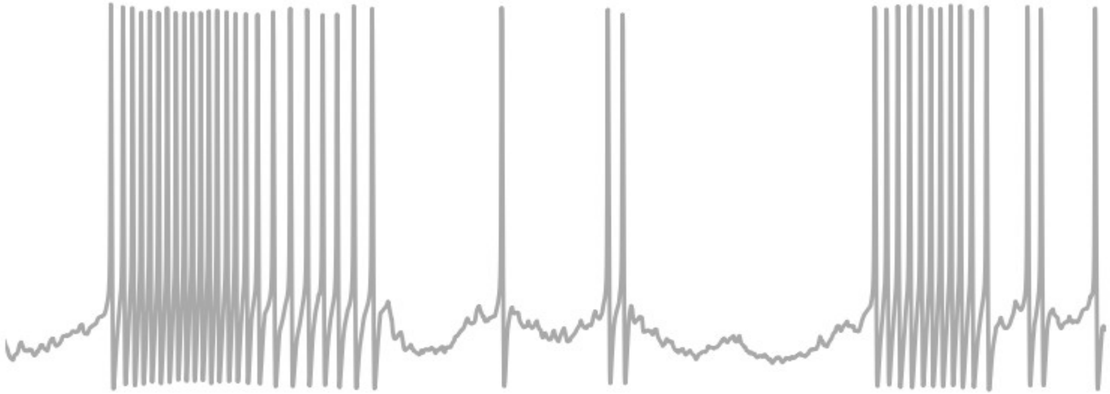

Single neuron simulations with Brian

Marcel Stimberg
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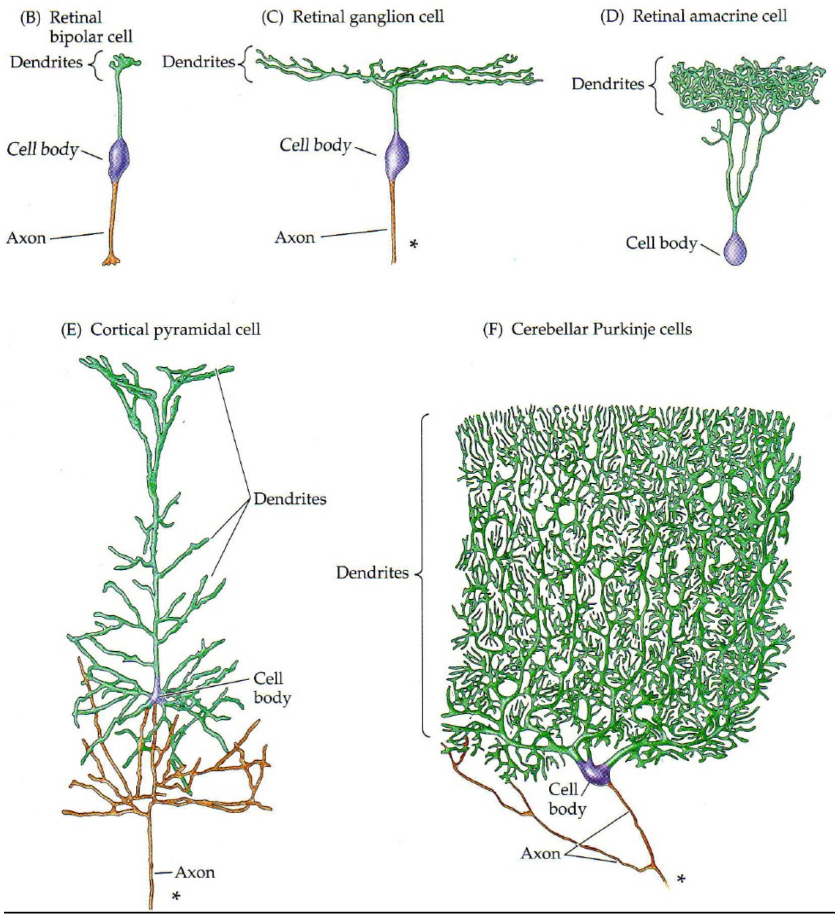
Neurons are basic units of computation



dendrites

soma

axon

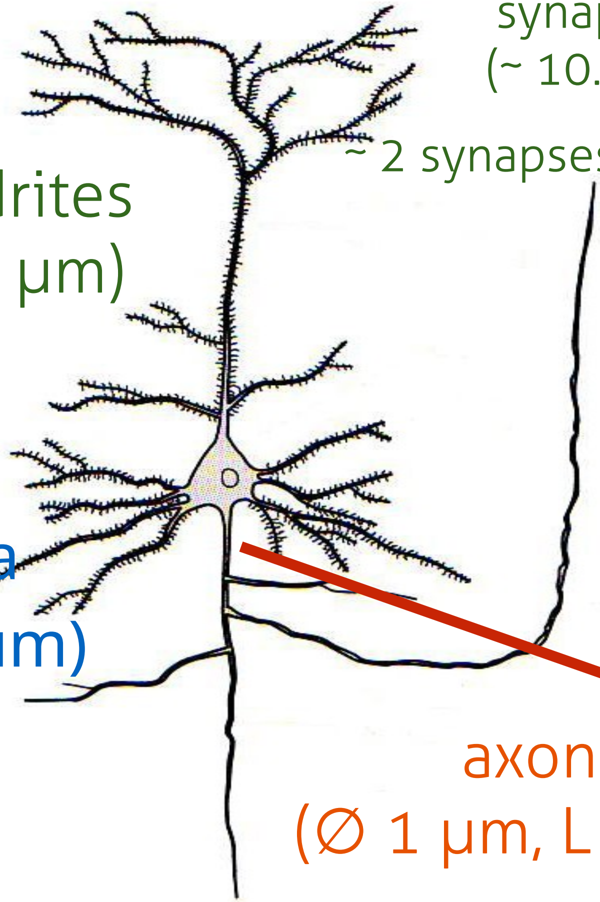
Signal flow

A typical cortical neuron

synapses
(~ 10.000)

~ 2 synapses/µm

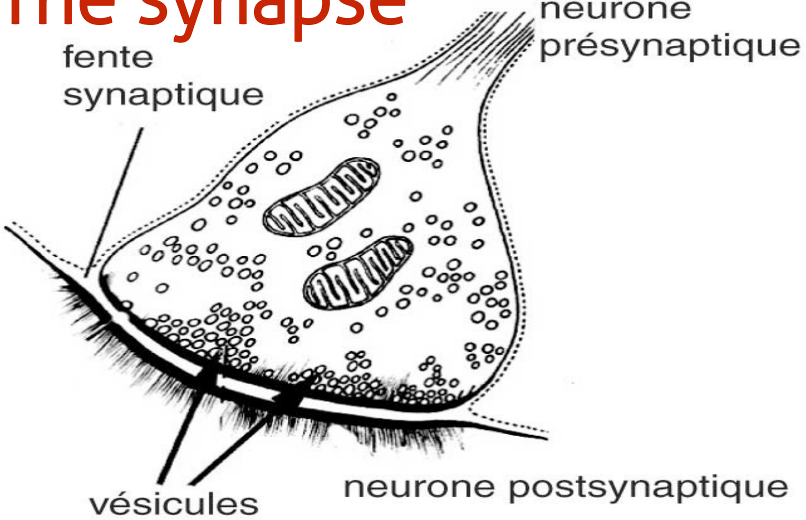
dendrites
(∅ 1 µm)



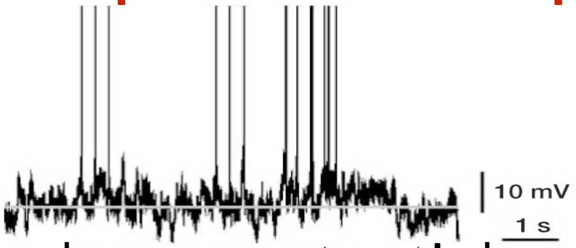
soma
(∅ 20 µm)

axon
(∅ 1 µm, L 4 cm)

The synapse

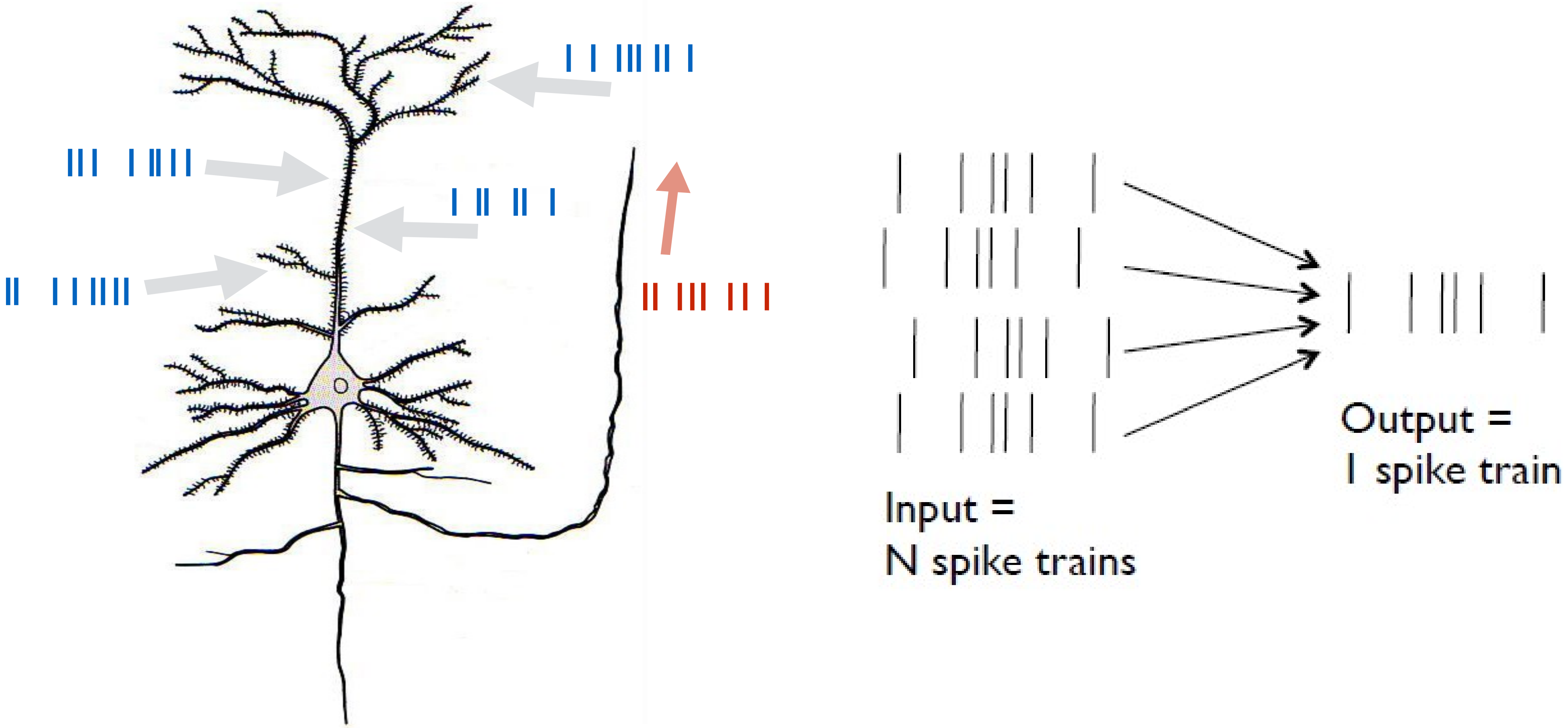


Action potential/s (spikes)

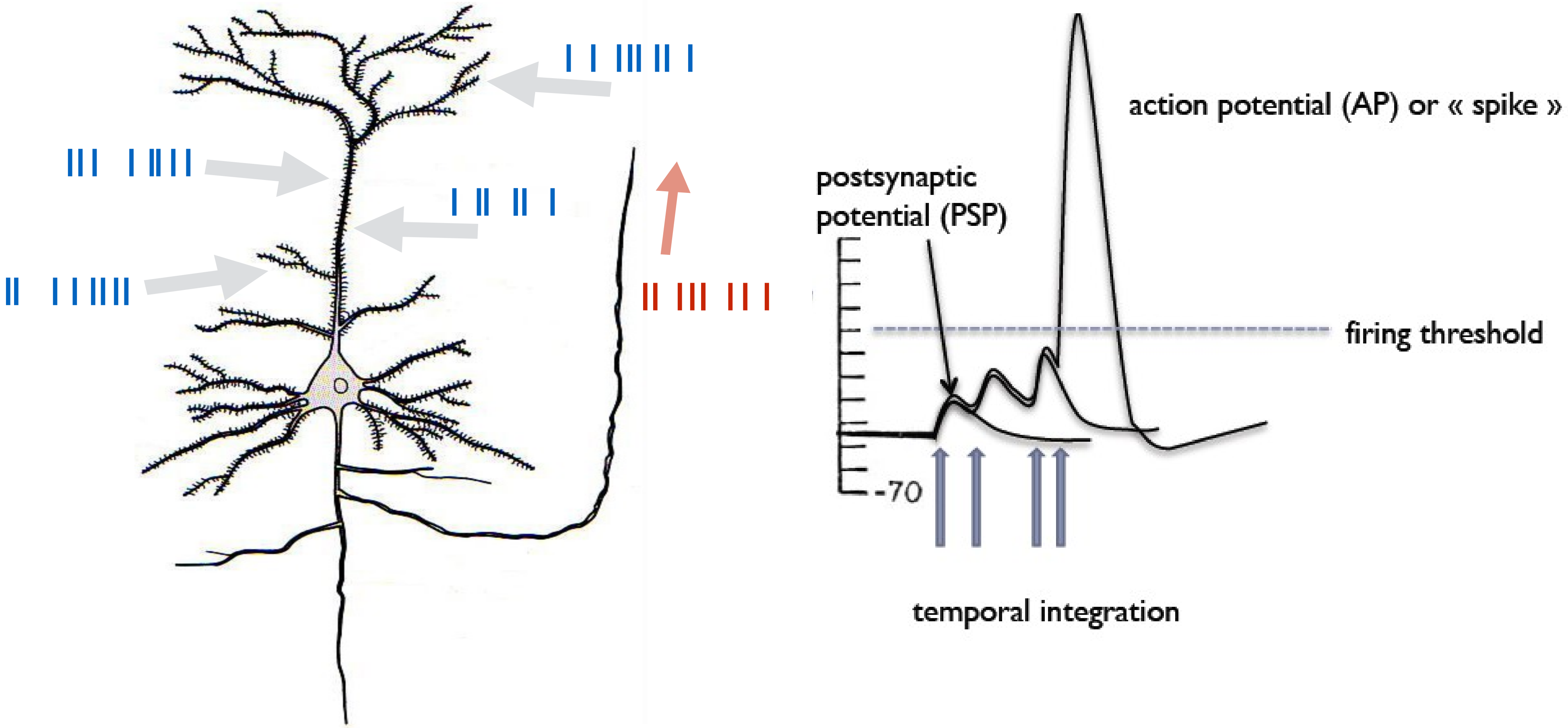


Membrane potential

Neurons are basic units of computation

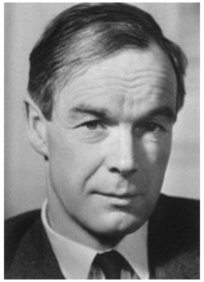


Neurons are basic units of computation

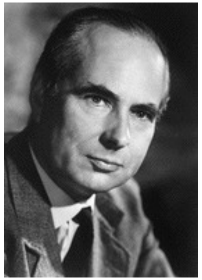


Neuron models

- **Hodgkin Huxley model** : description of ion channel dynamics (Hodgkin & Huxley, 1952)



Hogdkin



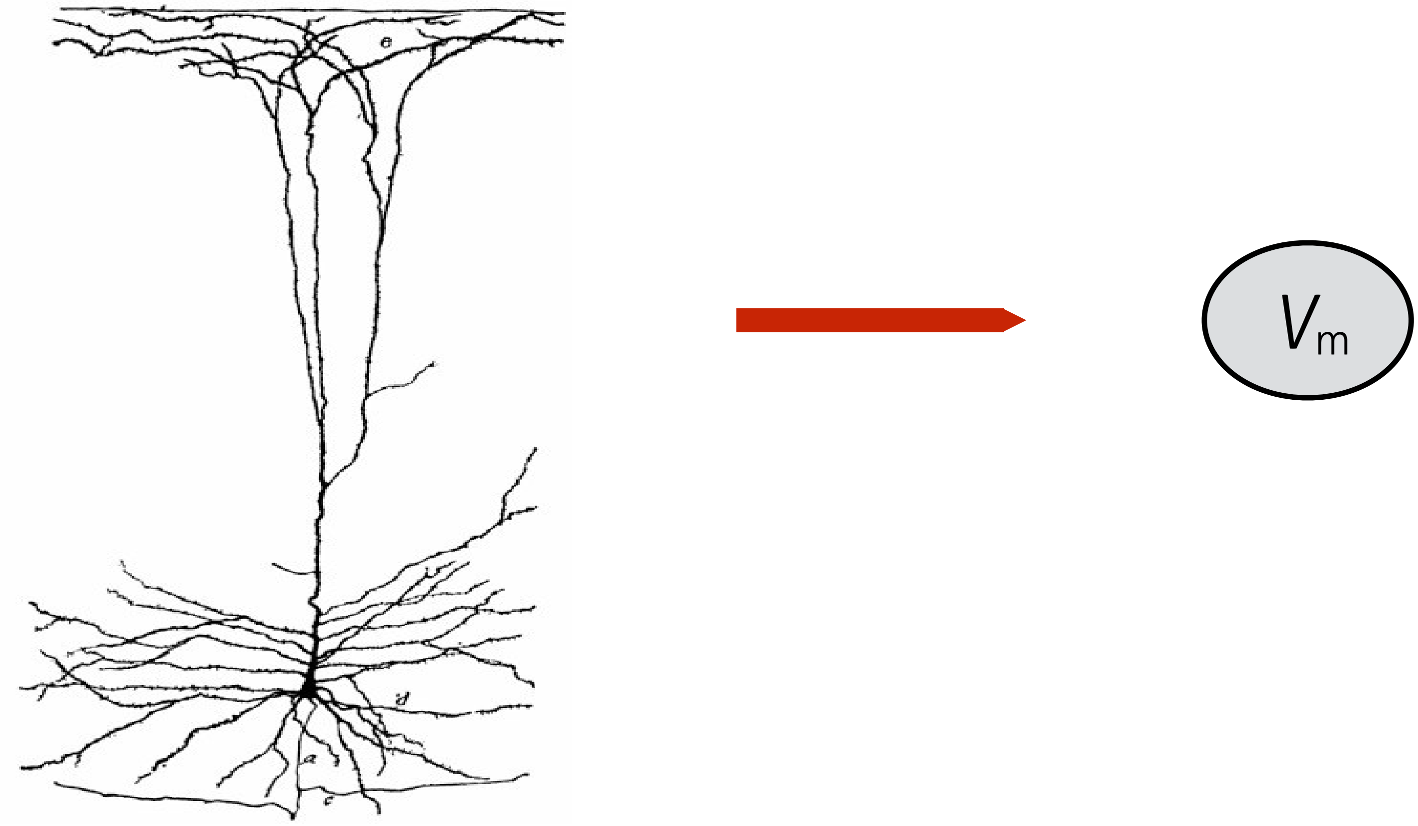
Huxley

- **integrate-and-fire model** : description of input integration membrane potential dynamics (LaPicque, 1907)

- **rate model** : description of the mean firing rate dynamics

- **cable theory** : description of input propagation along the dendrites (Rall, 1962)

Simplified models: point neuron

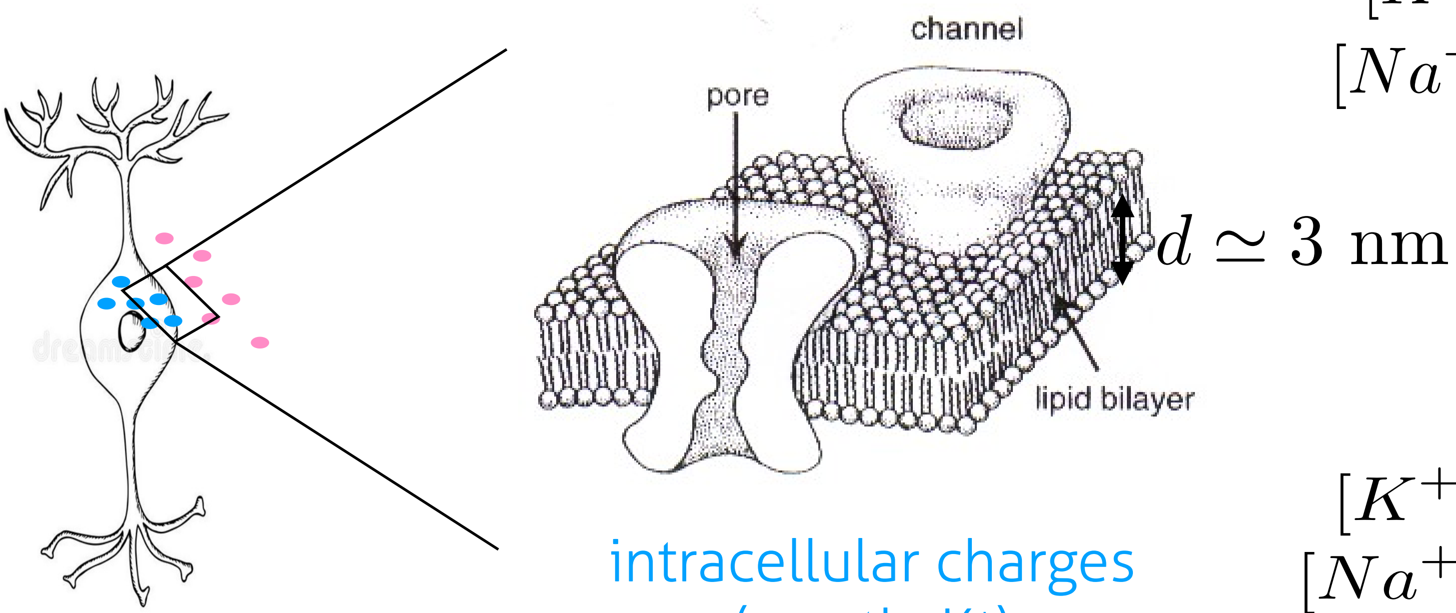


The neuronal cell membrane

The cell membrane is a lipid bilayer with protein inclusions

extracellular charges
(mostly Na^+, Cl^-)

$$[K^+]_{\mathrm{out}} \simeq 4\,\mathrm{mM}$$
$$[Na^+]_{\mathrm{out}} \simeq 144\,\mathrm{mM}$$

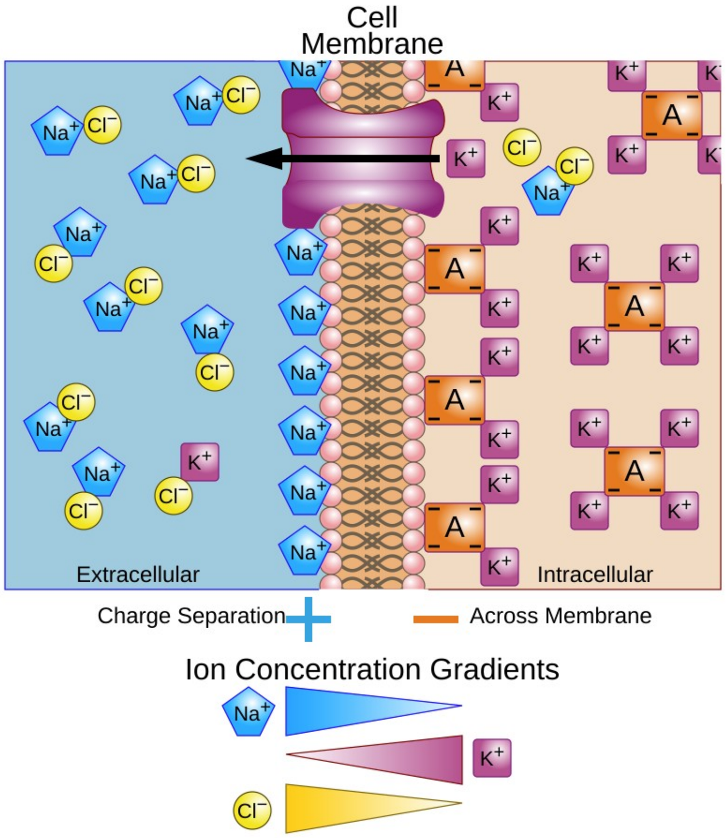


$$[K^+]_{\mathrm{in}} \simeq 160\,\mathrm{mM}$$
$$[Na^+]_{\mathrm{in}} \simeq 12\,\mathrm{mM}$$

intracellular charges
(mostly K^+)

The neuronal cell membrane

Each ion type has its **reversal potential**, where **concentration gradient** and **electrical gradient** cancel each other out

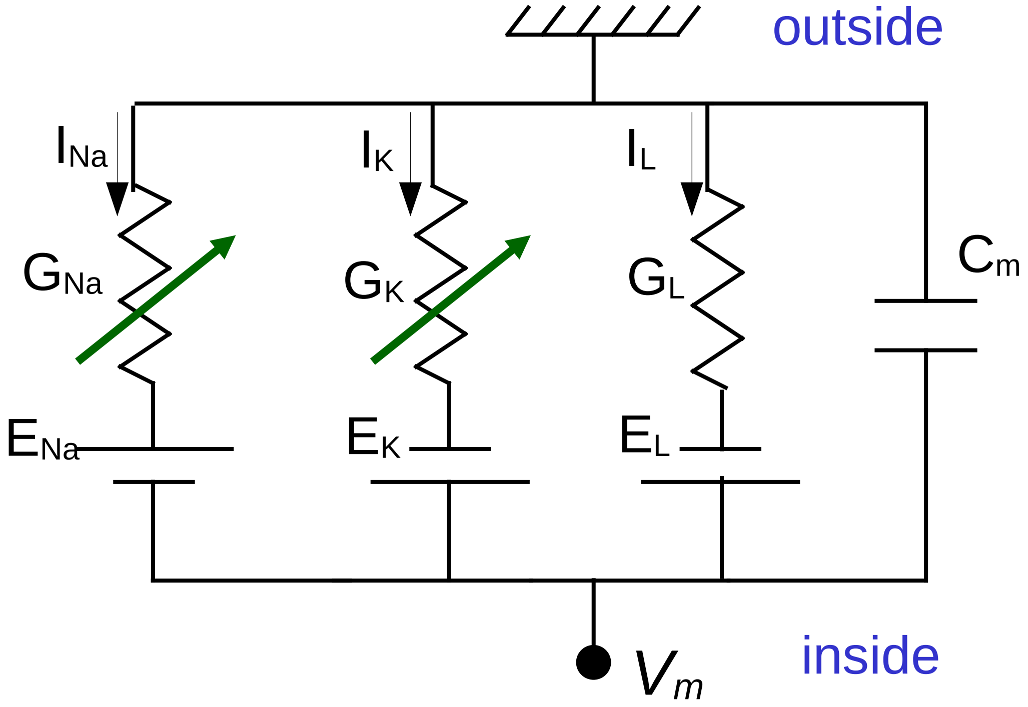


Reversal potentials (approx.)

Sodium (**Na**): +50mV
Potassium (**K**): -90mV
Chloride (**Cl**): -80mV

The neuronal cell membrane

Electrical equivalent circuit

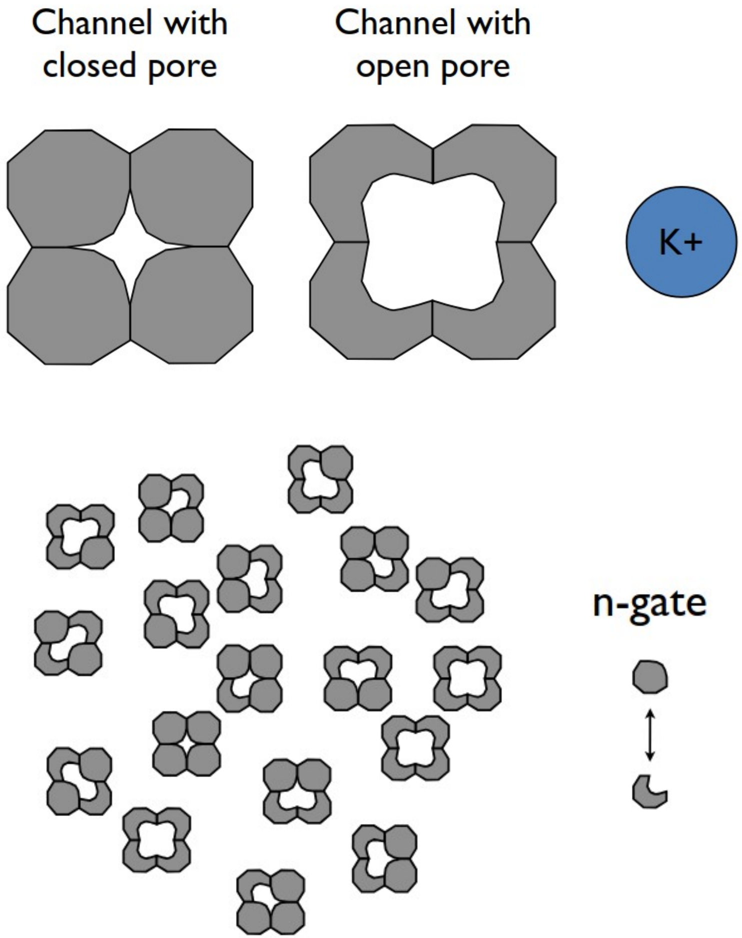


Kirchhoff's law:
$IC_m + I_{Na} + I_K + I_L = 0$

Definition of capacitance:
$IC_m = C^{dV}/_{dt}$

Ohm's law:
$I_x = g_x\underbrace{(V_m - E_x)}_{\text{Driving force}}$

HH model: the potassium channel

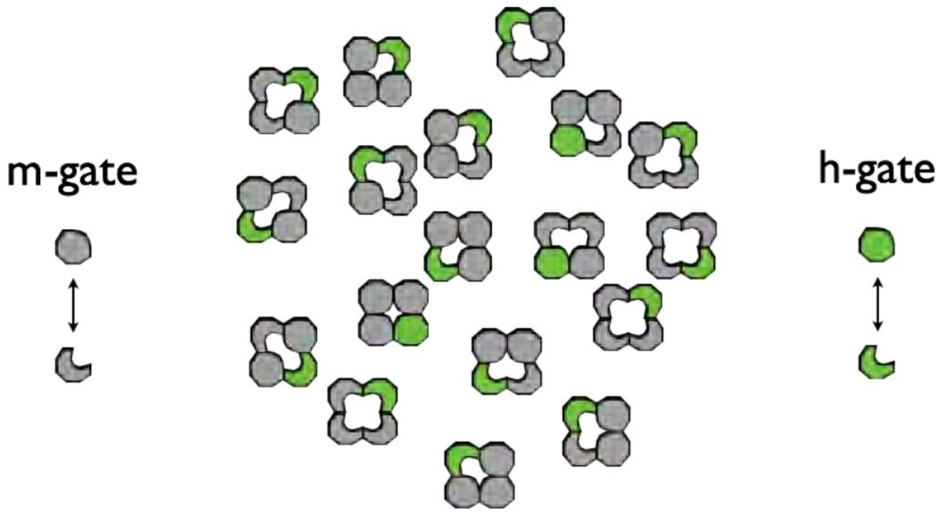


4 similar “subunits” need to be open

Probability of a subunit being open = n
Probability of channel being open = n^4

$$I_K = \bar{g}_K n^4 (V_m - E_K)$$

HH model: the sodium channel

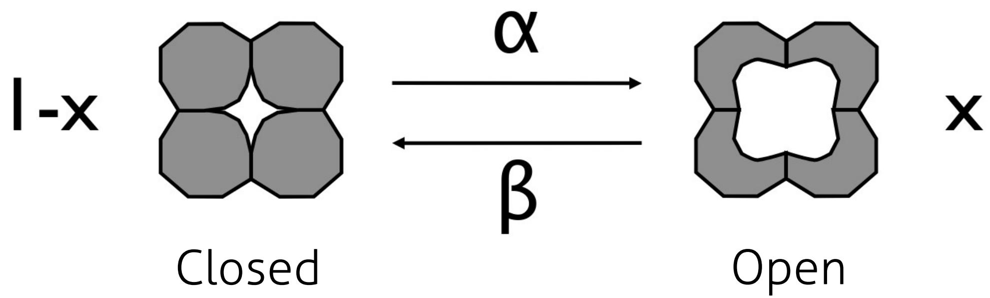


Two different types of subunits

$$I_{Na} = \bar{g}_{Na} m^3 h (V_m - E_{Na})$$

HH model: channel kinetics

x = probability that a channel is open
= fraction of channels that is open



$$dx/dt = \alpha\,(1 - x) - \beta x$$

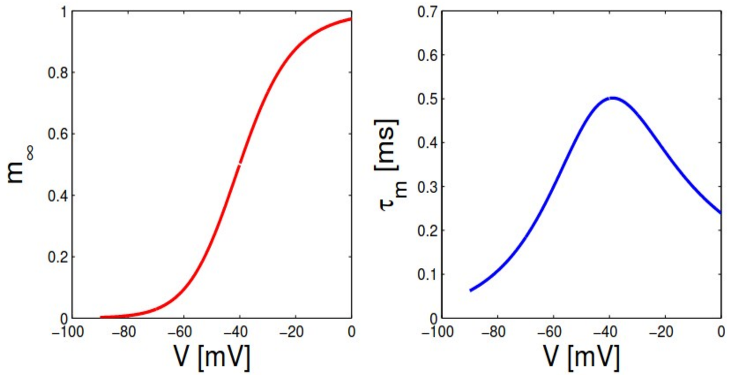
Re-arranging terms:

$$\underbrace{1/(\alpha + \beta)}_{\tau_x}\, dx/dt = \underbrace{\alpha/(\alpha + \beta)}_{x_\infty} - x$$

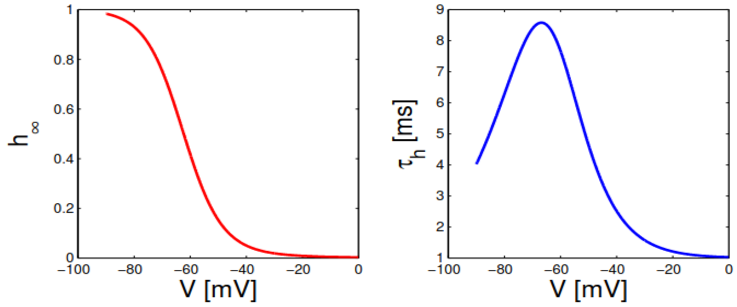
$$\boldsymbol{\tau_x\, dx/dt = x_\infty - x}$$

HH model: channel kinetics

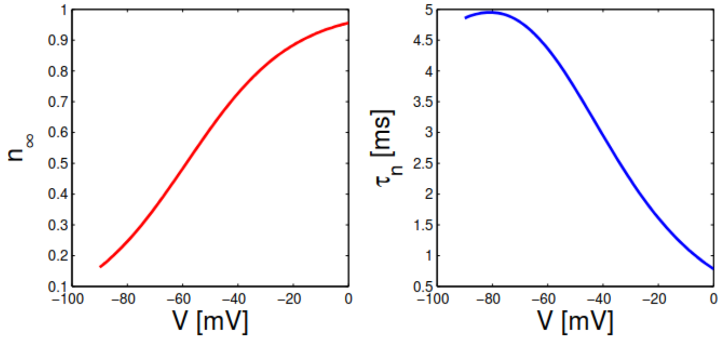
m-activation **increases rapidly**
with increasing membrane voltage.



h-activation **decreases slowly**
with increasing membrane voltage



n-activation **increases slowly**
with increasing membrane voltage.



The full Hodgkin-Huxley model equations

$$C\frac{\mathrm{d}V}{\mathrm{d}t} = g_L(E_L - V) + \bar{g}_{\mathrm{Na}}m(t)^3h(t)(E_{\mathrm{Na}} - V) + \bar{g}_{\mathrm{K}}n(t)^4(E_{\mathrm{K}} - V) + I_{\mathrm{stim}}$$

$$\tau_n\frac{\mathrm{d}n}{\mathrm{d}t} = n_\infty - n \qquad \tau_m\frac{\mathrm{d}m}{\mathrm{d}t} = m_\infty - m \qquad \tau_h\frac{\mathrm{d}h}{\mathrm{d}t} = h_\infty - h$$

$$\tau_n = \frac{1}{\alpha_n + \beta_n} \qquad \tau_m = \frac{1}{\alpha_m + \beta_m} \qquad \tau_h = \frac{1}{\alpha_h + \beta_h}$$

$$n_\infty = \frac{\alpha_n}{\alpha_n + \beta_n} \qquad m_\infty = \frac{\alpha_m}{\alpha_m + \beta_m} \qquad h_\infty = \frac{\alpha_h}{\alpha_h + \beta_h}$$

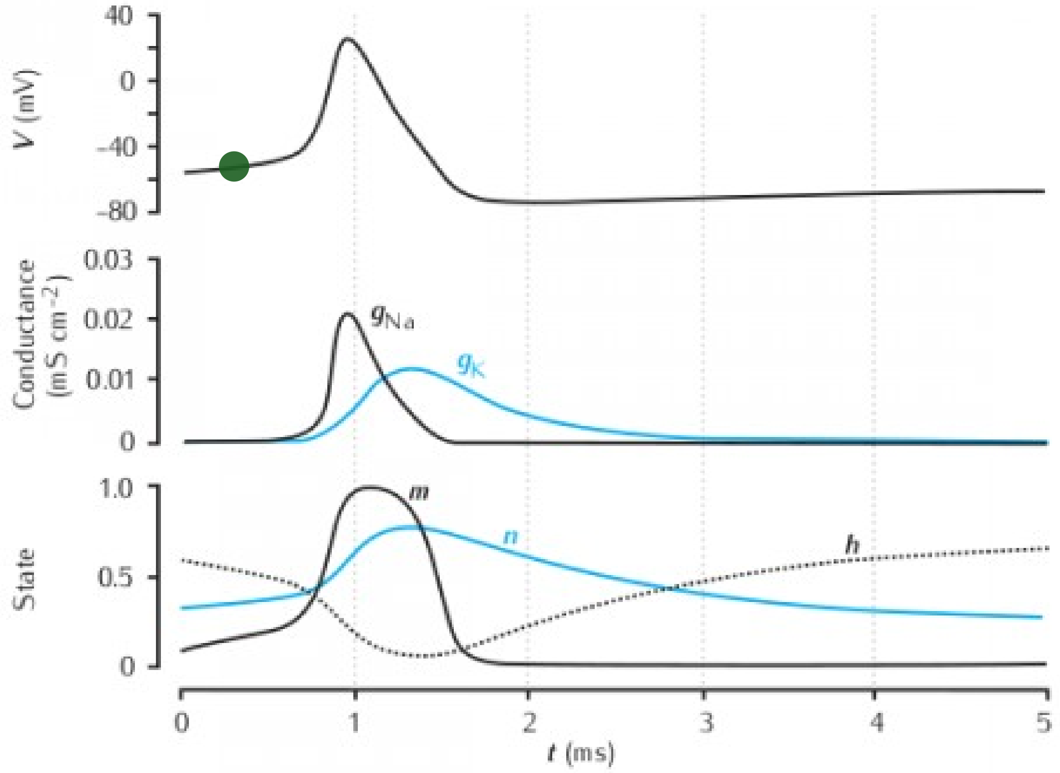
Transition rates (parameter values determined for squid giant axon):

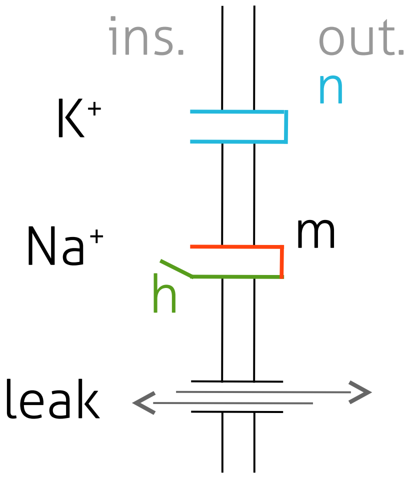
$$\alpha_n(V) = \frac{(0.1 - 0.01V)}{e^{1-0.1V} - 1} \qquad \alpha_m(V) = \frac{(2.5 - 0.1V)}{e^{2.5-0.1V} - 1} \qquad \alpha_h(V) = 0.07\,e^{-\frac{V}{20}}$$

$$\beta_n(V) = 0.125e^{-\frac{V}{80}} \qquad \beta_m(V) = 4e^{-\frac{V}{18}} \qquad \beta_h(V) = \frac{1}{e^{3-0.1V} + 1}$$

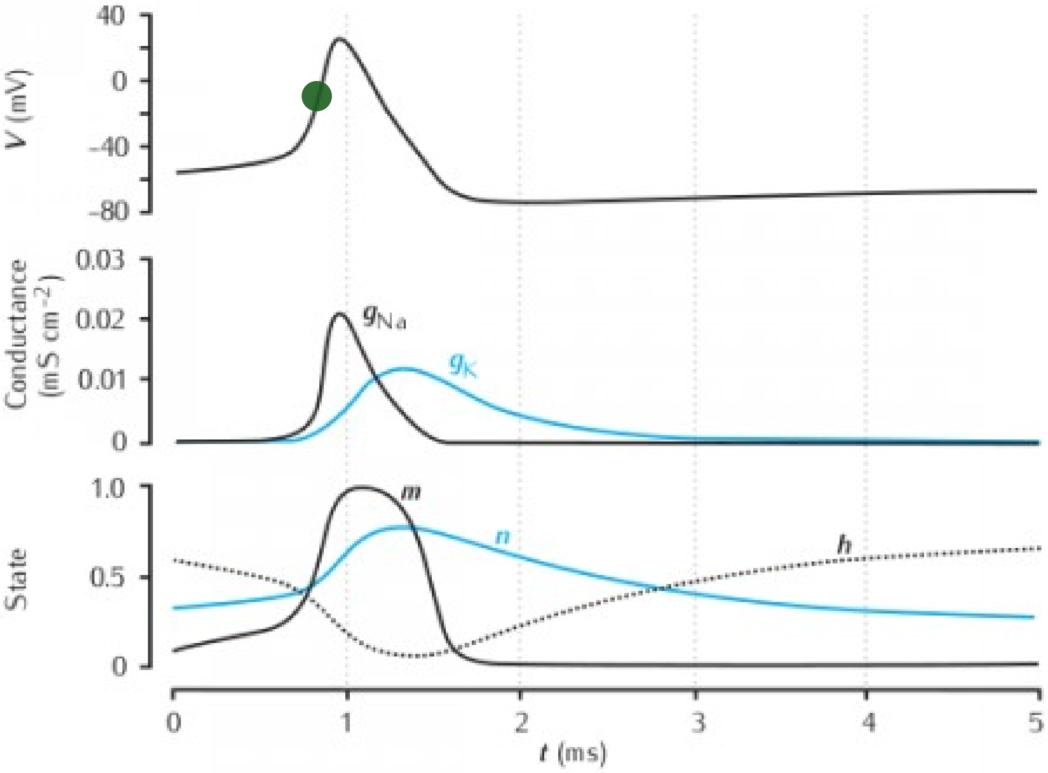
(**Rates** expressed as **/ms**, membrane potential **V in mV**)

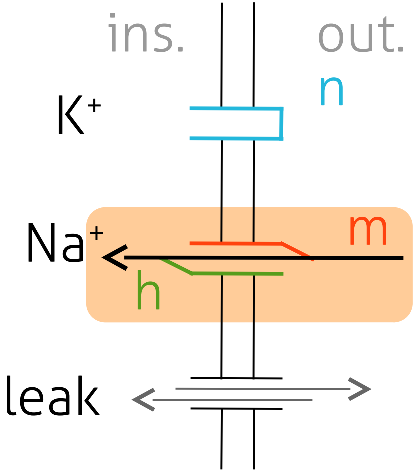
HH model: the action potential



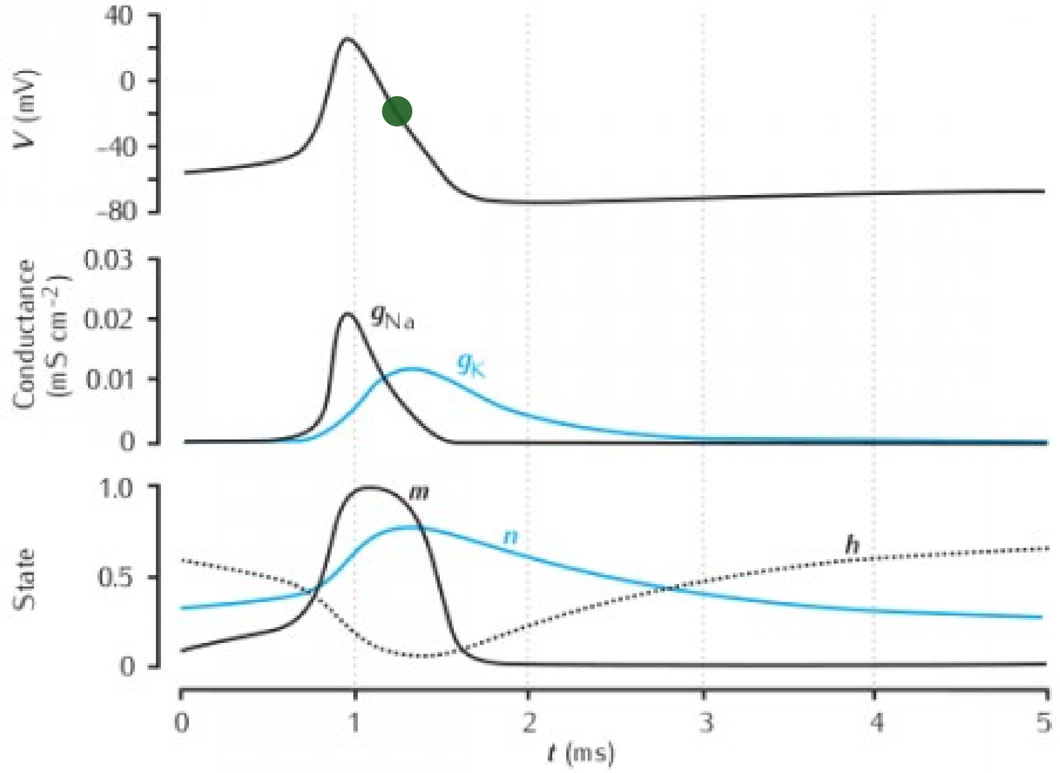


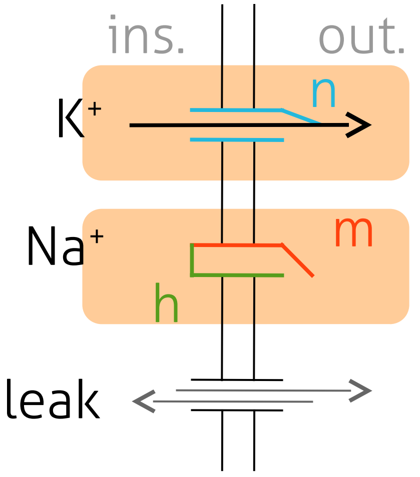
HH model: the action potential



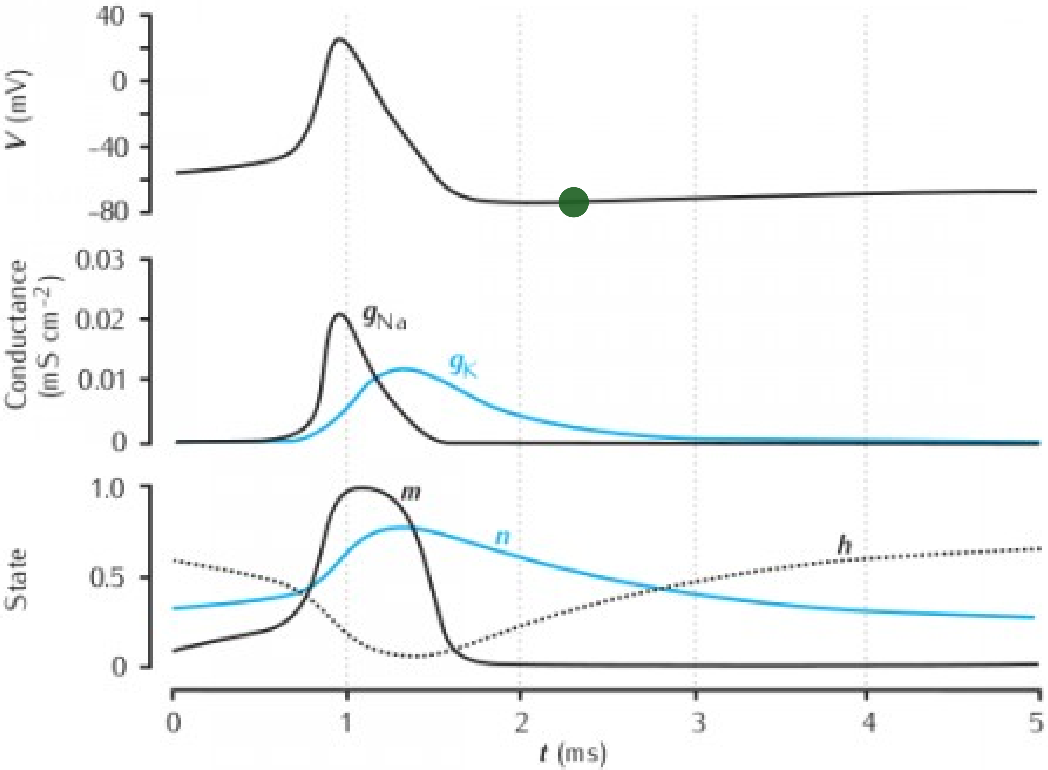


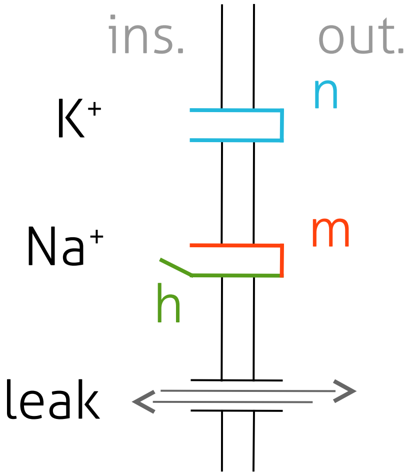
HH model: the action potential





HH model: the action potential





Leaky integrate-and-fire model

$$C\frac{\mathrm{d}V}{\mathrm{d}t} = g_L(E_L - V) + \bar{g}_{\mathrm{Na}} m(t)^3 h(t)(E_{\mathrm{Na}} - V) + \bar{g}_{\mathrm{K}} n(t)^4 (E_{\mathrm{K}} - V) + I_{\mathrm{stim}}$$

$$\tau_n \frac{\mathrm{d}n}{\mathrm{d}t} = n_\infty - n \qquad \tau_m \frac{\mathrm{d}m}{\mathrm{d}t} = m_\infty - m \qquad \tau_h \frac{\mathrm{d}h}{\mathrm{d}t} = h_\infty - h$$

$$\tau_n = \frac{1}{\alpha_n + \beta_n} \qquad \tau_m = \frac{1}{\alpha_m + \beta_m} \qquad \tau_h = \frac{1}{\alpha_h + \beta_h}$$

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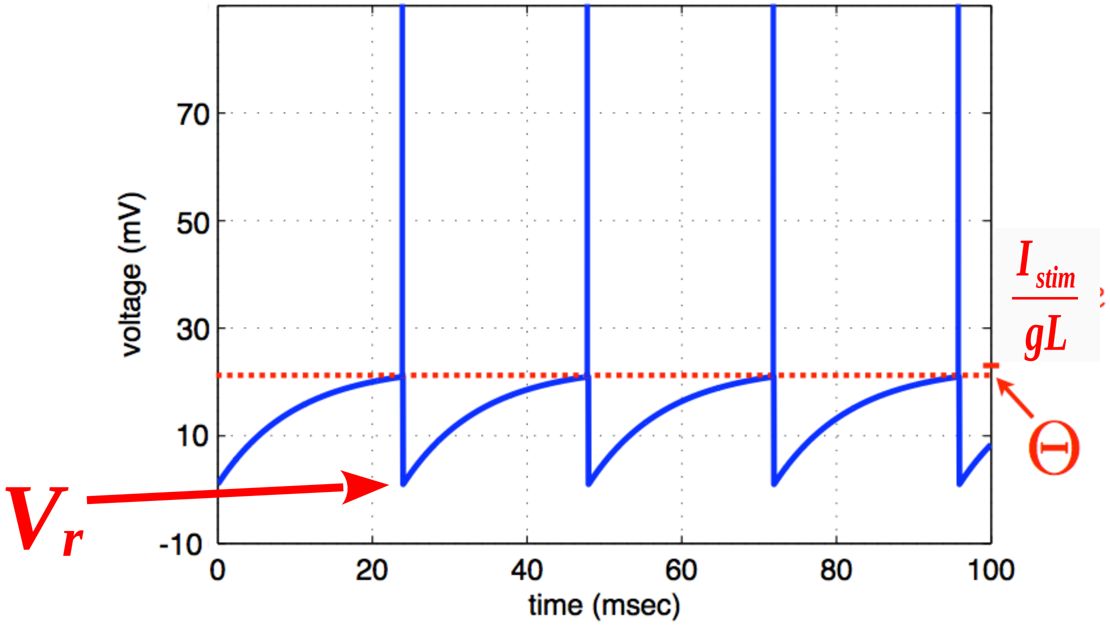
(**Rates** expressed as **/ms**, membrane potential **V in mV**)

Leaky integrate-and-fire model

$$C\frac{\mathrm{d}V}{\mathrm{d}t} = g_L(E_L - V) + I_{\mathrm{stim}}$$

+

Threshold: emit a spike if V crosses Θ
Reset: reset the membrane potential to V_r
(Refractoriness: do not allow spikes for *x* ms after a spike)



The

BRIAN

simulator

Brian's approach

- *Philosophy*: Mathematical model descriptions
  - Flexible system to define models with equations
  - Takes care of numerical integration / synaptic propagation
  - Physical units
- *Technology*: Code generation
  - High-level descriptions transformed into low-level code
  - Transparent to user

More info

Website: https://briansimulator.org

Documentation: https://brian2.readthedocs.io

Discussion forum: https://brian.discourse.group

Articles:

Stimberg, Marcel, Romain Brette, and Dan FM Goodman. "Brian 2, an Intuitive and Efficient Neural Simulator." ELife 8 (2019): e47314. https://doi.org/10.7554/eLife.47314.

Stimberg, Marcel, Dan F. M. Goodman, Victor Benichoux, and Romain Brette. "Equation-Oriented Specification of Neural Models for Simulations." Frontiers in Neuroinformatics 8 (2014). https://doi.org/10.3389/fninf.2014.00006

Tutorial

Deepnote notebook

http://tiny.cc/2025-T07