



Neural Data Science with **Python**

L12: Spectral analysis of analog signals

Michael Graupner

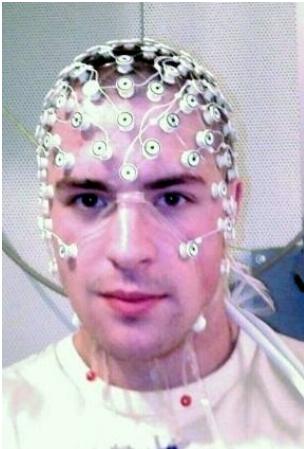
SPPIN – Saint-Pères Institute for the Neurosciences
Université Paris Cité, CNRS

Common analog signals in neuroscience

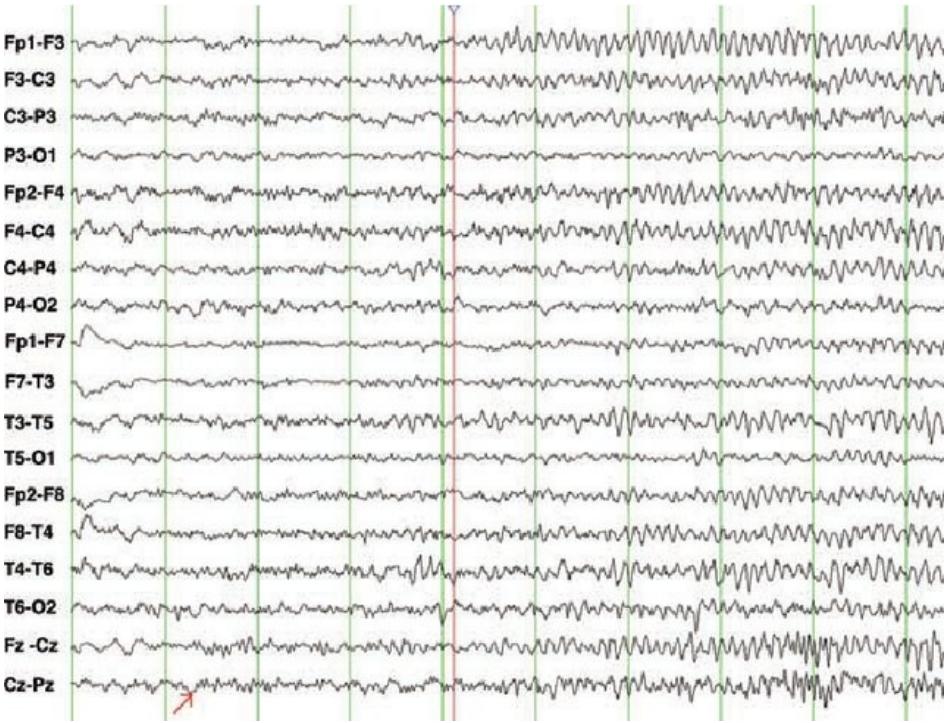
- membrane potential, transmembrane current recordings
- electroencephalogram - EEG
- magnetoencephalogram - MEG
- local field potential - LFP

Electroencephalogram - EEG

Electroencephalography (EEG) is an electrophysiological monitoring method to record electrical activity of the brain.



- typically noninvasive through electrodes on the scalp
- measures voltage fluctuations resulting from ionic current within the neurons in the brain
- focus on event-related potentials, or *spectral content* : exhibits prominent oscillations
- used for example in epilepsy, sleep disorder, anesthesia, coma research

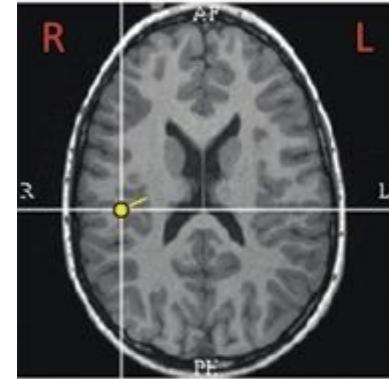


Magnetoencephalogram - MEG

Magnetoencephalography (MEG) is a functional neuroimaging technique for mapping brain activity by recording magnetic fields produced by electrical currents occurring naturally in the brain, using very sensitive magnetometers.



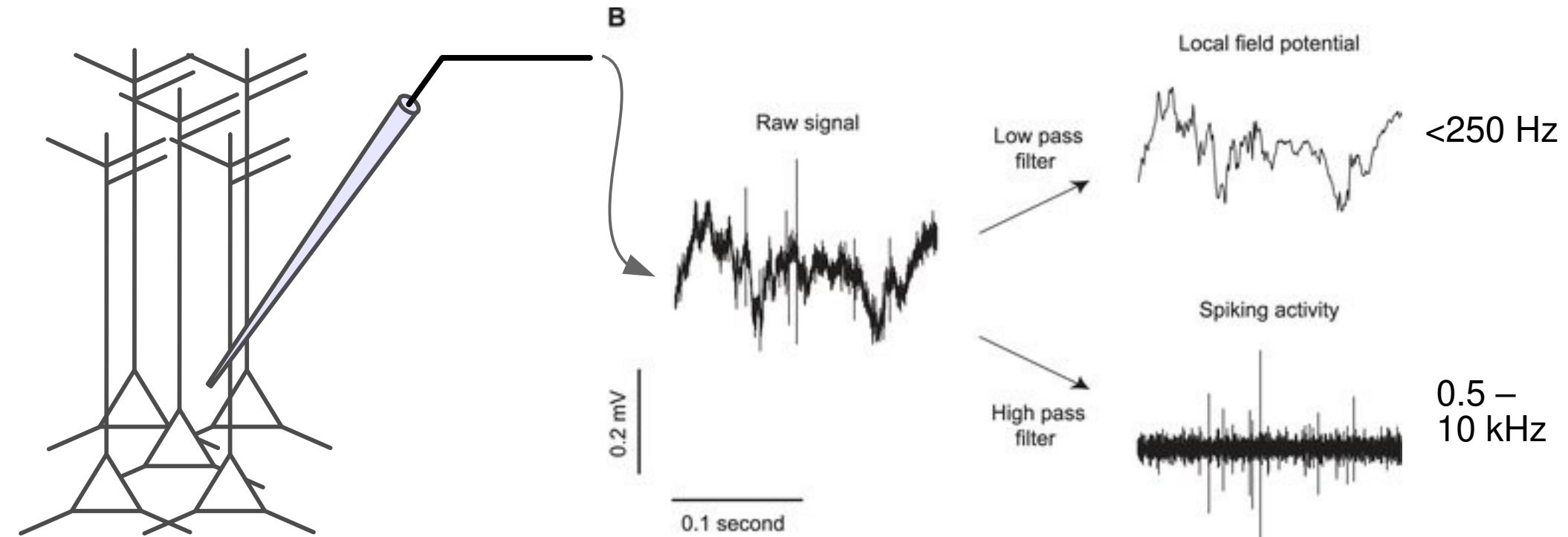
Magnetoencephalogram - MEG



- noninvasive through devices capable of measuring small magnetic fields
- magnetic fields are generated by ionic current flow: the effect of multiple neurons (50,000 – 100,000) excited together in a specific area generates a measurable magnetic field
- MEG provides timing as well as spatial information about brain activity
- used to study perceptual and cognitive brain processes : localizing regions affected by pathology; determining functions of parts of the brain

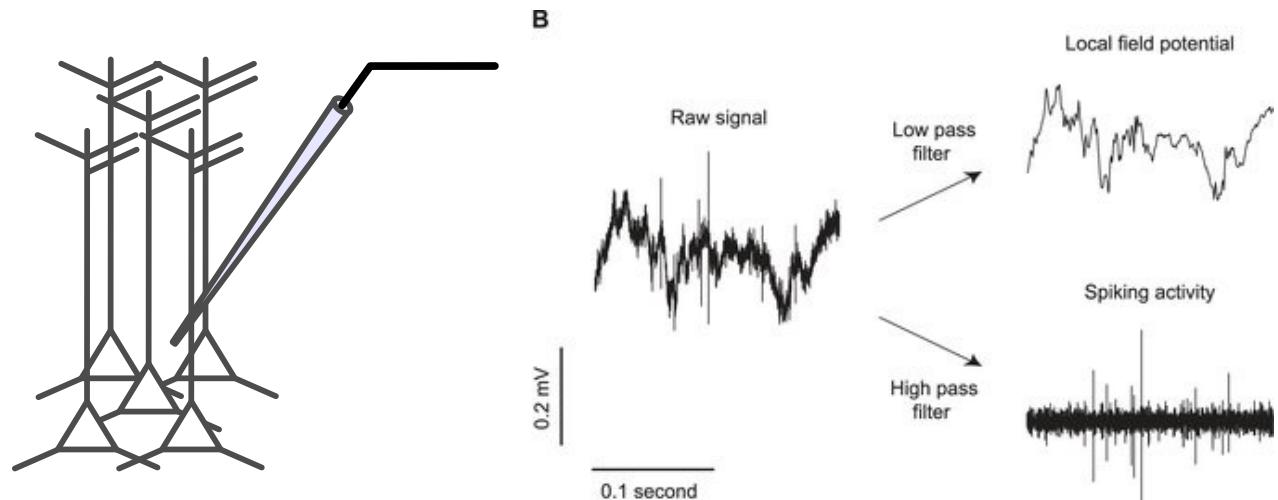
Local Field Potential - LFP

Local field potentials (LFP) are transient electrical signals generated in nervous tissue by the summed and synchronous electrical activity of the individual neurons in that tissue.



Local Field Potential - LFP

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- invasive through extracellular electrode placed nearby neurons generating signal
- LFP is the extracellular current flow that reflects the linearly summed postsynaptic potentials from local cell groups

Analog signals recorded from the brain

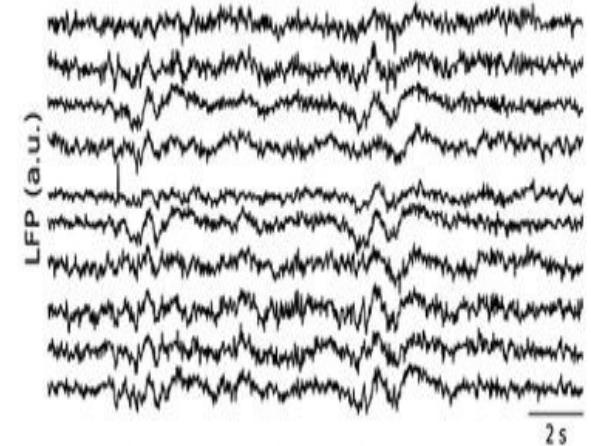
EEG



MEG



LFP



Oscillations in the brain

Delta oscillations (0.5-4 Hz)

- associated with deep sleep – also known as slow-wave sleep

Theta oscillations (4-8 Hz)

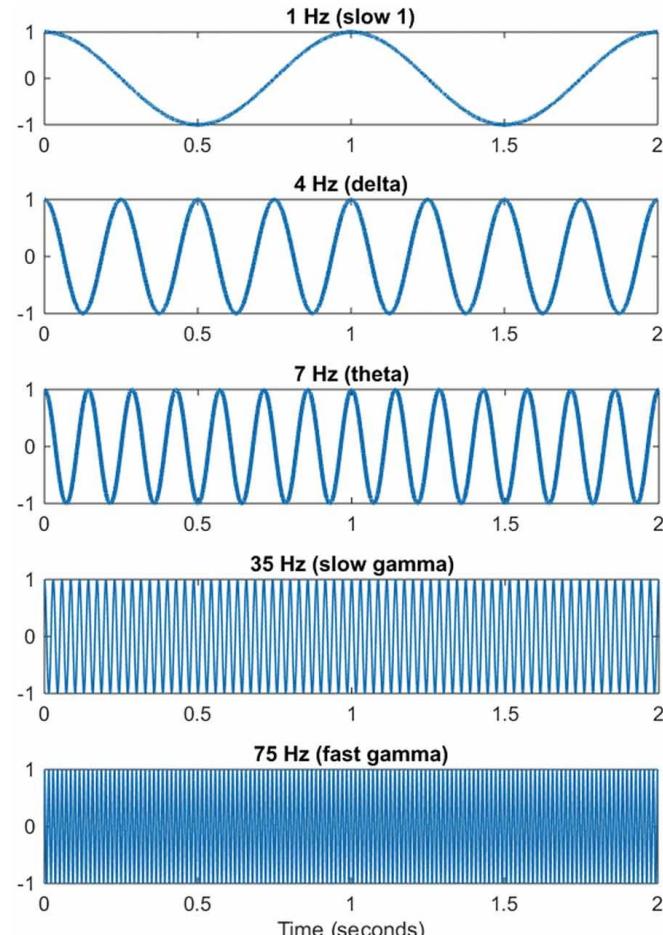
- observed during active motor behavior and REM sleep (rapid-eye movement sleep)
- prominently observed in the hippocampus

Beta oscillations (15-30 Hz)

- associated with normal waking consciousness and movement

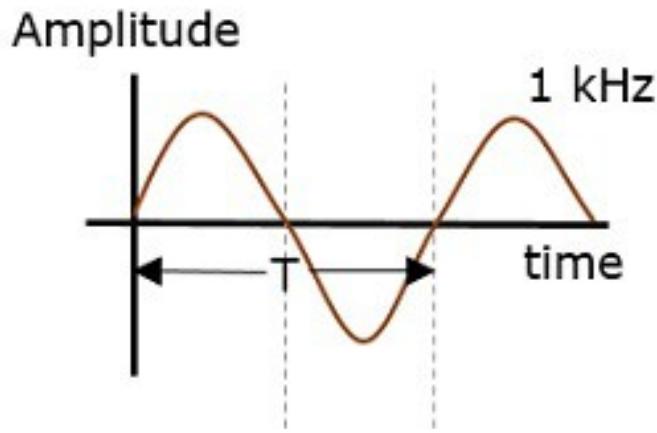
Gamma oscillations (30-150 Hz)

- occurring during conscious perception

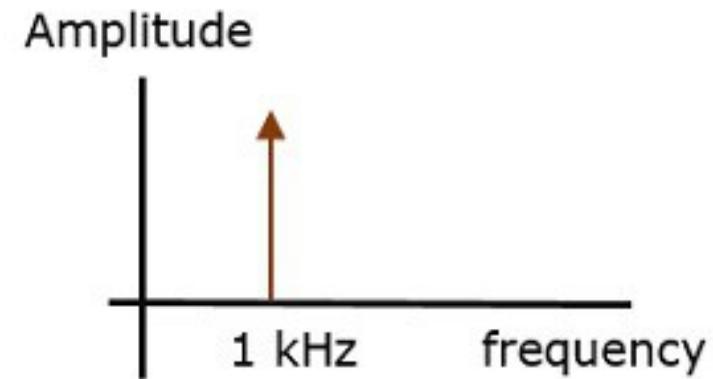


Starting point in analog signal processing

Time Domain
Representation



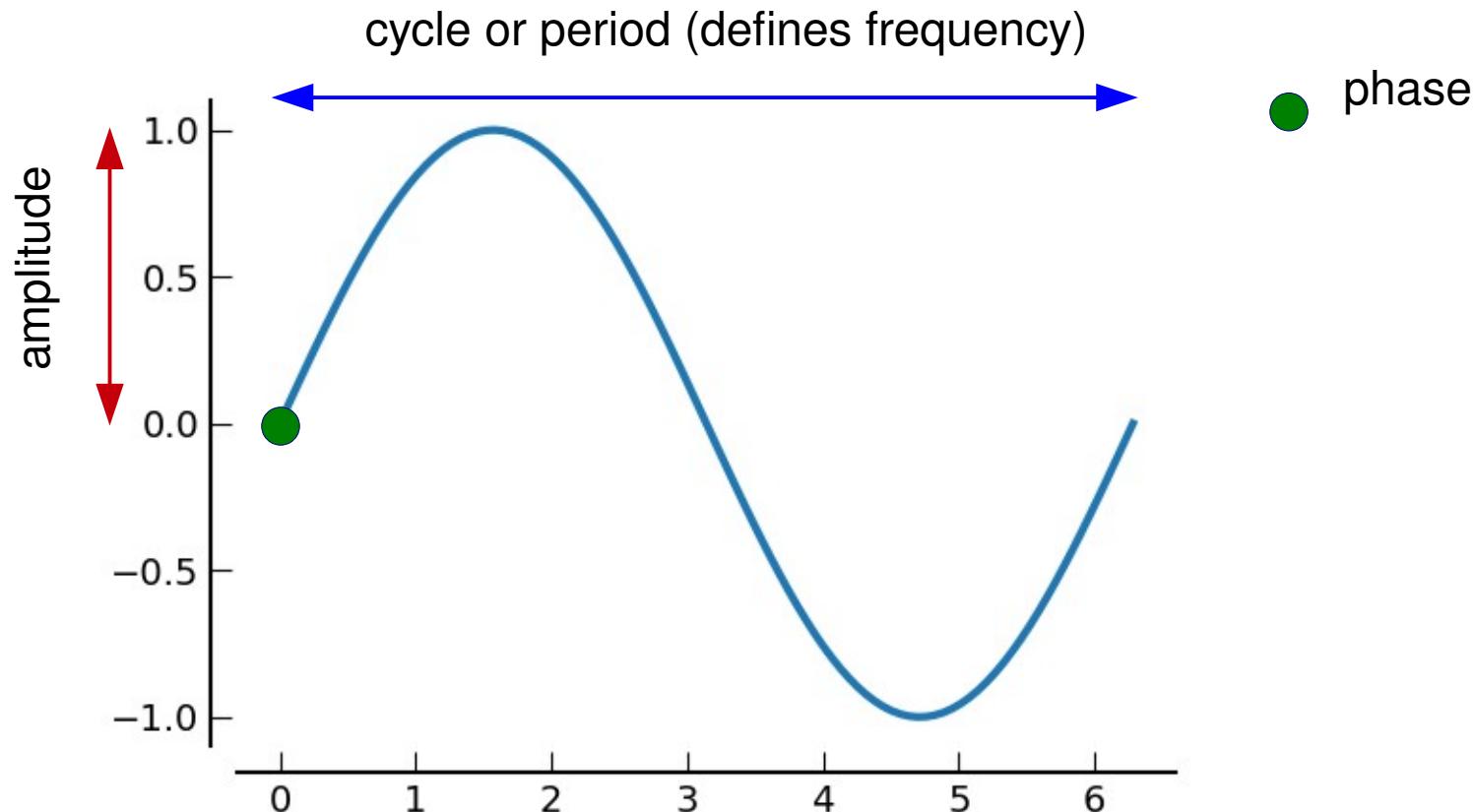
Frequency Domain
Representation



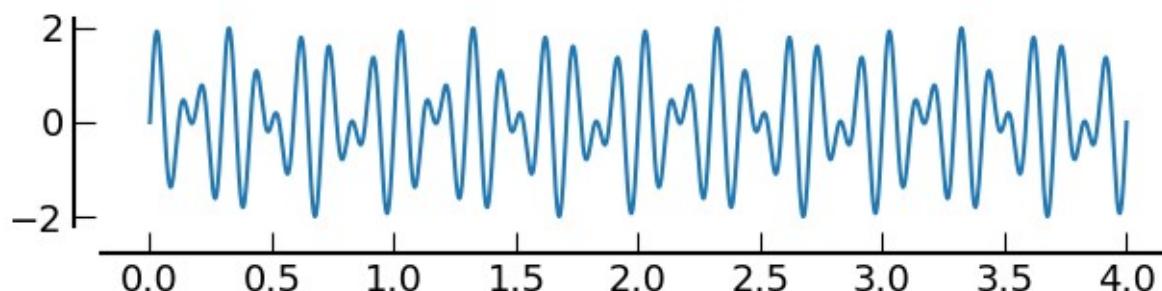
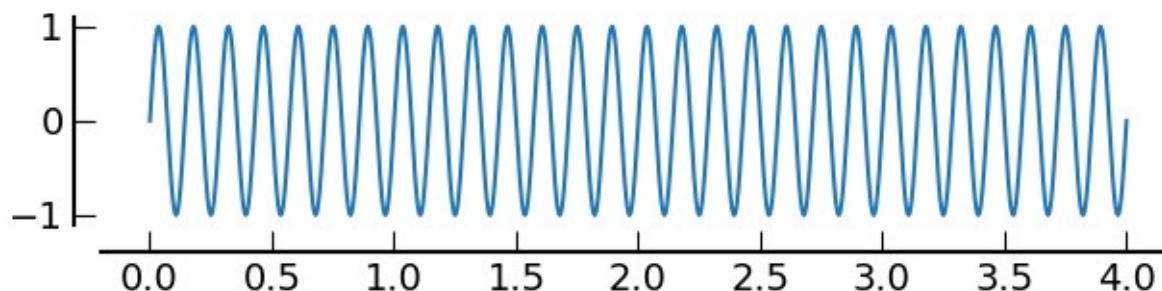
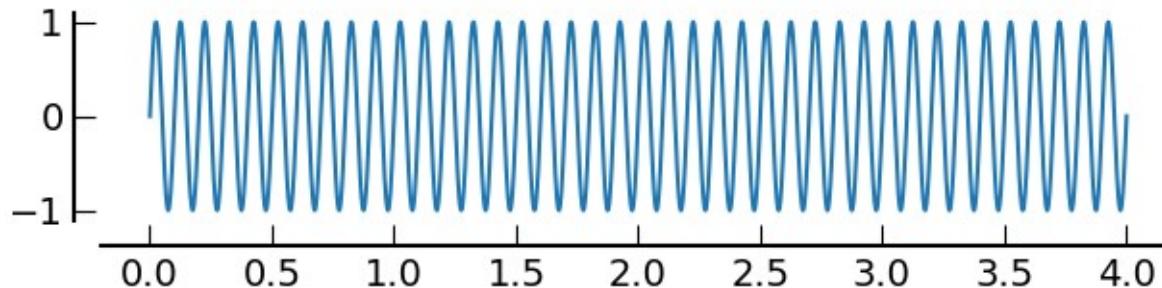
- show how a signal changes over time
- signals are recorded and often represented in time domain

- emphasizes periodic or repeating components of the signal
- decomposes a function of time (a signal) into its constituent frequencies

Sine wave : cyclically repeating signal

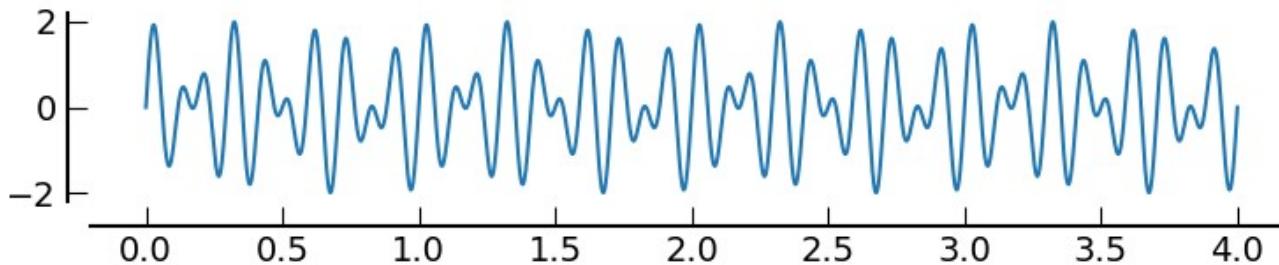


Example : Adding two sine waves

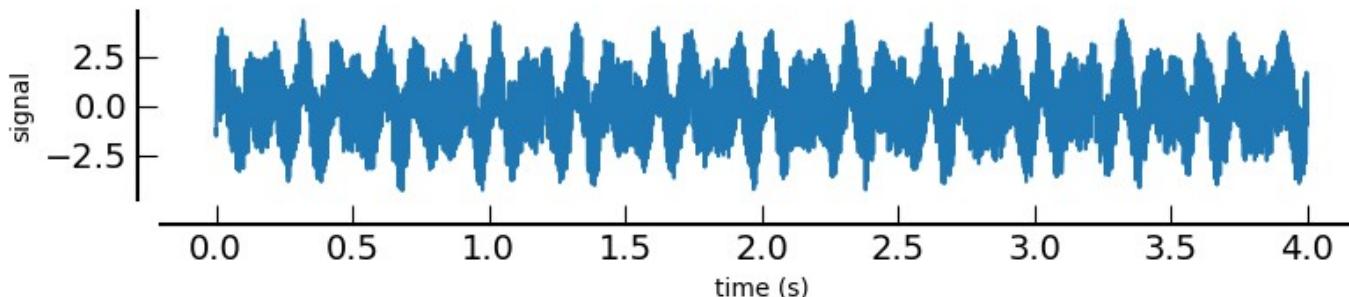


What is the frequency in each of the signals?

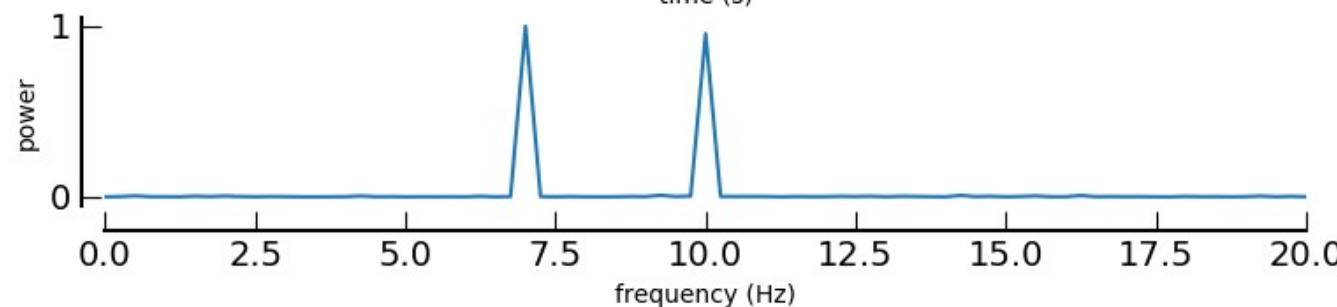
Signal with noise



2 added sine waves

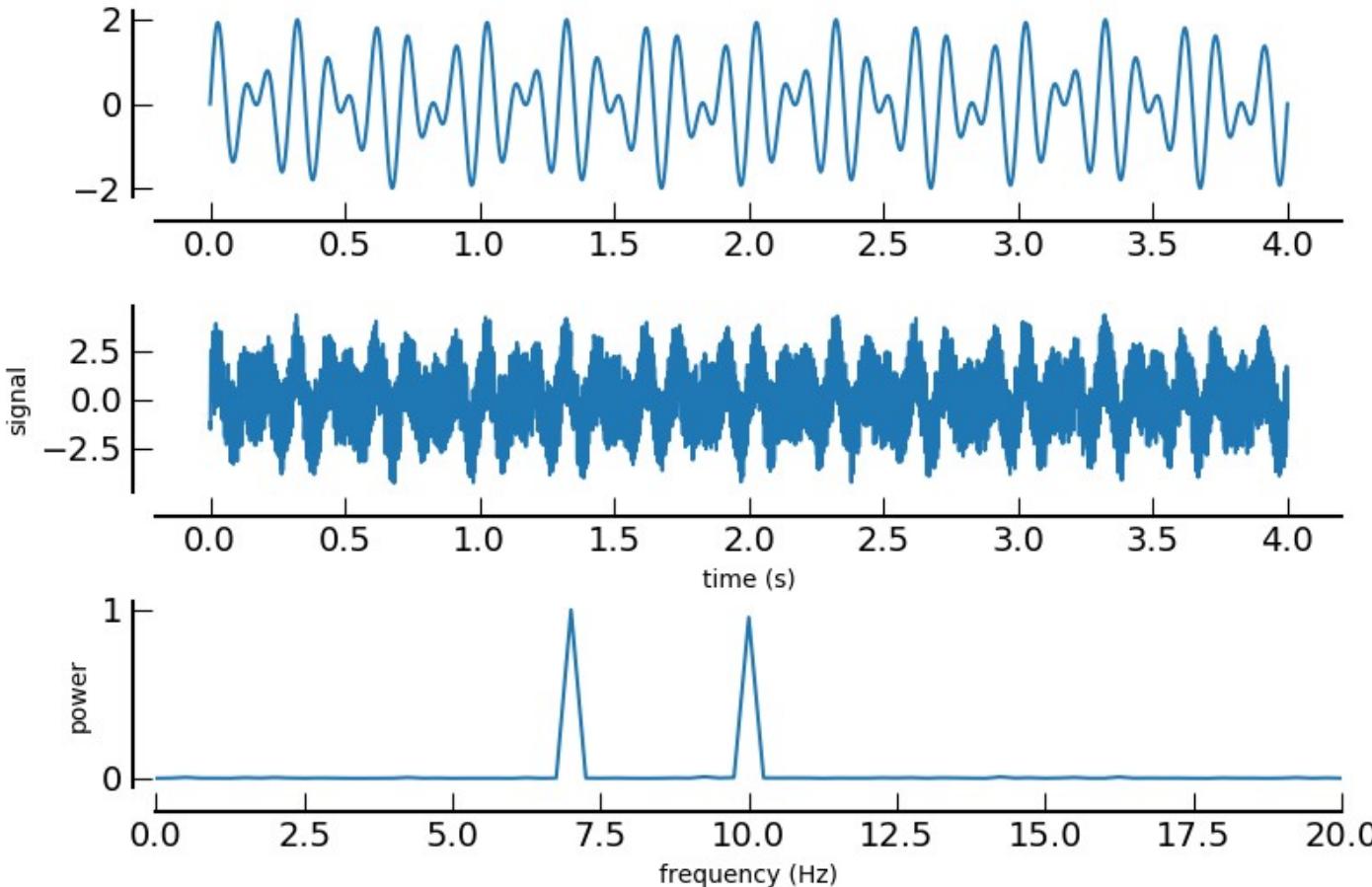


2 added sine waves + noise
time domain



2 added sine waves + noise
frequency domain

Signal with noise

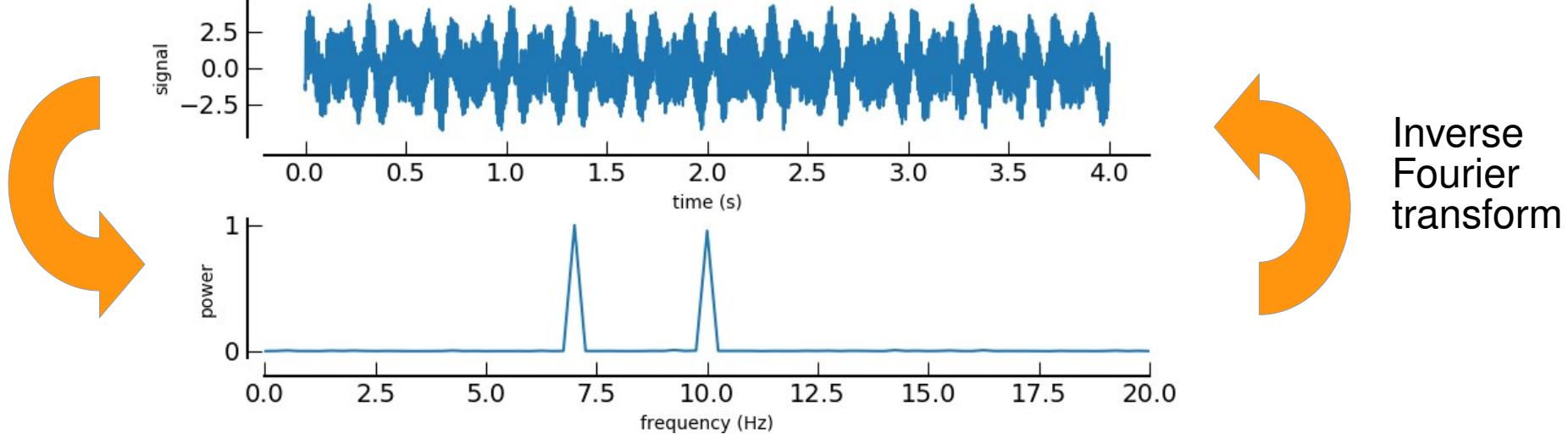


Noisy and compound signals :

hard to discern signal frequency in the time domain but not in the frequency domain.

Fourier transform

Fourier
transform



Inverse
Fourier
transform

- **Fourier transform** : transports the signal from the time domain into frequency space
- **Inverse Fourier transform** : transports any signal in the frequency domain back in the time domain

Fourier transform

the Fourier transform of a time series usually involves complex numbers representing **magnitude** and **phase** (we are only interested in magnitude here)

$$F\{x(t)\} = X(f) = \int_{-\infty}^{+\infty} x(t) e^{-i2\pi ft} dt$$

Fourier transform
of the signal $x(t)$

Fourier transform
itself, which is a
function of
frequencies f

inner-product of the signal
with the exponential term:
looking for the presence of
sine waves with certain
frequencies and outputs the
degree (power)

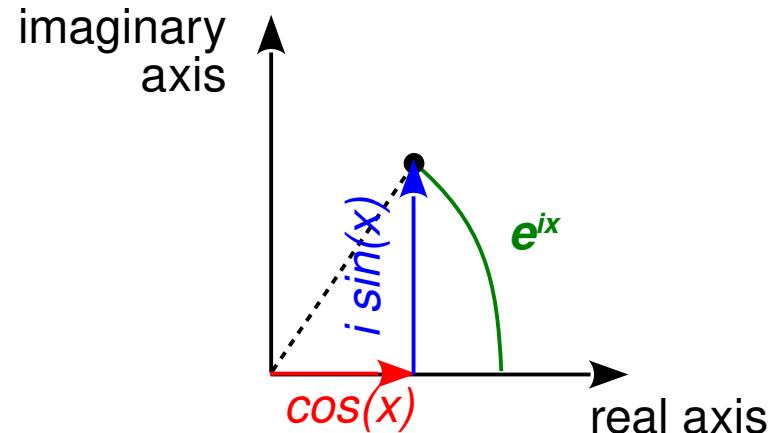


Jean-Baptiste Joseph Fourier
(1768-1830)

Euler's formula

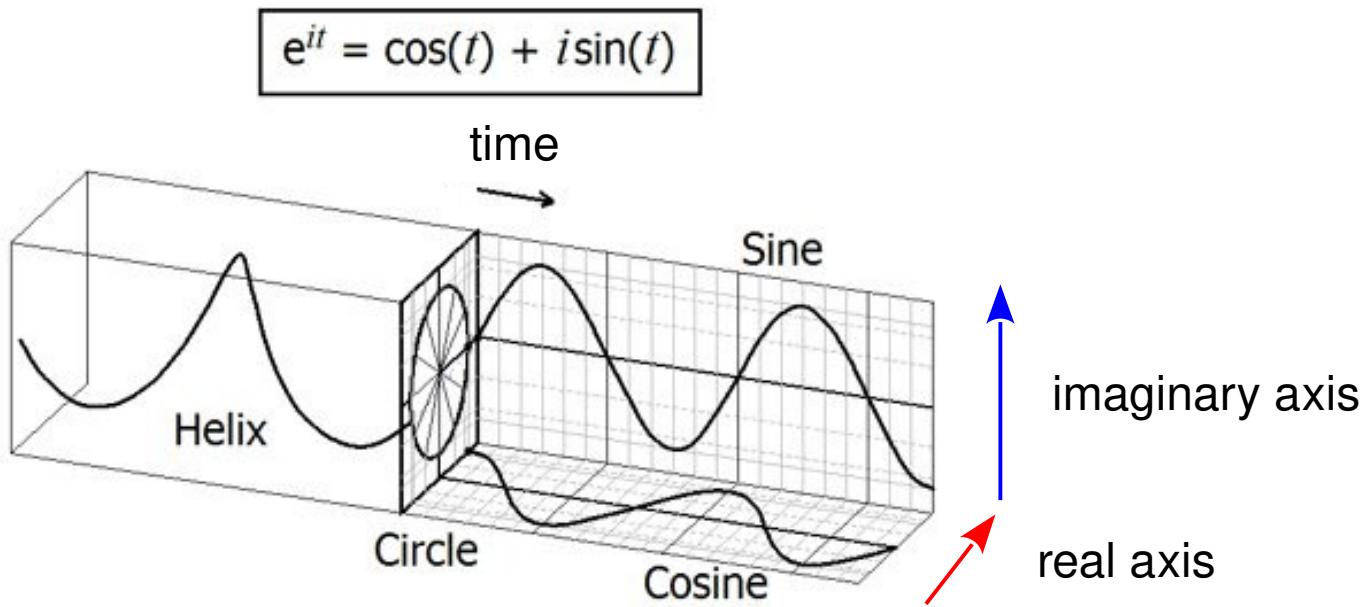
Euler's formula establishes relationship between trigonometric functions and complex exponential function.

$$e^{ix} = \cos x + i \sin x$$



- circular motion can be described by :
 1. decomposing Cartesian coordinates into an x- (cosine) and y-coordinate (sine)
 2. moving on the radius by the phase angle (angular distance)

Euler's formula describes circular motion

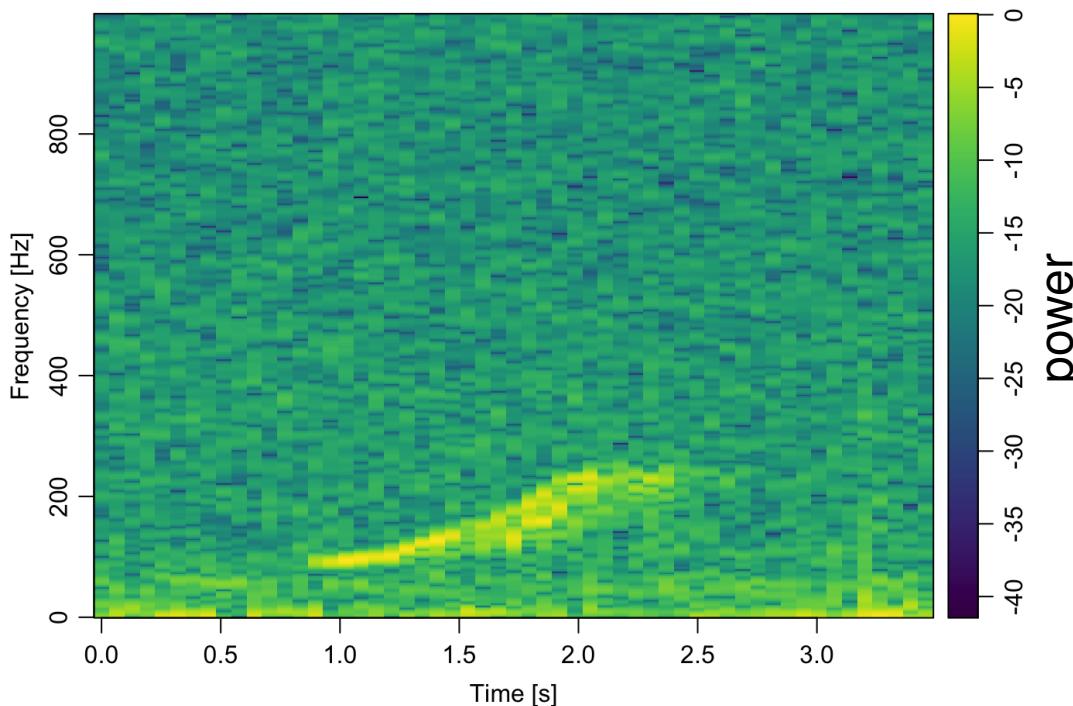


- adding time, creates a helix in the 3D space of complex numbers and time
- Euler's formula describes rotations

Fourier transform

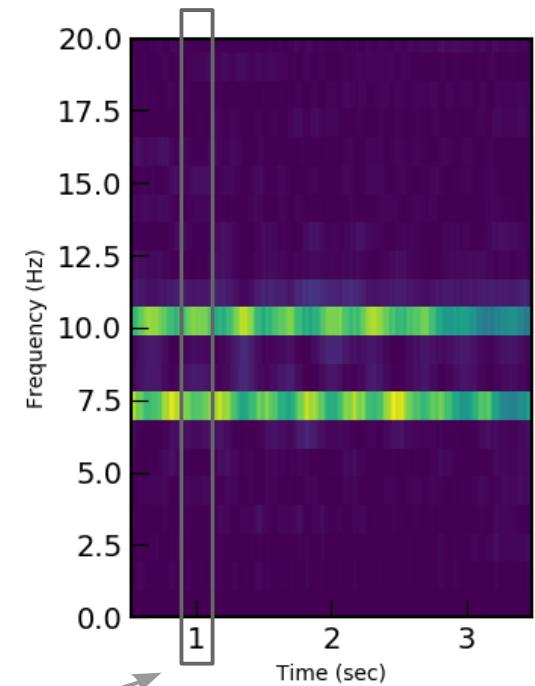
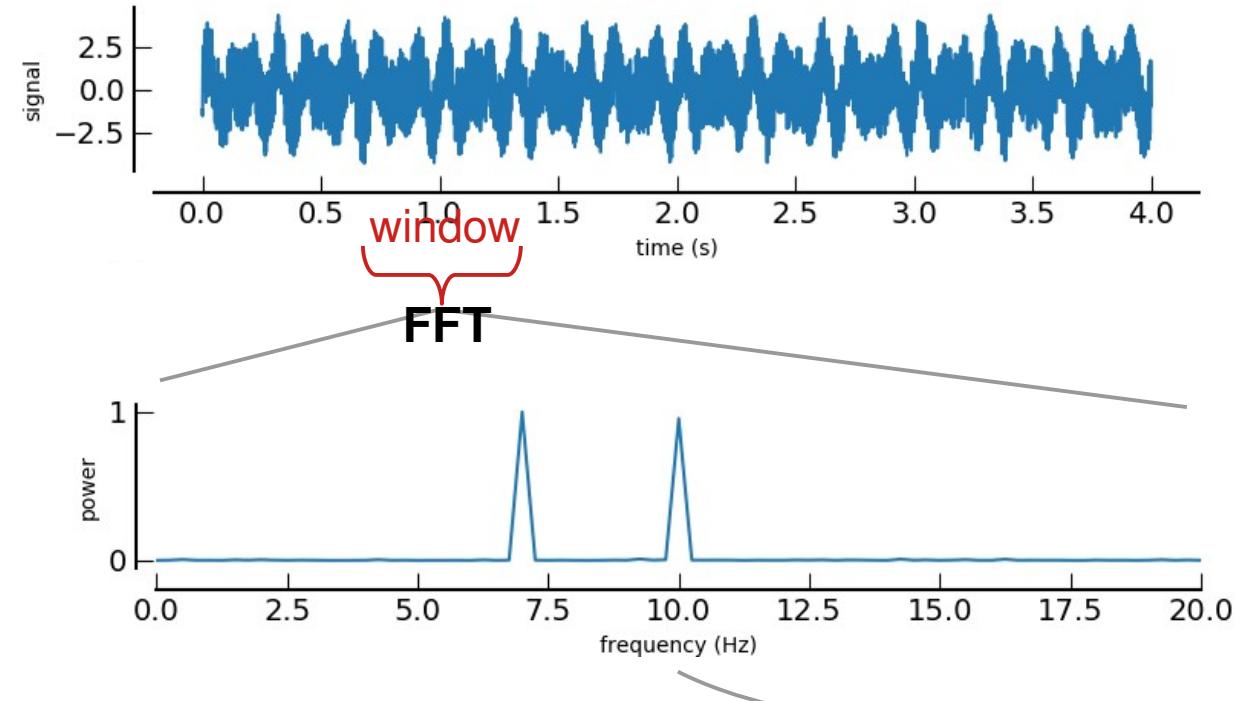
- power lies in it's generality : can be applied to analyze a wide variety of signals
- in neuroscience : signals include EEG, MEG, LFP, fMRI, visual signals, sound signals, etc.
- original formula defined over all time (-infinity → +infinity) : the **fast Fourier transform (FFT)** introduces a discrete Fourier transform on short time windows of a signal sampled at discrete time points

Visualization : spectrogram



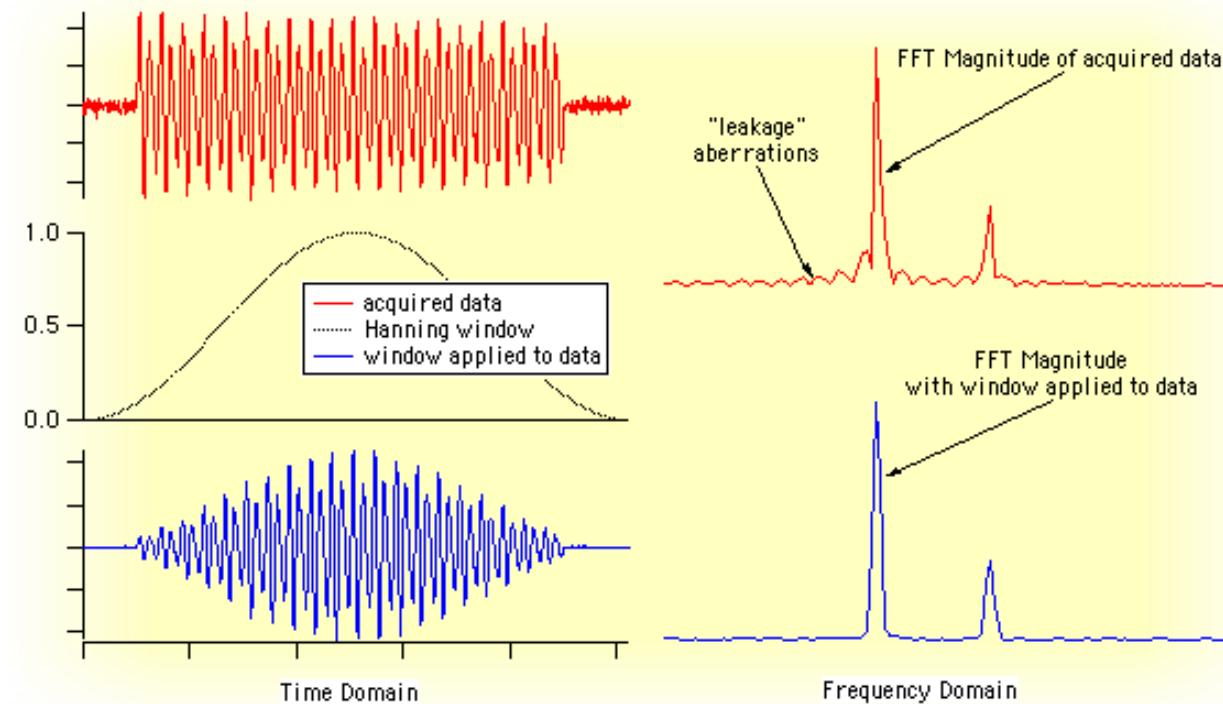
- visual representation of power at different frequencies over time (stack of FFT outputs)

Visualization : construction of a spectrogram



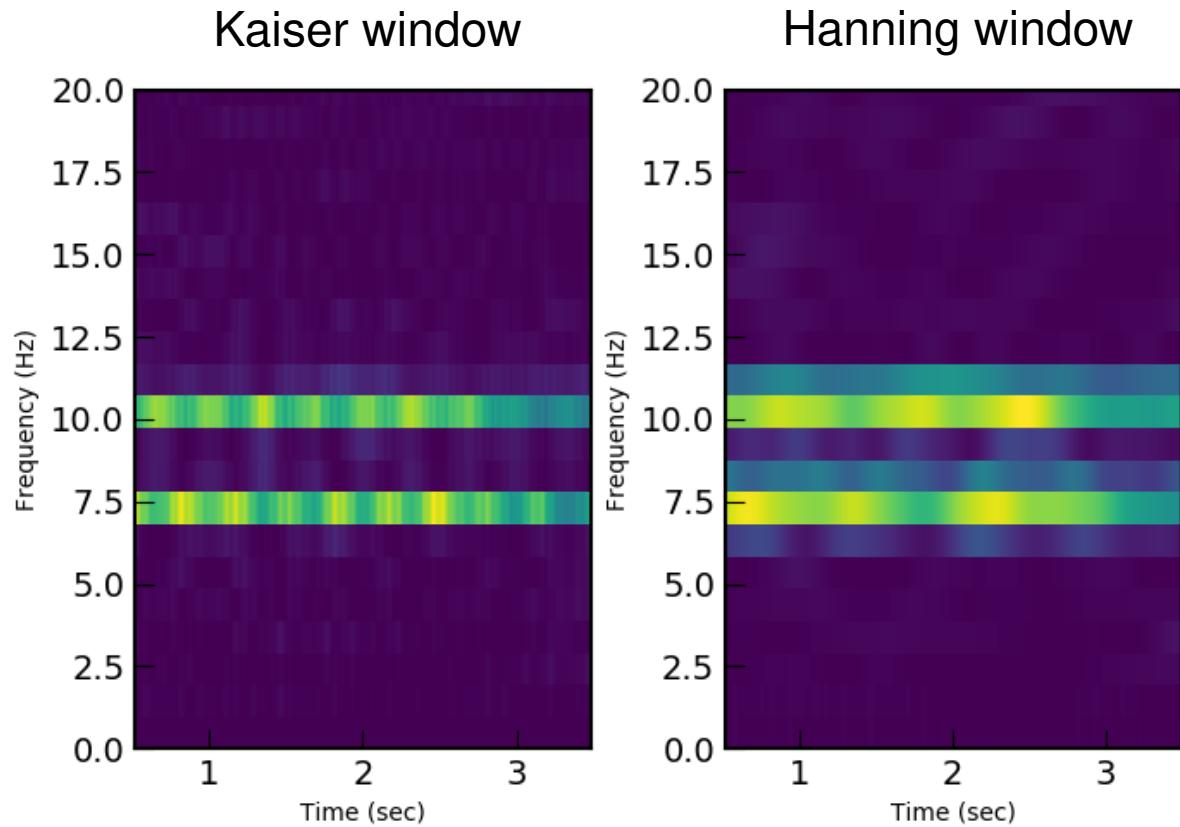
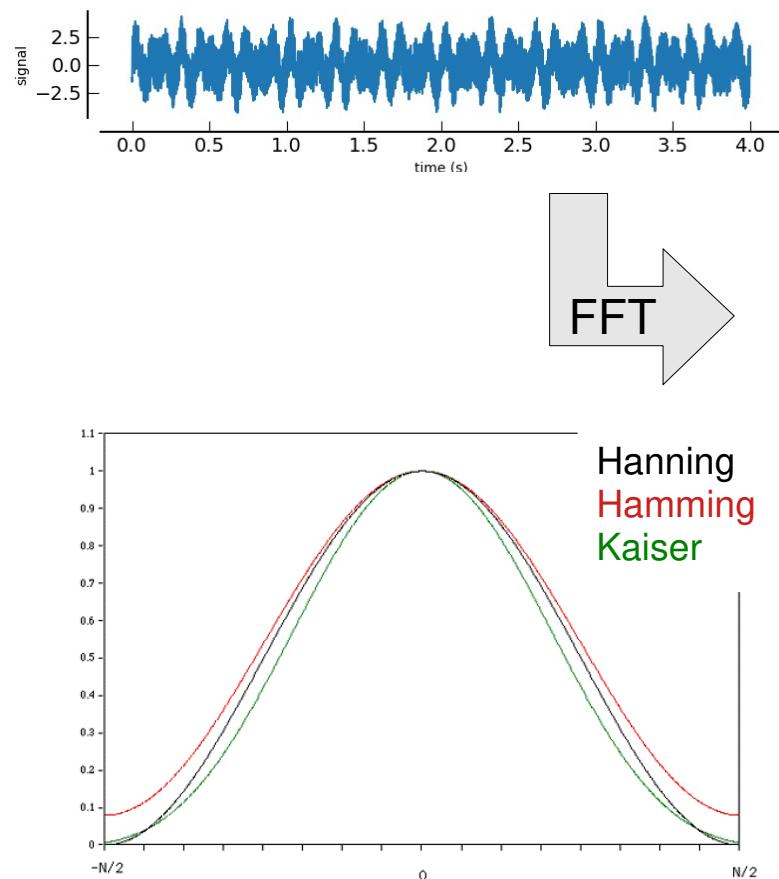
- the spectrogram results from doing the FFT on the part of the signal which falls into the window and plotting the frequency content in that window

Spectrogram : window function



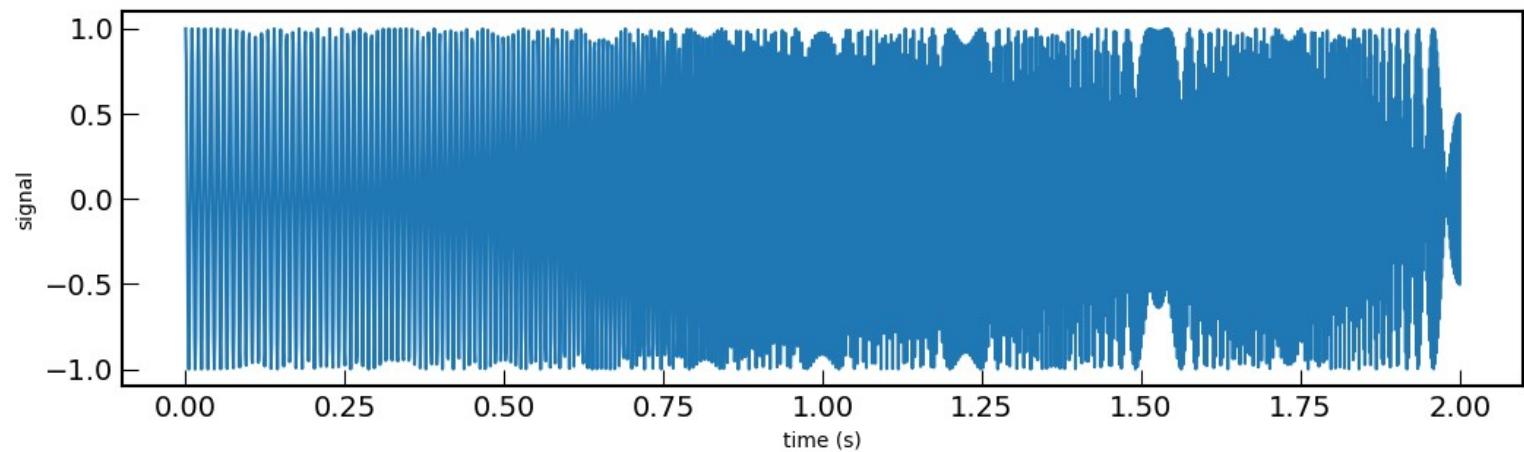
- Fourier transform is defined for infinite time, splitting the data into pieces introduces edge artifacts
- window functions try to avoid these edge artifacts as much as possible : done by amplifying the center and smooth out the edge of the window
- different window function emphasize different properties

Spectrogram : window functions

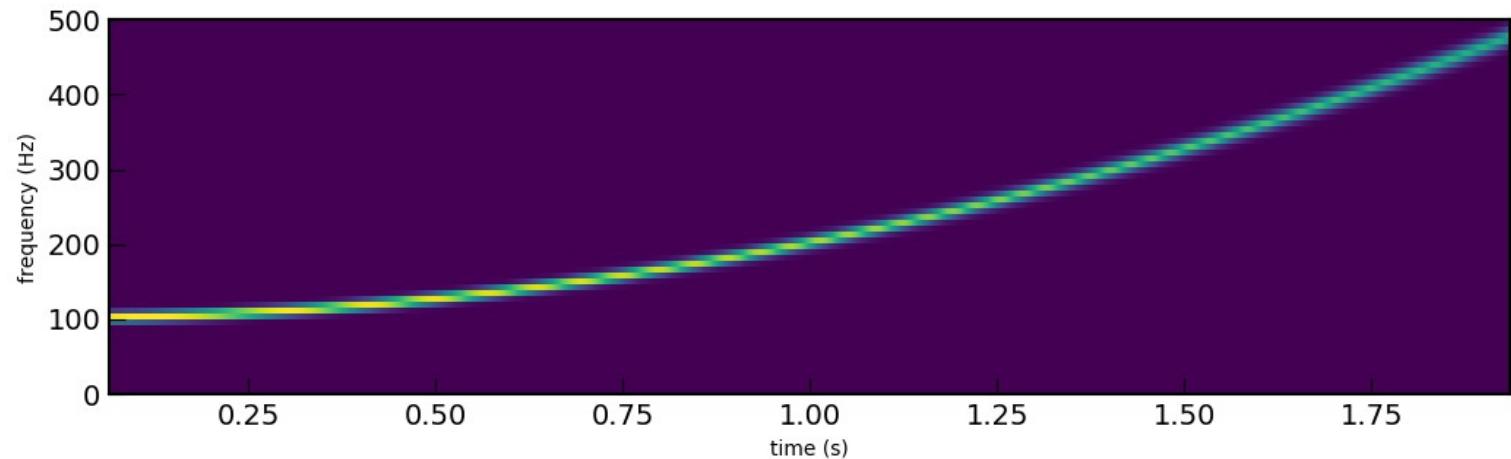


Spectrogram : useful insights

signal as function
of time

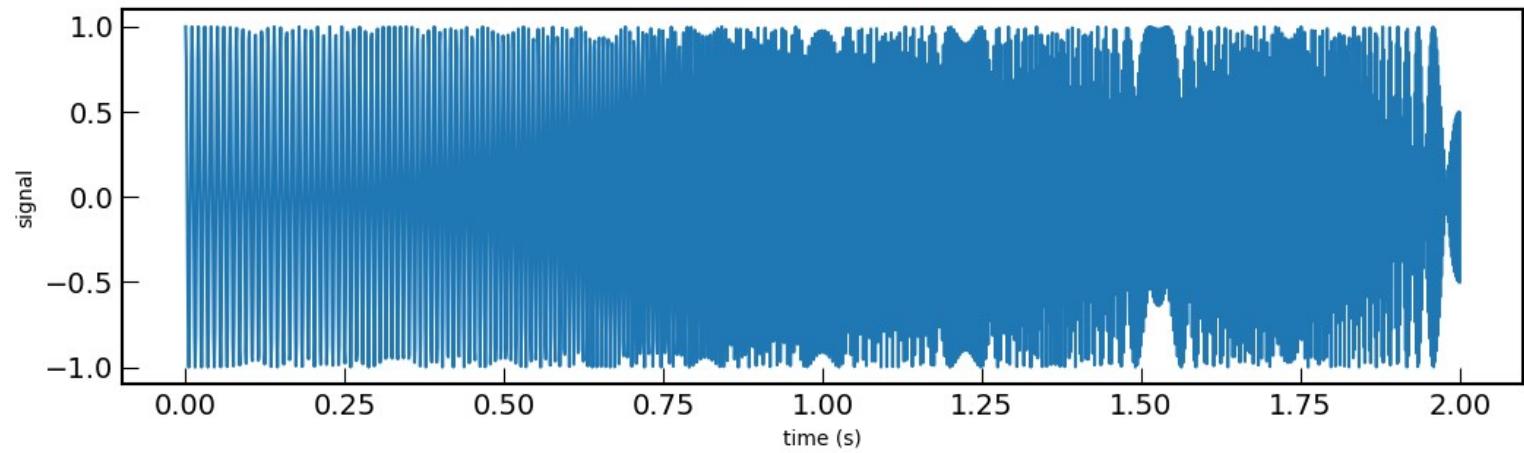


signal in frequency
domain



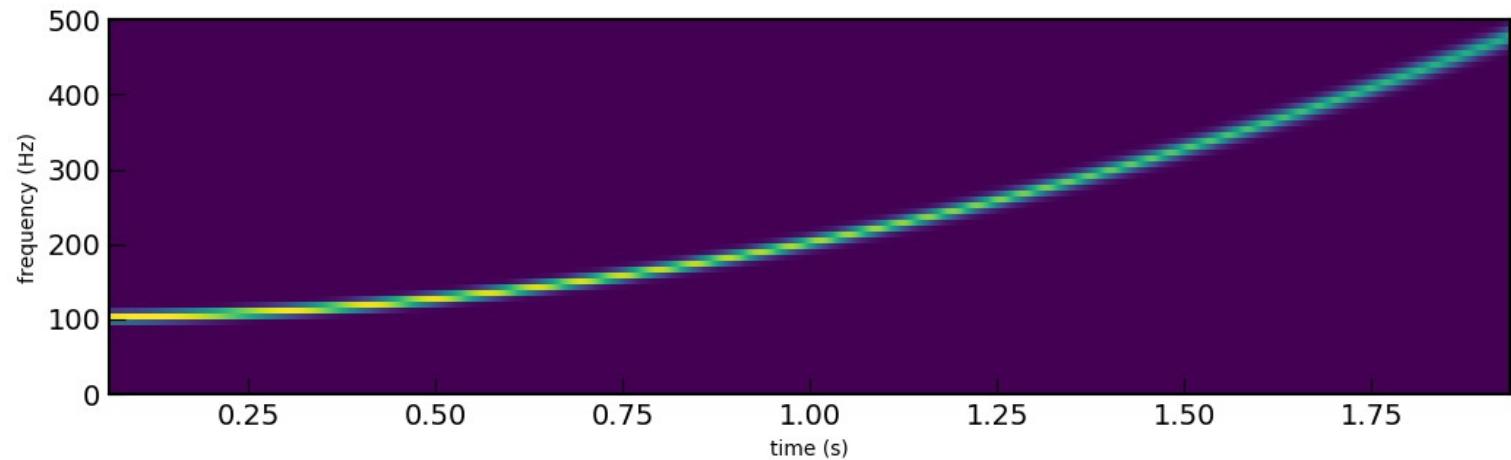
Spectrogram : useful insights

signal as function
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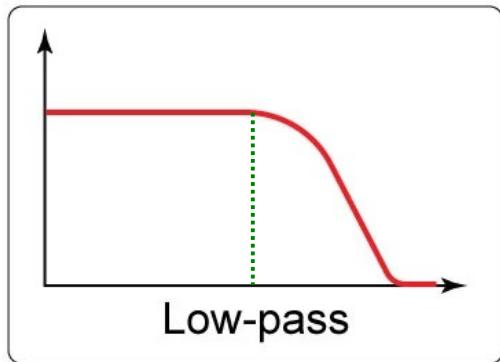


signal in frequency
domain

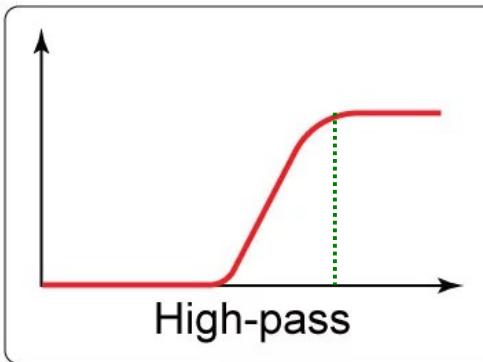
(exponential chirp)



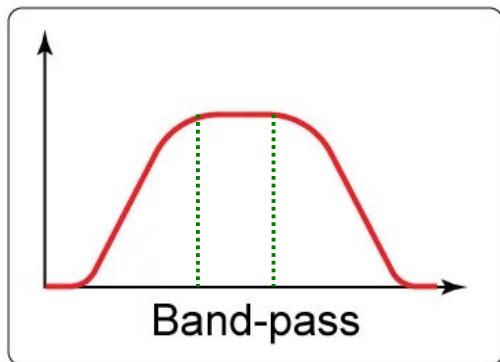
Signal filtering



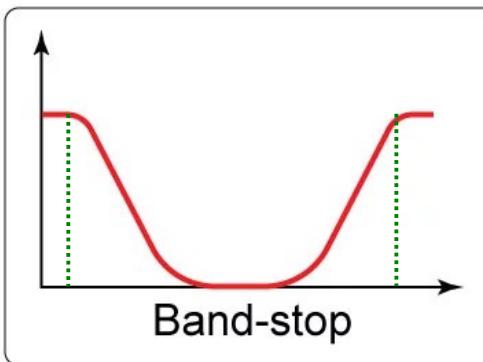
Low-pass



High-pass



Band-pass



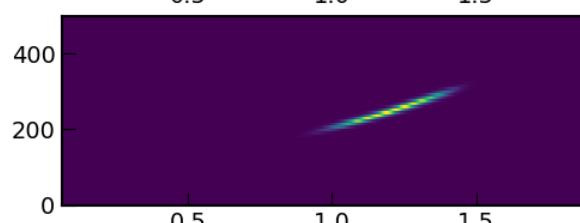
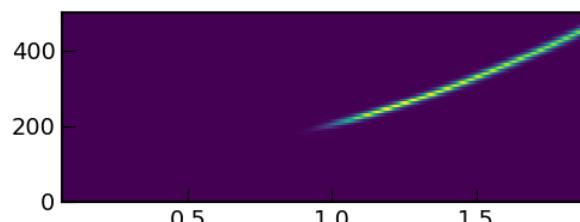
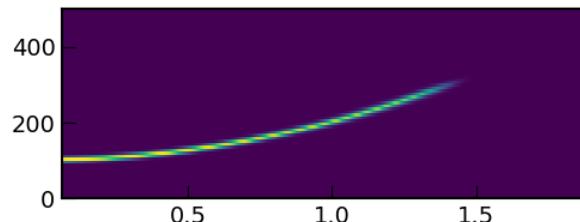
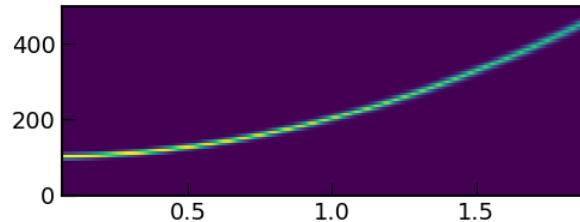
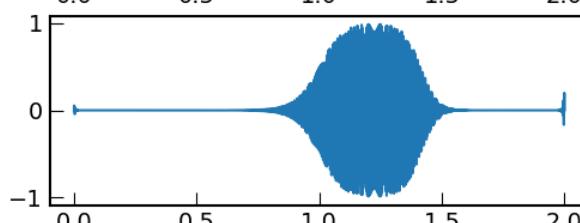
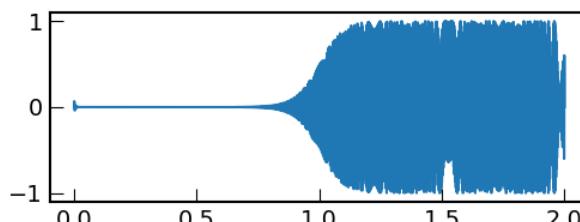
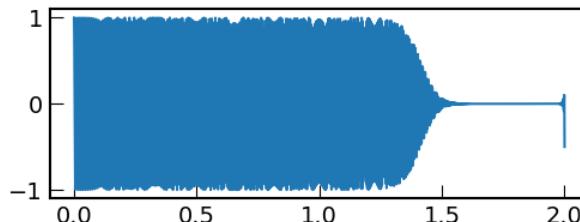
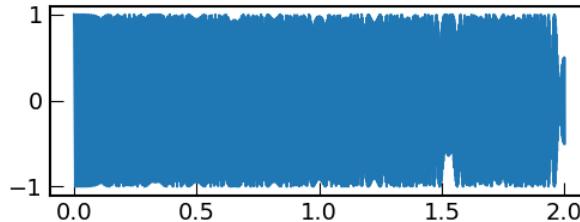
Band-stop

frequency

..... cut-off frequency(ies)

- Filtering is a process that removes frequencies or frequency bands from a signal.
- frequency below/above a cutoff frequency are attenuated (for low-pass/high-pass filters)
- band-pass and band-stop filters are determined by a frequency band

Signal filtering



original, full chirp

low-pass filtered chirp
- only low frequencies

high-pass filtered chirp
- only high frequencies

band-pass filtered chirp
- band of frequencies